The first Two Fermion Generations in Twisted Mass Lattice QCD

Karl Jansen

for the **European Twisted Mass Collaboration**

- **Maximally twisted mass fermions, $N_f = 2$ results**
- **Introducing $N_f = 2+1+1$ flavours**
- **Using $N_f = 2+1+1$ flavours of quarks**
  - light meson physics
  - Osterwalder-Seiler valence quarks
  - non-perturbative renormalization
  - overlap valence quarks

Why the first two generations?

- want to include as many quarks as possible: charm still realistic
- charm quark mass
- needed for charmed mesons, e.g. η, η', η_c and baryons
- decay constants f_D, f_{D_s}
- heavy quark effects in operator matrix elements
- running of α_s with $N_f = 4$
- very natural to include strange/charm doublet for twisted mass

- **Cyprus (Nicosia)**
  C. Alexandrou, M. Constantinou
- **France (Orsay, Grenoble)**
  R. Baron, B. Bloissier, Ph. Boucaud, M. Brinet, J. Carbonell, P. Guichon, P.A. Harraud, O. Pène
- **Italy (Rome I,II,III, Trento)**
  P. Dimopoulos, R. Frezzotti, V. Lubicz, G. Martinelli, G.C. Rossi, L. Scorzato, S. Simula, C. Tarantino
- **Netherlands (Groningen)**
  E. Pallante, S. Reker
- **Poland (Poznan)**
  K. Cichy, A. Kujawa
- **Spain (Huelva, Madrid, Valencia)**
  V. Gimenez, D. Palao, J. Rodriguez-Quintero, A. Shindler
- **Switzerland (Bern)**
  A. Deuzemann, U. Wenger
- **United Kingdom (Glasgow, Liverpool)**
  G. McNeile, C. Michael
- **Germany (Berlin/Zeuthen, Bonn, Hamburg, Münster)**
  V. Drach, F. Farchioni, J. González López, G. Herdoiza, K. Jansen, I. Montvay, G. Münster, M. Petschlies, T. Sudmann, C. Urbach, M. Wagner

Wilson (Frezzotti, Rossi) twisted mass QCD (Frezzotti, Grassi, Sint, Weisz)

Fermion action of twisted mass fermions

$$S_l = \sum_x^l \bar{\chi}_x \left[m_q + \tfrac{1}{2}\gamma_\mu \left[\nabla_\mu + \nabla_\mu^*\right] - ar\tfrac{1}{2}\nabla_\mu^*\nabla_\mu + i\mu_{\rm tm}\tau_3\gamma_5\right] \chi_x^l$$

$$S_h = \sum_x \bar{\chi}_x^h \left[m_q + \tfrac{1}{2}\gamma_\mu \left[\nabla_\mu + \nabla_\mu^*\right] - ar\tfrac{1}{2}\nabla_\mu^*\nabla_\mu i\gamma_5\tau_1\mu_\sigma + \tau_3\mu_\delta\right] \chi_x^h$$

- quark mass parameter m_q , twisted mass parameter $\mu_{\rm tm}$
- strange and charm quark masses

$$m_{s,c} = Z_P^{-1}\left(\mu_\sigma \pm Z_P/Z_S\mu_\delta\right)$$

simulation: $Z_P/Z_S \approx 0.65$

- note, m_q the same in S_l and S_h

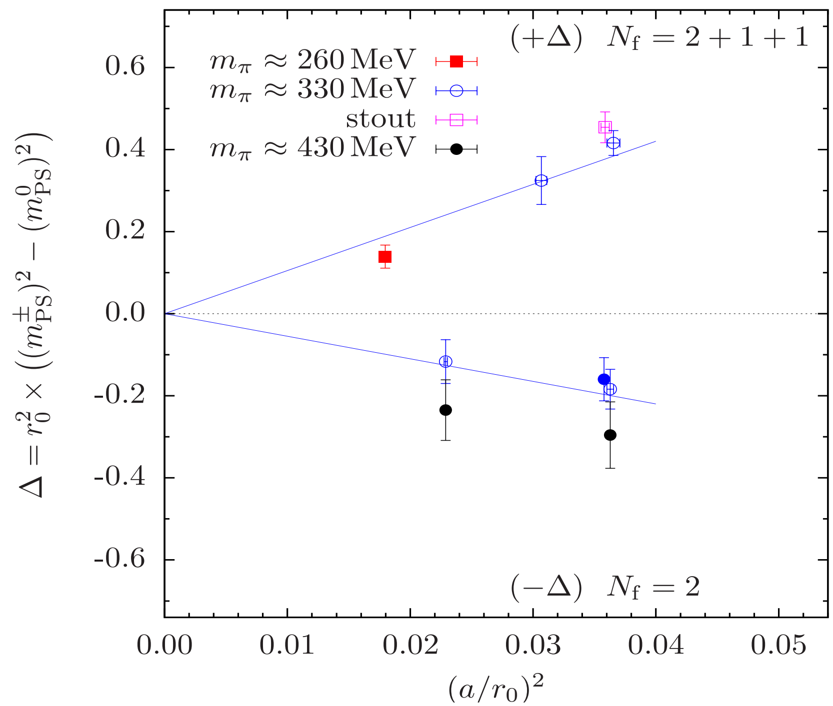
Pros and Cons of a generic fermion action

Pro maximal twisted mass:

- $O(a)$-improvement for all physical quantities *automatically*
- helps to simplify mixing patterns in non-perturbative renormalization
- explicit infrared regularization through $\mu_{\rm tm}$

Con twisted mass:

- isospin violation at any $a \neq 0$
- observe large $O(a^2)$ effect in neutral pion mass (a similar large $O(a^2)$ effect expected for Wilson fermions in another quantity)



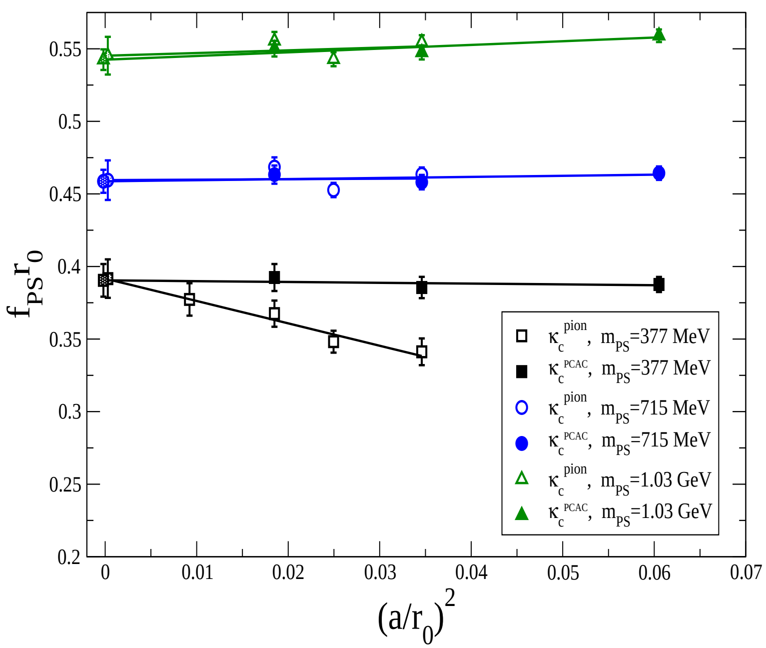
Controlling the effect theoretically:

Frezzotti, Rossi (2007), Dimopoulos et.al (2010), Colangelo, Wenger, Wu (2010), Bär (2010)

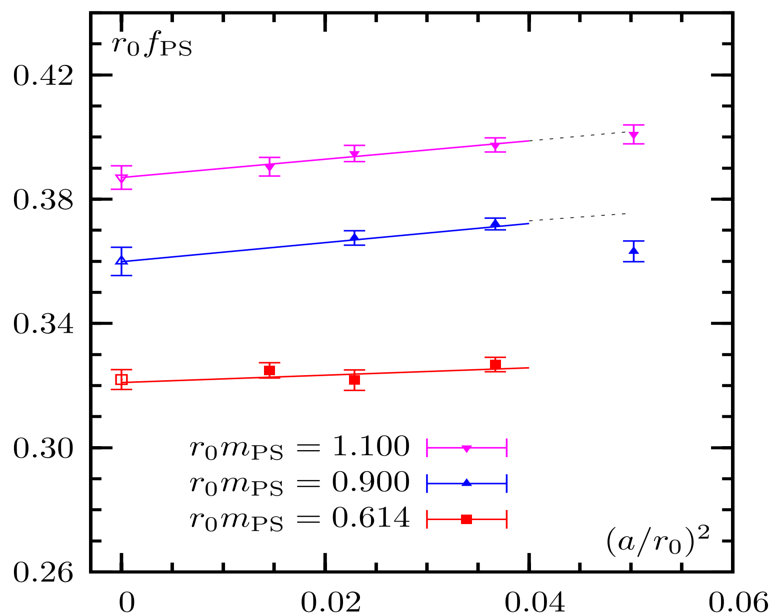
what counts in the end: Universality

Lattice spacing scaling of F_{PS}

- automatic $O(a)$-improvement
  - theoretically investigated (Frezzotti, Rossi, Aoki, Bär, Scorzato, ...)
  - numerically verified (ETMC, Lewis et.al.)



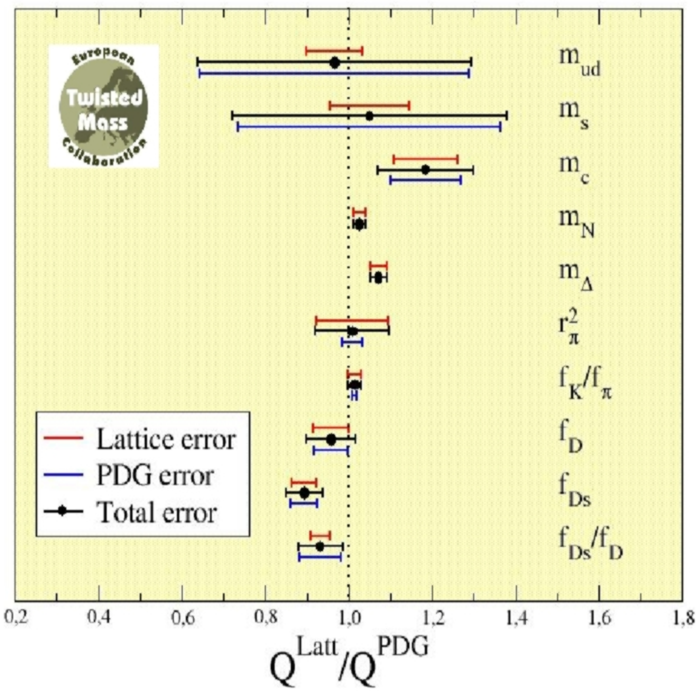
quenched



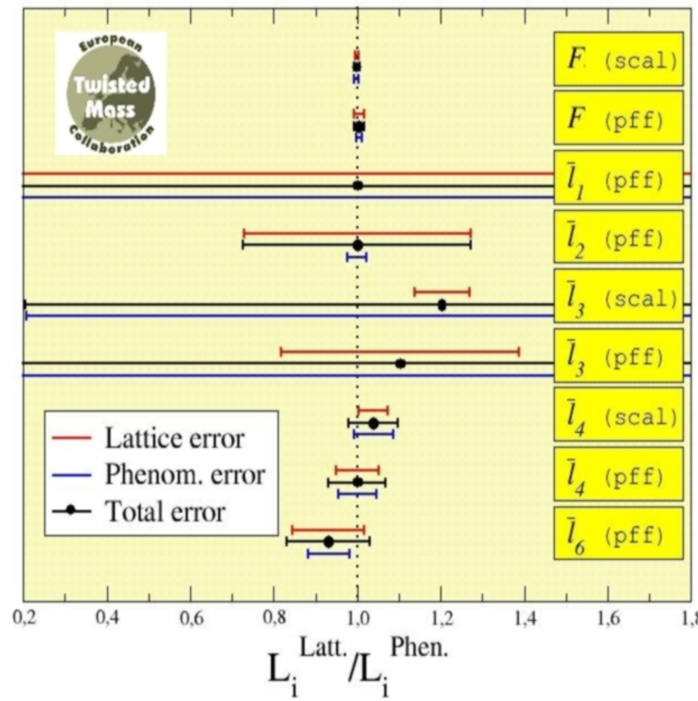
dynamical $(N_f = 2)$

Selected results for $N_f = 2$

Simulation results versus PDG

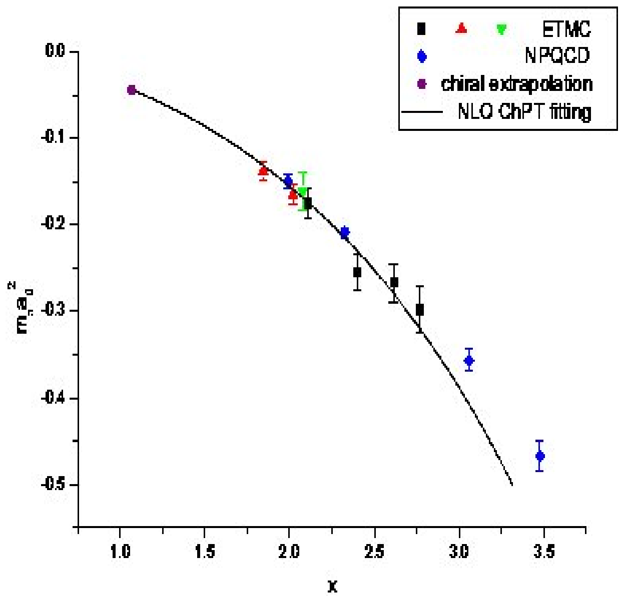


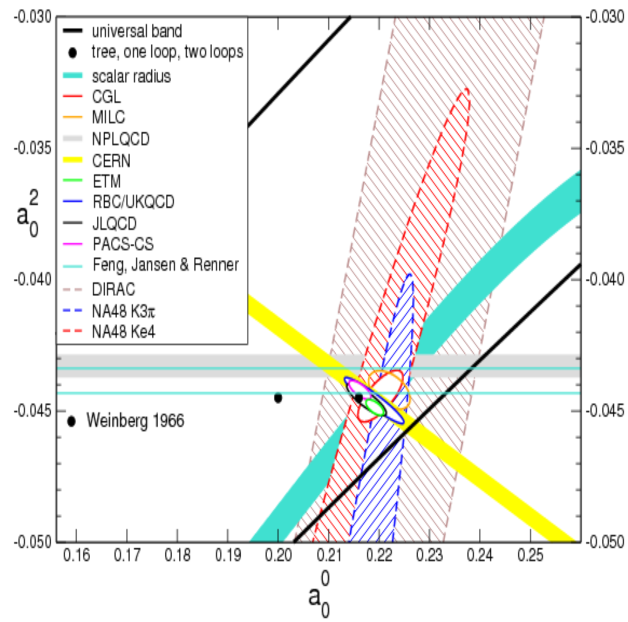
Low energy constants



I=2 Pion scattering length

(X. Feng, D. Renner, K.J.)





energy determined from

$R(t) = \langle(\pi^+\pi^+)^\dagger(t+t_s)(\pi^+\pi^+)(t_s)\rangle / \langle(\pi^+)^\dagger(t+t_s)\pi^+(t_s)\rangle^2$

$\rightarrow \quad \Delta E = c/L^3 \cdot a_{\pi\pi}^{I=2}(1+O(1/L))$

E865 (BNL) $m_\pi a_{\pi\pi}^{I=0} = 0.203\,(33)$ and $m_\pi a_{\pi\pi}^{I=2} = -0.055\,(23)$.

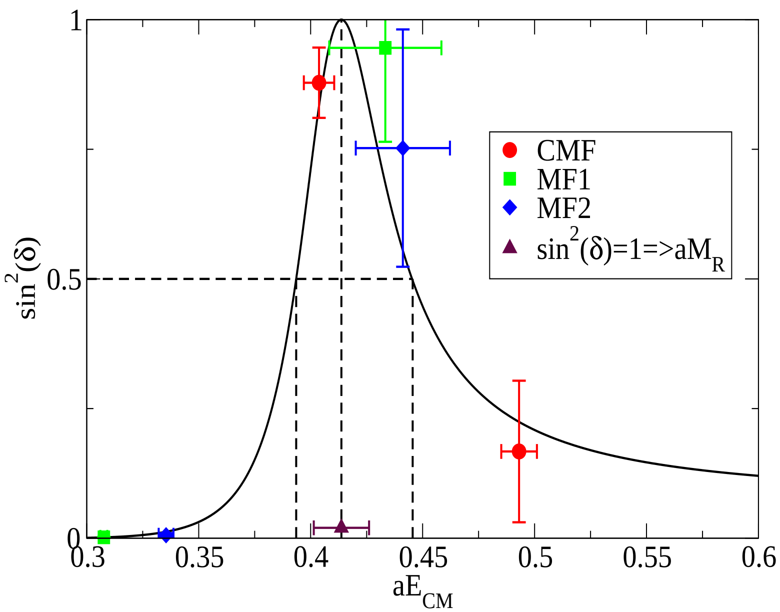
NA48/2 (CERN) $m_\pi a_{\pi\pi}^{I=0} = 0.221\,(5)$ and $m_\pi a_{\pi\pi}^{I=2} = -0.0429\,(47)$.

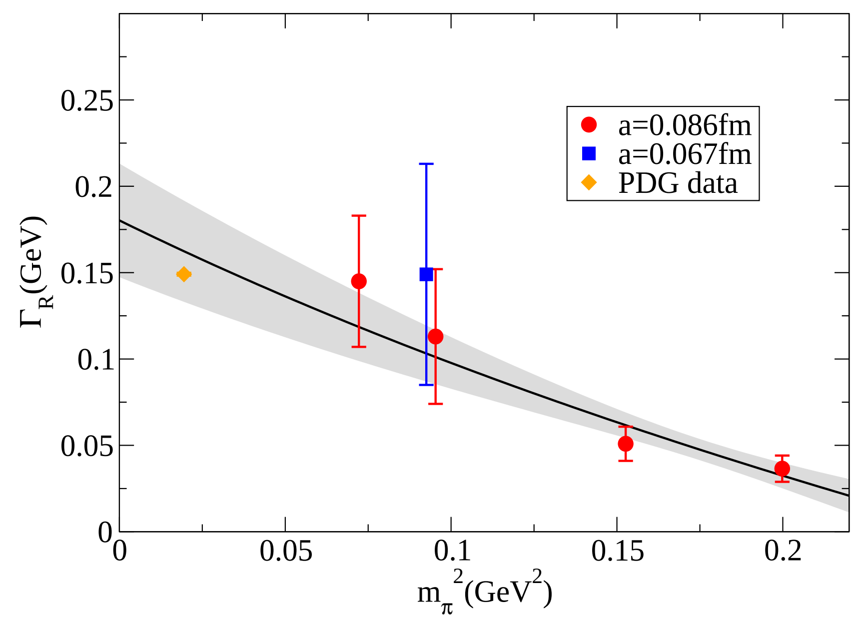
our work $m_\pi a_{\pi\pi}^{I=2} = -0.04385\,(28)(38)$

The ρ-meson resonance: dynamical quarks at work

(X. Feng, D. Renner, K.J.)

- usage of three Lorentz frames
- decay $\rho^0 \to \pi^+\pi^-$ (disconnected neglected)
- analysis of generalized eigenvalue problem to extract energies





$m_{\pi^+} = 330$ MeV, $a = 0.079$ fm, $L/a = 32$

fitting $z = (M_\rho + i\frac{1}{2}\Gamma_\rho)^2$

$m_\rho = 1033(31)$ MeV, $\Gamma_\rho = 123(43)$ MeV

Simulation setup for $N_f = 2+1+1$
Configurations available through ILDG

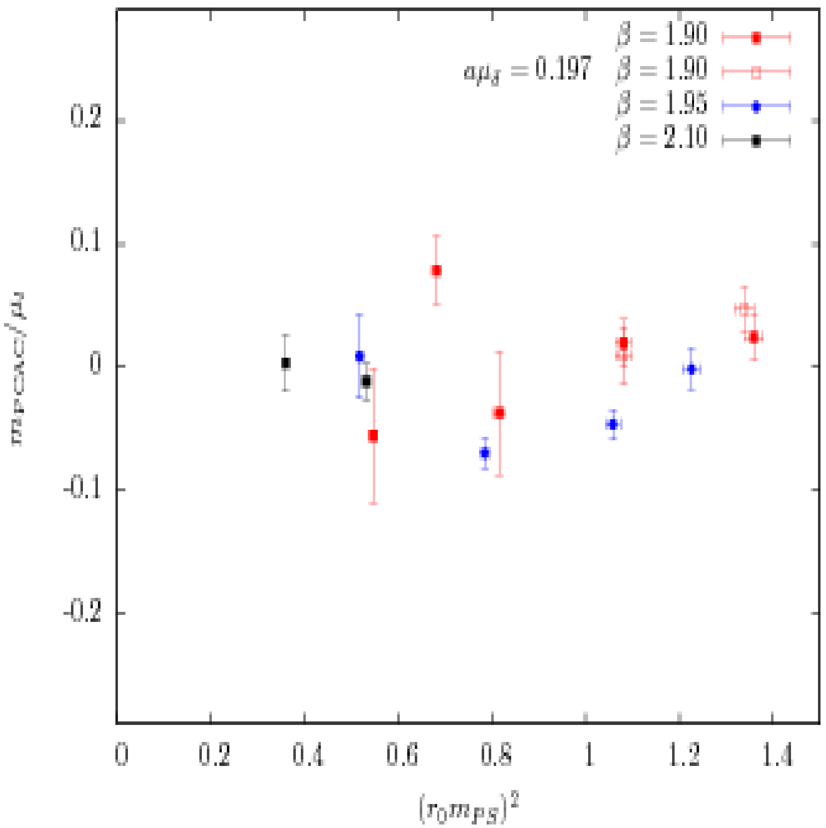
β	a[fm]	L^3T/a^4	m_π[MeV]	status
1.9	≈ 0.085	$24^3 48$	300 – 500	ready
1.95	≈ 0.075	$32^3 64$	300 –500	ready
2.0	≈ 0.065	$32^3 64$	300	ready
2.1	≈ 0.055	$48^3 96$	300 – 500	running/ready
		$64^3 128$	230	thermalizing
		$64^3 128$	200	planned
		$96^3 192$	160	planned

- trajectory length always one
- 1000 trajectores for thermalization
- $\geq$ 5000 trajectores for measurements

Tuning to maximal twist for $N_f = 2+1+1$

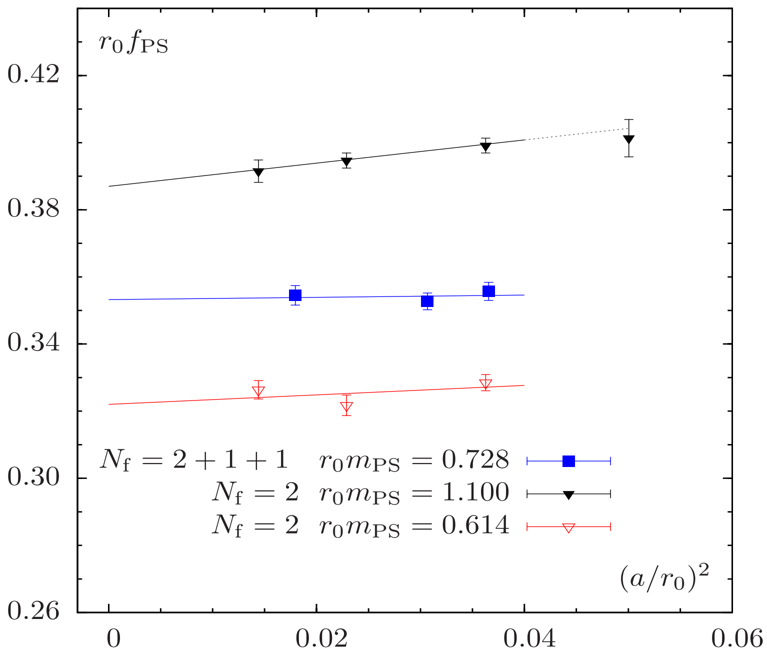
Maximal twist: tune m_q such that

$$m_{\rm PCAC} = \frac{\sum_{\mathbf{x}} \langle \partial_0 A_0^a(x) P^a(0) \rangle}{2 \sum_{\mathbf{x}} \langle P^a(x) P^a(0) \rangle} = 0$$

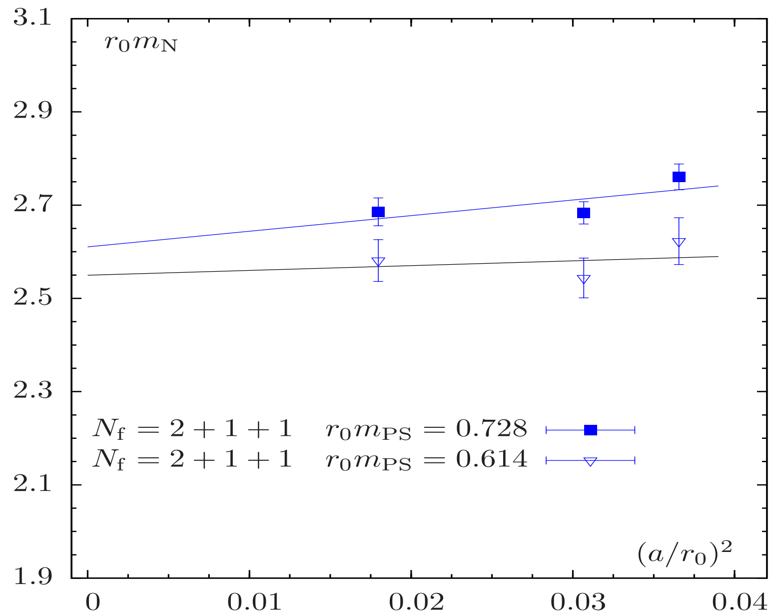


- tuning of m_q at *each* $\mu_{\rm tm}$ used
- demand $m_{\rm PCAC} \lesssim 0.1 \mu_{\rm tm}$
- demand $\Delta(m_{\rm PCAC}) \lesssim 0.1 \mu_{\rm tm}$

Lattice spacing scaling for $N_f = 2 + 1 + 1$



f_{PS}

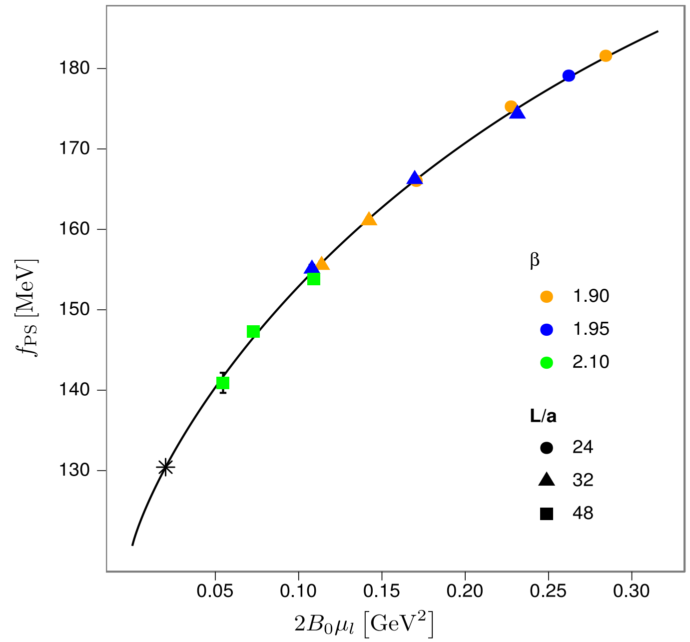


nucleon mass

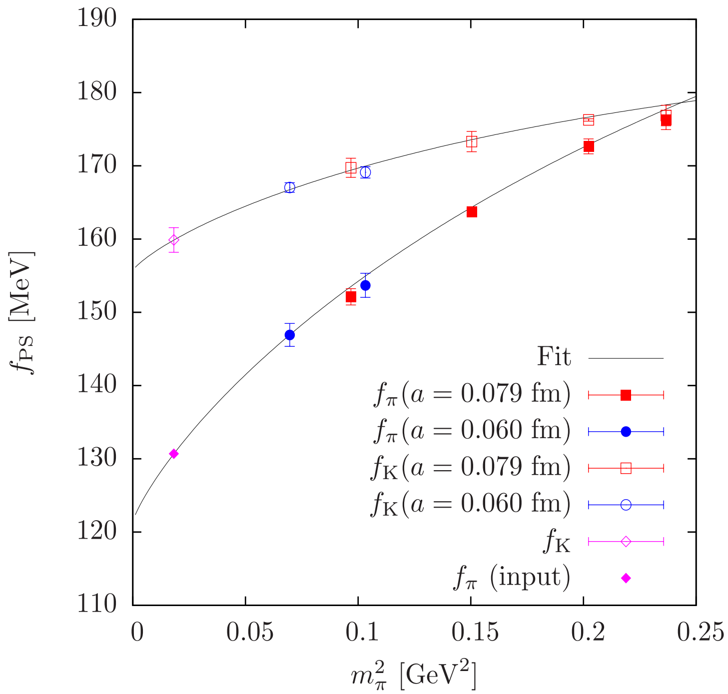
$N_f = 2+1+1$ light quark sector: χPT fit

- central values + stat. error : $f_\pi = 130.4(2)$ MeV $\rightsquigarrow$ scale
- estimate systematic effects : lattice artifacts, FSE

	$N_f = 2$	$N_f = 2+1+1$
$\bar{\ell}_3$	3.70(27)	3.50(31)
$\bar{\ell}_4$	4.67(10)	4.66(33)
f_π/f_0	1.076(3)	1.076(9)
B_0 [MeV]	2437(120)	2638(200)
$\langle r^2\rangle_s^{\mathrm{NLO}}$ [fm^2]	0.710(28)	0.715(77)



$N_f = 2+1+1$ light quark sector: adding strange quark



- fit $\beta = 1.95$ and $\beta = 2.10$ simultaneously
- from setting $m_{\mathrm{PS}}^2(\mu_\ell, \mu_s, \mu_s) = 2m_K^2 - m_\pi^2$
- $m_\pi = 135$ MeV, $f_\pi = 130.7$ MeV, $m_K = 497.7$ MeV

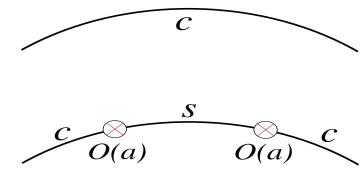
prelinimary fit results:

- $f_K/f_\pi = 1.224(13)$, $f_K = 160(2)$ MeV, $\bar{\ell}_4 = 4.78(2)$
- errors statistical only

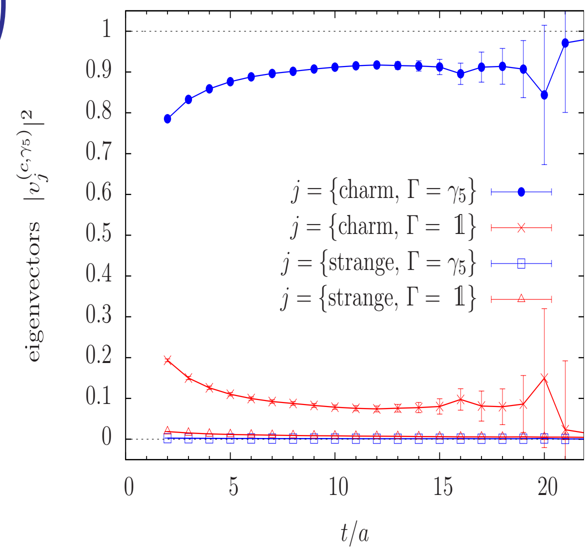
$N_f = 2+1+1$ heavy quark sector

Wilson twisted mass Dirac operator for (c, s) pair:

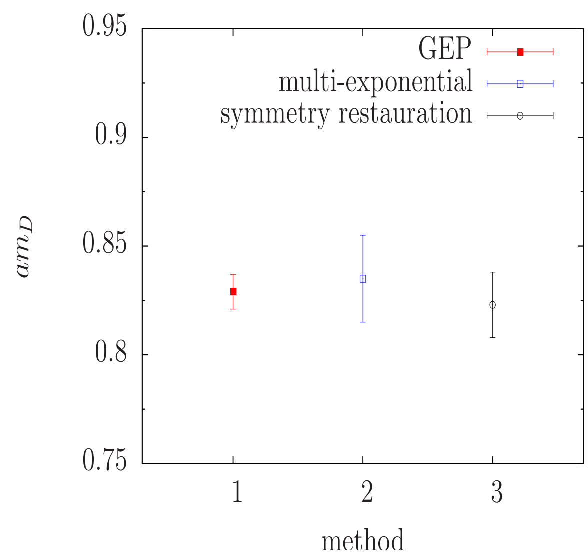
$$D_h = \begin{pmatrix} \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma + \mu_\delta & i\gamma_5 \left(\frac{a}{2}\nabla^*_\mu \nabla_\mu - m_q\right) \\ i\gamma_5 \left(\frac{a}{2}\nabla^*_\mu \nabla_\mu - m_q\right) & \gamma_\mu \tilde{\nabla}_\mu + \mu_\sigma - \mu_\delta \end{pmatrix}$$



- mixing of c and s flavour and of parity
- Kaon is the ground state : good precision
- D meson appears as an excited state
- three independent methods:
  - generalised eigenvalue problem
  - multi-exponential fits
  - imposing parity and flavour restoration at finite a



- they provide consistent results for m_D
- overcome mixing of flavour $\rightsquigarrow$ mixed action



$N_f = 2+1+1$ approaching the charm quark

- introduce Wilson twisted mass doublets in the valence sector

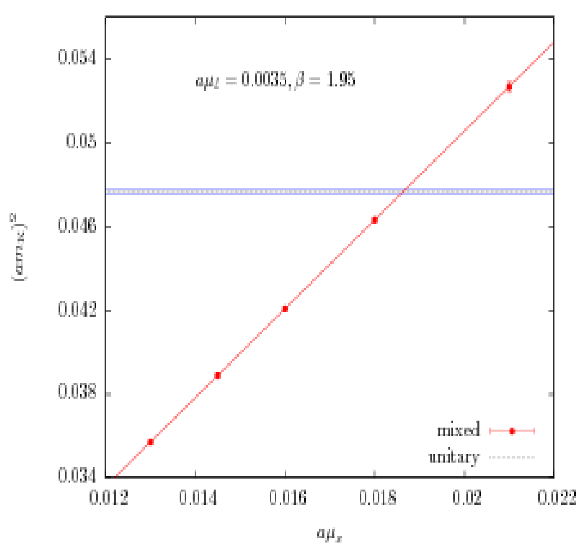
$$D_{tm}(\mu_{val}) = D + m_{\mathrm{crit}} + i\ \mu_{val}\gamma_5\tau^3$$

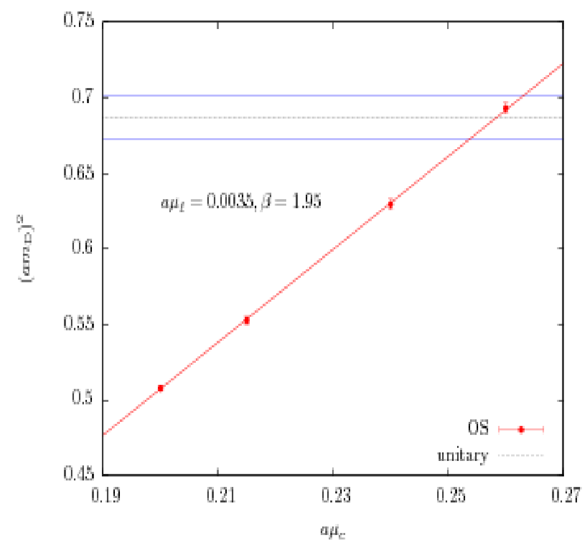
(Osterwalder, Seiler (1990), Pena et al. (2004); Frezzotti, Rossi (2004))

- m_{crit} from unitary set-up

- $4-6$ values for μ_{val} in the strange μ_s and the charm μ_c region
  inversions with multi-mass solver

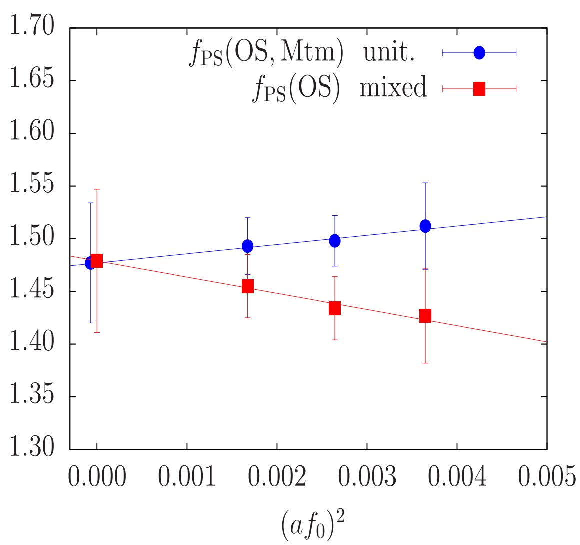
- matching to unitary set-up using m_K and m_D
  $\Rightarrow$ obtain simulated μ_s and μ_c



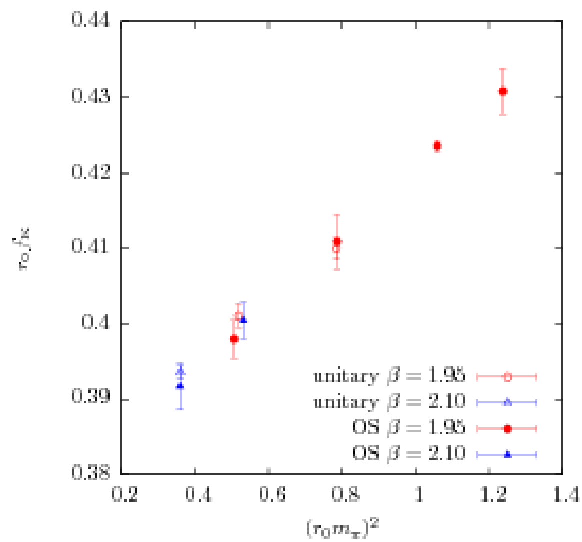


Unitary versus Osterwalder-Seiler: f_K

- the unitary f_K can be computed from: $f_K = (m_\ell + m_s)\frac{\langle 0|P_K|K\rangle}{m_K^2}$
  with $m_s = \mu_\sigma - (Z_{\rm P}/Z_{\rm S})\mu_\delta$
- similar formula for f_D
- P_K is the physical Kaon projecting operator
- the mixed action f_K computed from: $f_{\rm PS} = \left(\mu_{val}^{(1)} + \mu_{val}^{(2)}\right) \frac{|\langle 0|P|PS\rangle|}{m_{\rm PS}\,\sinh m_{\rm PS}}$,



Test for $N_f = 2$



situation for $N_f = 2 + 1 + 1$

Projection operator

- unitary kaon decay constant

$$f_K = \frac{\mu_\ell + \mu_\sigma - (Z_P/Z_S)\mu_\delta}{2m_K^2} \cdot \langle 0|(P_K - P_D) + i(Z_S/Z_P)(S_K + S_D)|K\rangle$$

- Kaon is lowest state, so flavour mixings should play no role

- mixing of scalar and pseudoscalar

Preliminary analysis of f_D and f_{D_s} in MA set-up

- $SU(2)$ heavy meson χPT fit to our data for $f_{D_s}\sqrt{m_{D_s}}$ and $f_{D_s}\sqrt{m_{D_s}}/(f_D\sqrt{m_D})$
  (ETMC, Blossier et al. (2009))

- including terms proportional to $a^2 m_{D_s}^2$ and $1/m_{D_s}$

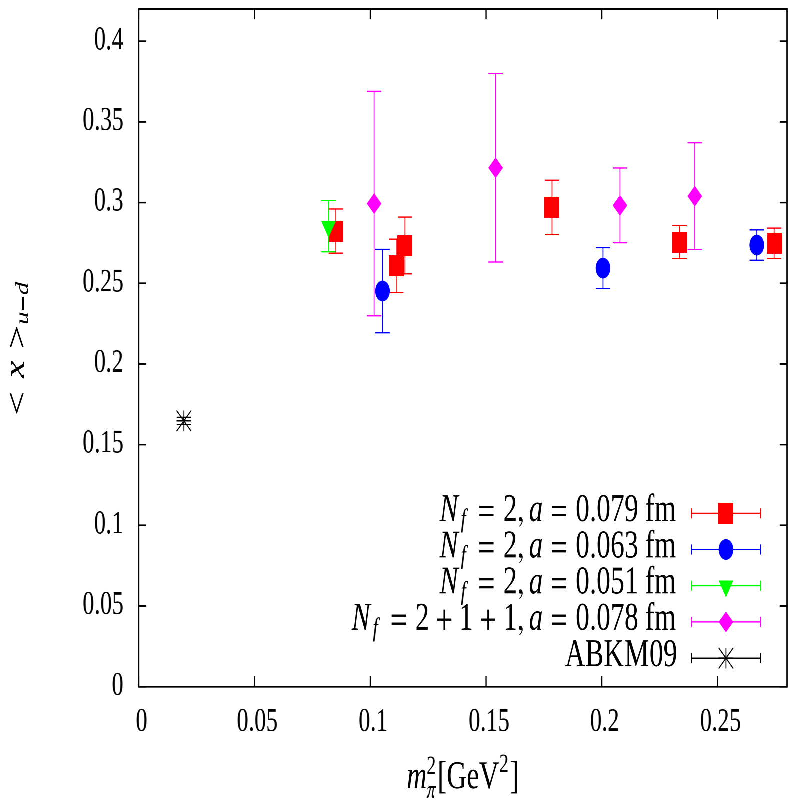
- results very encouraging
  $f_{D_s} = 250(3)\ \mathrm{MeV}, \quad f_D = 204(3)\ \mathrm{MeV}, \quad f_{D_s}/f_D = 1.230(6)$

- very preliminary but very first results from $N_f = 2+1+1$!

Nucleon structure for $N_f = 2+1+1$

(C. Alexandrou, M. Constantinou, S. Dinter, V. Drach, D. Renner, K.J.)

First calculation for $\langle x \rangle$ comparison to $N_f = 2$



same effect as for $N_f = 2$: need to explore smaller quark mass region

simulations are underway

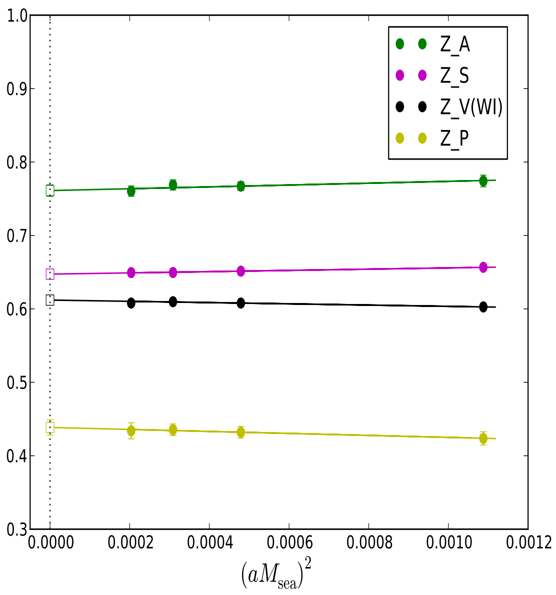
Non-perturbative renormalization for $N_f = 2+1+1$

renormalisation factors computed from dedicated $N_f = 4$ flavour simulations of Wilson fermions

- RI-MOM scheme at non zero values of both the standard and twisted mass parameters

$$M_R = \frac{1}{Z_P}\sqrt{(Z_A m_{PCAC})^2 + \mu_q^2} \quad \to \quad 0$$

- $O(a)$ improvement via average of simulations with $+m_{PCAC}$ and $-m_{PCAC}$



study at $\beta = 1.95$

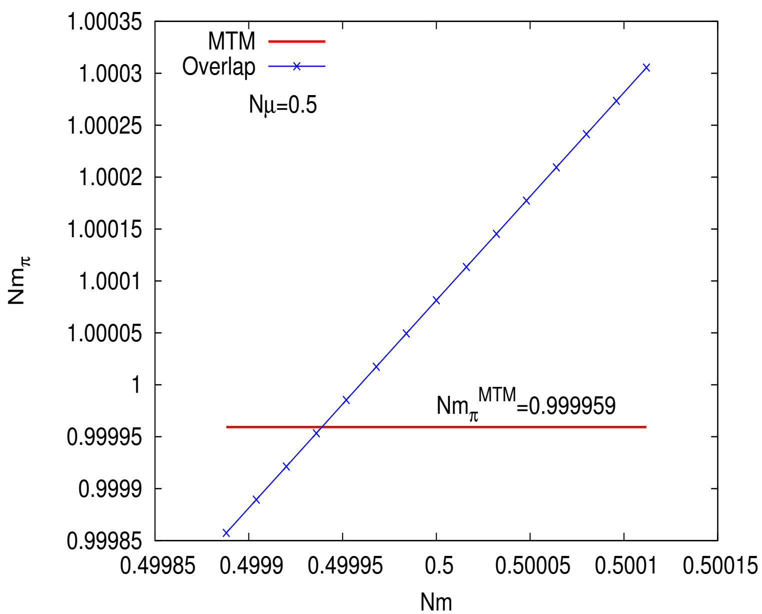
$a = 0.08$ fm, $L = 1.9$ fm

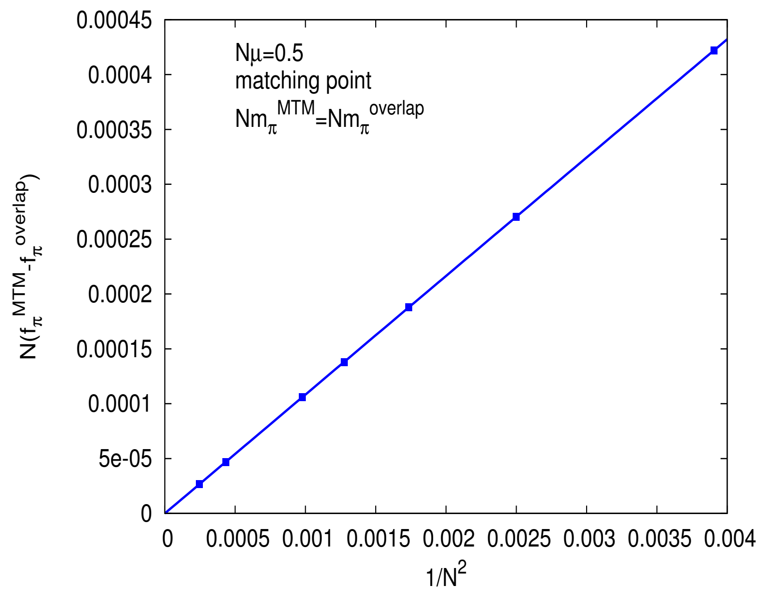
- linear mass dependence
- allows for chiral extrapolation

Alternative valence quarks: overlap fermions

K. Cichy, G. Herdoiza, K.J.

- illustration in free theory





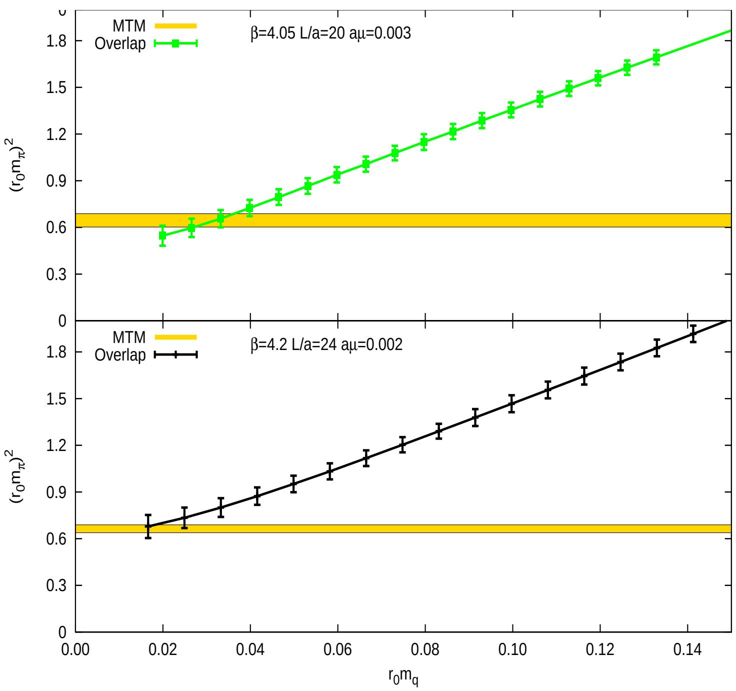
matching of pion mass **pion decay constant**

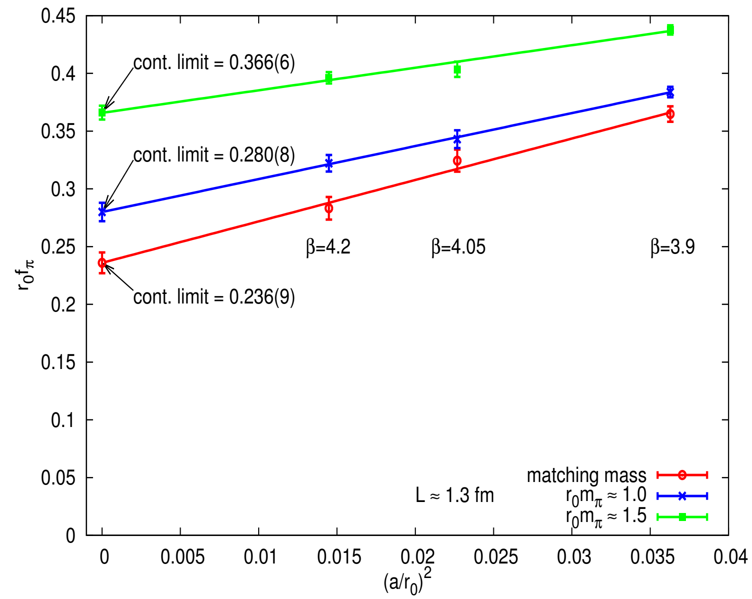
– lattice artefacts for $a \neq 0$

– difference vanishes with $O(a^2)$ towards continuum limit

In real life ...

- 3 lattice spacings, small volume $L = 1.3\text{fm}$, $M_\pi^\infty = 300\text{MeV}$



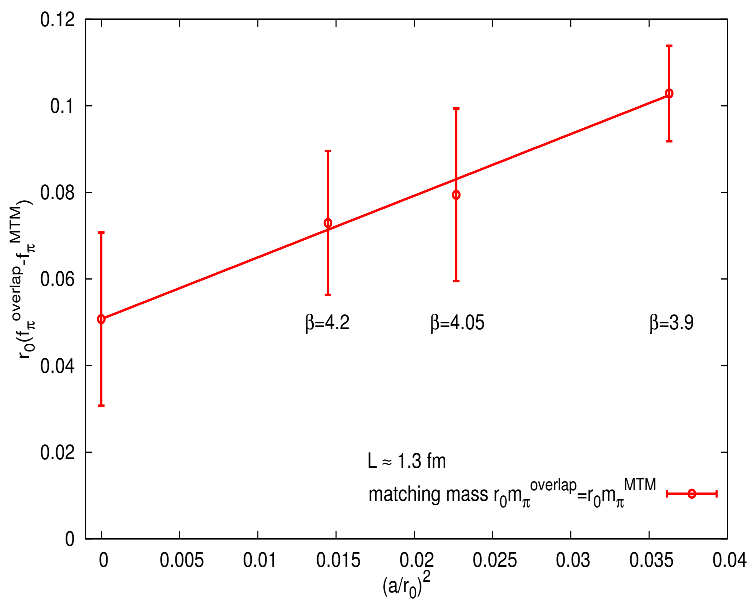


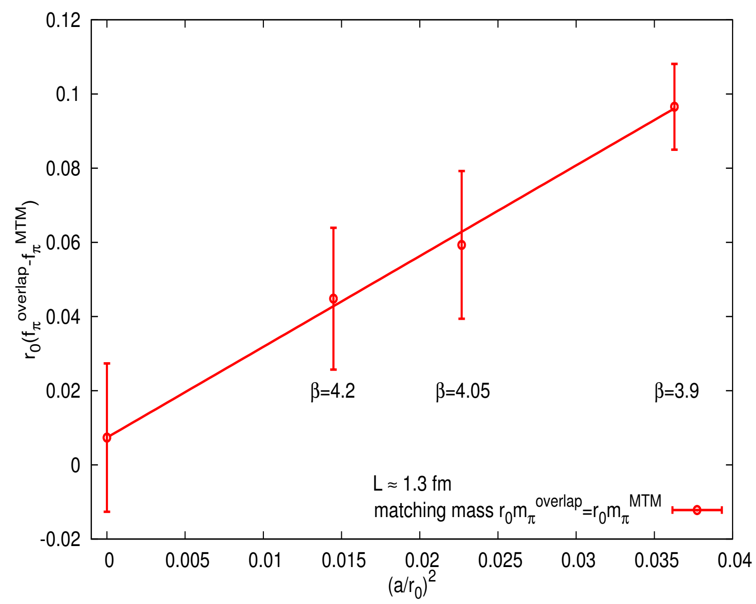
matching of pion mass **pion decay constant**

– valence overlap scaling with $O(a^2)$

The catch ...

- 3 lattice spacings, small volume $L = 1.3\text{fm}$, $M_\pi^\infty = 300\text{MeV}$



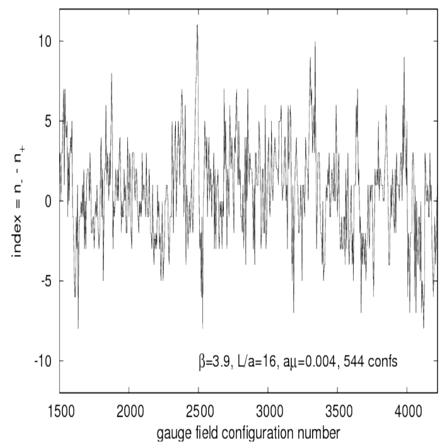


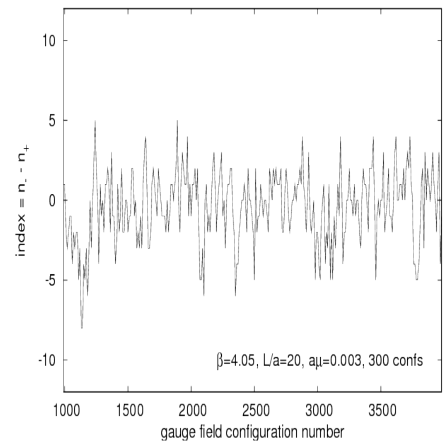
Analysis from PP correlator **analysis from $PP - SS$ correlator**

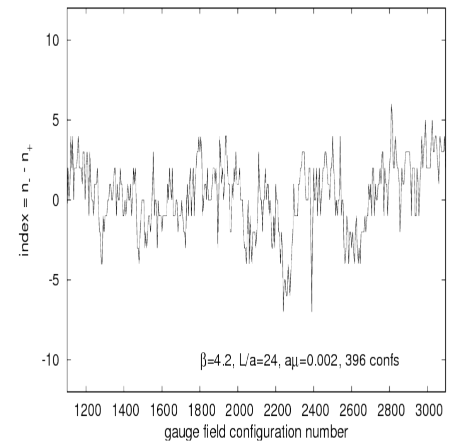
– strong effect of zeromodes, cancelled in $PP - SS$

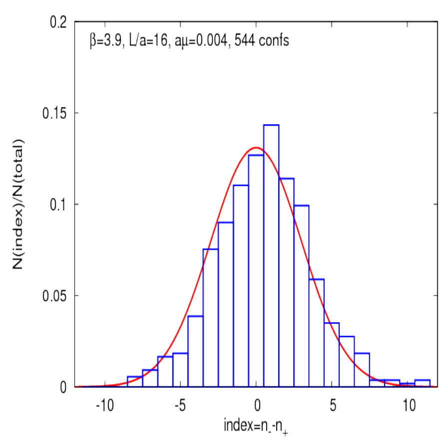
– warning for mixed actions

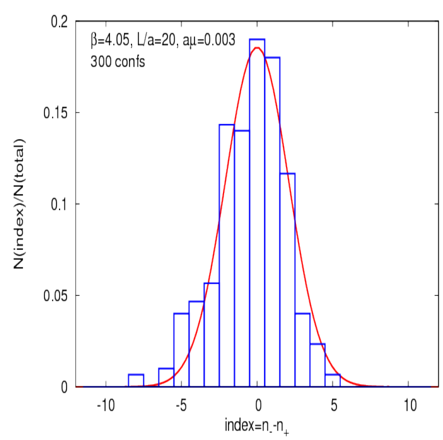
Topology

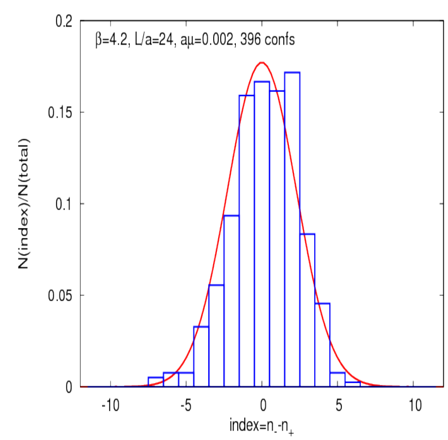

index = n_ - n_+
β=3.9, L/a=16, aμ=0.004, 544 confs
gauge field configuration number


index = n_ - n_+
β=4.05, L/a=20, aμ=0.003, 300 confs
gauge field configuration number


index = n_ - n_+
β=4.2, L/a=24, aμ=0.002, 396 confs
gauge field configuration number


N(index)/N(total)
β=3.9, L/a=16, aμ=0.004, 544 confs
index=n_-n_+


N(index)/N(total)
β=4.05, L/a=20, aμ=0.003
300 confs
index=n_-n_+


N(index)/N(total)
β=4.2, L/a=24, aμ=0.002, 396 confs
index=n_-n_+

Summary

- successful simulations with $N_f = 2$ flavours
  - using maximally twisted mass fermions
  - *automatic* $O(a)$*- improvement*

- First simulations with $N_f = 2 + 1 + 1$ flavours
  - using maximally twisted mass fermions in the sea
  - use Osterwalder-Seiler fermions in the heavy valence sector
  - already precise results for f_K/f_π, f_D, f_{D_s}
  - non-perturbative renormalization under way
  - exploration of overlap valence quarks

- our conclusion: adding stange and charm as dynamical degrees of freedom perfectly feasible