

# Lattice Problems for Particle Physics Phenomenology - A Review -

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# 1. Introduction

- In this review, I will not talk about the development of actual lattice calculations in flavor physics.
- Those were already beautifully covered at Lattice 2010 by C. Sachrajda (K physics) and J. Heitger (Heavy quark Physics).
- Also, newer developments will be presented
  - A. Juettner (New ideas for  $g-2$  on the lattice)
  - H. Wittig (Form factor calculations for mesons and baryons)
  - T. Kaneko (Hadron form factors from lattice QCD with exact chiral symmetry)
  - G. von Hippel ( Heavy-light meson properties from non-perturbative HQET)

- Instead, I will review the motivation of lattice calculation for particle physics phenomenology, keeping the present and future experiments in mind.
- For phenomenology-oriented lattice theorists, there is not something entirely new in my talk. Nevertheless, it may be worth to think again about what is important for phenomenology.

# Goal of Flavor Physics

## New Paradigm

Test of the Standard Model  
Kobayashi Maskawa theory



Search for New Physics

“Today’s signal is  
tomorrow’s background”

- Measure more precisely!
- Understand SM contribution better!

- Effective Hamiltonian

$$H_{\text{eff}} = \sum_n c_n(\mu) Q_n(\mu)$$

$\mu$ : Renormalization scale

Wilson coefficients

Decay  
(Mixing) Operators

Short distance effects:

Standard Model + **New Physics**

$$c_n(\mu) = c_n^{SM}(\mu) + c_n^{New}(\mu)$$

We look for this!

# Effective operators

$$(\bar{q}_1 \Gamma q_2) (\bar{q}_3 \Gamma q_4)$$

$$B - \bar{B} \quad \text{mixing}$$

$$K - \bar{K} \quad \text{mixing}$$

$$B \rightarrow J/\psi K_s, \phi K_s$$

$$B \rightarrow DK, \pi\pi$$

$$K \rightarrow \pi\pi$$

# Effective operators

$$(\bar{q}_1 \Gamma q_2) (\bar{l}_3 \Gamma l_4)$$

$$B \rightarrow X_{c,u} l \nu, D^{(*)} l \nu, \pi l \nu$$

$$B \rightarrow X_s l^+ l^-, K^{(*)} l^+ l^-$$

$$B \rightarrow \tau \nu, \mu \nu$$

$$D \rightarrow K l \nu, \pi l \nu$$

$$D \rightarrow \tau \nu, \mu \nu$$

# Effective operators

$$\bar{q}_1 \Gamma_{\mu\nu} F_{\mu\nu} q_2$$

$$B \rightarrow X_s \gamma$$

(Chromo-) EDM



# How to probe new physics?

1. Good Data from Experiment
2. Significant New Physics Effects in the Wilson Coefficients
3. Reliable Theoretical Prediction for Hadron Matrix Element

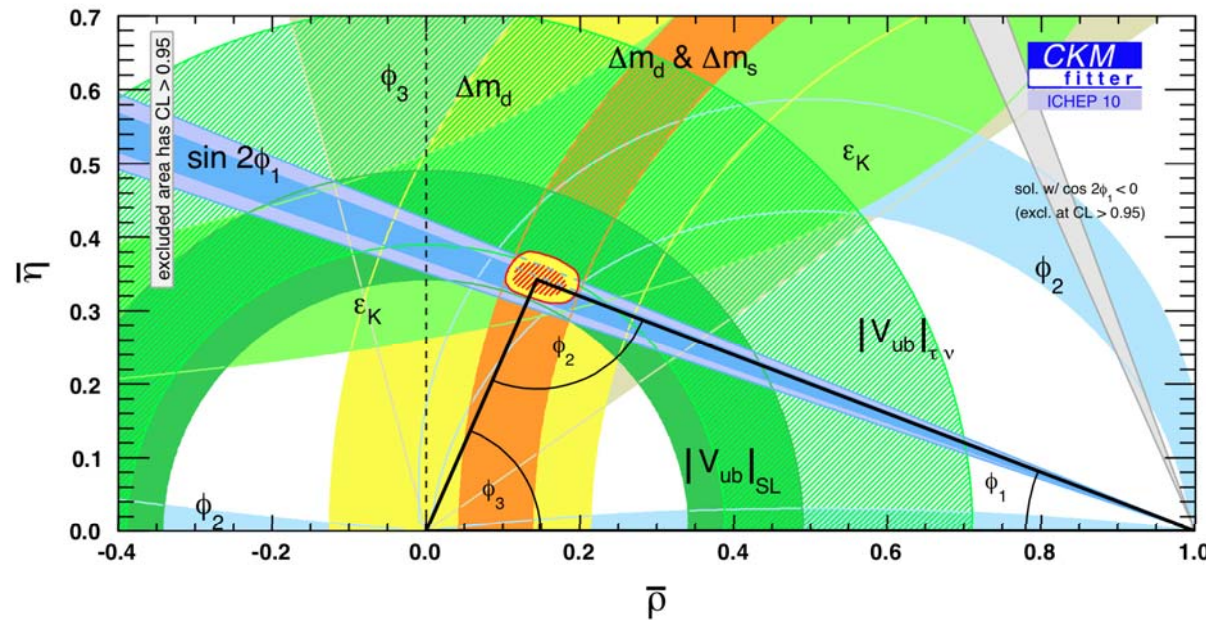
Lattice QCD, ....



# Topics in this review

1. B,D,K physics
2. EDM
3.  $g-2$
4. Summary

# What do we know now?



Preliminary results (ICHEP 2010),  
<http://ckmfitter.in2p3.fr/>

1. Kobayashi-Maskawa theory is established.
2. New Physics effect is not the dominant contribution.
3. There is still a room for new physics of order 10-20%.



Need for Super B factory

# Particle Physics in LHC era

## Complementary role of flavor physics

- LHC .... Discovery of new particles  
masses and main decay modes
- Flavor Physics .... Flavor Structure of new particles
  - ➔ We can learn about the origin of new particle masses  
(, just as we did for Standard Model particles).
  - e.g. SUSY breaking mechanism

# Constraints on new physics

flavor mixing and CP violation from new physics

$$[\text{Amplitude}] = \left( \frac{c_{SM} g_W^2}{M_W^2} + \frac{c_{new}}{M_{new}^2} \right) \times [\text{Hadron Matrix Element}]$$

Suppressed: 1-loop, GIM mechanism, ...


$B - \bar{B}$  mixing

- Tree  $M_{new} > 2 \times 10^4 \text{ TeV}$
- 1-loop  $M_{new} > 2 \times 10^3 \text{ TeV}$
- Tree Minimal flavor mixing  $M_{new} > 5 \text{ TeV}$
- 1-loop Minimal flavor mixing  $M_{new} > 0.5 \text{ TeV}$

# Super B factory

## Peak Luminosity

B factory ....  $2 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

 Super B factory ....  $7 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$   
 $\Leftrightarrow 10 \text{ ab}^{-1} / \text{year}$

Rare and/or Cleaner Process  
Spectrum and angular distribution } will be available

## Expected Performance of Super B factory for rare and clean processes

Measurement	3/ab	10/ab	50/ab
$S(B \rightarrow \phi K_s)$	16%	8.7%	3.9%
$\text{Br}(B \rightarrow D^* \tau \nu)$	10%	5%	2.5%
$A_{B \rightarrow K^* l+l-}^{CP}$ (high mass)	$\sim 12\%$	$\sim 6\%$	$\sim 3\%$
$( V_{ub}  f^+(q^2))_{B \rightarrow \pi l \nu}$	6%	3%	
$( V_{ub}  f_B)_{B \rightarrow \tau \nu}$		$< 10\%$	
$( V_{ub}  f_B)_{B \rightarrow \mu \nu}$		$< 10\%$	
$\phi_1(B \rightarrow J/\Psi K_S)$	$0.6^\circ$	$0.34^\circ$	$0.18^\circ$
$\phi_2(B \rightarrow \rho \rho)$	$2.9^\circ$	$1.5^\circ$	$0.72^\circ$
$\phi_3(B \rightarrow D^{(*)} K)$		1.2-2%	

With this impressive expected performance, we may detect new physics effect for several channels in Super B factory.

Moreover, there will be other experiments.

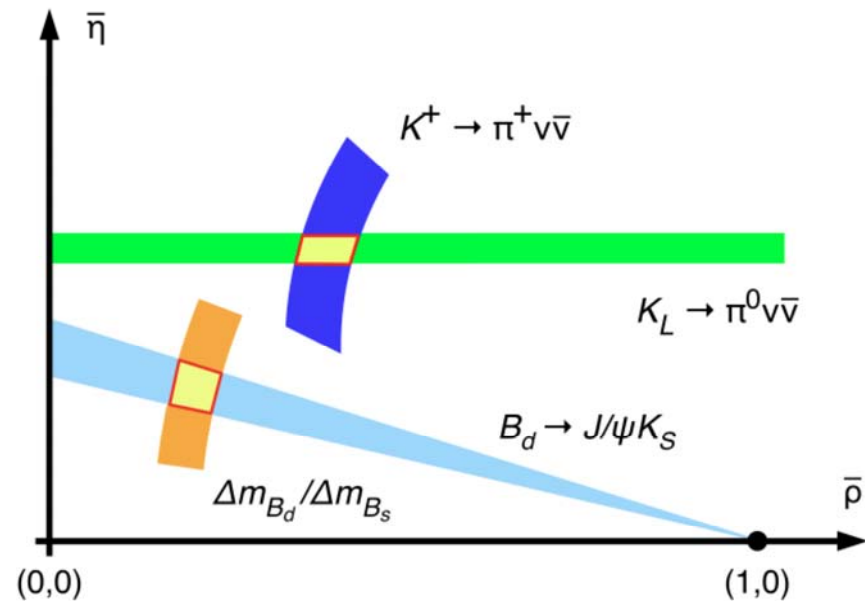


- $K^0 \rightarrow \pi^0 \nu \bar{\nu}$        $\text{Br}(K^0 \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} \simeq 3 \times 10^{-11}$

## KOTO @JPARC

- Step-1 2010-15 ... 1<sup>st</sup> measurement for SM case
- Step-2 2015-20 .... 100 events for SM case

Theoretically very clean



# Patterns of deviation from SM

	4-th gen.	2HDM (typeII)	SUSY	
			mSUGRA(MFV)	SU(5)( $\nu_R$ )
U.Tri. ( $\Delta m_{B_d},  V_{ub} , \phi_1$ )	open	open	closed	closed
$\epsilon_K$	large	large	small	large
$\Delta m_{B_s}$	large	large	small	large
$S_{\phi K_s}$	large	large	small	large
$S_{\psi\phi}$	large	large	small	large
$A_{b \rightarrow s\gamma}^{CP}$	large	large	small	small
$B \rightarrow \tau\nu$	large	large	large	large
$B \rightarrow D\tau\nu$	large	large	large	large
$B_{s,d} \rightarrow \mu^+\mu^-$	large	large	small	small
$K_L \rightarrow \pi^0\nu\nu$	large	large	small	large
EDM( $d_n$ )	large	large	small	large
EDM( $d_{Hg}$ )	large	large	small	large
EDM( $d_{Ti}$ )	large	large	small	large

# A strategy for constraining the new physics

1. Determine CKM parameter precisely from tree-level process.

$$|V_{ub}| \text{ from } B \rightarrow \pi l \nu, \quad |V_{cb}| \text{ from } B \rightarrow D^{(*)} l \nu, \quad \phi_3 \text{ from } B \rightarrow D^{(*)} K$$

2. Then study the loop process

$$K \rightarrow \pi \nu \bar{\nu} \text{ w.o. lattice}$$

$$K - \bar{K} \text{ mixing from lattice}$$

$$B_{d(s)} \rightarrow \psi K_s(\phi), \phi K_s \text{ w. o. lattice}$$

$$B \rightarrow \rho \rho \text{ w. o. lattice}$$

$$B_{d(s)} - \bar{B}_{d(s)} \text{ mixing from lattice}$$

3. Also study tree process involving the charged Higgs

$$Br(B \rightarrow D \tau \nu) / Br(B \rightarrow D \mu \nu) \text{ almost w.o. lattice}$$

$$Br(B \rightarrow \tau \nu) / Br(B \rightarrow \mu \nu) \text{ w.o. lattice}$$

- The lattice calculation in the above process can be done 1-5% accuracy.
- In order to identify new physics, form factors and Bag parameters for non-SM-type operator should also be computed.
- Matrix element of b-quark should be obtained by the interpolation of c-quark and static(HQET). Statistical accuracy of HQET and systematic error in charm is the key. (c.f. talk by Wittig, Hippel)
- K meson should be completely controlled by 1% level, but controlling the chiral extrapolation error may be the final challenge (c.f. Kaneko's talk).

How charm is controlled?  $f_{D_d}, f_{D_s}$

	FDd (MeV)	FDs (MeV)
CLEO'09	206(8)	257(6)
HPQCD'07	207(4)	241(3)
FNAL/MILC'09	207(11)	249(11)
ETMC'09	197(16)	244(12)
*HPQCD'10		247(2)
*FNAL/MILC'10	220(8)(5)	261(8)(5)
*ETMC'10	204(3)(?)	251(3)(?)

## HPQCD 2007

- 2+1-flavor MILC config. with improved staggered fermion
- Exact chiral symmetry
- Conserved axial vector current  $Z_A = 1$
- $m_\pi^{\text{lightest}} = 300 \text{ MeV}$
- Continuum limit taken with  $a=0.15, 0.12, 0.09 \text{ fm}$
- Scale input by  $\Upsilon$  1S-2S splitting

## HPQCD 2010 (update)

- Continuum limit taken with  $a=0.15, 0.12, 0.09, 0.06, 0.045 \text{ fm}$
- Scale input by  $\Upsilon, D_s$  splittings  $\rightarrow 3\% \text{ change!!!}$

## ETMC 2009

- 2-flavor config. with twisted mass Wilson fermion
- Exact chiral symmetry
- Conserved axial vector current  $Z_A = 1$
- $m_\pi^{\text{lightest}} = 300 \text{ MeV}$
- Continuum limit taken with  $a=0.10, 0.086, 0.067 \text{ fm}$

## ETMC 2010 (update)

- 2+1+1-flavor config with twisted mass Wilson fermion
- Continuum limit taken with  $a=0.079, 0.060\text{fm}$

## FNAL/MILC 2009

- 2+1-flavor MILC config. with improved staggered fermion
- Fermilab formalism for the valence charm
- Partially nonperturbative renormalization
- $m_{\pi}^{\text{lightest}} = 300 \text{ MeV}$
- Continuum limit taken with  $a=0.15, 0.12, 0.09 \text{ fm}$

## FNAL/MILC 2010 (update)

- New analysis with more statistics



## 2. Electric Dipole Moment

- Neutron EDM (Baker et al '06)

$$d_n < 2.9 \times 10^{-26} \text{ ecm}$$

$$d_n^{\text{SM}} \sim 10^{-32} \text{ ecm}$$

- Thallium EDM (Regan et al '02)

$$d_{\text{Tl}} < 9.0 \times 10^{-25} \text{ ecm}$$

- Mercury EDM (Griffith et al '09)

$$d_{\text{Hg}} < 3.1 \times 10^{-29} \text{ ecm}$$

The next generation EDM experiments can push

# Modified Peccei Quinn Mechanism :c.f. Bigi-Uraltsev (1991)

The theta term is removed by Peccei-Quinn mechanism at the minimum of axion potential  $U$ . But when there are additional CP-odd term, the situation is different.

$$L_{\text{CPodd}} = \theta \frac{\alpha}{4\pi} G\tilde{G} + c_{\text{odd}} O_{\text{odd}}$$

$$U = \frac{1}{2}(g_{11}\theta^2 + 2g_{12}\theta c_{\text{odd}} + g_{22}c_{\text{odd}}^2)$$

$$\text{where } g_{11} \equiv \int d^4x \langle \frac{\alpha}{4\pi} G\tilde{G}(x) \frac{\alpha}{4\pi} G\tilde{G}(0) \rangle, g_{12} \equiv \int d^4x \langle \frac{\alpha}{4\pi} G\tilde{G}(x) O_{\text{odd}}(0) \rangle$$

$$g_{22} \equiv \int d^4x \langle O_{\text{odd}}(x) O_{\text{odd}}(0) \rangle$$

Minimum of  
axion potential



$$\theta_{\text{min}} = -\frac{g_{12}}{g_{11}} c_{\text{odd}}$$

$$d_n \propto \theta_{\text{min}} \langle n | \frac{\alpha}{4\pi} G\tilde{G} | n \rangle + c_{\text{odd}} \langle n | O_{\text{odd}} | n \rangle$$

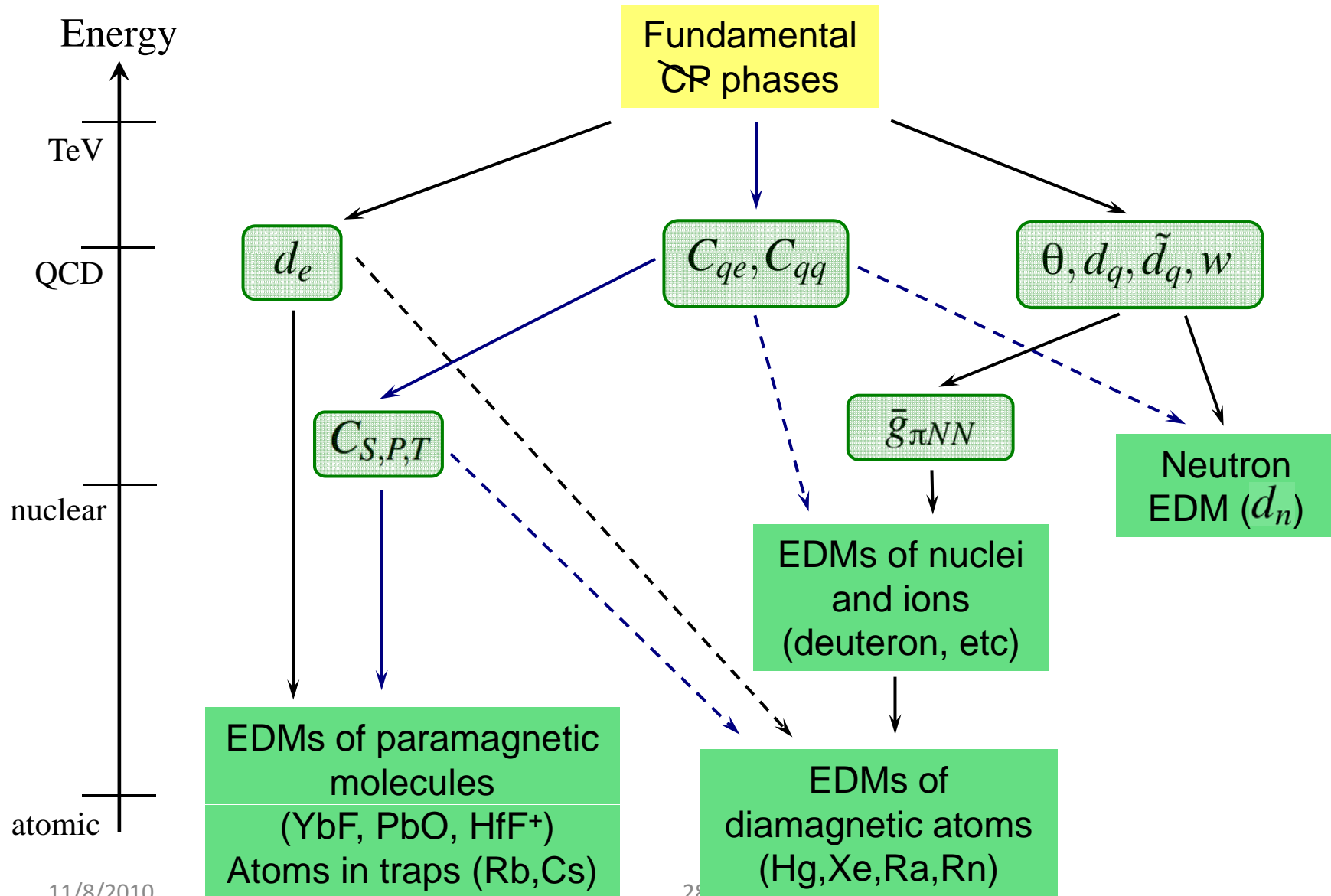
$$L_{CP\text{odd}} = \sum_i \frac{c_i}{M^{d-4}} O_d^{(i)}$$

$$\bar{\theta} \frac{\alpha_s}{8\pi} G\tilde{G}$$

$$\sum_{q=u,d,s} d_q \bar{q} F \sigma \gamma_5 q + \sum_{q=u,d,s} \tilde{d}_q \bar{q} G \sigma \gamma_5 q + d_e \bar{e} F \sigma \gamma_5 e + w g_s^3 G\tilde{G}$$

$$\sum_{q=u,d,s} C_{qq}(\bar{q}q)(\bar{q}i\gamma_5 q) + C_{qe}(\bar{q}q)(\bar{e}i\gamma_5 q)$$

# Origin of the EDMs (Future progress)



For Hg atom, main contribution to EDM comes from the Schiff moment  $S$  of nucleon potential:

$$d_{\text{HG}} = -2.8 \times 10^{-17} S \text{ ecm}$$

$$\text{where } V_{\text{eff}} = 4\pi S(\vec{I} \cdot \vec{\nabla})\delta(\vec{r})$$

$S$  arise from the CP violating N-N-pion(eta) interaction from Chromo-EDM, which gives a strong constraint on some of the SUSY models (Hisano-Shimizu)

$$g_{NN\pi(\eta)}^{\text{CPodd}} \sim \langle N | \bar{q}\sigma G\gamma_5 q | N \pi(\eta) \rangle \sim \langle N | \bar{q}\sigma G q | N \rangle$$

## Difficulty in chromo-EDM

chromo-EDM mixes with the power divergent mass term.

**Non-perturbative subtraction of the mass term is needed.**

(Similar situation as the Penguin operator in  $K \rightarrow \pi\pi$  decay)

In the continuum, the subtraction is perturbatively done by requiring that the renormalized quark masses after adding chromo-EDM operator take physical mass values.

### 3. muon $g - 2$

contribution	$a_\mu \times 10^{11}$
QED	116 584 718.09 (0.15)
EW	154 (2)
had(LO)	6955(40)
had(NLO)	7(26)
total SM	116591834(2)(41)(26)
E821(BNL)	116592089(54)(33)

3.2 sigma deviation

$$\Delta a = a_{\text{exp}} - a_{\text{SM}} = 255(63)(49) \times 10^{-11}$$

c.f.  $a^{\text{SUSY}} = \pm 130 \times 10^{-11} \times \left( \frac{100\text{GeV}}{m_{\text{SUSY}}} \right)^2 \tan \beta$

Next generation experiment is being planned at JPARC  
 → Factor 5 improvement?

- Vacuum Polarization from lattice QCD  
Accuracy better than 1% is needed.  
c.f. A. Juettner
- Light-by-Light scattering from lattice QCD  
100% error is still a big progress.
  - RBC/UKQCD, JLQCD:  $\pi^0 \rightarrow \gamma\gamma$  form factor  
assuming pi exchange dominance.
  - Hayakawa-Yamada-Blum et al.  
direct computation of muon in QCD+QED!



# Summary

- Owing to the progress in experiment theoretically cleaner channels and/or rare channels are accessible.
- Taking the ratios of processes, the appearing new processes which does not need lattice calculations. Lattice remains important for determining the CKM elements and predicting independent processes for testing the deviation patterns peculiar to new physics.
- EDM and  $g-2$  are getting more and more important. Order of magnitude calculation is quite important there.

# Backup Slides