Heavy quark potentials from lattice QCD

Miho Koma
(Numazu Coll. Tech.)

Lattice QCD confronts experiments ∼ *Japanese-German Seminar 2010* ∼
Mishima Chamber of Commerce and Industry, Mishima, Japan, 6 Nov. 2010

**[Y.Koma, M.Koma & H.Wittig, Phys.Rev.Lett.97('06)122003,
Y.Koma, M.Koma, Nucl.Phys.B769('07)79,
Y.Koma, M.Koma & H.Wittig, PoS(LATTICE 2007)111,
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Y.Koma & M.Koma, PoS(LATTICE 2009)122,
Y.Koma & M.Koma, New Frontiers in QCD 2010, PTP Suppl. in press,
Y.Koma & M.Koma, Chiral '10, AIP Conference Proc. in press]**

INTRODUCTION

- ▷ **Heavy quarkonium can be described as a non-relativistic two body system**
  $\Longrightarrow$ Schrödinger equation with inter-quark potential

- ▷ **Inter-quark potential?**
  - **Phenomenological potential (Coulomb + linear + Spin dependent corrections)**
  - **"Inter-quark potential" from (lattice) QCD ?**
    - ⋆ **QCD is not a quantum mechanics but a field theory...**
    - ⋆ **inter-quark potential $\neq$ static potential**

- ▷ **In the framework of an effective field theory for heavy quarkonium (potential NRQCD (pNRQCD)), we can study inter-quark potential including relativistic corrections from QCD**

[Brambilla,Pineda,Soto&Vairo('99-)]

POTENTIAL NRQCD

- ▷ **pNRQCD is an effective field theory for the heavy quarkonium.**

- ▷ **energy hierarchy**
  (m_q:quark mass, v: quark velocity in the CM frame, $v \ll 1$)
  $m_q > m_q v > m_q v^2$

- ▷ **integrating out the energy scale above $m_q v \Longrightarrow$ pNRQCD**

- ▷ **inter quark potential $V(r)$ appears as a low-energy matching factor dependent on the $q\bar{q}$ distance r**
  (cf. low energy constant of chiral perturbation theory)

- ▷ **$V(r)$ consists of static potential and relativistic corrections with $1/m_q$ expansion**

▷ **Effective Hamiltonian for quarkonium up to $O(1/m_q^2)$**

[Pineda&Vairo('01)]

$$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V^{(0)}(r) + \frac{1}{m_1}V^{(1,0)}(r) + \frac{1}{m_2}V^{(0,1)}(r) + \frac{1}{m_1^2}V^{(2,0)}(r) + \frac{1}{m_2^2}V^{(0,2)}(r) + \frac{1}{m_1 m_2}V^{(1,1)}(r) + O(1/m^3)$$

- $V^{(0)}(r)$: **static potential**
- $V^{(1)}(r)$: $O(1/m_q)$ **correction**
- $V^{(2,0)}(r)$, $V^{(0,2)}(r)$, $V^{(1,1)}(r)$: $O(1/m_q^2)$ **corrections**
  $\Rightarrow$ **incl. spin- and momentum-dependent corrections**

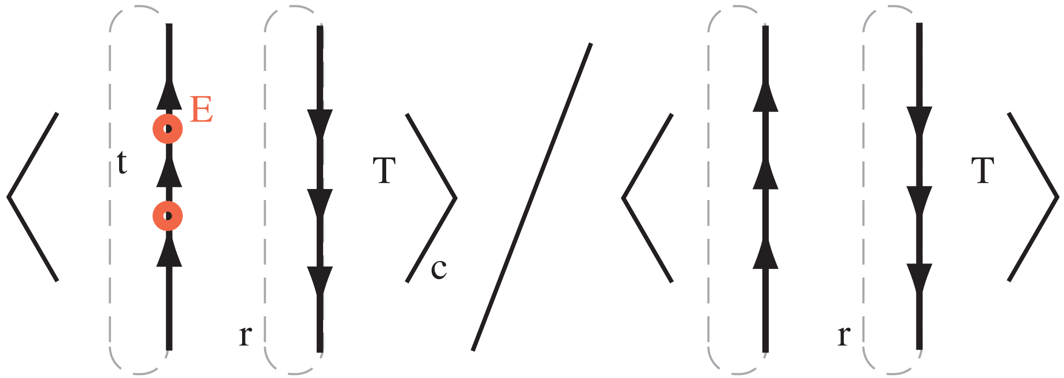
$O(1/m_q)$ POTENTIAL — DEFINITIONS

- ▷ **Nonperturbative expression** [Brambilla,Pineda,Soto&Vairo('01)]

$$V^{(1)}(r) = -\frac{1}{2}\lim_{\tau'\to\infty}\int_0^{\tau'} dt\ t\,\langle\langle g^2\vec{E}(0,0)\cdot\vec{E}(0,t)\rangle\rangle_c$$

- ▷ **Color electric field strength correlator on the Polyakov loop correlation function**

$$\langle\langle g^2 E^i(0,0)\, E^i(0,t)\rangle\rangle_c =$$



We measure this quantity accurately by utilizing the multilevel algorithm and fit it to its spectral representation.

- ▷ **for all technical details, see** [Koma, Koma, NPB769('07)79]

$O(1/m_q)$ POTENTIAL — DEFINITIONS

▷ **Nonperturbative expression** [Brambilla,Pineda,Soto&Vairo('01)]

$$V^{(1)}(r) = -\frac{1}{2}\lim_{\tau'\to\infty}\int_0^{\tau'} dt\; t\,\langle\langle g^2\vec{E}(0,0)\cdot\vec{E}(0,t)\rangle\rangle_c$$

▷ **Color electric field strength correlator on the Polyakov loop correlation function (spectral rep.)**

$$\langle\langle g^2 E^i(0,0)\,E^i(0,t)\rangle\rangle_c = 2g^2\sum_{n=1}^{\infty}\langle 0(r)|\mathcal{E}_i|n(r)\rangle\langle n(r)|\mathcal{E}_i|0(r)\rangle e^{-(\Delta E_{n0})\frac{T}{2}} \times \cosh\left((\Delta E_{n0})(\frac{T}{2}-t)\right)$$

The integration is performed analytically.

▷ **for all technical details, see** [Koma, Koma, NPB769('07)79]

$O(1/m_q^2)$ SPIN DEPENDENT POTENTIALS — DEFINITIONS

$$V_{\mathrm{SD}}(r)=\left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2}-\frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r}+2\frac{V_1'(r)}{r}\right)+\left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1m_2}-\frac{\vec{s}_1\cdot\vec{l}_2}{2m_1m_2}\right)\frac{V_2'(r)}{r}$$
$$+\frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2}-\frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r)+\frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r)$$

$O(1/m^2)$ SPIN DEPENDENT POTENTIALS — DEFINITIONS

$$V_{\rm SD}(r) = \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2} - \frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r} + 2\frac{V_1'(r)}{r}\right) + \left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1 m_2} - \frac{\vec{s}_1\cdot\vec{l}_2}{2m_1 m_2}\right)\frac{V_2'(r)}{r}$$
$$+\frac{1}{m_1 m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r) + \frac{\vec{s}_1\cdot\vec{s}_2}{3m_1 m_2}V_4(r)$$

▷ **Nonperturbative expressions** (set $\vec{r} = (r,0,0)$)

$$V_1'(r) = 2\int_0^\infty dt\ t\langle\langle gB_y(0,0)\ gE_z(0,t)\rangle\rangle$$
$$V_2'(r) = 2\int_0^\infty dt\ t\langle\langle gB_y(0,0)\ gE_z(r,t)\rangle\rangle$$
$$V_3(r) = 2\int_0^\infty dt\,\{\langle\langle gB_x(0,0)\ gB_x(r,t)\rangle\rangle - \langle\langle gB_y(0,0)\ gB_y(r,t)\rangle\rangle\}$$
$$V_4(r) = 2\int_0^\infty dt\,\{\langle\langle gB_x(0,0)\ gB_x(r,t)\rangle\rangle + 2\langle\langle gB_y(0,0)\ gB_y(r,t)\rangle\rangle\}$$

$O(1/m^2)$ SPIN DEPENDENT POTENTIALS — DEFINITIONS

$$V_{\rm SD}(r) = \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2} - \frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r} + 2\frac{V_1'(r)}{r}\right) + \left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1m_2} - \frac{\vec{s}_1\cdot\vec{l}_2}{2m_1m_2}\right)\frac{V_2'(r)}{r}$$
$$+\frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r) + \frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r)$$

▷ **Nonperturbative constraints**

(exploit Lorentz invariance of field strength correlators)

- **Gromes relation** [Gromes('84), Brambilla,Gromes&Vairo('01)]

  $$V_2' - V_1' = V^{(0)\prime}$$

  Relativistic corrections must be determined nonperturbatively

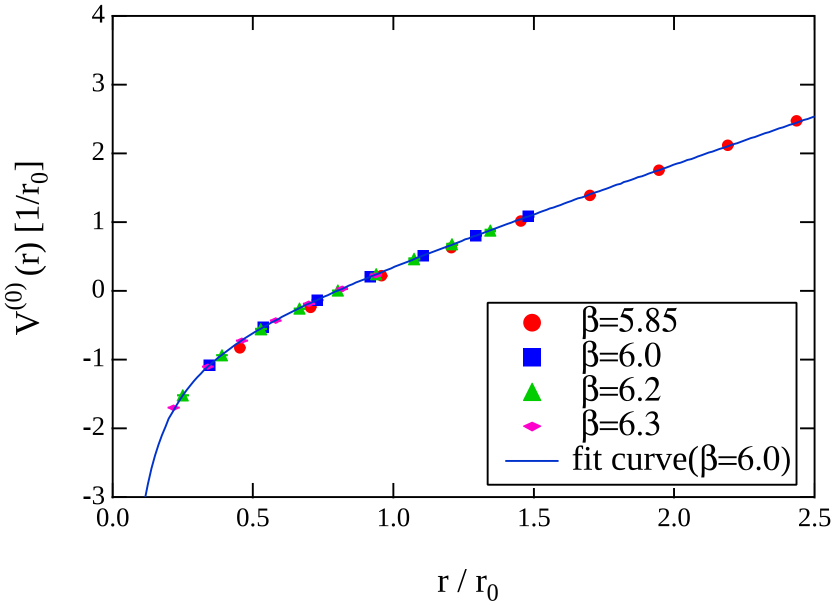
SIMULATION DETAILS

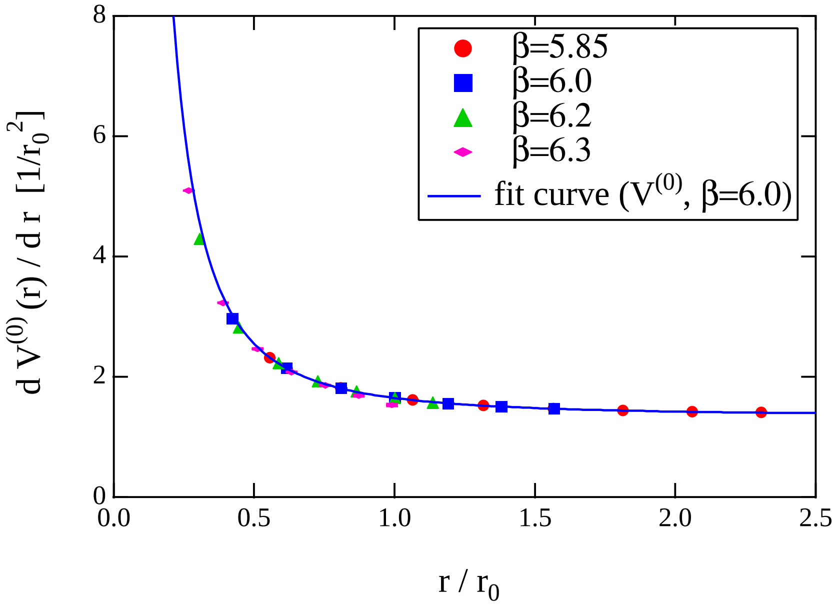
$\beta = 6/g^2$	a [fm]	$V = L^3T$	$N_{\rm tsl}$	$N_{\rm iupd}$	$N_{\rm conf}$
5.85	0.122	24^4	3	50000	77
6.0	0.093	$20^3 40$	4	7000	33
6.2	0.068	$24^3 32$	5	10000	33
6.3	0.059	24^4	6	6000	39
5.85	0.122	24^4	3	50000	133
6.0	0.093	$24^3 32$	4	50000	100
6.2	0.068	$30^3 40$	5	50000	33

- **Wilson gauge action**
- **set lattice spacing by Sommer scale** $r_0 = 0.5$ fm
- **NEC SX5, SX8, SX9 @ RCNP Osaka University, JAPAN**

STATIC POTENTIAL & FORCE

- $\triangleright$ $V^{(0)}(r_I) = -\frac{1}{T}\ln\langle P(0)P(r)^*\rangle + O(e^{-(\Delta E_{10})T})$
- $\triangleright$ $V^{(0)\prime}(\bar{r}) = \frac{1}{a}\{V^{(0)}(r) - V^{(0)}(r-a)\}$

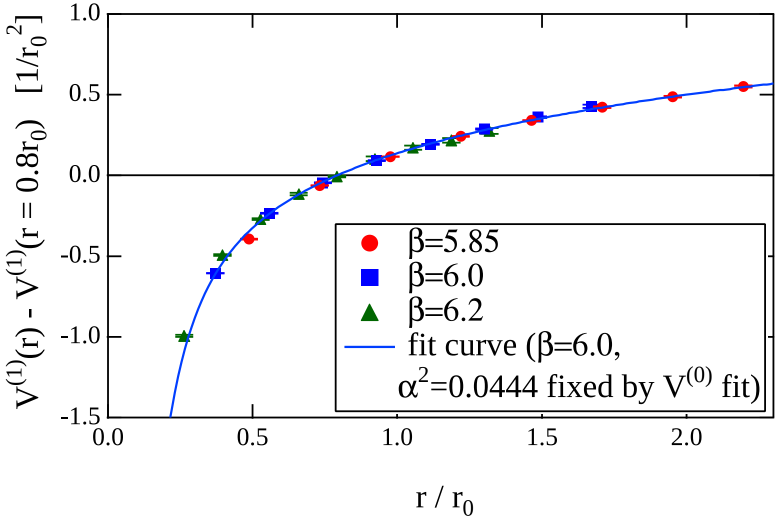


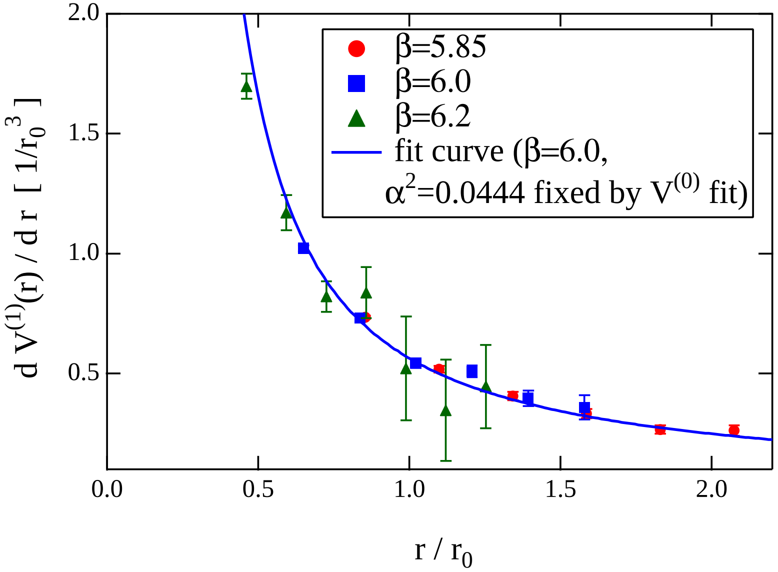


- $V^{(0)}(r) = -\frac{c}{r} + \sigma r + \mu \quad \Rightarrow \quad c = 0.2808(5),\ \sigma a^2 = 0.0468(1)$

$O(1/m_q)$ POTENTIAL — RESULTS

▷ $V(r) = V^{(0)}(r) + \left(\frac{1}{m_1} + \frac{1}{m_2}\right) V^{(1)}(r) + \cdots$



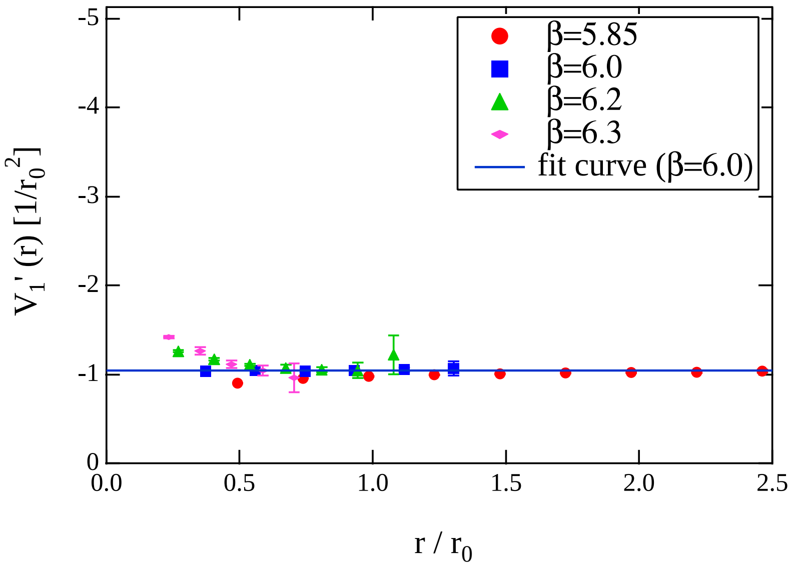


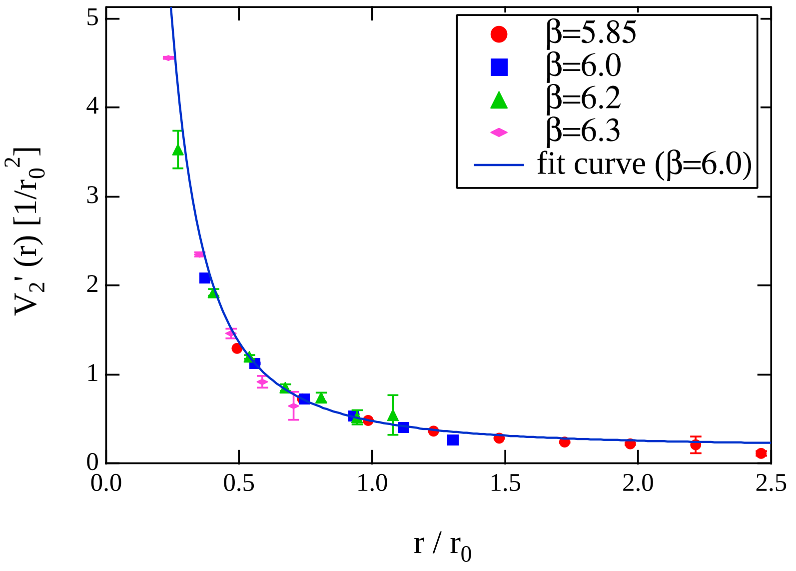
▷ **functional form**

$$V^{(1)}(r) = -\frac{\alpha^2}{r^2} + c' \ln r \quad \Rightarrow \quad c' r_0^2 = 0.475(4) \ \ (\alpha = (3/4)c)$$

$O(1/m_q^2)$ SPIN DEPENDENT POTENTIAL

$$V_{\rm SD}(r) = \left(\frac{\vec{s}_1\cdot\vec{l}_1}{2m_1^2} - \frac{\vec{s}_2\cdot\vec{l}_2}{2m_2^2}\right)\left(\frac{V^{(0)\prime}(r)}{r} + 2\frac{V_1'(r)}{r}\right) + \left(\frac{\vec{s}_2\cdot\vec{l}_1}{2m_1m_2} - \frac{\vec{s}_1\cdot\vec{l}_2}{2m_1m_2}\right)\frac{V_2'(r)}{r}$$
$$+ \frac{1}{m_1m_2}\left(\frac{(\vec{s}_1\cdot\vec{r})(\vec{s}_2\cdot\vec{r})}{r^2} - \frac{\vec{s}_1\cdot\vec{s}_2}{3}\right)V_3(r) + \frac{\vec{s}_1\cdot\vec{s}_2}{3m_1m_2}V_4(r)$$





spin-orbit

$O(1/m_q^2)$ SPIN DEPENDENT POTENTIAL

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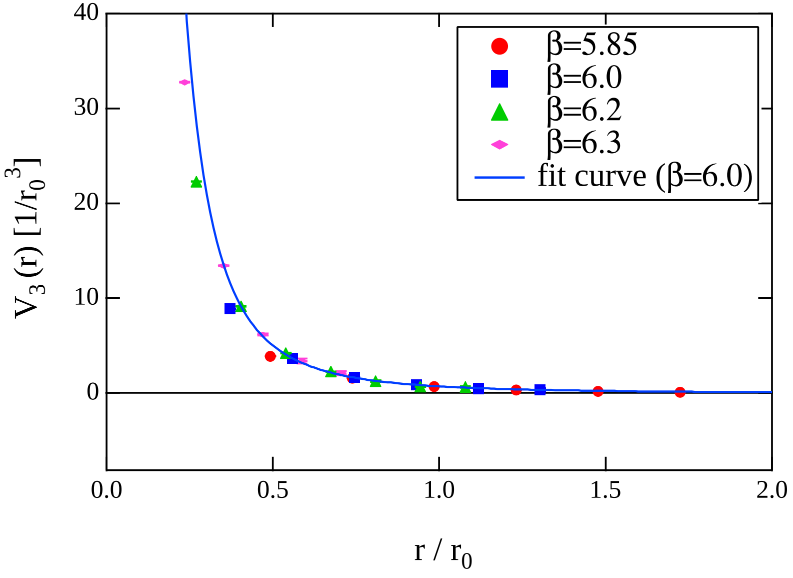
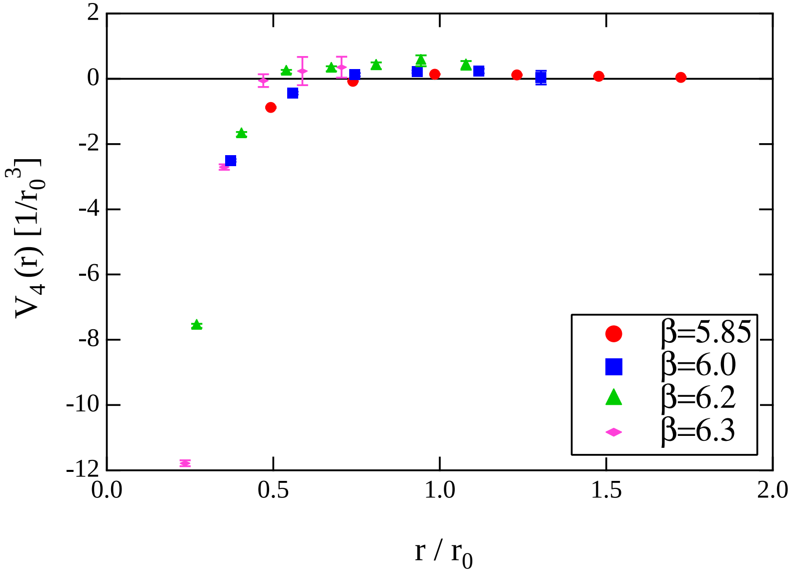


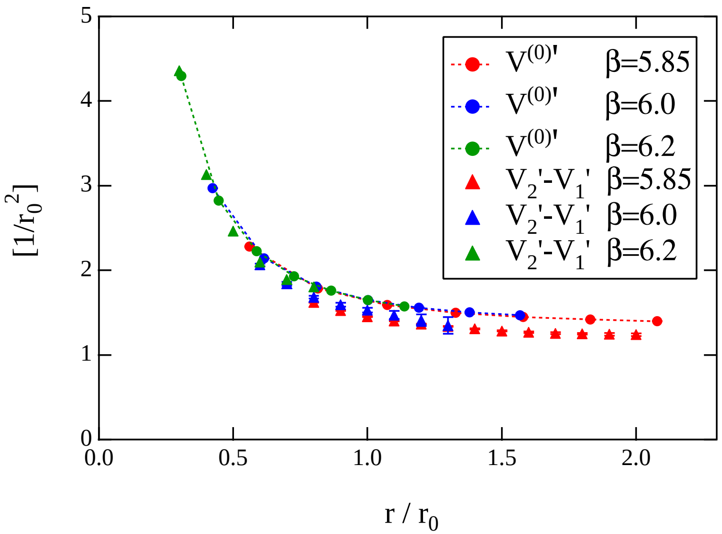


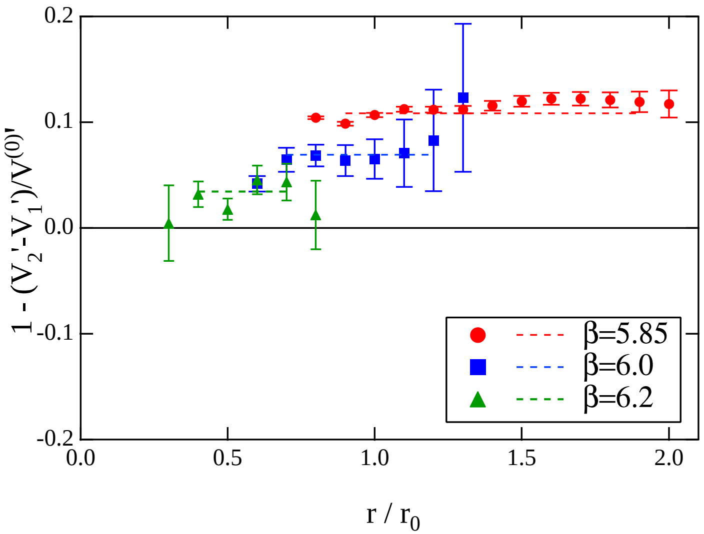
spin-spin

$O(1/m_q^2)$ SPIN DEPENDENT POTENTIAL

▷ **Gromes relation** [Gromes('84), Brambilla,Gromes&Vairo('01)]

$$V^{(0)\prime}(r) = V_2'(r) - V_1'(r)$$





⇒ **existence of nonperturbative contribution to spin-dep corr.**

⇒ **restoration of Lorentz invariance**

SUMMARY

▷ We have investigated the relativistic corrections to the heavy quark potential at $O(1/m_q)$ and $O(1/m_q^2)$

Current observation...

▷ Potentials are measured at $0.2 \lesssim r \lesssim 1.2$ fm

▷ $O(1/m_q)$ potential

- $V^{(1)}$ increases as a function of $r \Longrightarrow$ flavor dependent correction
- functional form $\Longrightarrow -\alpha^2/r^2 + c' \ln r$?

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- ▷ $O(1/m_q)$ potential
  - $V^{(1)}$ increases as a function of $r \Longrightarrow$ flavor dependent correction
  - functional form $\Longrightarrow -\alpha^2/r^2 + c' \ln r$?
- ▷ Spin-dependent potentials at $O(1/m_q^2)$
  - V_1' and V_2' are long ranged (V_2' has finite tail up to 1.2 fm)
  - V_3 and V_4 are short ranged
  - Gromes relation, satisfied in the continuum limit

SUMMARY

- ▷ **We have investigated the relativistic corrections to the heavy quark potential at $O(1/m_q)$ and $O(1/m_q^2)$**

Further applications. . .

- ▷ **Spectroscopy**

- ▷ **Comparison with the QCD Vacuum models**