Hadron form factors from lattice QCD with exact chiral symmetry

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1. introduction

hadron form factors in this talk

- pion form factors : $F_V^\pi(q^2)$ and $F_S^\pi(q^2)$
  - good testing ground for convergence of ChPT expansion
  - determination of LECs : $F_V^\pi(q^2) \to L_9, \quad F_S^\pi(q^2) \to 2L_4 + L_5$
- kaon EM form factors : $F_V^{K^+}(q^2)$ and $F_V^{K^0}(q^2)$
  - share (many) LECs with $F_V^{\pi^+}(q^2)$ (at NNLO) *(cf. Bijnens-Talavera, 2002)*
- $K \to \pi$ form factor : $f_+(0)$
  - determination of CKM element $|V_{us}|$ $\Rightarrow$ test of SM
- nucleon strange quark content : f_{T_s}
  - direct experimental searches for dark matter

lattice calculation

- 3-pt. function : (much) noisier than 2-pt. functions
- calculation with various choices of initial / final hadrons; hadron momenta; ...
- disconnected diagrams for $F_S^\pi(q^2)$ and f_{T_s}

1. introduction

this talk

JLQCD's studies of hadron form factors in $N_f = 2+1$ QCD

- overlap quarks $\Rightarrow$ straightforward comparison w/ ChPT ($a=0$)
- all-to-all quark propagator $\Rightarrow$ precise calculation of various 3-pt. functions

outline

- simulation method
- pion and kaon EM form factors
- pion scalar form factors
- kaon weak decay form factors
- nucleon strange quark content

2. simulation method

2.1 simulation setup

configurations

- $N_f = 2+1$ QCD w/ Iwasaki gauge + overlap quarks

  determinant to suppress zero modes: $\det[H_W^2]/\det[H_W^2 + \mu^2]$ $(\mu = 0.2)$
- $a =$ 0.112(1) fm $(\beta = 2.30)$ $\Leftarrow$ M_Ω as input *(poster by Noaki)*

status for meson form factors

- $L \sim$ 1.8 fm $(16^3 \times 48)$ in $Q = 0$ sector

  effects of finite V / fixed Q to $F_V^{\pi^+}$: small (few %) for $N_f = 2$ *(JLQCD/TWQCD, 2009)*
- 4 m_{ud} : $M_\pi \simeq$ 310 – 560 MeV; $m_s =$ 0.080 : $m_{s,\text{phys}} =$ 0.081
- 50 conf $\times$ 50 HMC traj. for each (m_{ud}, m_s)
- periodic boundary condition $\Rightarrow$ 0.5 GeV$^2 \lesssim |q^2| \lesssim$ 2.0 GeV^2

for nucleon f_{T_s}

- $L \sim$ 2.7 fm $(16^3 \times 48)$; $m_s = 0.100$

on-going :

- $24^3 \times 48$, twisted boundary conditions (TBCs), reweighting w.r.t. m_s, ...

  (this talk = preliminary analysis)

2.2 measurement method

all-to-all quark propagator *(TrinLat, 2005)*

$$D = \sum_{k=1}^{12VT} \frac{1}{\lambda_k} u_k u_k^\dagger = \sum_{k=1}^{N_{\rm low}} \frac{1}{\lambda_k} u_k u_k^\dagger + (1 - P_{\rm low}) D^{-1}$$

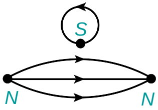
- low-mode contribution
  - dominate low-energy observables $\ni$ form factors
  - evaluated exactly using $N_{\rm low} = 160$ ($16^3 \times 48$) and 240 modes ($24^3 \times 48$)
- high-mode contribution
  - (possibly) small $\Rightarrow$ estimated by noise method

    a single noise vector / conf + dilution w.r.t color/spinor/t

$\Rightarrow$ work well for light mesons

low-mode averaging (LMA) *(DeGrand-Schäfer, 2004; Giusti et al., 2004)*

- low-mode contribution : calculated using eigenmodes
- high-mode contribution : calculated using point-to-all prop.
- for nucleon corr. : LMA for nucleon piece / all-to-all for scalar loop

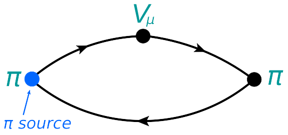


2.2 measurement method

pros and cons

- once all-to-all propagator is prepared, we can calculate
  - connected / disconnected corr.
  - w/ different (Fourier mode) momenta of hadrons
  - w/ different smearing functions

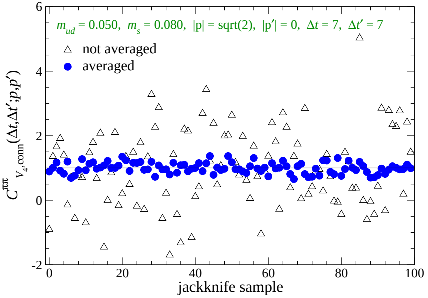
  very helpful to study various form factors



- (remarkably) improve statistical accuracy

  dominant low-mode contribution

  $\Leftarrow$ average over hadron source location



- have to re-calculate all-to-all propagator for different boundary conditions

3. EM form factors

3.1.1 pion EM form factor : calculation

$$\langle \pi^+(p') | j_\mu | \pi^+(p) \rangle = (p+p')_\mu F_V^{\pi^+}(q^2)$$

ratio method (S. Hashimoto, et al., 2000)

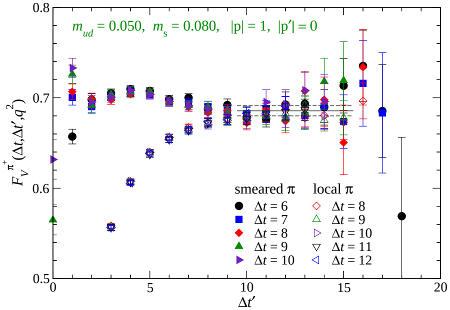
$$C_{V_4}^{\pi\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')$$

$$\to \frac{Z_\pi^*(|\mathbf{p}|)\, Z_\pi(|\mathbf{p}'|)}{4E(p)E(p')\, Z_V} e^{-E(p)\Delta t} e^{-E(p')\Delta t'}$$

$$\times \langle \pi(p') \,| V_4 |\, \pi(p) \rangle$$

$$C^\pi(\Delta t; \mathbf{p}) \;\to\; \frac{Z^*(|\mathbf{p}|)\, Z_\pi(|\mathbf{p}'|)}{2E(p)} e^{-E(p)\Delta t}$$

$$(Z_\pi(|\mathbf{p}|) = \langle O_\pi(\mathbf{p}) | \pi(p) \rangle)$$

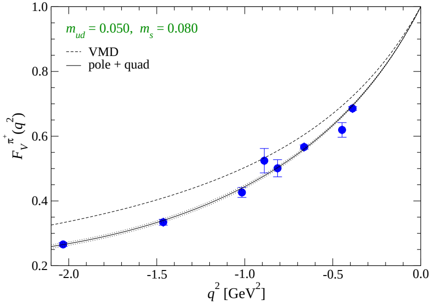


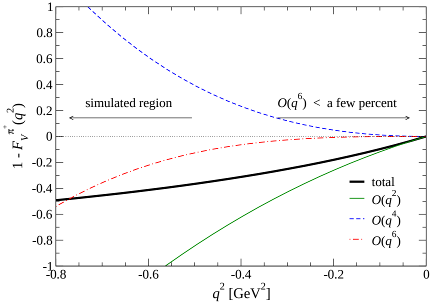
$$R_4^\pi(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{C_{V_4}^{\pi\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C^\pi(\Delta t; \mathbf{p})\, C^\pi(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p') \,| V_4 |\, \pi(p) \rangle}{|Z_\pi(\mathbf{0})|^2\, Z_V}$$

$$F_V(\Delta t, \Delta t'; q^2) = \frac{2M_\pi}{E_\pi(p) + E_\pi(p')} \frac{R_4^\pi(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_4^\pi(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}$$

all-to-all propagator ⇒ statistical accuracy ∼ a few %

3.1.2 pion EM form factor : q^2 dependence





- close to VMD near $q^2 = 0$ $\Rightarrow$ include ρ meson pole w/ measured mass

  $\Rightarrow$ approximate deviation (higher poles/cuts) by generic polynomial form

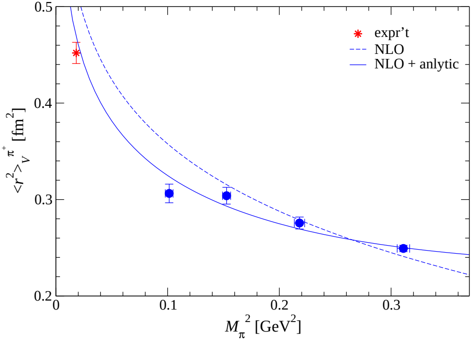
$$F_V(q^2) = \frac{1}{1 - q^2/M_\rho^2} + c_1\, q^2 + c_2\,(q^2)^2 + c_3\,(q^2)^3 = 1 + \frac{\langle r^2 \rangle_V}{6}\, q^2 + c_V\,(q^2)^2 + \cdots$$

- do not fit based on ChPT : $O(q^6)$ (NNNLO) contribu. is small at $|q^2| \lesssim$ (0.550 GeV)^2
- simulated pion masses : $M_\pi^2 \lesssim$ "(0.550 GeV)^2"

  m_q dependence of $\langle r^2 \rangle_V^{\pi^+}$ may be described by ChPT up to NNLO

3.1.3 pion EM form factor : chiral fit of radius

chiral fit of $\langle r^2 \rangle_V^{\pi^+}$ in NLO $SU(3)$ ChPT



- at NLO *(Gasser-Leutwyler, 1985)*

$$\langle r^2 \rangle_V^{\pi^+} = \frac{1}{2NF_0^2}(-3 + 24NL_9^r) - 2\nu_\pi - \nu_K$$

$$\nu_X = (1/2NF_0^2)\ln[M_X^2/\mu^2]$$

  - $N=(4\pi)^2; \quad \mu=4\pi F_0$
  - use $F_0 = 52$ MeV from $F_{\pi,K}$ *(poster by Noaki)*

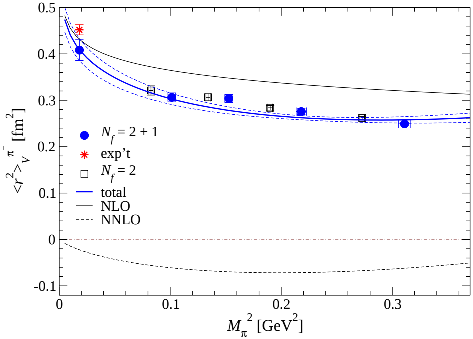
    $\Leftrightarrow \quad F_0 = 88$ MeV *(Bijnens, 2009)*

- small $F_0 \Rightarrow$ enhance NLO log $\Leftrightarrow N_f=2$
- NLO fit : large $\chi^2/\mathrm{dof} \sim 11$ $\quad (L_9 = 2.02(0.02)\times 10^{-3} \Rightarrow L_9 = 5.99(0.43)\times 10^{-3}$ *(Bijnens, 2009))*
- $M_\pi^2/F^2 \Rightarrow \xi = M_\pi^2/F_\pi^2$: does not help ...
- NLO + analyt. : reduce χ^2/dof to ~ 2.8, $\langle r^2 \rangle_V^{\pi^+} = 0.469(11)$ fm^2 $\quad (L_9 = 1.71(0.07))$

3.1.3 pion EM form factor : chiral fit of radius

chiral fit of $\langle r^2 \rangle_V^{\pi^+}$ in NNLO $SU(2)$ ChPT

small effects of sea strange quarks $\Rightarrow$ try NNLO analysis in $SU(2)$ ChPT



- c_V as (additional) fit data
- fix $F = 75(6)$ MeV *(JLQCD/TWQCD, 2009)*

  $\Leftrightarrow$ $F = 86$ MeV *(Colangelo-Dürr, 2004)*
- NNLO formulae *(Gasser-Meißner, 1991)*

$$\langle r^2 \rangle_V^{\pi^+} \ni \bar{l}_6, \bar{l}_1 - \bar{l}_2, r_{V,r}$$
$$c_V \ni \bar{l}_6, \bar{l}_1 - \bar{l}_2, r_{V,c}$$

$\Rightarrow$ fit parameters

- $\langle r^2 \rangle_V^{\pi^+} = 0.408(26)(37)$ fm^2, $c_V = 3.51(32)(51)$ GeV^{-4} : consistent w/ exp't
- $\bar{l}_6 = 12.5(1.2)(2.0)$, $\bar{l}_1 - \bar{l}_2 = -3.5(0.9)(1.3)$ $\Leftrightarrow$ $\bar{l}_6 = 16(1)$ $\bar{l}_1 - \bar{l}_2 = -4.7(6)$ *(pheno.)*
- extension to NNLO $SU(3)$ ChPT?

  many $O(p^4)$ and $O(p^6)$ LECs $\Rightarrow$ need additional constraints (=observables)

3.2.1 kaon EM form factor

$\underline{K^+}$

$$\langle K^+(p')|j_\mu|K^+(p)\rangle = (p+p')_\mu \, F_V^{K^+}(q^2)$$

- ChPT : $SU(3)$, NNLO *(Bijnens-Talavera, 2002)*
  $\Rightarrow$ test NLO in this talk
- experiments : rather old
  - $K\,e$ scattering
  - *FNAL, 1980; NA7, 1986*

$\underline{K^0}$

$$\langle K^0(p')|j_\mu|K^0(p)\rangle = (p+p')_\mu \, F_V^{K^0}(q^2)$$

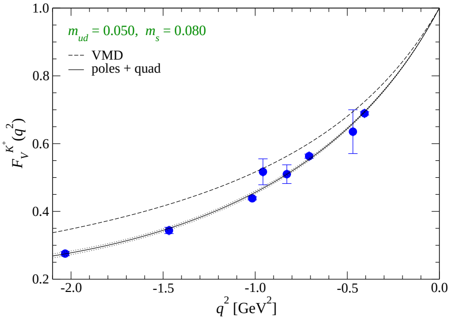
- ChPT : $SU(3)$, NNLO *(Bijnens-Talavera, 2002)*
  $\Rightarrow$ test NLO in this talk
- experiments : updated recently
  - from rare K decay $K \to \pi^+\pi^- e^+ e^-$
  - *NA48, 2003; KTeV, 2006*
- small difference of $\bar{d}\gamma_\mu d$ and $\bar{s}\gamma_\mu s$ contributions

$$R_4^K(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{C_{V_4}^{KK}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C^K(\Delta t; \mathbf{p})\, C^K(\Delta t'; \mathbf{p}')} = \frac{\langle K(p')\,|V_4|\,K(p)\rangle}{|Z_K(\mathbf{0})|^2\, Z_V}$$

$$F_V^K(\Delta t, \Delta t'; q^2) = \frac{2M_K}{E_K(p) + E_K(p')} \, \frac{R_4^K(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_4^K(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}$$

3.2.2 kaon EM form factor : q^2 dependence

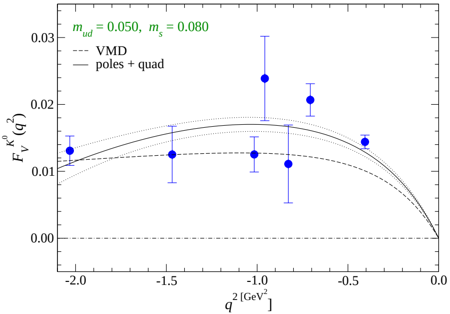
$F_V^{K^+}(q^2)$



- close to VMD

$$\frac{2}{3}\frac{1}{1-q^2/M_\rho^2}+\frac{1}{3}\frac{1}{1-q^2/M_\phi^2}+\ldots$$

$F_V^{K^0}(q^2)$



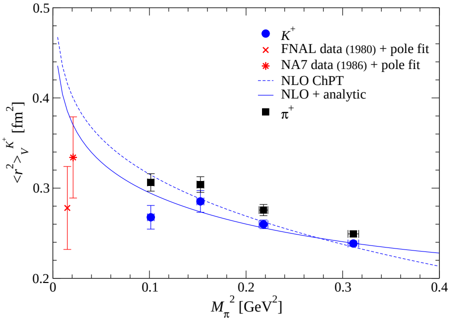
- small but nonzero signal using all-to-all propagator
- close to VMD

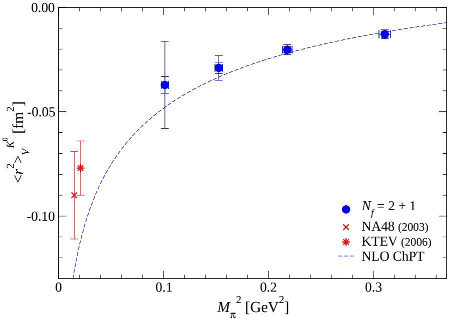
$$-\frac{1}{3}\frac{1}{1-q^2/M_\rho^2}+\frac{1}{3}\frac{1}{1-q^2/M_\phi^2}+\ldots$$

$\Rightarrow$ use vector poles + quadratic correction to calculate radii $\langle r^2\rangle_V^{K^+}$ and $\langle r^2\rangle_V^{K^0}$

3.2.3 kaon EM form factor : chiral fit of radii

$\langle r^2\rangle_V^{K^+} = (1/2NF^2)(-3+24NL_9^r) - \nu_\pi - 2\nu_K$ $\quad$ $\langle r^2\rangle_V^{K^0} = \nu_\pi - \nu_K$





- slightly smaller than $\langle r^2\rangle_V^{\pi^+}$
- NLO : $\chi^2/\mathrm{dof} \sim 3.2$
- + analyt. : $\chi^2/\mathrm{dof} \sim 0.6$
  $\langle r^2\rangle_V^{K^+} = 0.377(10)$

- no $O(p^4)$ coupling at NLO
- consistent w/ NLO ChPT within large sys. err. $\Rightarrow$ TBCs could be helpful

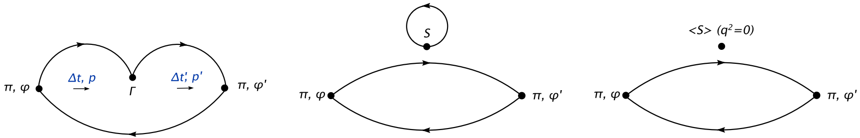
analysis to be extended to NNLO : $\langle r^2\rangle_V^{\pi^+}$, $\langle r^2\rangle_V^{K^+}$, $\langle r^2\rangle_V^{K^0}$ share $O(p^6)$ LECs

4. pion scalar form factor

4.1 pion scalar form factor

$$\langle \pi(p') | S | \pi(p) \rangle = F_S^\pi(q^2)$$

- determination of LECs : l_4 $(N_f = 2)$, $2L_5 + L_4$ $(N_f = 3)$
- 6 times larger NLO chiral log than $\langle r^2 \rangle_V^{\pi^+}$ $\Leftrightarrow$ can we observe at $M_\pi \gtrsim 310$ MeV?
- not directly accessible to experiments $\Rightarrow$ LQCD
- need connected, disconnected, VEV contributions $\Rightarrow$ all-to-all propagator



ratio method

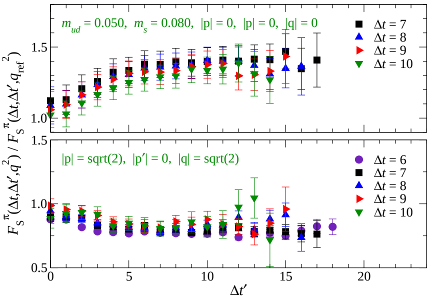
$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; q^2_{\rm ref})} = \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{p}_{\rm ref}, \mathbf{p}'_{\rm ref})} \Rightarrow \langle r^2 \rangle_S^\pi$$

$$R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{C_S^{\pi\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C^\pi(\Delta t; \mathbf{p})\, C^\pi(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p') | S | \pi(p) \rangle}{|Z_\pi(\mathbf{0})|^2 Z_S}$$

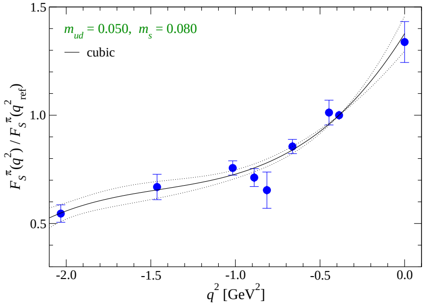
$$F_S(\Delta t, \Delta t'; q^2_{\rm ref}) \Leftarrow C^{\pi\pi}_{S,\rm VEV} = 0 \quad \text{at } q^2_{\rm ref} \neq 0$$

4.2 effective plot / q^2 dependence

effective value



q^2 dependence



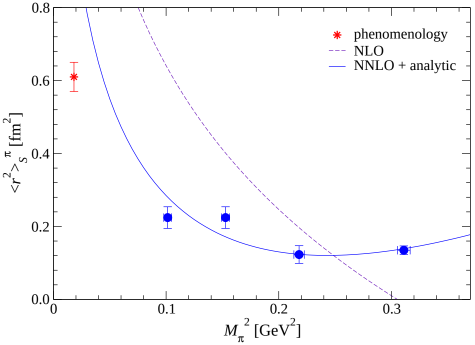
- statistical accuracy $\sim$ 5 – 10 % $\Rightarrow$ $C^{\pi\pi}_{S,\mathrm{disc}}$, $C^{\pi\pi}_{S,\mathrm{VEV}}$
- q^2 dependence : lack of knowledge of scalar resonances at simulated m_q

  $\Rightarrow$ use simple / generic polynomial form $\Rightarrow$ $\chi^2/\mathrm{dof} \sim 1$

$$F_S(q^2) \quad = \quad F_s(0)\left\{1 + \frac{\langle r^2\rangle^\pi_S}{6}\, q^2 + c_S\,(q^2)^2 + d_3\,(q^2)^3 + d_4\,(q^2)^4\right\}$$

4.3 chiral fit of radius

chiral fit of $\langle r^2\rangle_S^\pi$ in NLO $SU(3)$ ChPT



- in NLO ChPT *(Gasser-Leutwyler, 1985)*

$$\langle r^2\rangle_S^\pi = \frac{1}{NF_0^2}\{-8+24N(2L_4^r+L_5^r)\} -12\,\nu_\pi - 3\,\nu_K$$

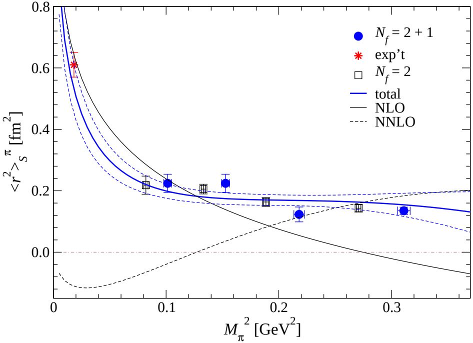
  - $N=(4\pi)^2$; $\mu=4\pi F_0$
  - use $F_0=52$ MeV

- $N_f=2,\ 2+1$: $\langle r^2\rangle_S^\pi$ has 6 times larger π-loop log than $\langle r^2\rangle_V^\pi$
- $N_f=2+1$: small F_0 further enhances chiral log

  $\Rightarrow$ fail to reproduce lattice data ($\chi^2/\mathrm{dof}\sim 100$)
- should extend to NNLO $\Leftarrow$ largely reduced $\chi^2/\mathrm{dof}\sim 7$ by including analytic

4.3.3 chiral fit of radius

chiral fit of $\langle r^2 \rangle_S^\pi$ in NNLO $SU(2)$ ChPT



- small effects of sea strange
- NNLO formula *(Bijnens et al., 1998)*

$$\langle r^2 \rangle_S^\pi \quad \ni \quad l^r_{\{4,1,2,3,6\}}, r_{S,r}$$

- F, l_3^r from M_π, F_π *(JLQCD/TWQCD, 2008)*
- l_6^r, $l_1^r - l_2^r$ from $\langle r^2 \rangle_V^{\pi^+}$, c_V
- $\bar{l}_2 = 4.3(1)$ *(Colangelo et al., 2001)* for NNLO
- free param : l_4^r (NLO), $r_{S,r}$ (poorly known)

- $\langle r^2 \rangle_S^\pi = 0.531(32)(41)$ fm^2 : consistent w/ phenomenology (*Colangelo et al., 2002;* from $\pi\pi$)
- $\bar{l}_4 = 3.9(1.4)(2.1)$ $\Leftrightarrow$ $5.5(0.7)$ $(N_f = 3)$ *(JLQCD/TWQCD, 2009)*; $4.4(0.2)$ *(Colangelo et al., 2001)*
- extension to NNLO $SU(3)$ ChPT $\Leftrightarrow$ $SU(2)$ ChPT : valid at $M_\pi \ll M_K$

5. kaon weak decay form factor

5.1 $K \to \pi$ form factor

$$\langle \pi^+(p')|V_\mu|K^0(p)\rangle = (p+p')_\mu f_+(q^2) + (p-p')_\mu f_-(q^2), \quad f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$\Gamma \propto |V_{us} f_+(0)|^2$: theoretical calc. of $f_+(0)$ $(= f_0(0))$ + Γ from exp't $\Rightarrow$ $|V_{us}|$

double ratios *(Bećirević et al., 2005; JLQCD, 2006; RBC, 2006)*

$$R = \frac{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C_{V_4}^{\pi K}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}{C_{V_4}^{KK}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C_{V_4}^{\pi\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})} \to \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} f_0(q^2_{\rm max})^2 \quad (q^2_{\rm max} = (M_K - M_\pi)^2)$$

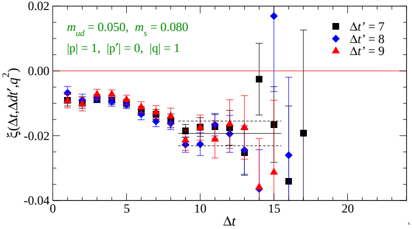
$$\tilde{R} = \frac{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C^\pi(\Delta t, \mathbf{0})\, C^\pi(\Delta t', \mathbf{0})}{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C^\pi(\Delta t, \mathbf{p})\, C^\pi(\Delta t', \mathbf{p}')} \to \left\{ 1 + \frac{E_K(\mathbf{p}) - E_\pi(\mathbf{p}')}{E_K(\mathbf{p}) + E_\pi(\mathbf{p}')} \xi(q^2) \right\} \frac{f_+(q^2)}{f_0(q^2_{\rm max})}$$

$$R_k = \frac{C_{V_k}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C_{V_4}^{KK}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C_{V_k}^{KK}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}$$

$\to$ a function of $\xi(q^2)$ $\quad (\xi(q^2) = f_-(q^2)/f_+(q^2))$

$\leftarrow$ average over source locations / $(\mathbf{p}, \mathbf{p}')$

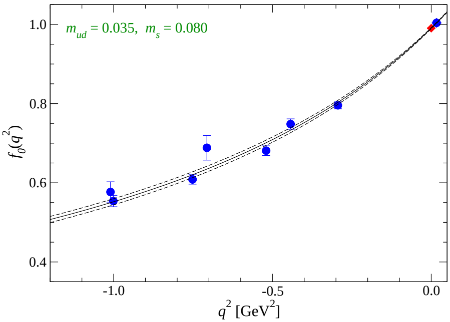
$\Rightarrow$ can construct $f_+(q^2)$ and $f_0(q^2)$



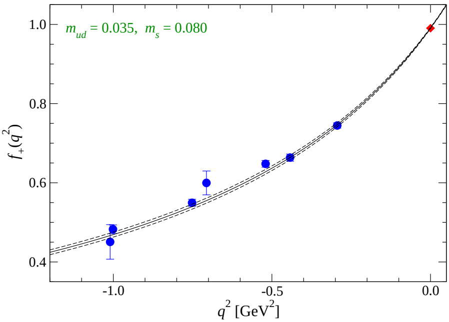
5.2 q^2 dependence

this talk : consistency of form factor shape w/ exp't $\Rightarrow$ $f_+(0)$

$f_0(q^2)$ vs q^2



$f_+(q^2)$ vs q^2



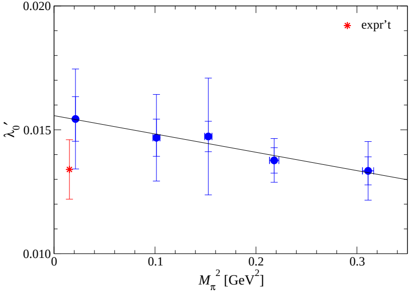
- well described by polynomial / single pole form (similar to exp't data)

$$f_X(q^2) = f_X(0)\left\{1 + c_{X,1}q^2 + c_{X,2}(q^2)^2 + [c_{X,3}(q^2)^3]\right\}, \quad \frac{f_X(0)}{1 - q^2/M_{X,\mathrm{pole}}^2} \quad (X = 0, +)$$

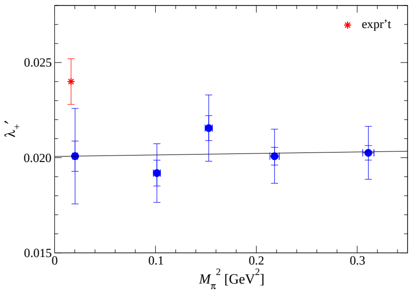
5.3 q^2 dependence

$$f_X(q^2) = f_X(0) + c_{X,1}q^2 + c_{X,2}(q^2)^2, \qquad \lambda'_X = M_\pi^2 c_{X,1} \quad (X = 0, +)$$

λ'_0



λ'_+

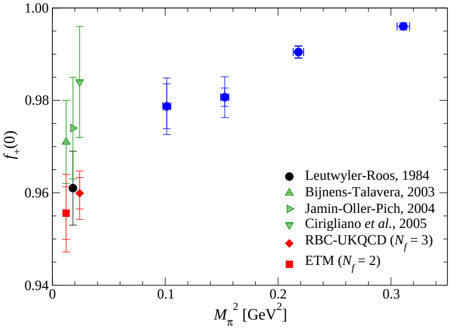


- mild quark mass dependence : $m_{s,\mathrm{sim}} - m_{s,\mathrm{phys}} \Rightarrow$ not large effect (?)
- consistent with experiment *(PDG,2008)*
- curvature

$$\lambda''_+ = 2c_{+,2}M_\pi^4 = 0.08\,(0.10) \times 10^{-2} \quad \Leftrightarrow \quad 0.20(0.05) \times 10^{-2} \text{ (exp't)}$$

5.4 chiral behavior

$f_+(0)$ vs M_π^2



- NLO ChPT + quark model
  - *Leutwyler-Roos, 1984*
- NNLO ChPT
  - *Bijnens-Talavera, 2003*
  - *Jamin-Oller-Pich, 2004*
  - *Cirigliano et al., 2005*
- LQCD
  - *RBC-UKQCD, 2008 ($N_f = 3$)*
  - *ETM, 2009 ($N_f = 2$)*
  - and old studies w/ heavier m_{ud}
- NNLO ChPT $>$ LQCD ?

- smaller M_π^2 $\Rightarrow$ q^2_{max} deviates from 0 $\Rightarrow$ larger uncertainty of $f_+(0)$
- being improved by using TBCs / on larger volume

6. nucleon strange quark content

6.1 nucleon strange quark content

- scalar form factor at zero momentum transfer

$$\langle N|\bar{s}s|N\rangle \;\Rightarrow\; f_{T_s} \equiv \frac{m_s\langle N|\bar{s}s|N\rangle}{M_N}, \quad y \equiv \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{l}l|N\rangle}$$

- phenomenologically important :
  - fundamental parameter on nucleon structure
  - important parameter in experimental searches for dark matter

    $m_s\langle N|\bar{s}s|N\rangle \;\Rightarrow\;$ neutralino-nucleon cross section

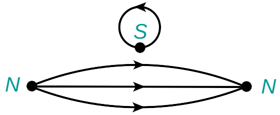
- not directly accessible to exp't $\Rightarrow$ lattice calculation $\sim$ challenging
  - purely disconnected / VEV subtraction $\Leftarrow$ all-to-all propagator

cf. indirect method : Feynman-Hellmann theorem

$$\langle N|\bar{s}s|N\rangle = \frac{\partial M_N}{\partial m_s}$$

$\Leftrightarrow$ chiral expansion of M_N in terms of m_s

only for scalar form factor



$\Rightarrow$ direct determination from disconnected 3-pt. function *(Takeda et al. @ Lat'10)*

6.2 renormalization

chiral symmetry is crucial to avoid unwanted op. mixing also for $\langle N|\bar{s}s|N\rangle$

$$(\bar{s}s)_r = \frac{1}{3}\left\{(\bar{q}q)_r - \sqrt{3}(\bar{q}\lambda_8 q)_r\right\} \qquad (q=(u,d,s)^T)$$

$$= \frac{1}{3}\left\{(Z_0+2Z_8)\bar{s}s + (Z_0-Z_8)(\bar{u}u+\bar{d}d) + \frac{b}{a^3} + \cdots\right\}$$

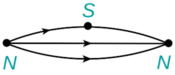
$$(\bar{q}q)_r = Z_0\bar{q}q, \quad (\bar{q}\lambda_8 q)_r = Z_8\bar{q}\lambda_8 q$$

$\rightarrow \quad Z_S\,\bar{s}s \quad$ (chiral symmetry)

- mixing with $\bar{u}u+\bar{d}d$

  $\bar{u}u+\bar{d}d \ni$ connected diagram $\gg$ disconnected = $\bar{s}s$

  $\Rightarrow$ possibly large contamination in $(\bar{s}s)_r$

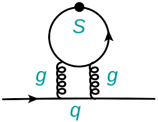


- mixing with lower dimensional operators

  $O(1/a^3)$ term : canceled by VEV subtraction

  $\Rightarrow$ increasingly severe cancellation toward $a\rightarrow 0$

  $\Rightarrow$ need more and more precise calcu. of 3pt-func

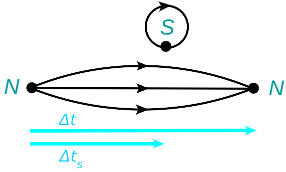


- similar contamination also in indirect method *(UKQCD, 2001)*

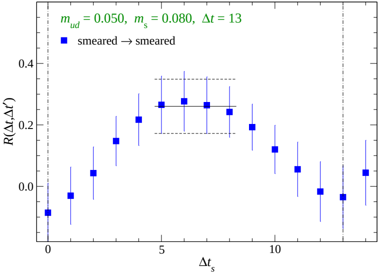
cf. $$\frac{\partial M_N}{\partial m_s} = Z_0\left\{\langle N|\bar{s}s|N\rangle + \frac{\partial\Delta m}{\partial m_{s,\mathrm{sea}}}\langle N|\bar{u}u+\bar{d}d+\bar{s}s|N\rangle\right\} \qquad (m_q = Z_m(m_q+\Delta m))$$

6.3 ratio method

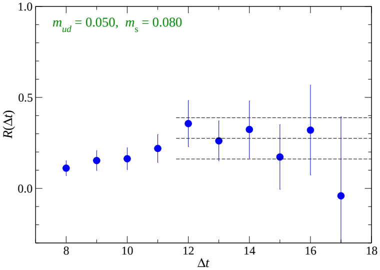
ratio method



$$R(\Delta t, \Delta t_s) = \frac{C_{3pt}(\Delta t, \Delta t_s)}{C_{2pt}(\Delta t)}$$
$$\rightarrow \langle N|\bar{s}s|N\rangle \quad (\Delta t_{(s)} \rightarrow \infty)$$



⇓ constant fit in terms of Δt_s



- use Gaussian smeared source and sink

  local op. ⇔ large excited state contamination ($N_f = 2$)

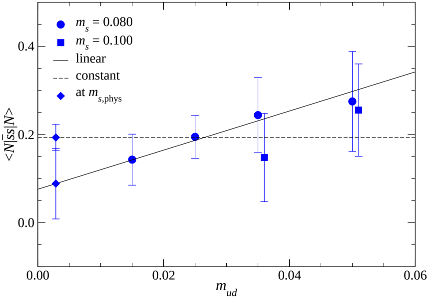
- constant fit w.r.t. Δt_s ⇒ $R(\Delta t)$

  ⇒ no significant Δt-dependence

  ⇒ ground state ME

6.4 chiral fit

$\langle N|\bar{s}s|N\rangle$ vs m_{ud}



- mild dependence on $m_{ud,s}$
- simple polynomial fit

$$\langle N|\bar{s}s|N\rangle = c_0 + d_{1,ud}\, m_{ud} + d_{1,s}\, m_s$$

$\Rightarrow$ $\chi^2 \sim 0.1$ (lin.), $\chi^2 \sim 0.5$ (const.)

- HBChPT fit w/ phenomenological F, D
  - LO + NLO + NNLO analyt. $\Rightarrow$ $\chi^2 \sim 0.5$
  - bad convergence : LO $\sim$ –NLO $\lesssim$ NNLO

$\Leftrightarrow$ not so good even @ phys. point

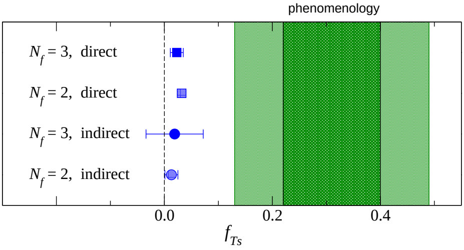
(M_B @ $O(p^4)$: Borasoy-Meißner, 1996; ...)

$$\langle N|\bar{s}s|N\rangle_{\mathrm{bare}} = 0.086(81)_{\mathrm{stat.}}(107)_{\mathrm{extrap.}}(30)_{\mathrm{disc.}} \Rightarrow f_{T_s} = 0.013(12)(16)(5)$$

- other sys. err. : not large
  - finite volume : $M_\pi L \gtrsim 4$ $\Rightarrow$ not large
  - fixed Q : (at most) a few % level for other MEs

6.4 chiral fit

summary of JLQCD's results



- indirect calculations *(JLQCD: Ohki et al.)*
  - $N_f = 2$: PQ ChPT at $O(p^3)$
    $m_{q,\mathrm{val}} = m_{q,\mathrm{sea}} = m_s$
  - $N_f = 3$: reweighting w.r.t. m_s
    polynomial chiral fit

- all studies consistently favor small strange quark content $f_{T_s} \approx 0.02$

- smaller than phenomenology $f_{T_s} = 0.31(9)$, $y = 0.44(13)$?

  $y = 1 - \hat{\sigma}/\sigma_{\pi N}(0)$, $\sigma_{\pi N}(t) = \langle N(p')|\bar{u}u + \bar{d}d|N(p)\rangle$, $\hat{\sigma} = \langle N(0)|\bar{u}u + \bar{d}d - 2\bar{s}s|N(0)\rangle$

  - shifted up by recent exp't *(George Washington Univ/TRIUNF, 2010)*

    $\sigma_{\pi N}(t_{\mathrm{CD}}) = F_\pi^2 D^+(t_{\mathrm{CD}}) - \Delta_R = 79(7)$ MeV ($\Leftrightarrow y = 0.27(13)$ w/ *Kosh et al., 1982*)

  - involves $O(p^4)$ HBChPT estimates

    $\Delta_\sigma = \sigma_{\pi N}(t_{\mathrm{CD}}) - \sigma_{\pi N}(0) = 15$ MeV, $\hat{\sigma} = 36(7)$ MeV

7. summary

JLQCD's studies of hadron form factors in $N_f = 2+1$ QCD

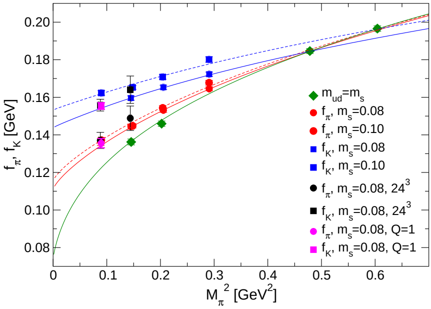
- all-to-all propagators / LMA
  - various form factors with small additional cost
- overlap action
  - direct comparison w/ ChPT at $a=0$ / avoid contamination due to op. mixing
- EM and scalar form factors
  - NLO ChPT fits : fail to reproduce our data
  - to be extended to NNLO (*cf.* $N_f=2$: JLQCD/TWQCD, 2009)

    very complicated form w/ many $O(p^4, p^6)$ couplings

    $\Rightarrow$ simultaneous fit to various observables : *cf.* $\langle r^2 \rangle_V^{\{\pi^+, K^+, K^0\}}$
- $K \to \pi$ form factor
  - form factor shape : good agreement w/ exp't
  - chiral extrap. of $f_+(0)$: to be done w/ controlled sys. error to obtain $|V_{ud}|$
- nucleon strange quark content :
  - small strange quark content $f_{T_s} \sim 0.02$ $\Leftrightarrow$ pheno. : 0.31(9)
- extending to larger volume $24^3 \times 48$ (meson form factors) / TBCs

8.0.4 F_0

$F_{\pi,K}$



- phenomenology *(Bijnens, 2009)*

  $F_0 = 88$ MeV $\Rightarrow$ $f_0 = 124$ MeV

- JLQCD/TWQCD *(Noaki, Lattice 2010)*

  $F_0 = 52(2)$ MeV $\Rightarrow$ $f_0 = 74(2)$ MeV