Progress and challenges in

Hadron structure calculations on the lattice

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Overview

- form factors – nucleon rms radii and (anomalous) magnetic moment — $F_1, F_2(Q^2) \rightarrow \langle r^2 \rangle_{1,2},\ \mu$
- axial vector (and tensor) coupling constants — $g_A(, g_T)$
- GPDs – generalized form factors and the form factors of the energy momentum tensor — $H, E(x, \xi, t) \rightarrow A(t),\ B(t),\ C(t)$
- decomposition of the nucleon spin — $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$
- conclusions

QCDSF/UKQCD n_f=2 improved Wilson action parameters

- $N_f = 2$ dynamical Wilson - fermions with (NP) clover - improvement
- only connected contributions

- lattice spacing fixed using $m_N \leftrightarrow r_0 = 0.467\mathrm{fm}$
- three projectors

$$\tilde{\Gamma}_{\mathrm{unpol}} = \frac{1}{2}(1+\gamma_0),$$

$$\tilde{\Gamma}_{1,2} = \frac{1}{2}(1+\gamma_0)\gamma_5\gamma_{1,2}$$

- three sink - momenta $p' = (0,0,0),(1,0,0),(0,1,0)$
- non – perturbative operator renormalization

#	b	k	L	a (fm)	L (fm)	m_p (GeV)	$m_p L$
1	5.20	0.13420	16	0.0856	1.37	1.348	9.4
2	5.20	0.13500	16	0.0856	1.37	0.956	6.6
3	5.20	0.13550	16	0.0856	1.37	0.67	4.7
6	5.25	0.13460	16	0.0794	1.27	1.225	7.9
7	5.25	0.13520	16	0.0794	1.27	0.949	6.1
8	5.25	0.13575	24	0.0794	1.91	0.635	6.1
9	5.25	0.13600	24	0.0794	1.91	0.457	4.4
11	5.29	0.13400	16	0.0753	1.2	1.511	9.2
12	5.29	0.13500	16	0.0753	1.2	1.102	6.7
13	5.29	0.13550	12	0.0753	0.9	0.945	4.3
14	5.29	0.13550	16	0.0753	1.2	0.871	5.3
15	5.29	0.13550	24	0.0753	1.81	0.857	7.8
16	5.29	0.13590	12	0.0753	0.9	0.883	4.
17	5.29	0.13590	16	0.0753	1.2	0.66	4.
18	5.29	0.13590	24	0.0753	1.81	0.627	5.7
19	5.29	0.13620	24	0.0753	1.81	0.407	3.7
21	5.29	0.13632	32	0.0753	2.41	0.282	3.4
22	5.29	0.13632	40	0.0753	3.01	0.271	4.1
23	5.29	0.13640	40	0.0753	3.01	0.17	2.6
24	5.40	0.13500	24	0.0672	1.61	1.183	9.7
25	5.40	0.13560	24	0.0672	1.61	0.917	7.5
26	5.40	0.13610	24	0.0672	1.61	0.648	5.3
27	5.40	0.13625	24	0.0672	1.61	0.558	4.6
28	5.40	0.13640	24	0.0672	1.61	0.451	3.7
29	5.40	0.13640	32	0.0672	2.15	0.441	4.8
30	5.40	0.13660	32	0.0672	2.15	0.255	2.8
31	5.40	0.13660	48	0.0672	3.23	0.233	3.8

LHPC simulation details

- mixed action approach: DW fermions on a Asqtad staggered sea (MILC) for N_f=2+1; including HYP-smearing
- L_s=16, m_{res} ⊙ $0.1 m_q$
- a ~ 0.124 fm; volumes of ~(2.5 and ~3.5 fm)^3
- two sink momenta P'=(0,0,0), (-1,0,0)

operator renormalization: $Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{pert}}{Z_A^{pert}} Z_A^{nonpert}$

$\overline{\text{MS}}$ at 4 GeV^2

estimates of $Z_{\mathcal{O}}^{nonpert}$ avail.

Light m_{sea}^{asqtad}	Volume Ω	$(am)_\pi$	$(af)_\pi$	$(am)_N$	m_π [MeV]	f_π [MeV]	m_N [MeV]
0.007	$20^3 \times 64$	0.1842(7)	0.0657(3)	0.696(7)	292.99(111)	104.49(45)	1107.1(111)
0.010	$28^3 \times 64$	0.2238(5)	0.0681(2)	0.726(5)	355.98(80)	108.31(34)	1154.8(80)
0.010	$20^3 \times 64$	0.2238(5)	0.0681(2)	0.726(5)	355.98(80)	108.31(34)	1154.8(80)
0.020	$20^3 \times 64$	0.3113(4)	0.0725(1)	0.810(5)	495.15(64)	115.40(23)	1288.4(80)
0.030	$20^3 \times 64$	0.3752(5)	0.0761(2)	0.878(5)	596.79(80)	121.02(34)	1396.5(80)
0.040	$20^3 \times 32$	0.4325(12)	0.0800(5)	0.941(6)	687.94(191)	127.21(78)	1496.8(95)
0.050	$20^3 \times 32$	0.4767(10)	0.0822(4)	0.991(5)	758.24(159)	130.70(67)	1576.3(80)

- N_f=2+1 DW fermions (RBC/UKQCD configs)
- volume of ~(2.7 fm)^3

NP-operator renormalization

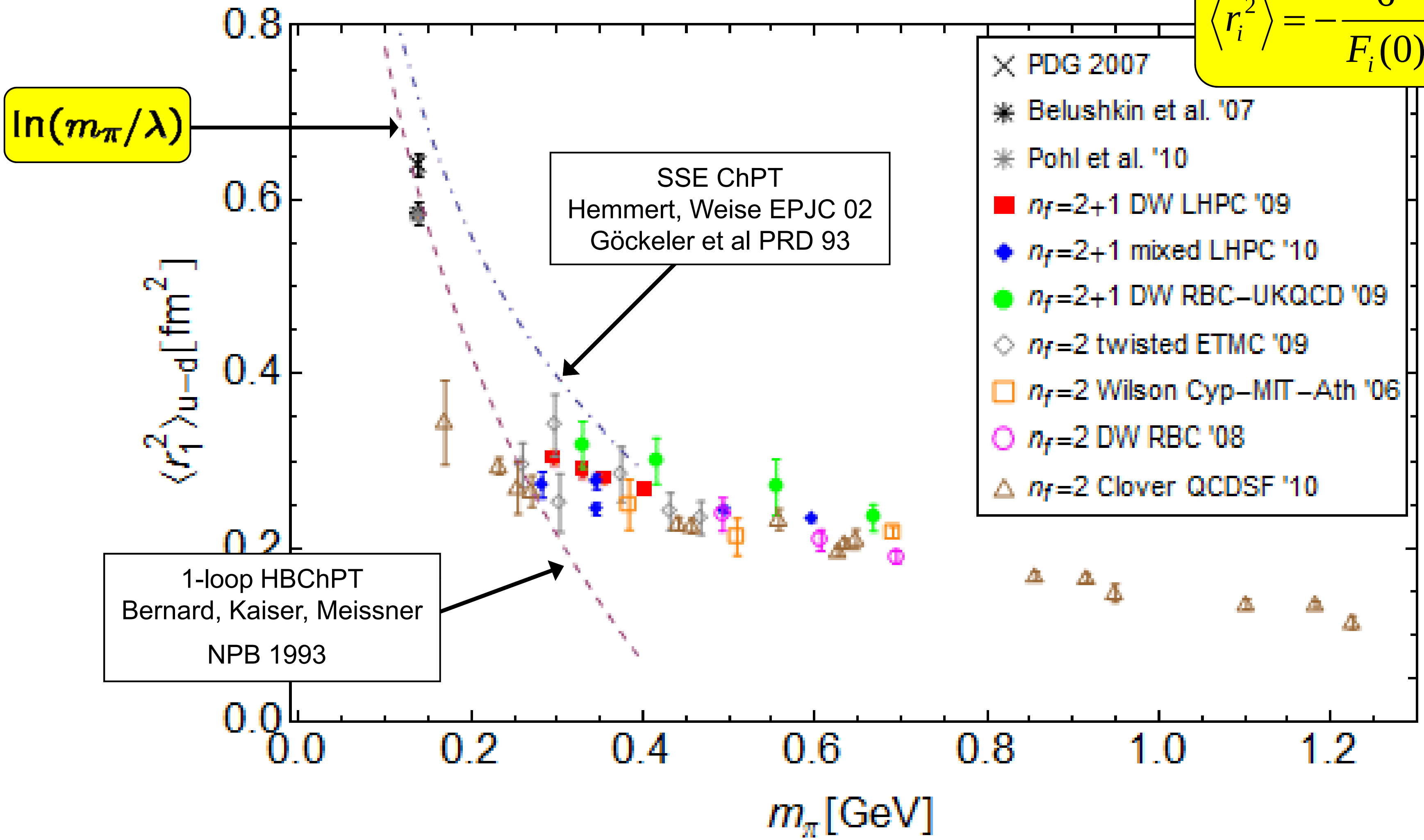
$L_s^3 \times L_t$	a [fm]	T	#	am_l/am_h	$am'_{res} \times 10^3$	am_π	m_π [MeV]	aF_π	F_π [MeV]	aM_N	M_N [MeV]
$24^3 \times 64$	0.114	9	3208	0.005/0.04	3.15(1)	0.1901(3)	329(5)	0.06100(11)	105.5(1.7)	0.657(4)	1136(20)
$32^3 \times 64$	0.084	12	4928	0.004/0.03	0.665(3)	0.1268(3)	297(5)	0.04400(15)	102.9(1.8)	0.474(4)	1109(21)
$32^3 \times 64$	0.084	12	7064	0.006/0.03	0.663(2)	0.1519(3)	355(6)	0.04571(09)	107.0(1.8)	0.501(2)	1172(21)
$32^3 \times 64$	0.084	12	4224	0.008/0.03	0.668(3)	0.1724(3)	403(7)	0.04755(18)	111.3(2.0)	0.522(2)	1221(21)

Proton mean square radii – Dirac isovector radius

Dirac and Pauli FFs

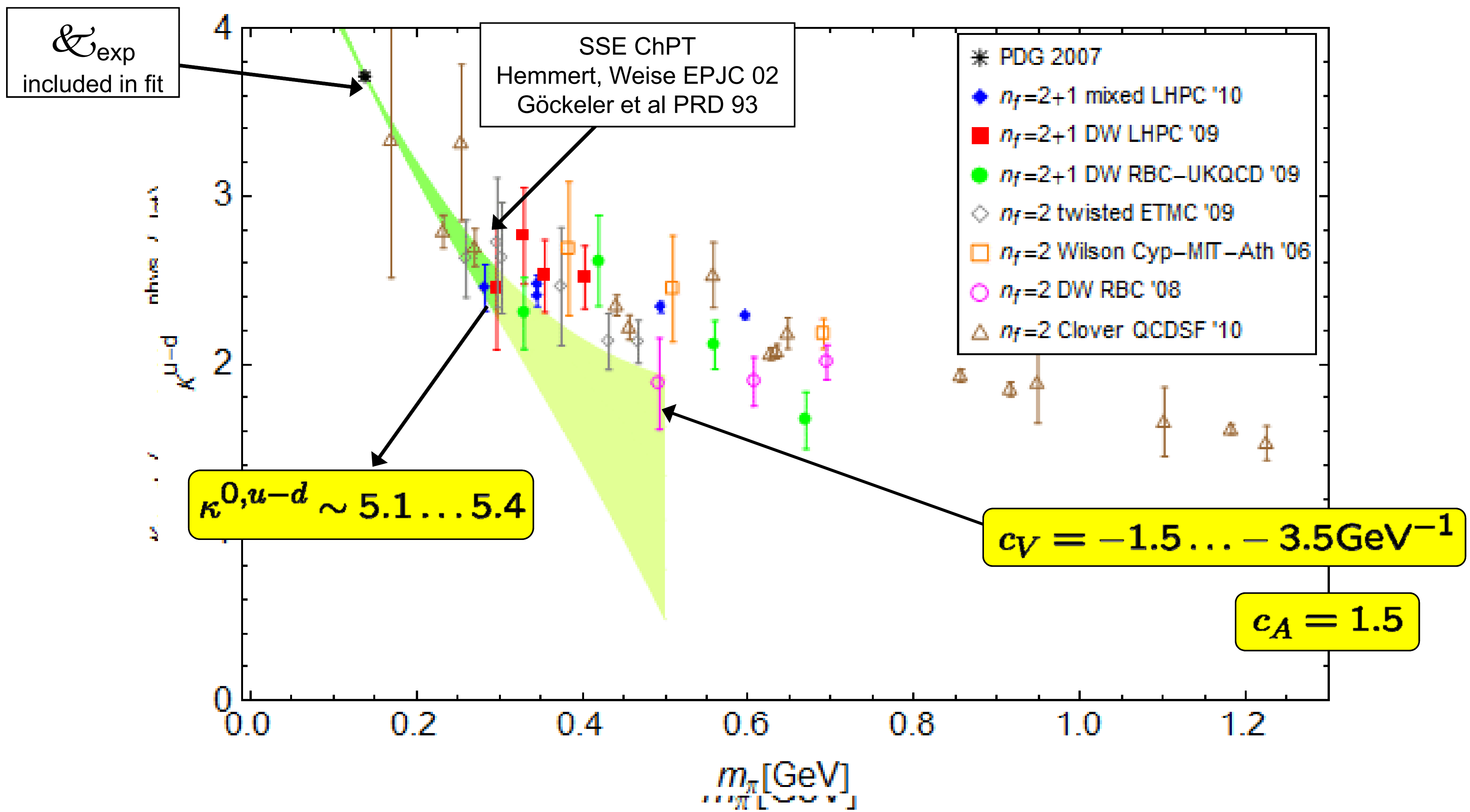
$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\Big\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\Big\}U(P)$$

$$\langle r_i^2\rangle = -\frac{6}{F_i(0)}\frac{d}{dt}F_i(t)\Big|_{t=0}$$



Nucleon isovector anomalous magnetic moment

(anomalous) magnetic moment → $\kappa = F_2(Q^2 \to 0) = \mu - F_1(0)$



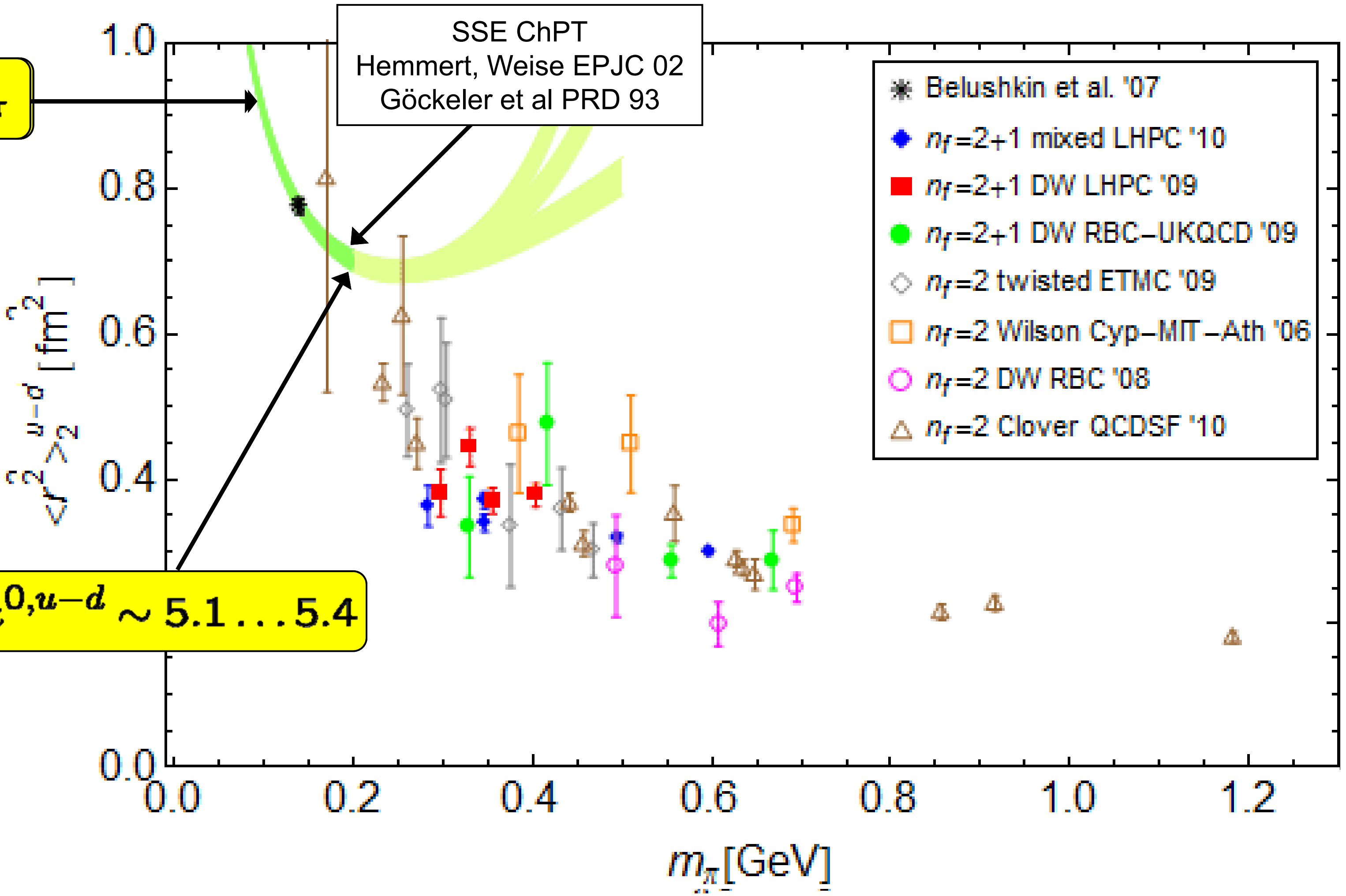
Proton mean square radii – Pauli isovector radius

Dirac and Pauli FFs

$$\langle P'|\overline{q}\gamma_\mu q|P\rangle = \overline{U}(P')\Big\{\gamma_\mu F_1(t) + i\frac{\sigma_{\mu\nu}\Delta^\nu}{2m_N}F_2(t)\Big\}U(P)$$

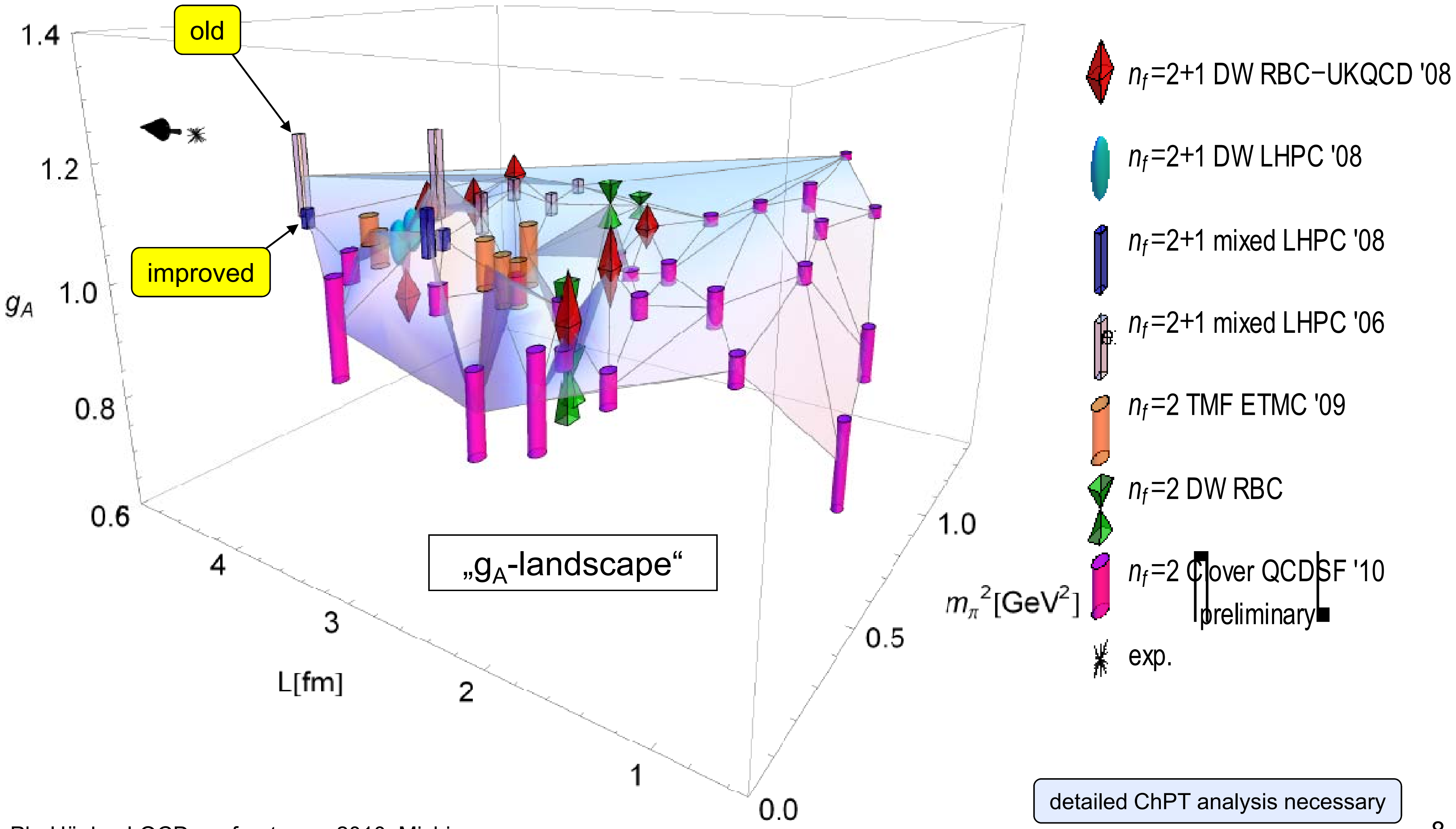
$1/m_\pi$

$\kappa^{0,u-d} \sim 5.1\ldots5.4$



Nucleon axial vector coupling constant

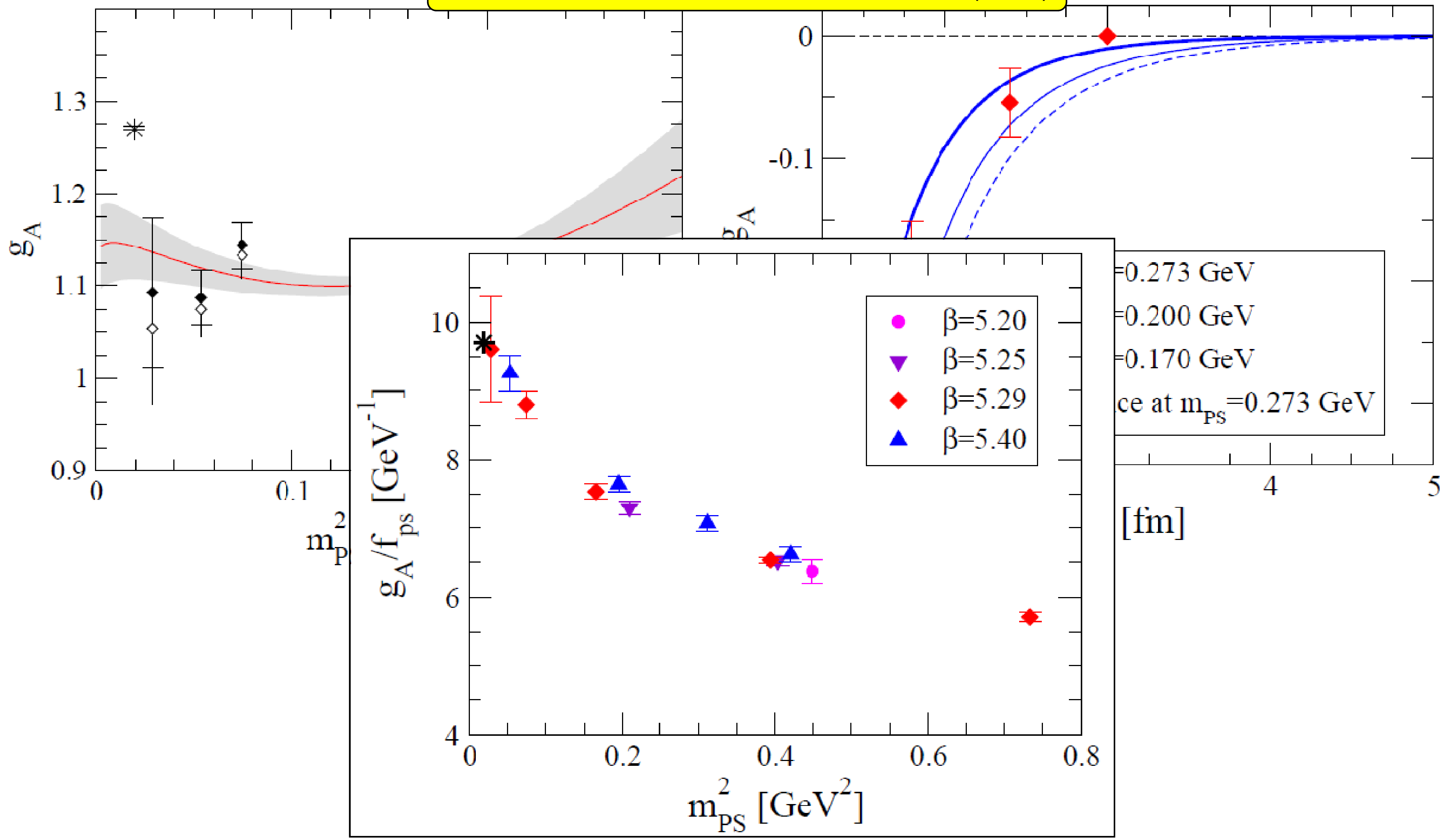
$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A \bar{U}(P)\gamma_\mu\gamma_5 U(P)$$



detailed ChPT analysis necessary

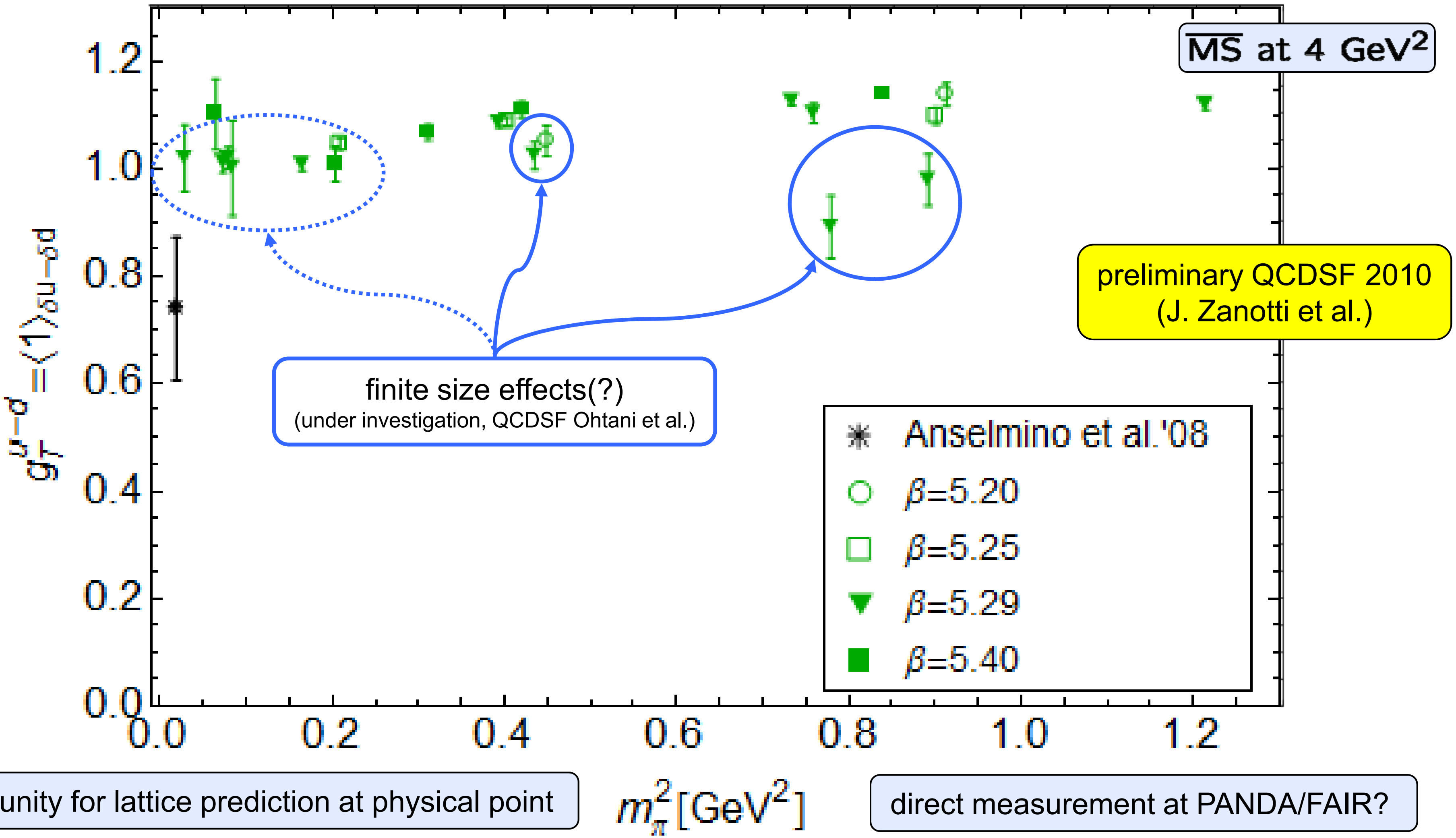
Nucleon axial vector coupling constant

QCDSF/UKQCD, Pleiter et al. PoSLat 153 (2010)



Tensor charge

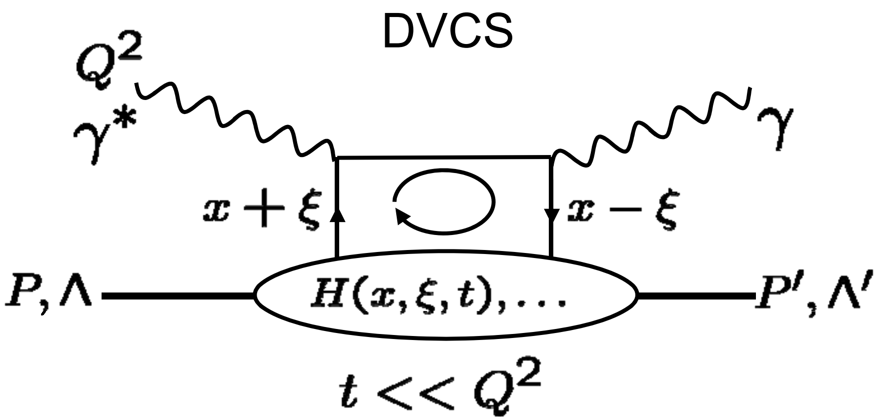
$$\langle P|\bar{q} i\sigma_{\mu\nu} q|P\rangle = g_T \bar{U}(P) i\sigma_{\mu\nu} U(P) \quad \longleftrightarrow \quad g_T = A_{T10}(0) = \int_{-1}^{+1} dx\, \delta q(x) = \langle 1\rangle_{\delta q} - \langle 1\rangle_{\delta\bar{q}}$$



opportunity for lattice prediction at physical point

direct measurement at PANDA/FAIR?

Generalized parton distributions (GPDs) in experiment



DVCS – factorization
[Collins `99,`01]

Müller et al. (1994); Ji, Radyushkin (1996); …

$$H(x,\xi,t), E, \tilde{H}, \tilde{E}, H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

measurements at HERMES/DESY, COMPASS/CERN, Jefferson Lab

$$\text{Im}\mathcal{H}(\xi,t) = \pi\Big\{H(\xi,\xi,t) - H(-\xi,\xi,t)\Big\} \equiv \pi H^+(\xi,\xi,t)$$

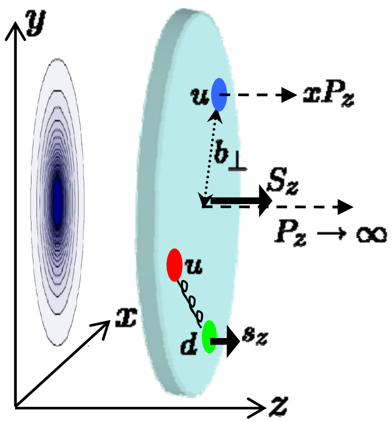
Dispersion relations [Anikin, Teryaev 2007; Diehl, Ivanov 2007;...]

$$\text{Re}\mathcal{H}(\xi,t) = \text{PV}\int_{-1}^{1} dx H^+(x,x,t)\left\{\frac{1}{\xi-x} - \frac{1}{\xi+x}\right\} + C(t)$$

(almost) no access to separate x,⊠-dependencies from DVCS and DV meson production at LO

excellent opportunity for lattice QCD to provide independent/complimentary information

Higher (x^{n-1}-) moments of GPDs



$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n-1} (-2\xi)^i A_{n,i}(t) + (-2\xi)^n C_{n,0}(t)|_{n\ \text{even}}$$

$$\int_{-1}^{1} dx\, x^{n-1} E(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n-1} (-2\xi)^i B_{n,i}(t) - (-2\xi)^n C_{n,0}(t)|_{n\ \text{even}}$$

GFFs

Ji&Lebed PRD 2000
Ph.H. PLB 2004

$t = \Delta^2 = (P' - P)^2,\ \xi = -n\cdot\Delta/2$

$n = 1$

$F_1(Q^2 = -t^2) = A_{10}(t)$
$F_2(Q^2 = -t^2) = B_{10}(t)$

$n = 2$

form factors of the EM-tensor

$\langle x^{n-1}\rangle_q = A^q_{n0}(0)$
$\langle x^{n-1}\rangle_{\Delta q} = \tilde{A}^q_{n0}(0)$

Decomposition of the nucleon spin

Ji's nucleon spin sum rule (X. Ji, PRL 1997)

$$\frac{1}{2} = S_z = \sum_{q,g} \frac{1}{2}\left\{A_{20}(0) + B_{20}(0)\right\}$$
$$= \sum_q J_q + J_g = \sum_q \frac{1}{2}\Delta\Sigma_q + \sum_q L_q + J_g$$

$$L_q \equiv J_q - \frac{1}{2}\Delta\Sigma_q$$
$$(L_g \equiv J_g - \Delta G)$$

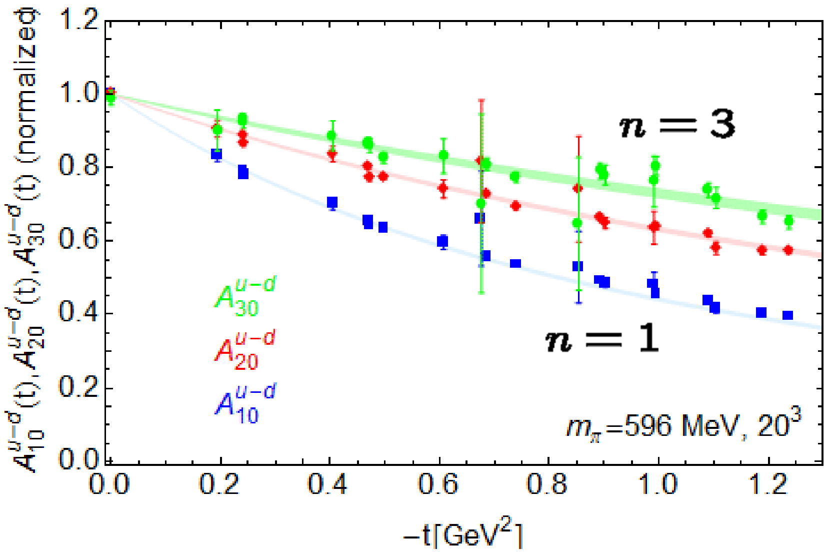
everything is:
-gauge-invariant
-scale & scheme dependent
-measurable

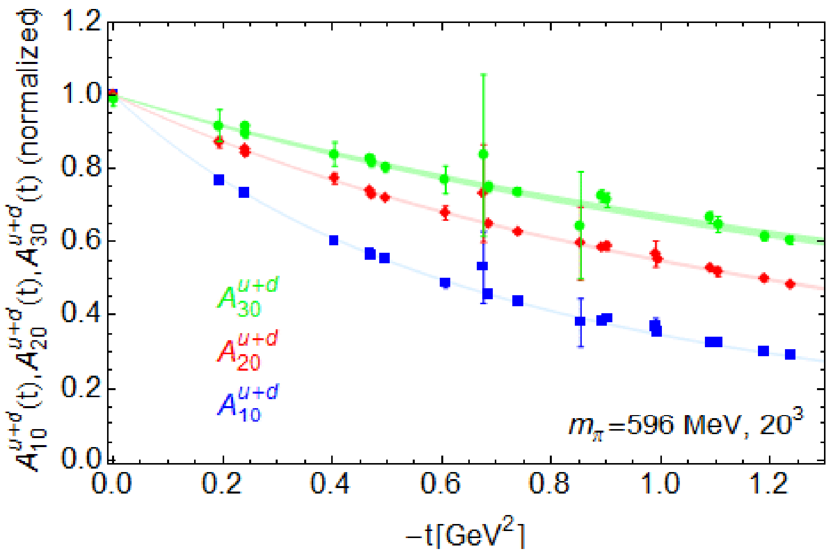
correlations in x and t

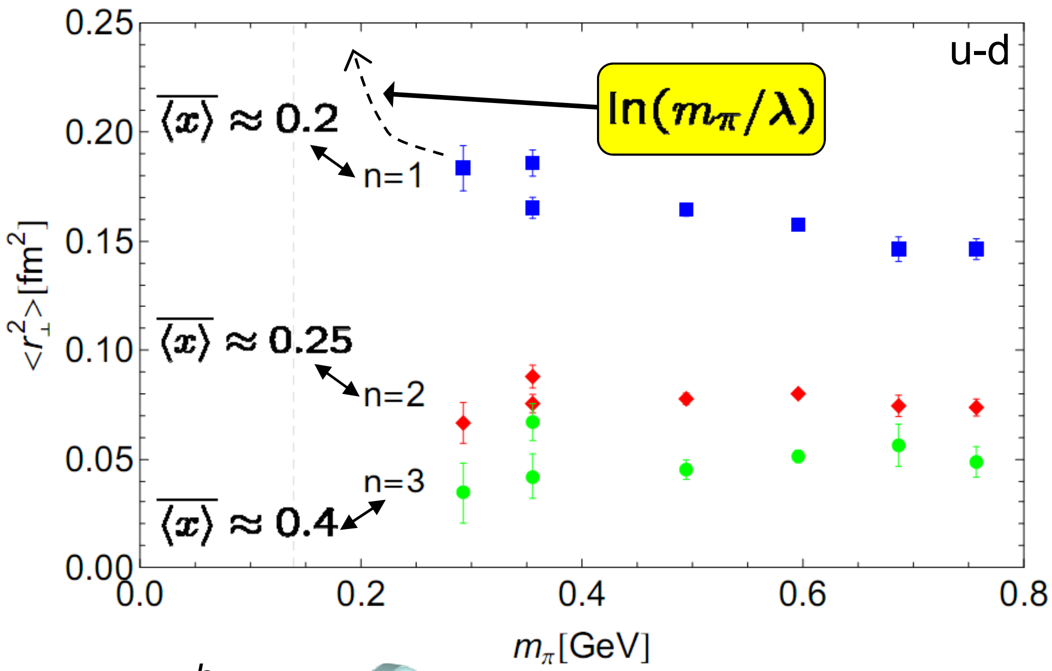
LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

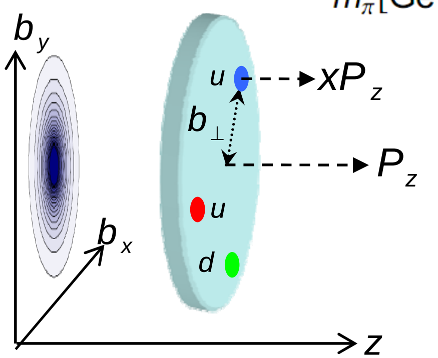
$$\bar{x} \rightarrow 1 \Leftrightarrow n \rightarrow \infty$$

$$\langle r_\perp^2 \rangle_n = \frac{\int d^2b_\perp b_\perp^2 \int dx x^{n-1} H(x, b_\perp)}{\int d^2b_\perp \int dx x^{n-1} H(x, b_\perp)}$$









$$\xrightarrow{x \rightarrow 1} \delta^2(b_\perp)$$

Pion mass dependence of generalized radii for n>1

$$\langle r_\perp^2 \rangle_{(n>1)} = \left(\frac{-4}{A_{n0}(t)} \frac{d}{dt} A_{n0}(t) \right)_{t=0} \approx \frac{\text{const}}{\langle x^{n-1} \rangle (m_\pi)}$$

Diehl, Manashov, Schäfer
EPJ A31 (2007)

no (logarithmic) divergence in m_π for n>1

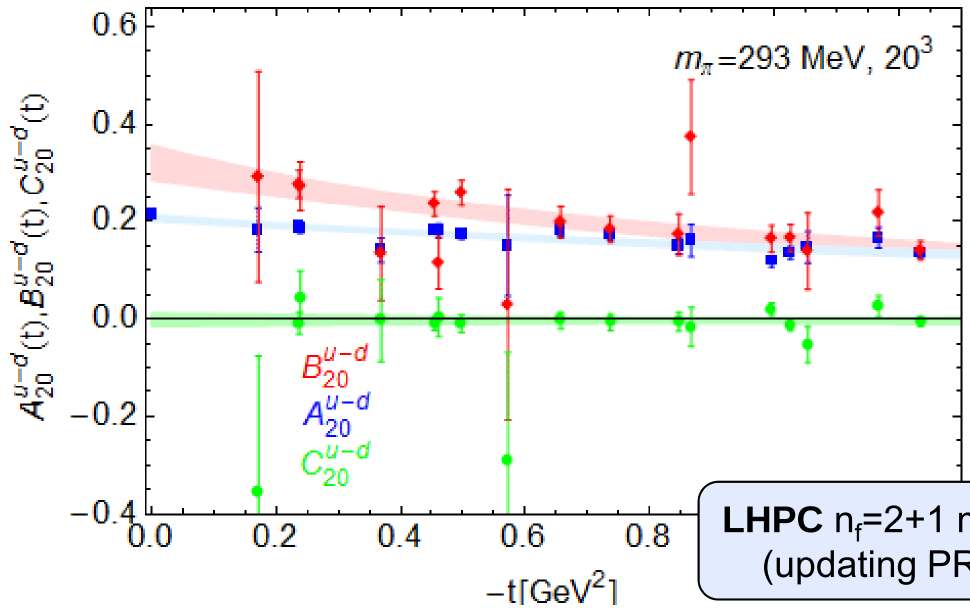
strong effect seen for n=1,...,3 will most likely persist when $m_\pi \to m_\pi^{phys}$

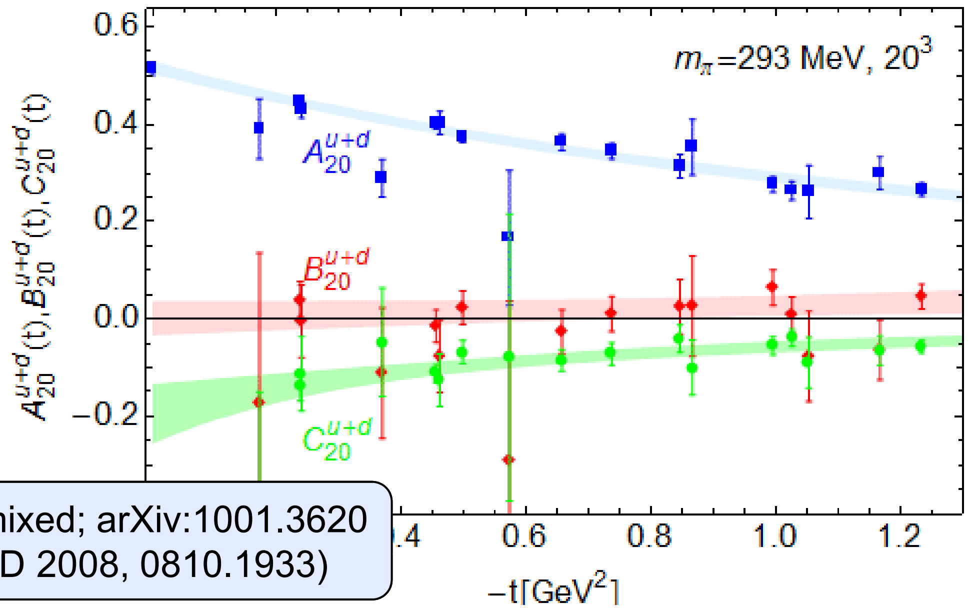
in any case

$$\frac{\langle r_\perp^2 \rangle^{u-d}_{(n=1)}}{\langle r_\perp^2 \rangle^{u-d}_{(n>1)}} \xrightarrow{m_\pi \to 0} \infty$$

n=2 - A, B, C - Form factors of the energy momentum tensor

$$\langle P'|\bar{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \langle P'|T_q^{\mu\nu}|P\rangle = U(P')\Big\{\gamma^{\{\mu}P^{\nu\}}A_{20}(t) - \frac{i\Delta_\rho\sigma^{\rho\{\mu}P^{\nu\}}}{2m_N}B_{20}(t) + \frac{\Delta^\mu\Delta^\nu}{m_N}C_{20}(t)\Big\}U(P)$$





LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

$B_{20}^{u-d} > A_{20}^{u-d}$
$C_{20}^{u-d} \approx 0$

$A_{20}^{u+d} > B_{20}^{u+d}$
$C_{20}^{u+d} < 0$

seems to be compatible with large N_c limit
– see e.g. Goeke, Polyakov, Vanderhaeghen PiPaNP 2001

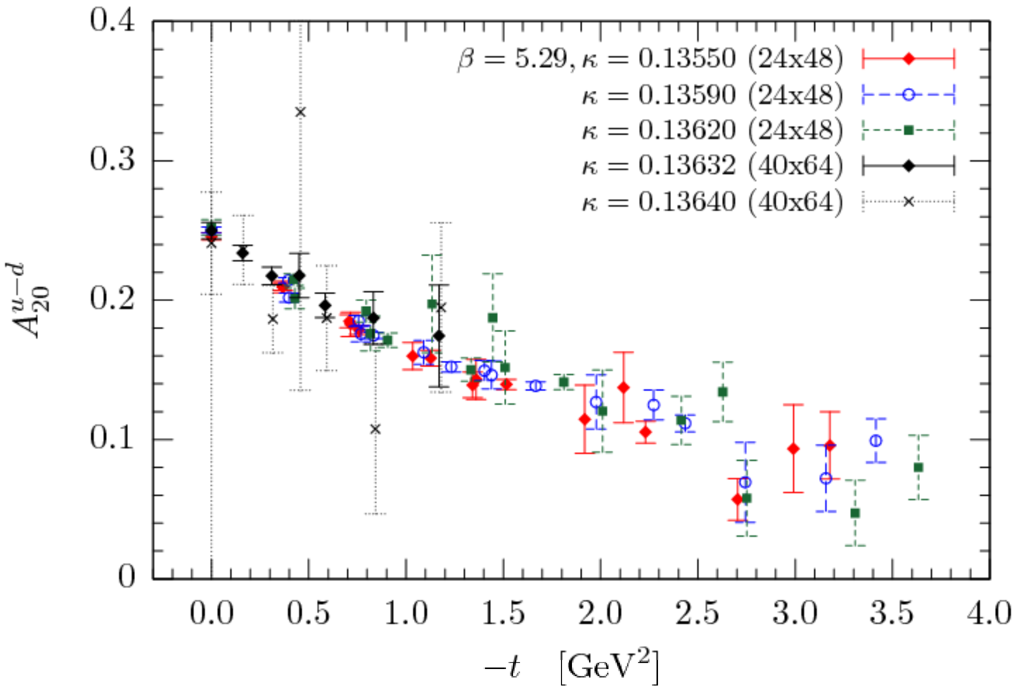
disconnected contributions
are not included↔
only u-d is „exact"

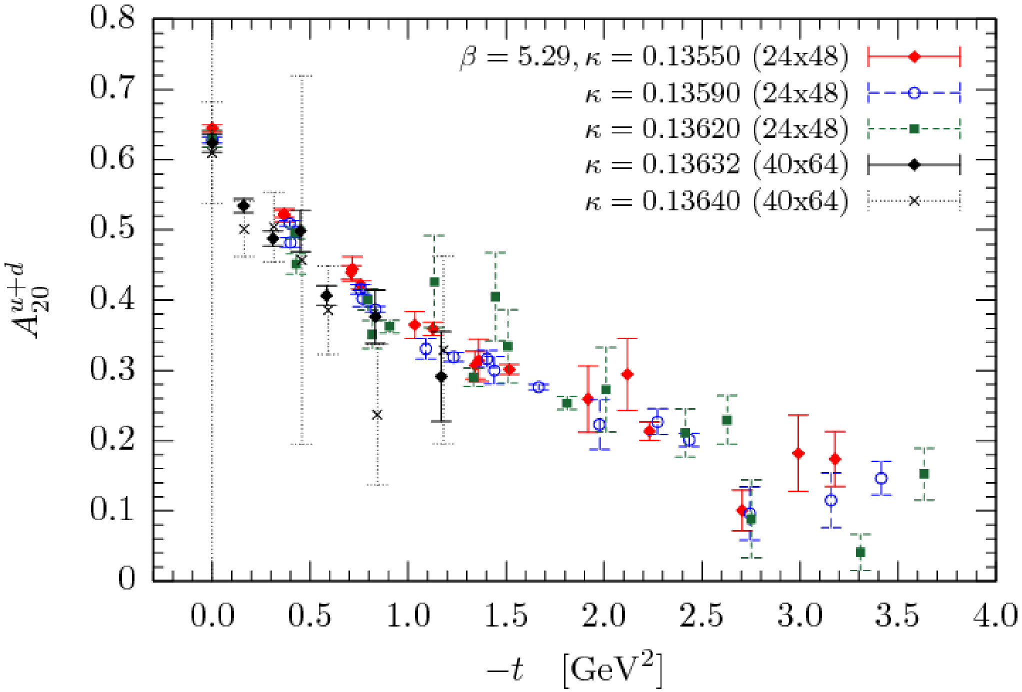
$\overline{MS}$ at 4 GeV^2

n=2 - A, B, C - Form factors of the energy momentum tensor

$$\langle P'|\bar{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \langle P'|T_q^{\mu\nu}|P\rangle = U(P')\left\{\gamma^{\{\mu}P^{\nu\}}A_{20}(t) - \frac{i\Delta_\rho\sigma^{\rho\{\mu}P^{\nu\}}}{2m_N}B_{20}(t) + \frac{\Delta^\mu\Delta^\nu}{m_N}C_{20}(t)\right\}U(P)$$

preliminary QCDSF 2010
(A. Sternbeck, F. Winter, D. Pleiter, J. Zanotti et al.)





n=2 - A, B, C - Form factors of the energy momentum tensor

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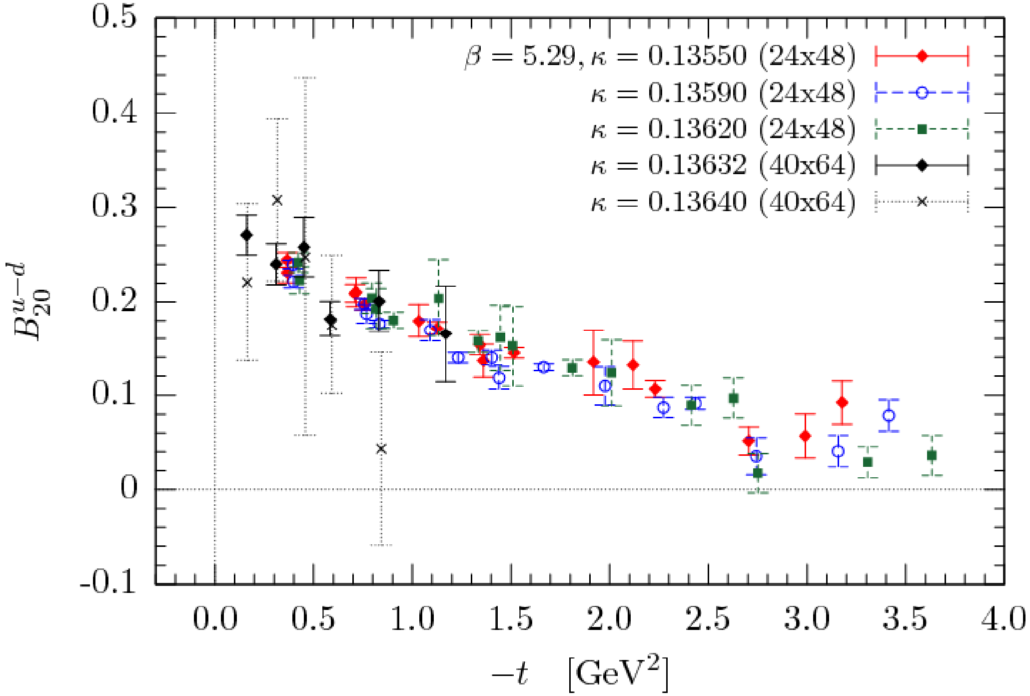
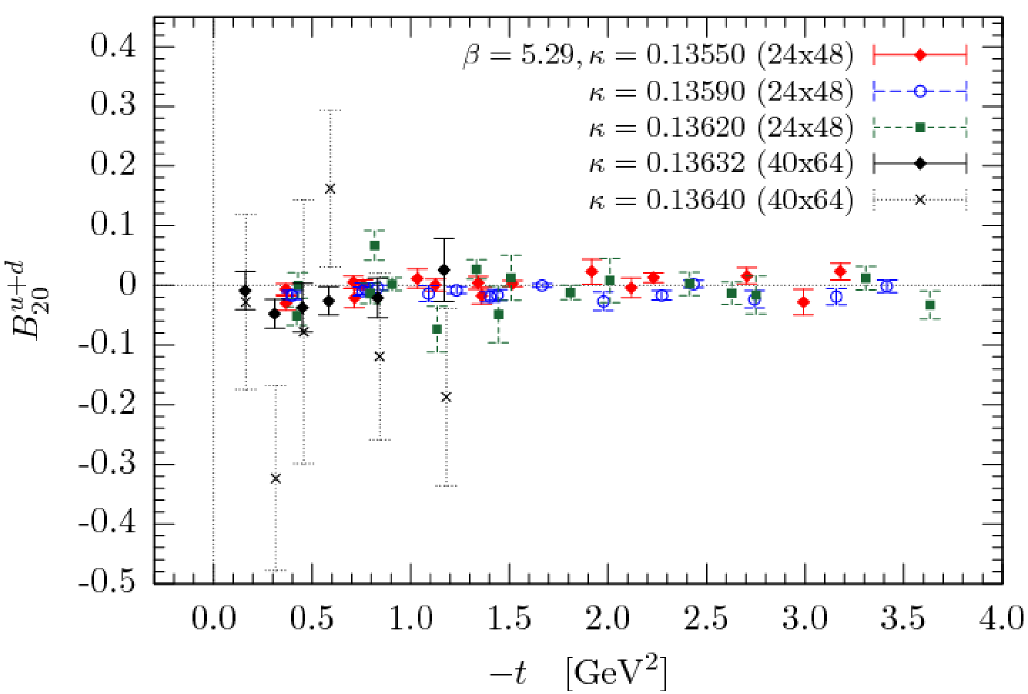
(A. Sternbeck, F. Winter, D. Pleiter, J. Zanotti et al.)





n=2 - A, B, C - Form factors of the energy momentum tensor

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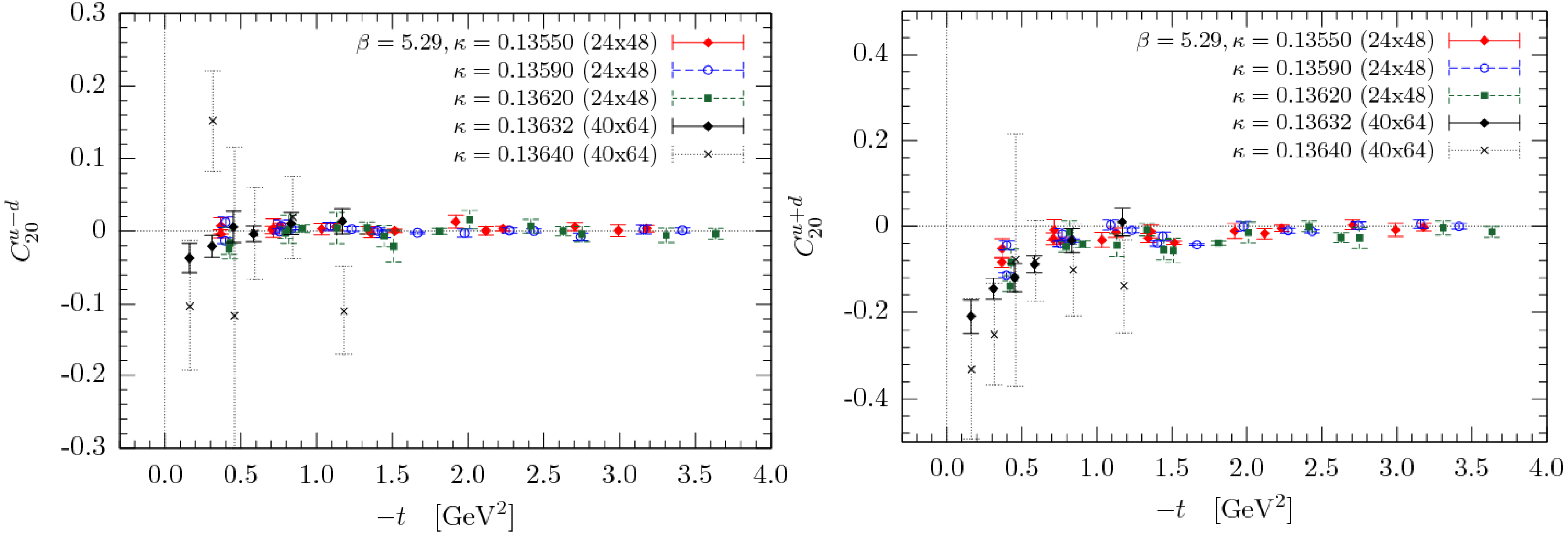
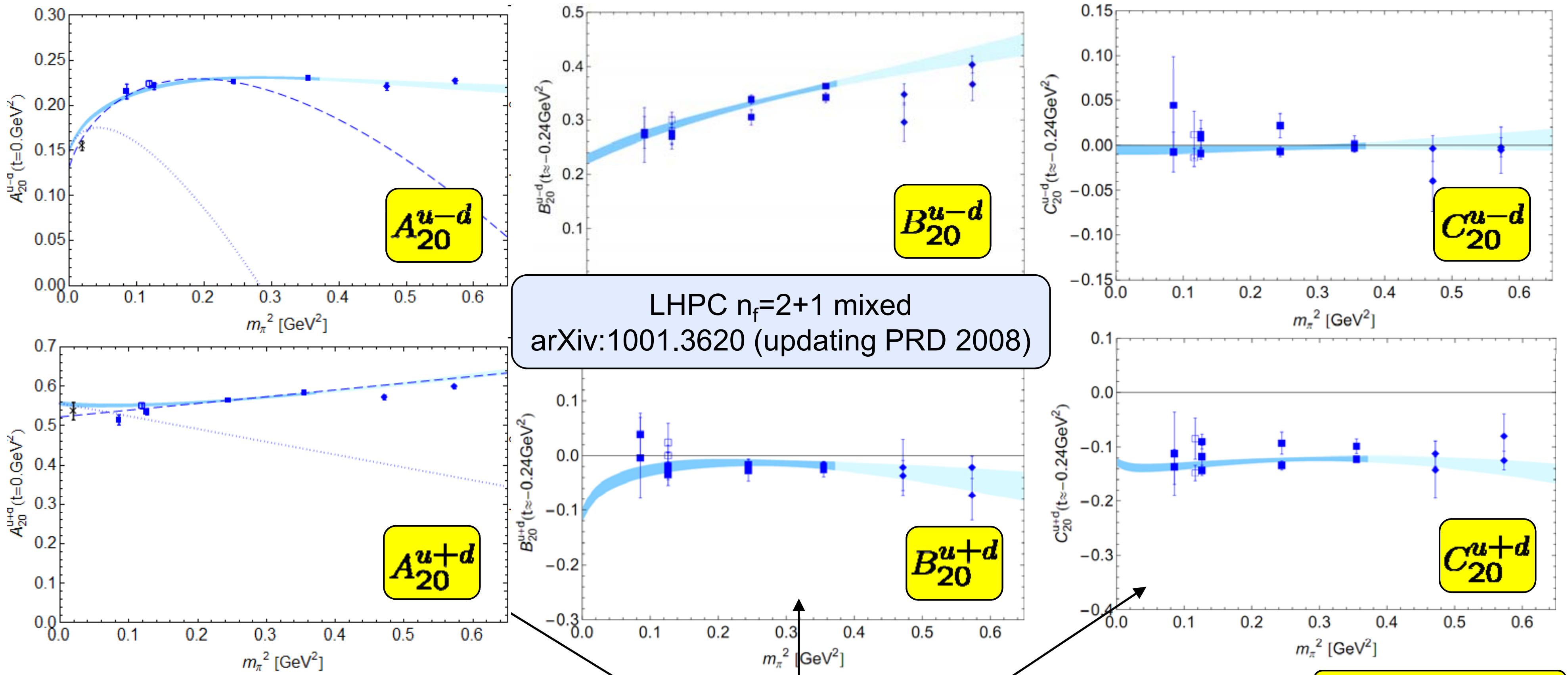


Form factors of the energy momentum tensor

Covariant BChPT calculation by Dorati, Gail, Hemmert NPA 2008

global simultaneous fits of A, B, C with common parameter <x>
+ 8 additional free parameters/LECs, to >80 lattice data points in each case (u-d and u+d)



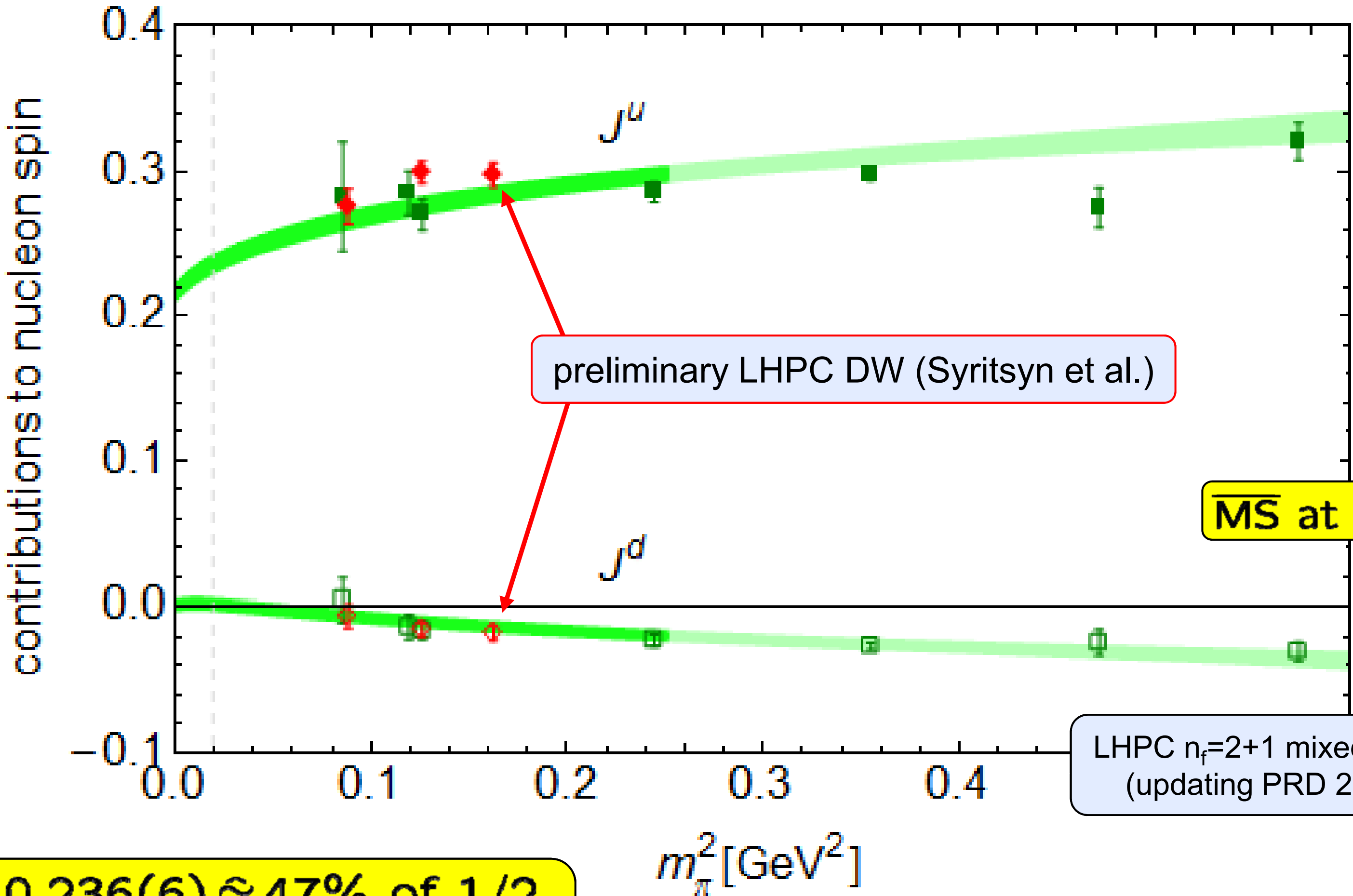
LHPC n_f=2+1 mixed
arXiv:1001.3620 (updating PRD 2008)

only quark line connected contributions

$\overline{\text{MS}}$ at 4 GeV^2

Quark angular momentum

$J_q = \frac{1}{2}\left(A_{20}^q(0) + B_{20}^q(0)\right)$ — from covariant BChPT extrapolations — Dorati, Gail, Hemmert NPA 2008



LHPC n_f=2+1 mixed; arXiv:1001.3620 (updating PRD 2008, 0810.1933)

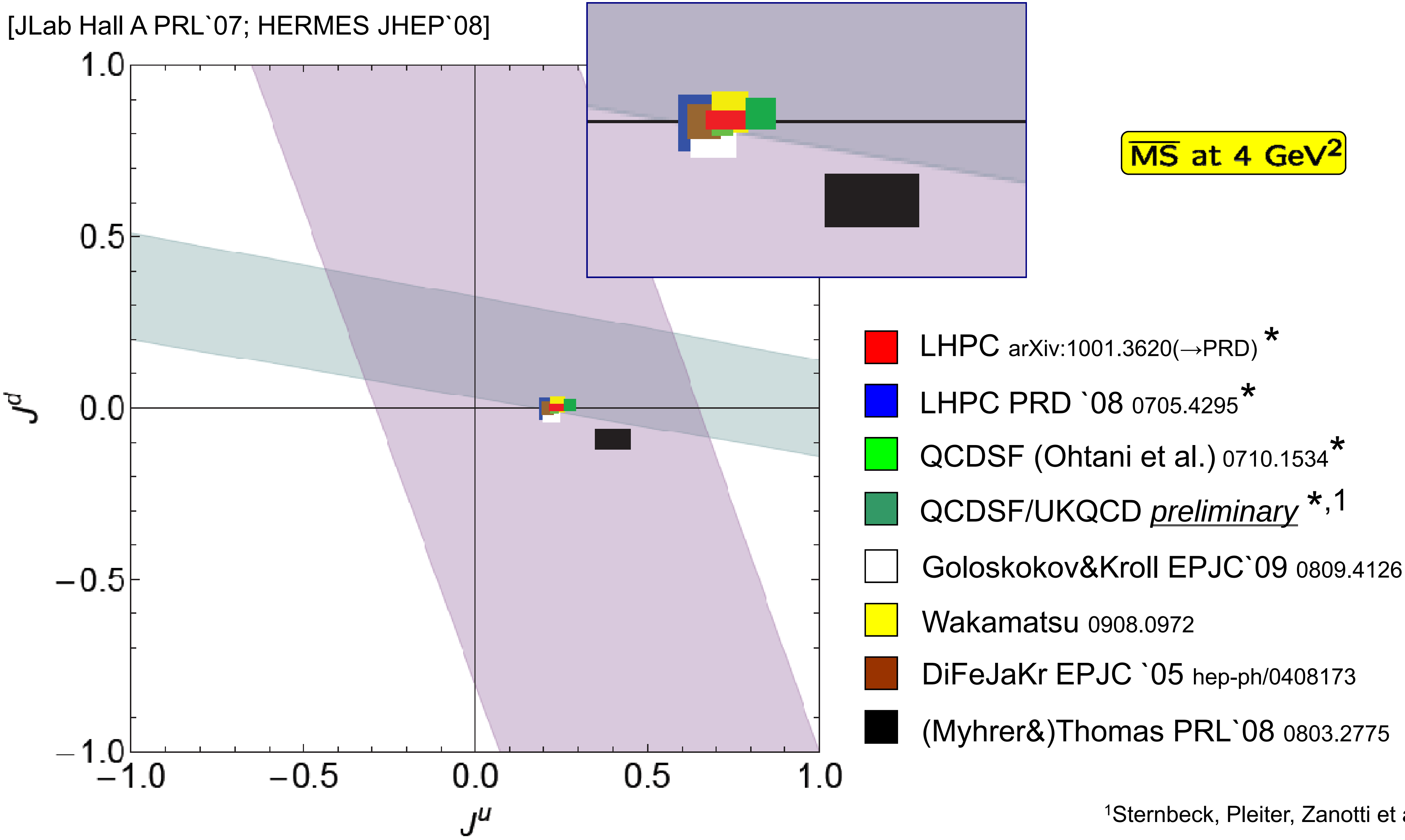
$J^u = 0.236(6) \approx 47\%$ of 1/2

$J^d = 0.0018(37) \approx 1\%$ of 1/2

$J^{u+d} \approx 0.238 \pm 0.008 \approx 48\%$ of 1/2

pioneering lattice calculations by Mathur et al. 1999 and Gadiyak, Ji and Jung in 2001

J_u, J_d template figure

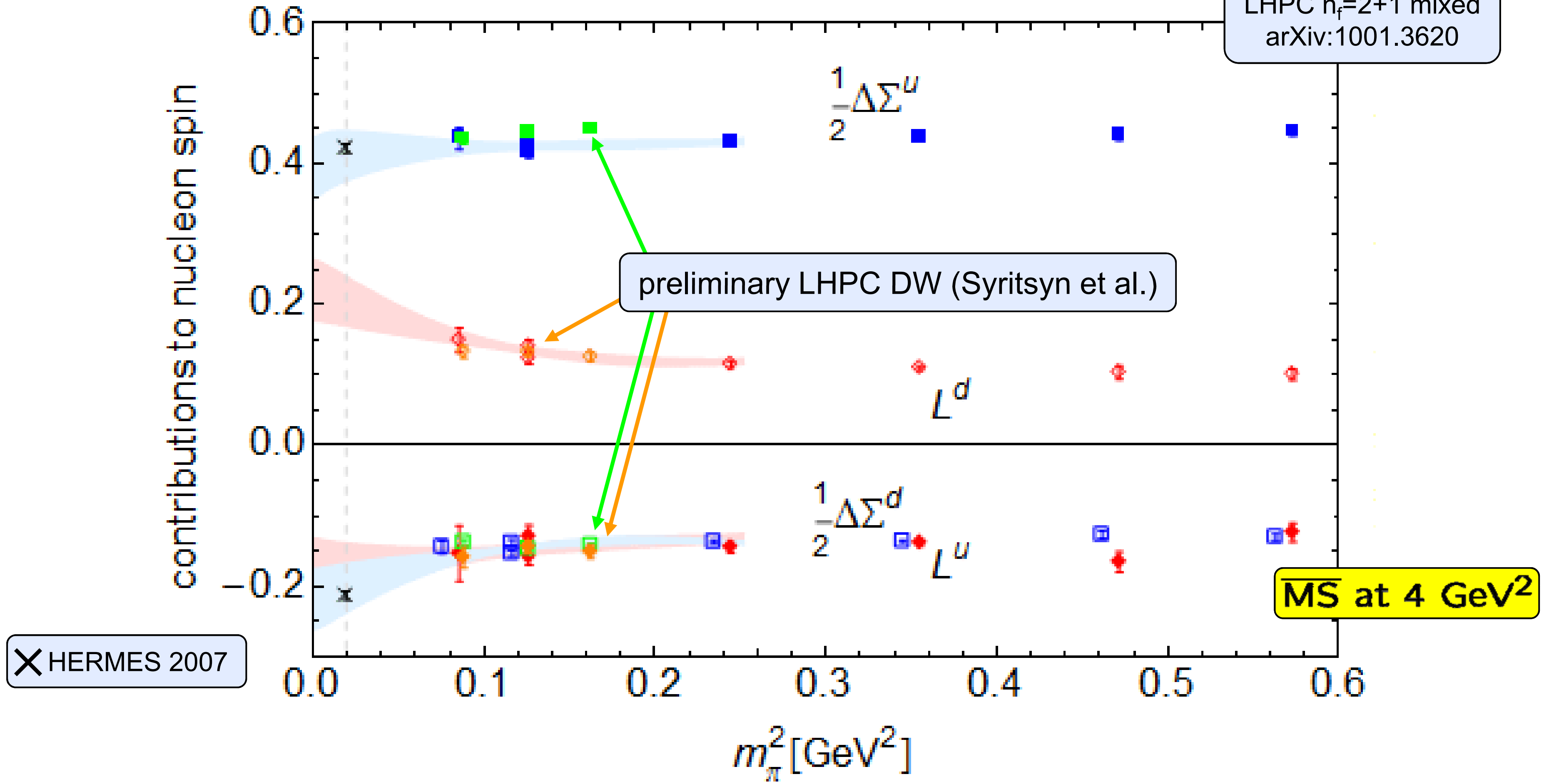


[1] Sternbeck, Pleiter, Zanotti et al.

*[non-singlet, connected only; add. uncertainties due to chiral extrapolations, renormalization]

Quark spin and orbital angular momentum

$L_q = J_q - \Delta\Sigma_q/2$



$J^u \approx 0.236 \pm 0.006 \mathrel{\hat{=}} 48\%$ of $1/2$

$J^d \approx 0.002 \pm 0.004$

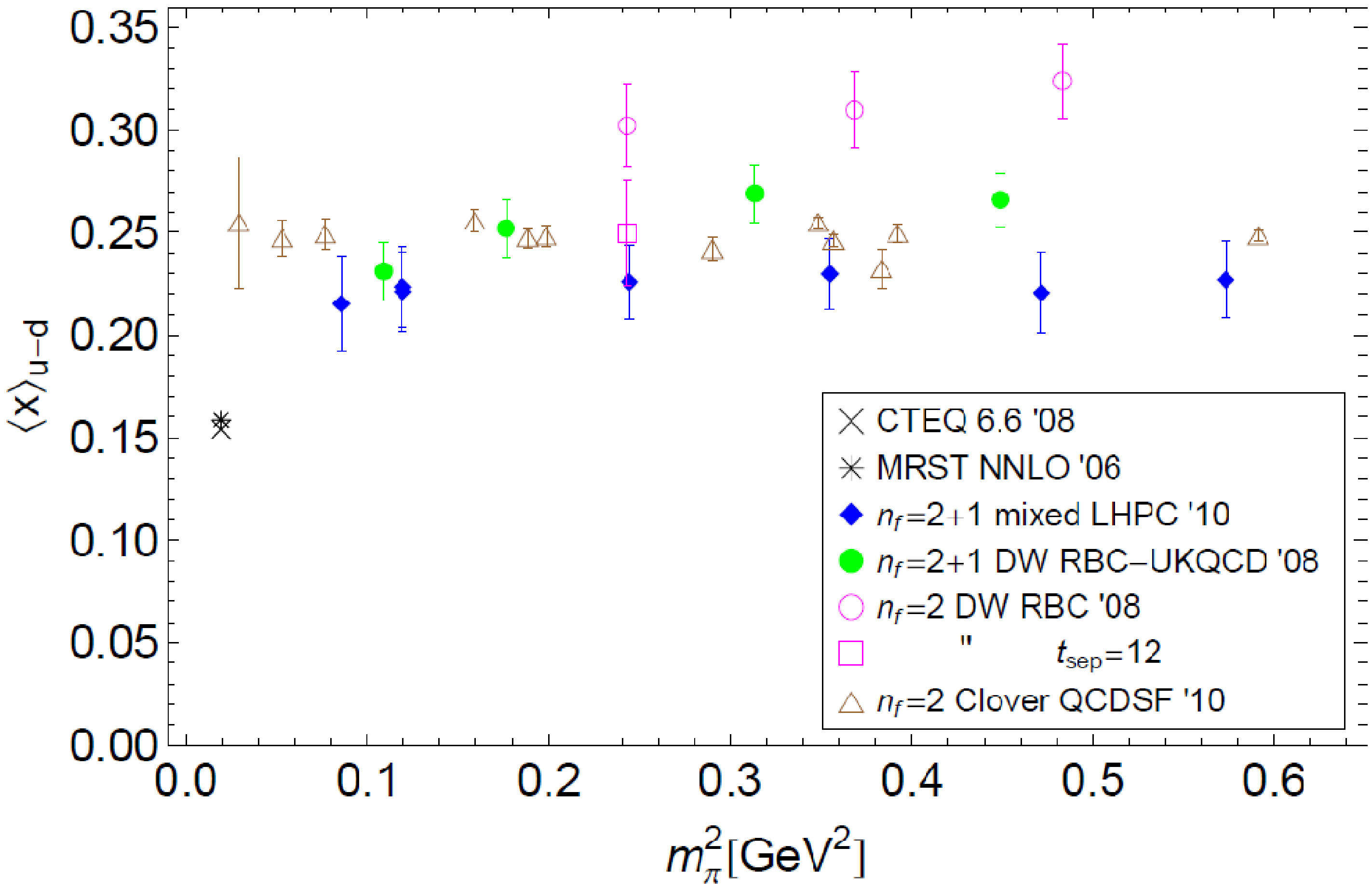
$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\%$ of $1/2$

$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

$\kappa^{u+d} = 3\kappa^{p+n} = -0.36$

Momentum fraction of quarks in the nucleon

$$\langle P|\overline{q}\gamma^{\{\mu}D^{\nu\}}q|P\rangle = \overline{U}(P)\gamma^{\{\mu}P^{\nu\}}U(P)\langle x\rangle \longleftrightarrow \langle x\rangle = A_{20}(0) = \int_{-1}^{+1} dx\, x\, q(x) = \langle x\rangle_q + \langle x\rangle_{\overline{q}}$$



substantial systematic uncertainties: renormalization, discretization errors, excited state contamination?

Conclusions

despite significant progress in lattice QCD studies of hadron structure still a long way to go

small statistical errors at $m_{\square}$~250...350 MeV for many observables

- have to address systematic effects and uncertainties
  - contaminations from excited states
  - (dipole, chiral,...) extrapolations in $t=Q^2$; **ptbcs**
  - disconnected diagrams
  - gluon contributions

most chiral extrapolations still not quantitatively reliable

- need high statistics lattice results for $m_{\square}$⦿250 MeV
  - try global, simultaneous chiral fits

already now qualitative and semi-quantitative predictions from the lattice available

mostly *complementary* to experimental efforts at HERMES, COMPASS, JLab

work done in collaboration with/based on results from

B. Bistrovic, J. Bratt, J.W. Negele,
A. Pochinsky, S. Syritsyn (MIT)
R.G. Edwards, B. Musch, D.G. Richards (JLab)
K. Orginos (W&M)
M. Engelhardt (New Mexico)
G. Fleming, M. Lin (Yale),
H.-W. Lin (INT),
H. Meyer (Mainz),
D.B. Renner (DESY Zeuthen),
M. Procura (TUM), W. Schroers

(LHPC)

D. Brömmel (Southampton),
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