Chiral logs in twisted mass Lattice QCD with large isospin breaking

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Introduction

- Twisted mass term: $m + i\sigma_3\mu$ — Frezzotti *et al* `01
- Advantages
  - Dirac operator bounded from below: $D^\dagger D \geq \mu^2$
  - Twisted mass renormalizes multiplicatively only
  - Simplified renormalization in some cases — Frezzotti, Rossi `04
  - Automatic O(a) improvement at maximal twist — Frezzotti, Rossi `04
- Drawbacks
  - Tuning to maximal twist needed
  - Parity and isospin breaking ➞ leads to pion mass splitting

$$\Delta_\pi^2 = M_{\pi^0}^2 - M_{\pi^\pm}^2 = \mathrm{O}(a^2)$$

Introduction

- Splitting is of $O(a^2)$ and vanishes in the continuum limit
- In practice rather large
  - $N_f = 2, \; a \approx 0.086\text{fm}$
    Urbach (ETMC) `07

$M_{\pi^0}/M_{\pi^\pm}$	$M_{\pi^\pm}$
≈ 0.86	$\approx 450\text{MeV}$
≈ 0.82	$\approx 310\text{MeV}$

  - $N_f = 2+1+1, \; a \approx 0.078\text{fm}$
    Baron *et al* (ETMC) '10

$M_{\pi^0}/M_{\pi^\pm}$	$M_{\pi^\pm}$
≈ 0.77	$\approx 400\text{MeV}$
≈ 0.54	$\approx 320\text{MeV}$

Introduction

- Worry: A large mass splitting might affect the chiral extrapolation
  - $N_f = 2$ continuum ChPT result for decay constant — Gasser, Leutwyler`83

$$f_{\pi,\mathrm{NLO}} = f\left(1 - \frac{1}{16\pi^2 f^2}\left(M_0^2 \ln\frac{M_0^2}{\Lambda_4^2}\right)\right) \qquad M_0^2 = 2Bm$$

  - Question: What pion mass in case of a large mass splitting ?

a) $M_0 \stackrel{?}{\longrightarrow} M_{\pi^\pm}$ b) $M_0 \stackrel{?}{\longrightarrow} M_{\pi^0}$ c) more complicated?

  - Chiral extrapolation may depend substantially on the exact answer !
  - Same question for other observables

- Fitresults for the GL coefficients may also depend substantially on the precise 1-loop result

Introduction

- Another aspect: Finite volume corrections
  - 1-loop continuum ChPT predicts FV corrections $\propto \exp(-M_0 L)$
  - Same question here: What mass ?
  - Using the "wrong" mass may underestimate the FV corrections significantly

Common wisdom: Corrections small for $M_0 L \sim 4$

large for $M_0 L \sim 2$

Introduction

- Desirable: Analytic results taking into account different masses in the logs
- Framework: twisted mass Wilson ChPT (tmWChPT)

Sharpe, Singleton '98
...

- In the following: 1-loop results for
  - the charged pion mass
  - the mass splitting
  - decay constant of the charged pion
- Last part of the talk: Confront these results with recent data from ETMC
- Reference: arXiv:1008.0784 [hep-lat] (to be published in PRD)

Twisted mass Wilson ChPT basics

- tm WChPT = low-energy effective theory of twisted mass lattice QCD
- LO Effective Lagrangian $\mathcal{L}_{\mathrm{LO}} = \mathcal{L}_2 + \mathcal{L}_{a^2}$

$$\mathcal{L}_2 = \frac{f^2}{4}\langle \partial_\mu \Sigma \partial_\mu \Sigma^\dagger \rangle - \frac{f^2 B}{2}\langle \Sigma M^\dagger + M \Sigma^\dagger \rangle$$

$$\mathcal{L}_{a^2} = \frac{f^2}{16} c_2 a^2 \langle \Sigma + \Sigma^\dagger \rangle^2$$

$\langle \ldots \rangle$: $\mathrm{tr}(\ldots)$

f, B, c_2 : low-energy constants (LECs)

M : quark mass matrix

- Why O(a^2) term at LO? little later ...

Mass matrix and maximal twist

- Quark mass matrix: $M = m + i\sigma_3\mu$ $\quad N_f = 2$ ChPT
- Tuning to maximal twist:

  PCAC mass condition $\quad m_{\rm PCAC} = 0 \quad \rightarrow \quad m = 0 \quad$ at LO

  (used by ETMC)

Aoki, OB `04
Sharpe, Wu `04
Aoki, OB `06

- In the following: Maximal twist only
  Arbitrary twist angles more complicated

Tree-level pion masses

charged pion $M_{\pi^\pm}^2 = 2B\mu$

neutral pion $M_{\pi^0}^2 = 2B\mu + 2c_2a^2$

Scorzato `04

mass splitting $\Delta_\pi^2 \equiv M_{\pi^0}^2 - M_{\pi^\pm}^2 = 2c_2a^2$

- neutral pion lighter than charged one for $c_2 < 0$
  (1^{st} order phase transition scenario)

Münster `04
Sharpe, Wu `04

- $N_f = 2+1+1$ ETMC data shows

$$\frac{|2c_2a^2|}{2B\mu} = \frac{M_{\pi^\pm}^2 - M_{\pi^0}^2}{M_{\pi^\pm}^2} \approx \begin{cases} 0.41 \\ 0.71 \end{cases}$$

➥ O(a^2) term as important as mass term ➞ take O(a^2) term at LO (so-called LCE regime)

1-loop ingredients

- Propagators

$$G^{ab}(p^2) = \frac{\delta^{ab}}{p^2 + M_{\pm}^2} \qquad a,b = 1,2$$

$$G^{33}(p^2) = \frac{1}{p^2 + M_0^2}$$

1-loop ingredients

- Vertices: Expand LO Lagrangian in the pion fields

$\mathcal{L}_{p^2,4\pi}$ ➞ same as in continuum

$\mathcal{L}_{M,4\pi} + \mathcal{L}_{c_2 a^2,4\pi}$ ➞ $-\frac{M_\pm^2}{24 f^2}\pi^4 - \frac{2c_2 a^2}{6 f^2}\pi^2\pi_3^2$

continuum vertex | new O(a^2) vertex

➥ 1- loop calculation almost as in continuum, but

- keep track of different masses in propagators
- one more (trivial) diagram

Results

- Charged pion mass

$$M^2_{\pi^\pm,\mathrm{NLO}} = M^2_\pm\left(1 + \frac{M_0^2}{32\pi^2 f^2}\ln\frac{M_0^2}{\Lambda_3^2} + C_{M_\pm}a^2\right)$$

↑ ↑

LECs of the NLO Lagrangian

- Properties
  - Neutral pion mass in the chiral log !!!
  - Standard Gasser-Leutwyler result recovered in the continuum limit

Results

Ξ_3, C_Δ : LECs

- Mass splitting

$$\Delta M^2_{\pi,\mathrm{NLO}} = 2c_2 a^2 \left(1 - \frac{M_0^2}{8\pi^2 f^2} \ln \frac{M_0^2}{\Xi_3^2} + C_\Delta a^2\right)$$

$$+ \frac{M_\pm^2}{16\pi^2 f^2} \left(M_\pm^2 \ln \frac{M_\pm^2}{\Lambda_3^2} - M_0^2 \ln \frac{M_0^2}{\Lambda_3^2}\right)$$

- Properties
  - Chiral logs with charged and neutral pion mass
  - Vanishes in the continuum limit (as it should)

Results

- Decay constant of the charged pion

$$f_{\pi,\mathrm{NLO}} = f\left(1 - \frac{1}{32\pi^2 f^2}\left(M_{\pm}^2 \ln\frac{M_{\pm}^2}{\Lambda_4^2} + M_0^2 \ln\frac{M_0^2}{\Lambda_4^2}\right) + C_f a^2\right)$$

LECs of the NLO Lagrangian

- Obtained with (“*indirect method* ”)

$$f_\pi = \frac{2\mu}{M_{\pi^\pm}^2} G_\pi \qquad G_\pi = |\langle 0|P^1(0)|\pi^1(\vec{p})\rangle|$$

- Properties
  - Arithmetic mean of the two chiral logs
  - Standard result in the continuum limit
- Same result if matrix element of the (physical) axial vector is computed

Finite volume corrections

- So far all results in infinite volume
- Finite volume corrections are easily included
  Modifications in the propagators only

Gasser, Leutwyler `87

- Example: Finite spatial volume with extent L needs replacement

Bernard `02

$$\ln \frac{M^2}{\Lambda^2} \quad \rightarrow \quad \ln \frac{M^2}{\Lambda^2} + \tilde{g}_1(ML)$$

modified
Bessel function

$$\tilde{g}_1(ML) = \frac{4}{ML} \sum_{\vec{n} \neq 0} \frac{K_1(|\vec{n}|ML)}{|\vec{n}|}$$

$$|\vec{n}| = \sqrt{n_1^2 + n_2^2 + n_3^2}$$

Similarities with Staggered ChPT

- Taste symmetry breaking ➞ Mass splitting of O(a^2)
- Large for MILC lattices with Asqtad quarks: $m \sim a^2$
- Take O(a^2) terms at LO in staggered ChPT ➞ Modified chiral logs
  - 1-loop mass for the *Goldstone pion* involves chiral log with the *taste singlet pion*
  - 1-loop decay constant of the *Goldstone pion* involves the average chiral log with *all taste partners*

Aubin, Bernard `03

Question

Are these new results relevant in practice ?

➥ Analyze recent ETMC data with new formulae

The ETMC data

Baron *et al* (ETMC) '10

- $N_f = 2+1+1$ twisted mass LQCD at maximal twist
- Data for decay constant and charged pion mass at
  - 2 lattice spacings $a \approx 0.086\text{fm}$, $a \approx 0.078\text{fm}$
  - 6 resp. 5 quark masses such that $270\,\text{MeV} \lesssim M_\pm \lesssim 510\,\text{MeV}$
  - ➥ 22 data points in total
- Lattice size: $1.9\,\text{fm} \lesssim L \lesssim 2.8\,\text{fm}$
- Data for pion mass splitting gives $\frac{|2c_2a^2|}{2B\mu} \approx \begin{cases} 0.41 \\ 0.71 \end{cases}$ at $a \approx 0.078\text{fm}$

➥ expect non-neglibible effect

Results of a combined chiral fit

Fit parameter:

4 continuum ChPT params

3 nonzero a corrections

Fit (both β values)	I	
Fit range: $a\mu_{0,\min}$	0.0025	
$a\mu_{0,\max}$	0.01	
maximal $M_{\pi^\pm}$ (MeV)	512	
$2B_0a$	4.57(11)	4.39(11)
f (MeV)	111.3(2.2)	116.2(2.5)
$\bar{l}_3$	3.44(7)	3.09(13)
$\bar{l}_4$	4.69(4)	4.62(5)
$-2c_2a^2$ (MeV^2)	- ★	$[187(19)]^2$
$C_{M_\pm}a^2$	0.19(2)	0.19(3)
C_fa^2	0.10(2)	0.13(2)
n_{data}	22	22
χ^2/n_{dof}	27.6/16	20.7/15
Q	0.12	0.42

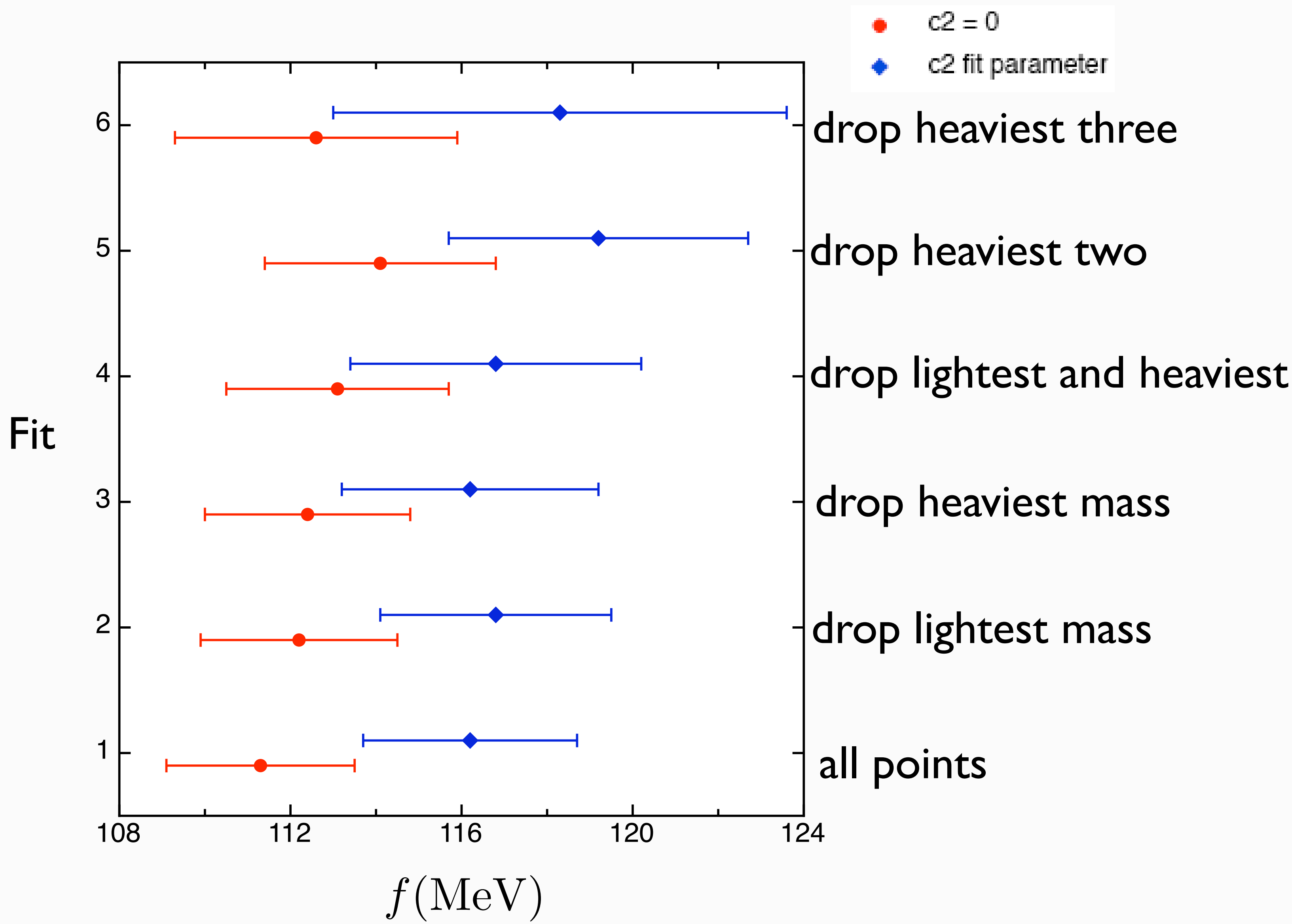
$^\star c_2 = 0$

Results of a combined chiral fit

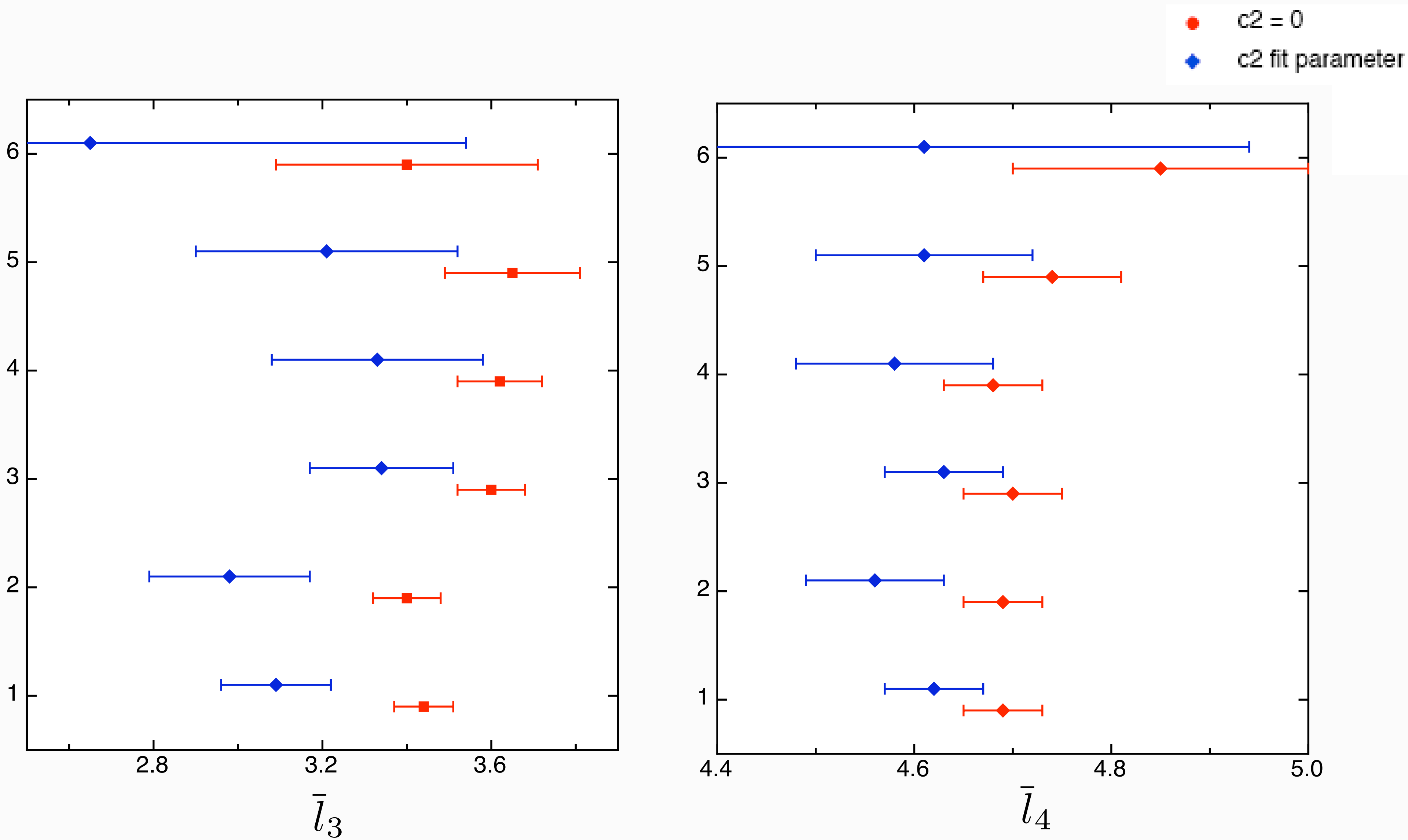
- Observations
  - Quality of the fit slightly better (lower χ^2/dof)
  - negative value for $2c_2a^2 \approx -(200\ \text{MeV})^2$
    ➥ data prefers neutral pion lighter than the charged one
  - From the fit: $0.15 \lesssim \frac{-2c_2a^2}{2B_0\mu_0} \lesssim 0.60$

    ➥ agrees well with estimate from the measured values
- Results for the LECs depends on the data included in the fit ...

Results for decay constant



Results for the GL coefficients



$$\bar{l}_k = \ln\left(\frac{\Lambda_k^2}{M_{\pi,\mathrm{phys}}^2}\right)$$

Results of a combined fit

- Observations 2: Central values for
  - f seem systematically higher
  - NLO LECs seem systematically smaller

  if pion mass splitting is included in the chiral logs
- Needs to be corroborated (improve error analysis !)

Conclusions and Outlook

- Current twisted mass simulations have a sizable mass splitting
- WChPT predicts modified chiral logs in this case
- Analytic expressions available for
  - Pseudoscalar masses
  - Decay constant
- Modifications affect chiral extrapolation and extraction of LECs
- Similar modifications in other observables?
  - Kaon mass and decay constant
  - ...