Form factor calculations for mesons and baryons

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In collaboration with:

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Motivation & Outline

Form factors:

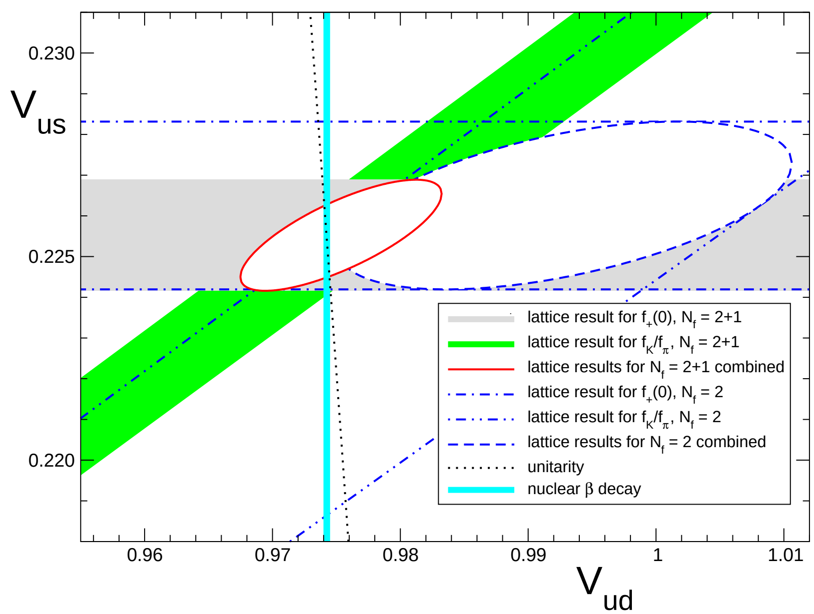
- provide information on hadron structure:

  $\longrightarrow$ distribution of electric charge and magnetisation; charge radii

- accurate experimental data available

- relatively simple to compute on the lattice:

  $\longrightarrow$ precise lattice estimates for $K_{\ell 3}$-decays *[FLAG Working Group of FLAVIAnet]*



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- Large systematic uncertainties remain for baryonic form factors

$\Rightarrow$ "Next-generation benchmark" for lattice QCD

Outline:

1. Lattice Set-up
2. Pion electromagnetic form factor
3. Form factors and axial charge of the nucleon
4. Summary & Outlook

1. Lattice Set-up

- Perform a systematic study of mesonic and baryonic form factors with controlled uncertainties:
  - lattice artefacts
  - finite-volume effects
  - chiral extrapolations
  - excited state contamination
- Determine form factors with fine momentum resolution
- Eventually: include quark disconnected diagrams

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- **C**oordinated **L**attice **S**imulations: *[https://twiki.cern.ch/twiki/bin/view/CLS/WebHome]*

  Berlin – CERN – Madrid – Mainz – Milan – Rome – Valencia – Wuppertal – Zeuthen

- Share configurations and technology

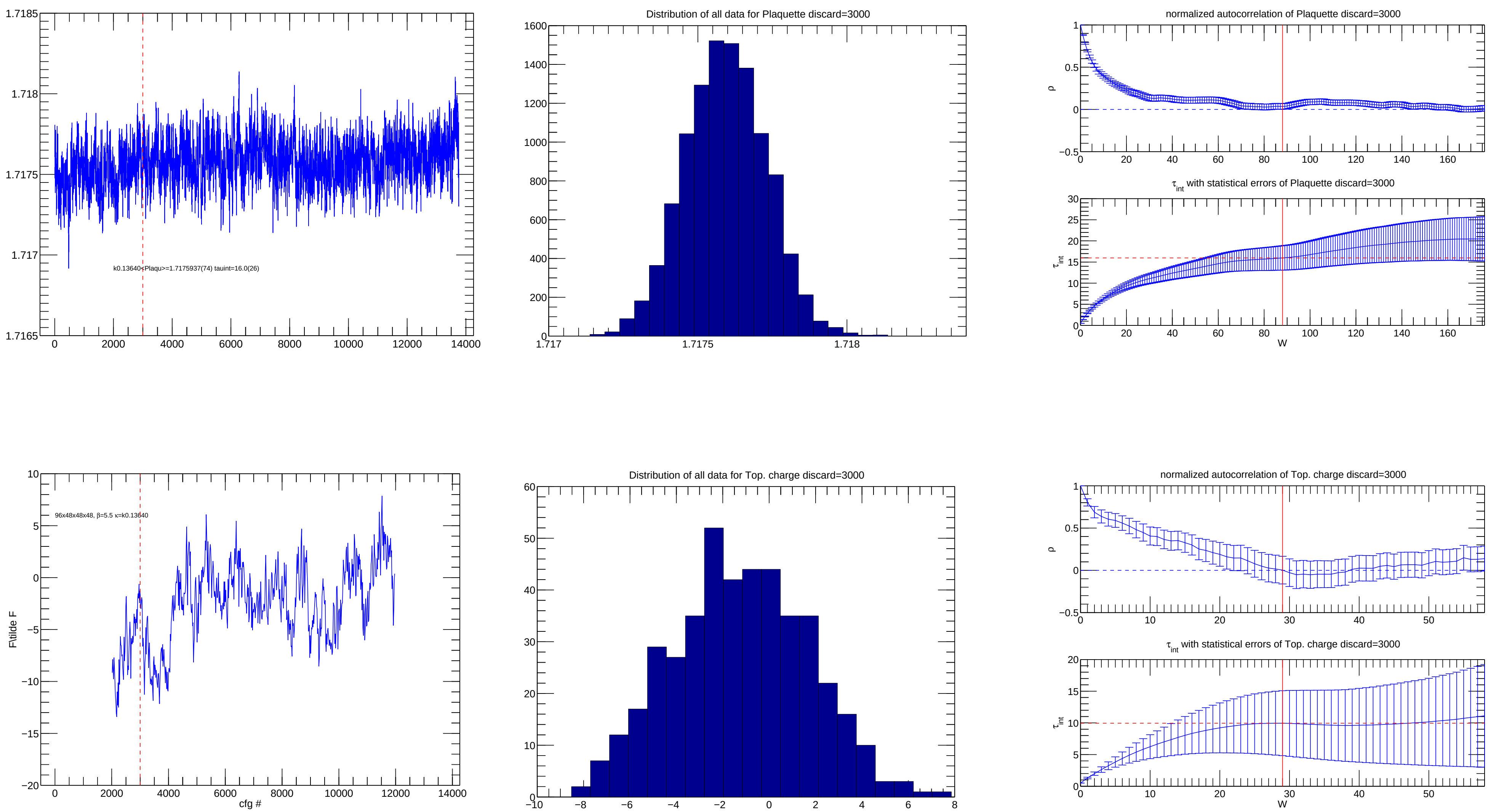
CLS run tables

- $N_\mathrm{f} = 2$ flavours of non-perturbatively $\mathrm{O}(a)$ improved Wilson quarks
- Use deflation accelerated DD-HMC algorithm *[Lüscher 2003–07]*
- Generated ensembles without serious topology problems:

β	$a[\mathrm{fm}]$	lattice	$L[\mathrm{fm}]$	masses	$m_\pi L$	Labels
5.20	0.08	64×32^3	2.6	4 masses	4.8 – 9.0	A1 — A5
5.20	0.08	96×48^3	3.8	1 mass	—	B6
5.30	0.07	48×24^3	1.7	3 masses	4.6 – 7.9	D1 — D3
5.30	0.07	64×32^3	2.2	3 masses	4.7 – 7.9	E3 — E5
5.30	0.07	96×48^3	3.4	2 masses	5.0, 4.2	F6, F7
5.50	0.05	96×48^3	2.5	3 masses	5.3 – 7.7	N3 — N5
5.50	0.05	128×64^3	3.4	1 mass	4.7	O6

[Capitani et al., arXiv:0910.5578, Brandt et al., arXiv:1010.2390]

Autocorrelation times @ $\beta = 5.5$ N3: $\kappa = 0.13640$



The “Wilson” cluster at Mainz

- 280 nodes: 2 AMD “Barcelona” processors @ 2.3 GHz: 2240 cores
- Infiniband network & switch
- Peak speed: ~ 20 TFlop/s
- Sustained speed: 3.7 TFlops/s $\Rightarrow$ 0.30 €/MFlops/s
- Waste heat: 20 kW/TFlops/s (Water-cooled server racks)

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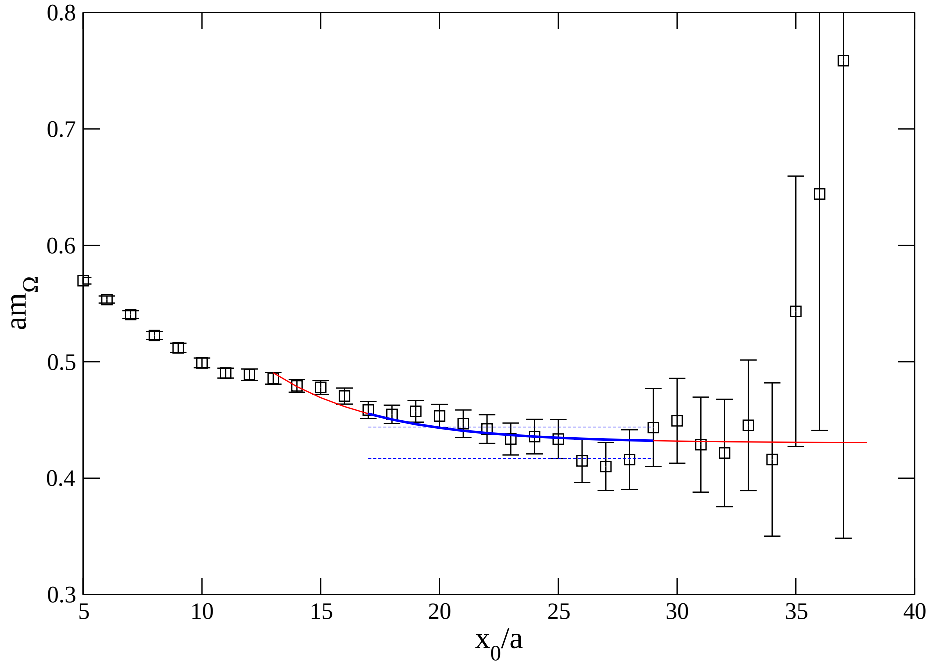
S

Scale setting

- Use mass of Ω-baryon to set the scale

- Effective masses from smeared-local correlator (Jacobi smearing)

  N5 : $\kappa_l = 0.1366, \quad \kappa_{\text{val}} = 0.1363, \quad \kappa_s = 0.13629(2)$



- Fit to ground plus 1^{st} excited state: $m_1 \equiv m_\Omega, \; m_2 \equiv m_\Omega + 2m_\pi$

Scale setting

- Observe weak sea quark mass dependence of m_Ω

$\rightarrow \quad \beta = 5.5: \qquad a_\Omega = 0.053(1)\,\mathrm{fm} \qquad$ (preliminary)

- Determination of am_Ω still on-going at $\beta = 5.3$

$$\beta = 5.3: \quad \text{“}a_\Omega\text{”} = \left(\frac{a_{\mathsf{ref}}(\beta = 5.3)}{a_{\mathsf{ref}}(\beta = 5.5)}\right) 0.053(1)\,\mathrm{fm} = 0.069(2)\,\mathrm{fm} \quad \text{(preliminary)}$$

- To come: comparison with r_0, f_K, t_0

4. The pion form factor

- Provides information on pion structure:

$$\langle \pi^+(\vec{p}_f)|\tfrac{2}{3}\overline{u}\gamma_\mu u - \tfrac{1}{3}\overline{d}\gamma_\mu d|\pi^+(\vec{p}_i)\rangle = (p_f + p_i)_\mu\, f_\pi(q^2)$$

$$q^2 = (p_f - p_i)^2 : \qquad \text{momentum transfer}$$

- Pion charge radius derived from form factor at zero q^2:

$$f_\pi(q^2) = 1 - \frac{1}{6}\langle r^2\rangle q^2 + \mathrm{O}(q^4) \quad \Rightarrow \quad \langle r^2\rangle = 6\,\frac{\mathrm{d}f_\pi(q^2)}{\mathrm{d}q^2}\bigg|_{q^2=0}$$

- Restriction on minimum accessible momentum transfer:

$$\vec{p}_{i,f} = \vec{n}\frac{2\pi}{L} \quad \Rightarrow \quad |q^2| \geq 2m_\pi\Big(m_\pi - \sqrt{m_\pi^2 + (2\pi/L)^2}\Big)$$

$$L = 2.5\,\mathrm{fm}, \quad m_\pi = 300\,\mathrm{MeV} \quad \Rightarrow \quad |q^2| \geq 0.17\,\mathrm{GeV}^2 = (0.41\,\mathrm{GeV})^2$$

$\longrightarrow$ Lack of accurate data points near $q^2 = 0$

Twisted boundary conditions

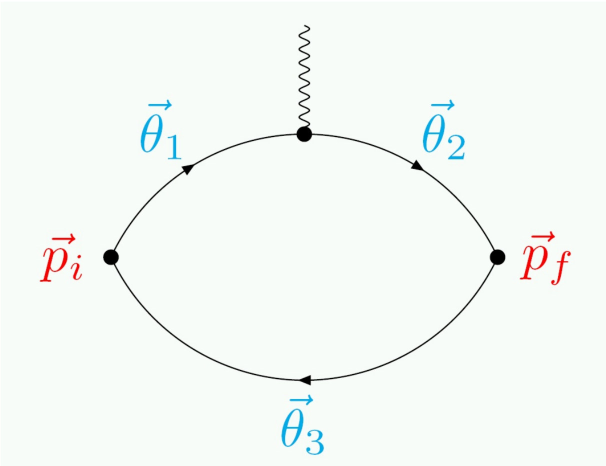
[Bedaque 2004; de Divitiis, Petronzio & Tantalo 2004; Flynn, Jüttner & Sachrajda 2005]

- Apply "twisted" spatial boundary conditions;

  Impose periodicity up to a phase $\vec{\theta}$:

$$\psi(x + L\hat{e}_k) = \mathrm{e}^{i\theta_k}\psi(x) \quad \Rightarrow \quad p_k = n_k\frac{2\pi}{L} + \frac{\theta_k}{L}, \quad k = 1, 2, 3$$

- Can tune $|q^2|$ to any desired value *[Boyle, Flynn, Jüttner, Sachrajda, Zanotti, hep-lat/0703005]*



$$\vec{\theta}_i = \vec{\theta}_1 - \vec{\theta}_3,$$
$$\vec{\theta}_f = \vec{\theta}_2 - \vec{\theta}_3$$

$$\Rightarrow q^2 = (p_i - p_f)^2 = \Big(E_\pi(\vec{p}_i) - E_\pi(\vec{p}_f)\Big)^2 - \left[\left(\vec{p}_i + \frac{\vec{\theta}_i}{L}\right) - \left(\vec{p}_f + \frac{\vec{\theta}_f}{L}\right)\right]^2$$

Preliminary results

[Brandt, Della Morte, Djukanovic, Endreß, Gegelia, Jüttner, H.W.]

β	$L^3 \cdot T$	a[fm]	L[fm]	m_π [MeV]	Lm_π
5.50	$48^3 \cdot 96$	0.053	2.5	600	7.7
5.50	$48^3 \cdot 96$	0.053	2.5	510	6.5
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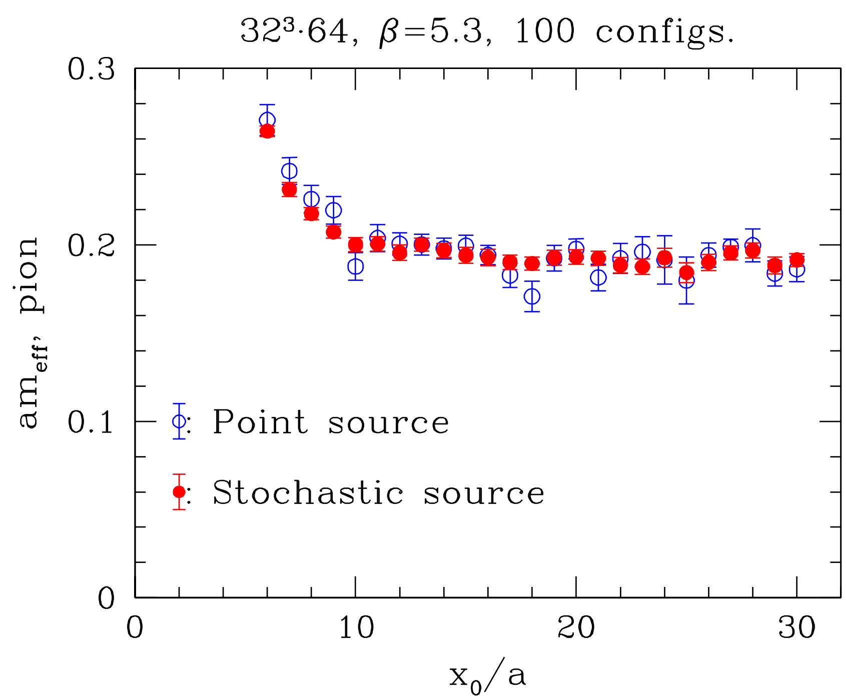
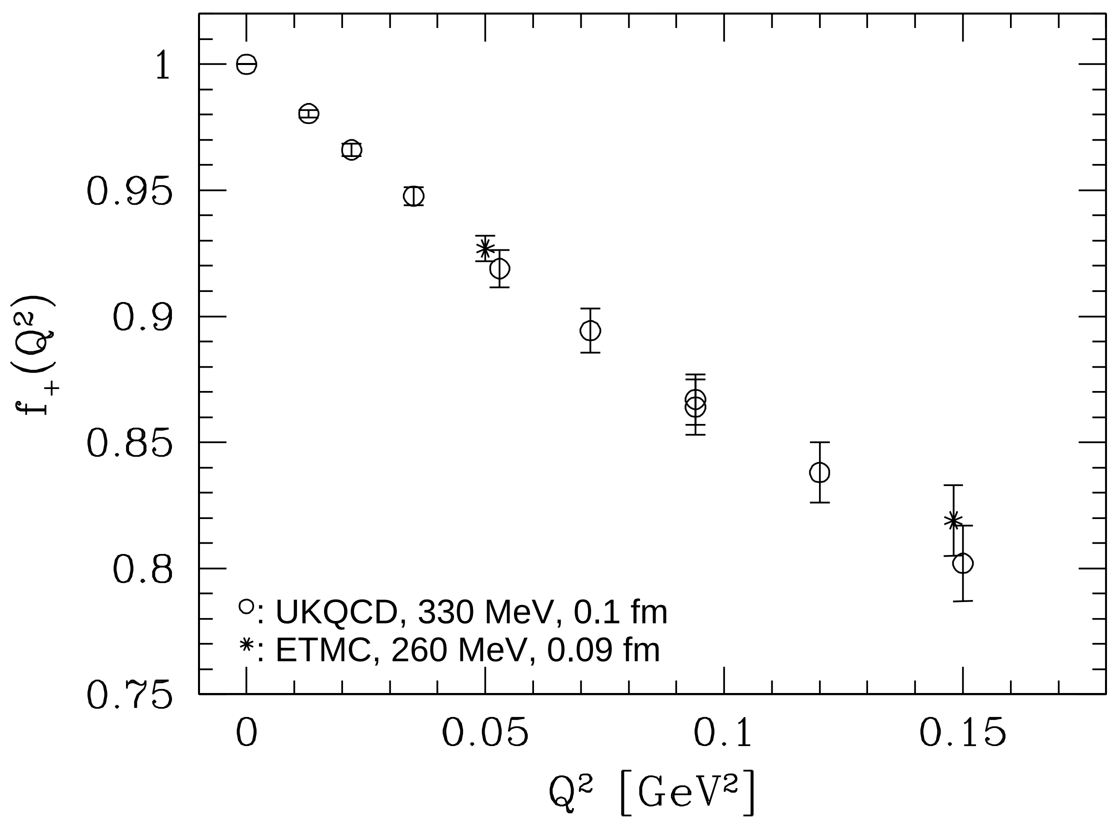
- Use stochastic noise source ("one-end trick") *[E. Endreß, Diploma thesis, 2009]*



Pion form factor

[Brandt, Della Morte, Djukanovic, Endreß, Gegelia, Jüttner, H.W.]

- Recently published results



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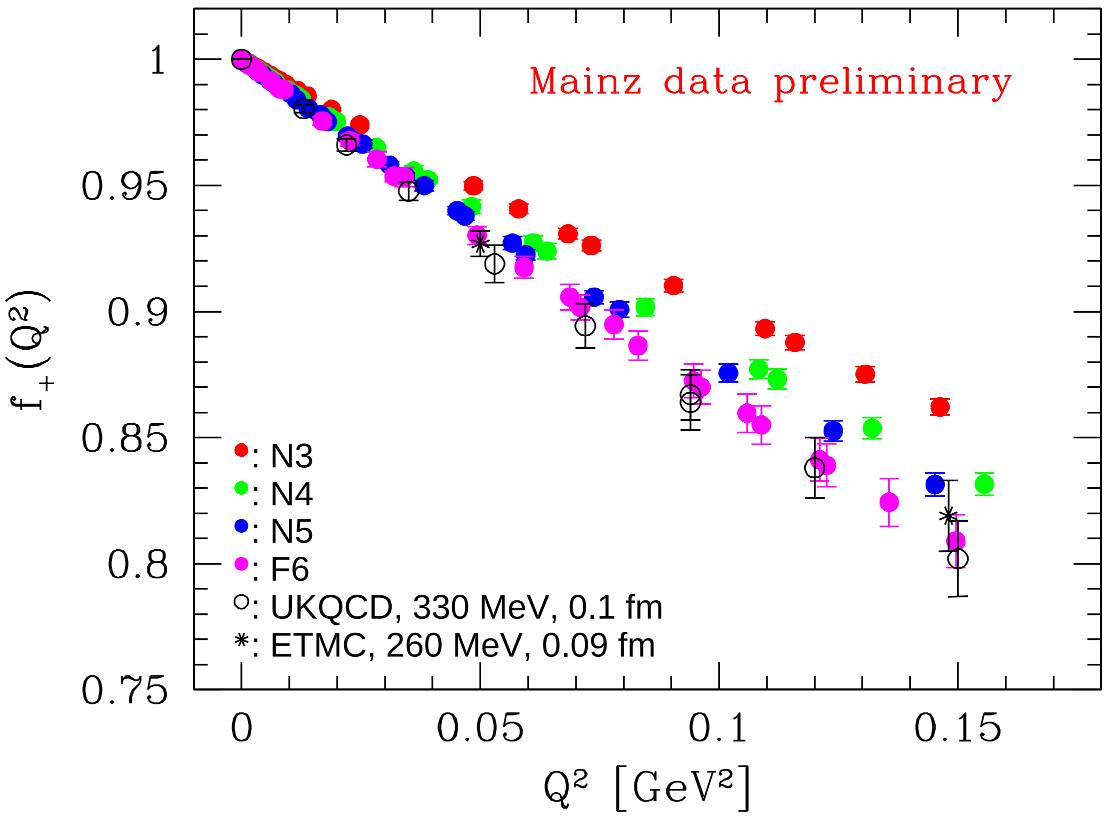

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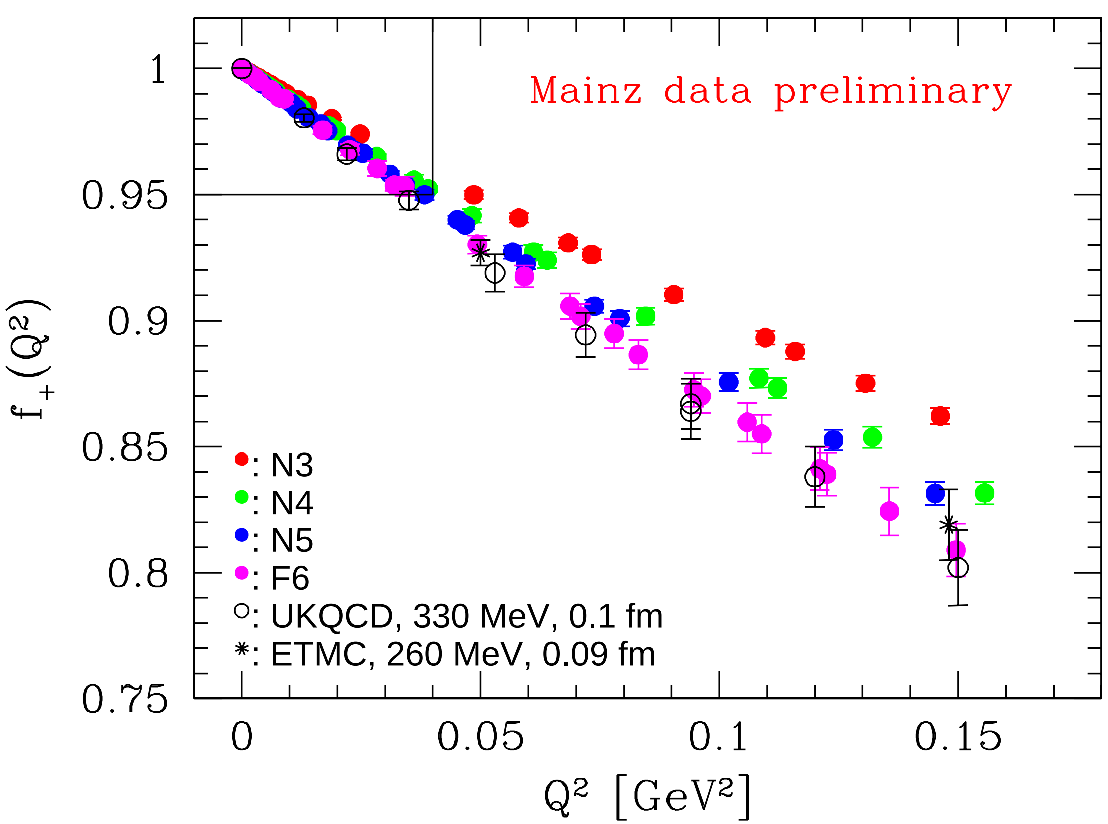

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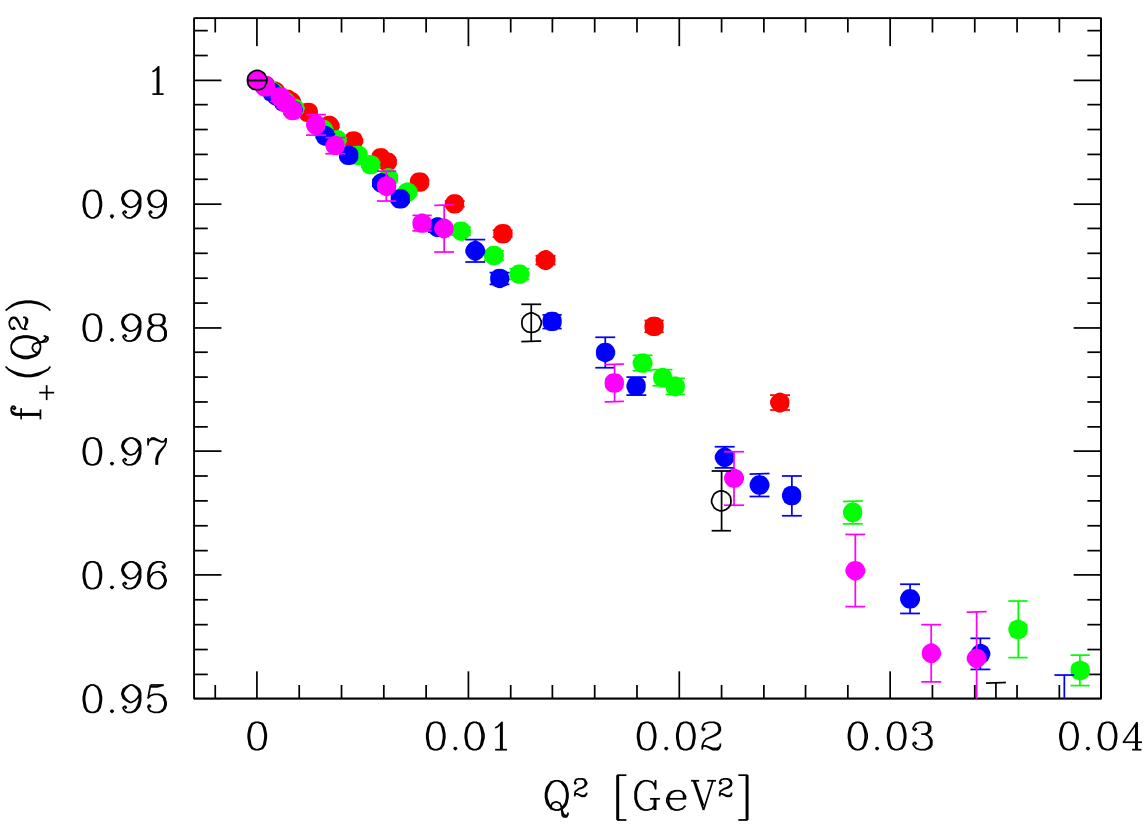

Pion charge radius

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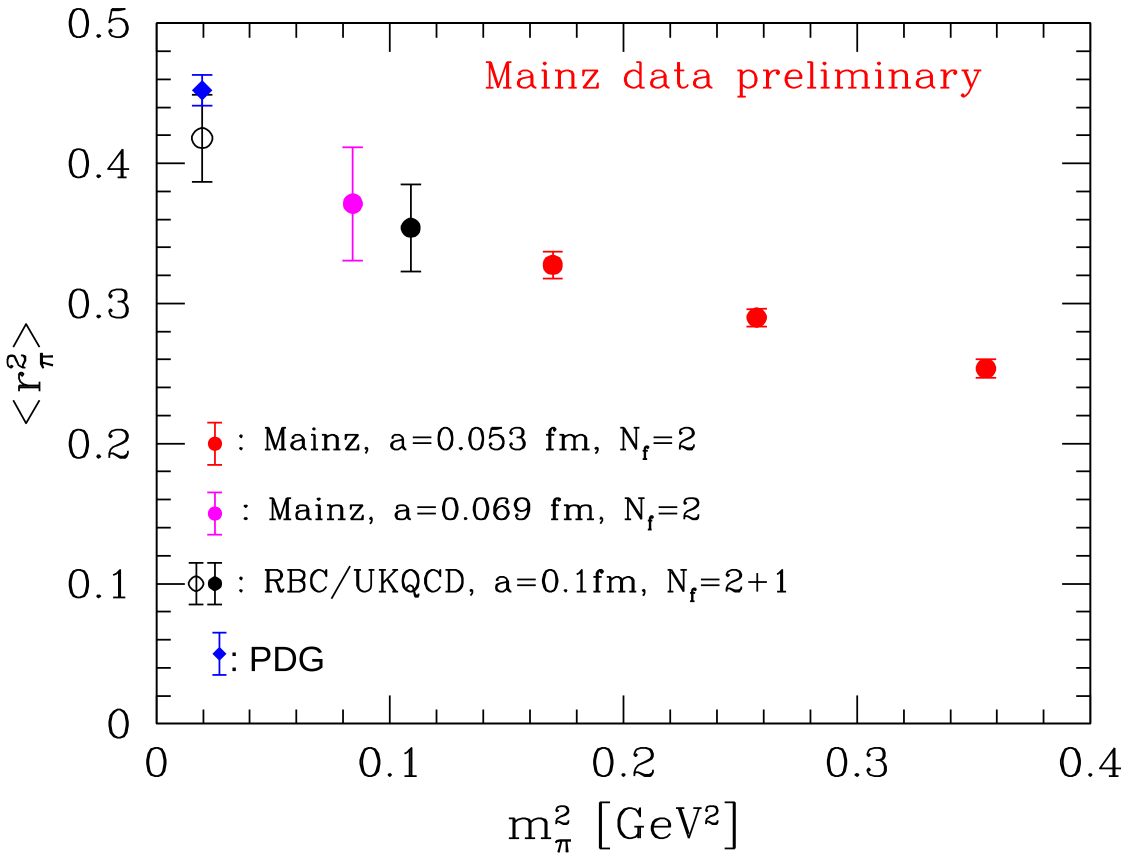

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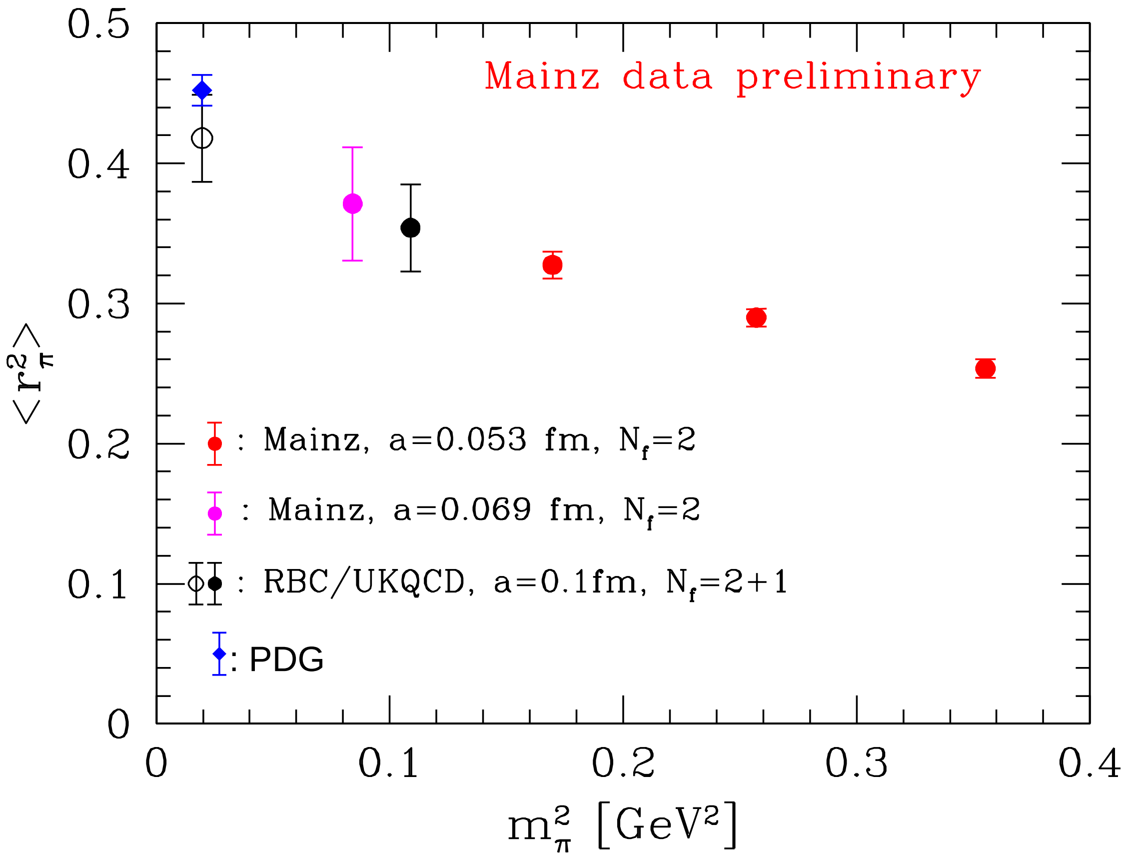


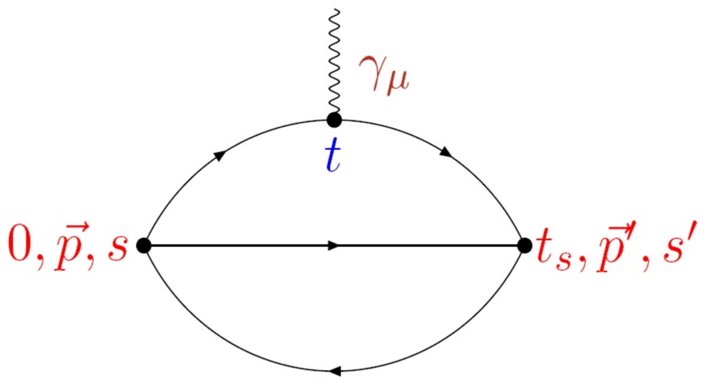
- Still to come: Fits to ChPT including vector degrees of freedom

3. Form factors and axial charge of the nucleon

- Dirac and Pauli form factors

$$\langle N(p',s')\,|V_\mu(x)|\,N(p,s)\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2m_{\rm N}}F_2(q^2)\right]u(p,s)$$

$$G_{\rm E}(q^2) = F_1(q^2) - \frac{q^2}{(2m_{\rm N})^2}F_2(q^2), \qquad G_{\rm M}(q^2) = F_1(q^2) + F_2(q^2)$$



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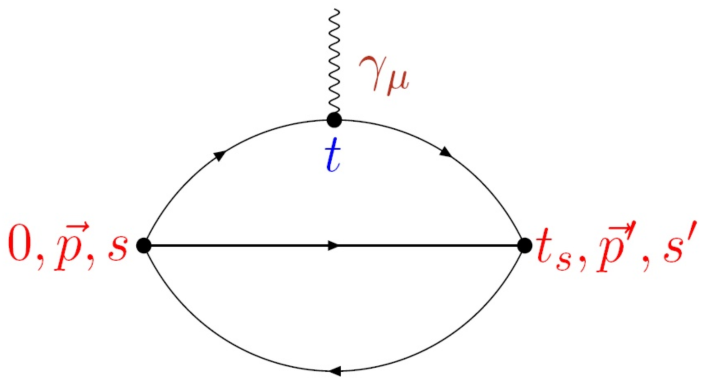


$\sim \langle N(p',s')\,|V_\mu(x)|\,N(p,s)\rangle$

- Disconnected diagrams contribute to iso-scalar form factors

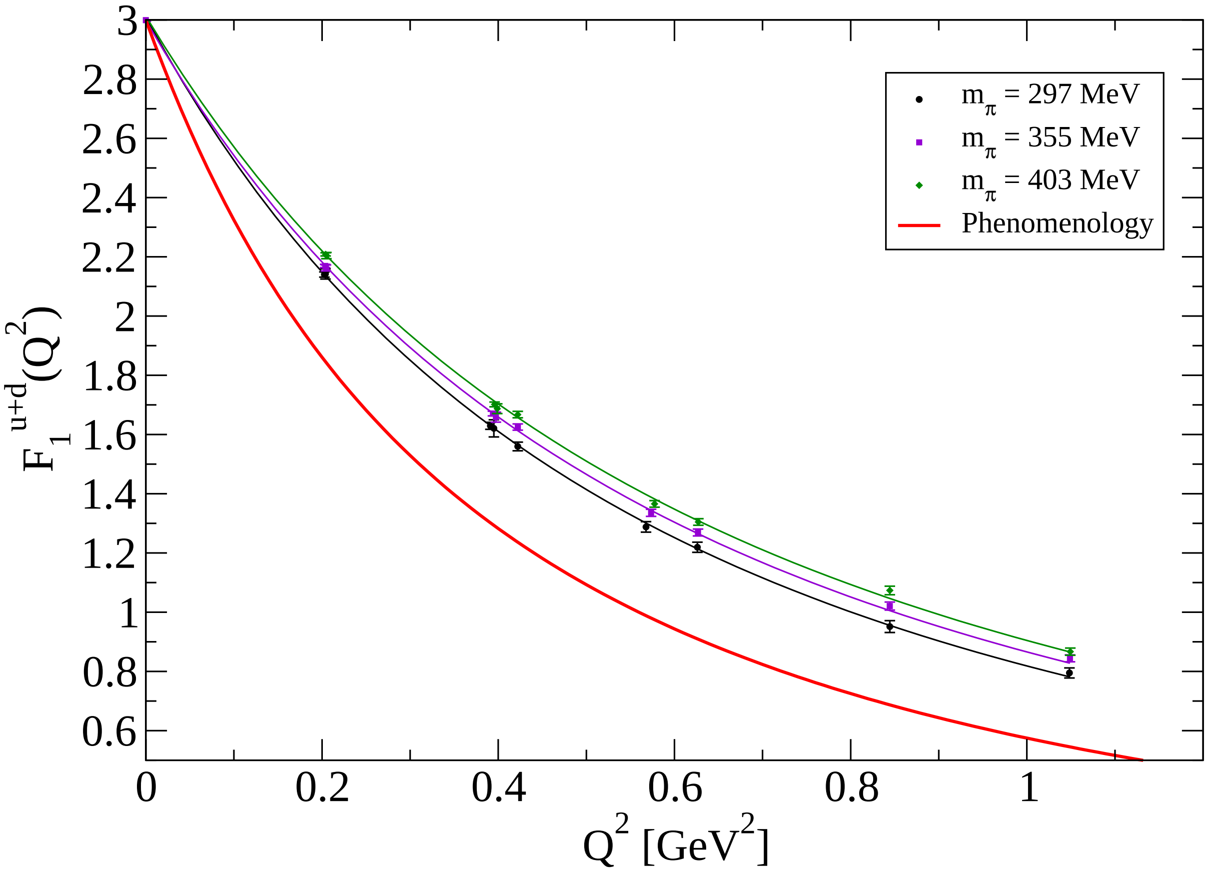
- Twisted boundary conditions: may incur large finite-volume effects?

Current Status

[Dru Renner @ Lattice 2009, Dina Alexandrou @ Lattice 2010]

- Form factors: experimental Q^2-dependence & charge radii not reproduced

[LHPC (Syritsin et al.), arXiv:0907.4194]



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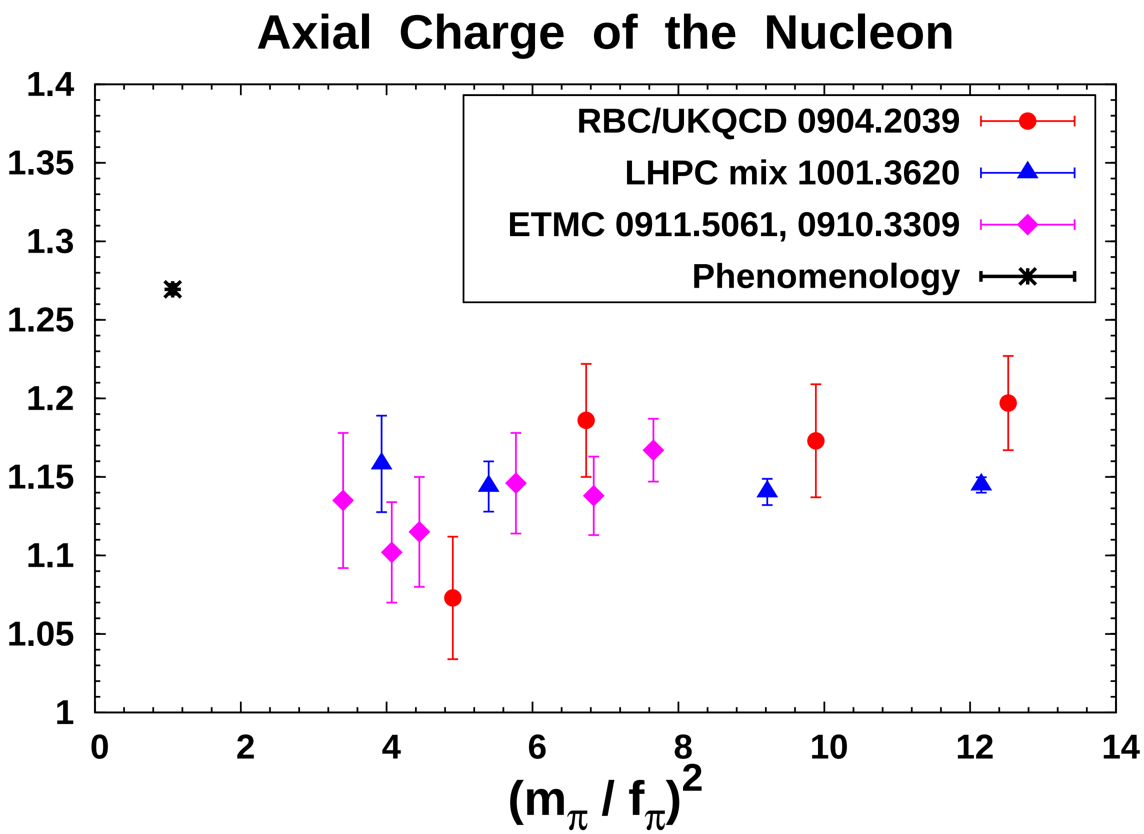

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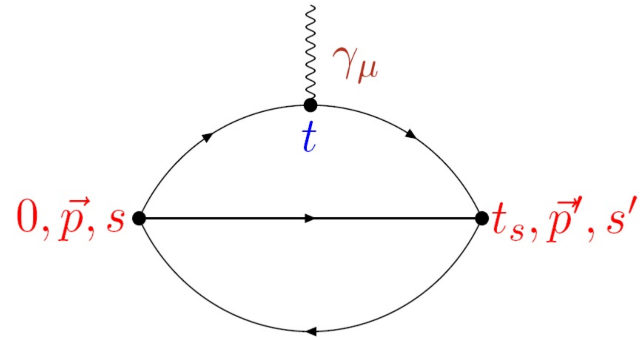
- Possible origin:
  - Lattice artefacts
  - Chiral extrapolations (pion masses too large)
  - Finite-volume effects
  - Contamination from excited states

Standard method

- Extract nucleon form factors from ratios of three- and two-point functions:



$$R(\vec{q}; t, t_s) = \frac{C_3(\vec{q}, t, t_s)}{C_2(\vec{0}, t_s)} \cdot \left\{ \frac{C_2(\vec{q}, t_s - t)\, C_2(\vec{0}, t)\, C_2(\vec{0}, t_s)}{C_2(\vec{0}, t_s - t)\, C_2(\vec{q}, t)\, C_2(\vec{q}, t_s)} \right\}^{1/2} \propto G_{\mathrm{E}}(q^2),\ G_{\mathrm{M}}(q^2)$$

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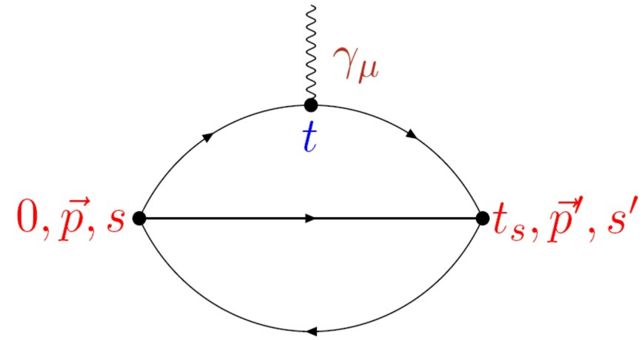

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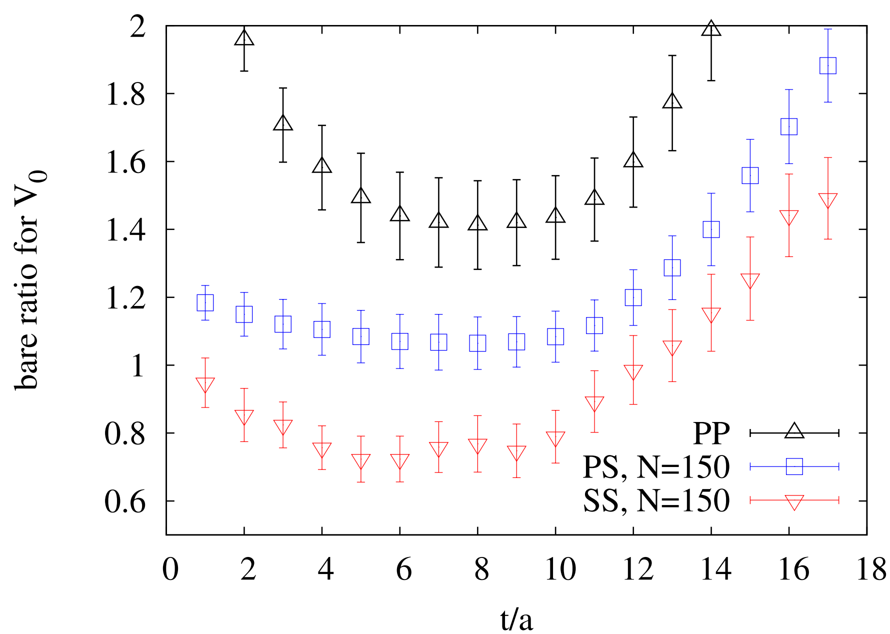
- $\beta = 5.3, \quad 32^3 \cdot 64, \quad a = 0.069\,\mathrm{fm}$
- $R(\vec{q};t,t_s)$, iso-scalar V_0 (connected) at $Q^2 = 0.87\,\mathrm{GeV}$, $t_s = 18$
- Several source/sink combinations



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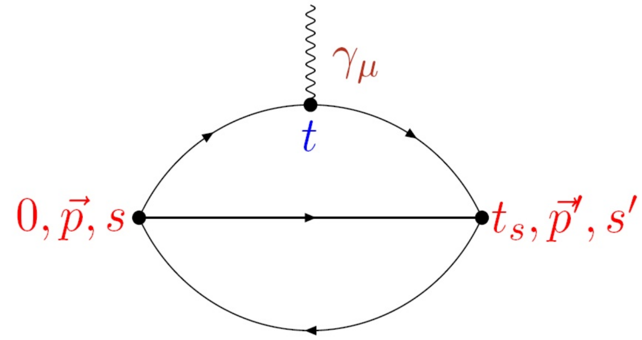

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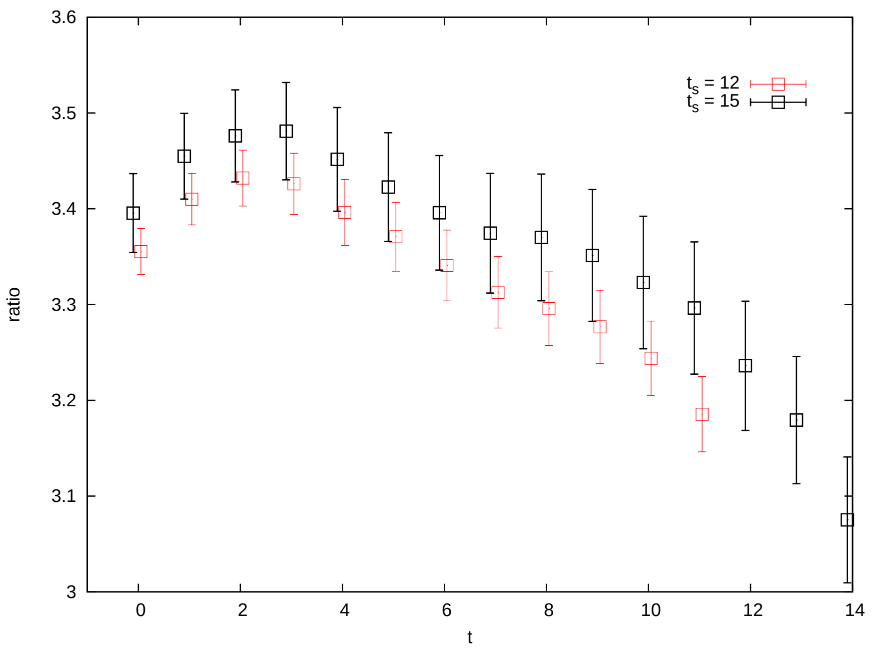
- Smeared-local correlator



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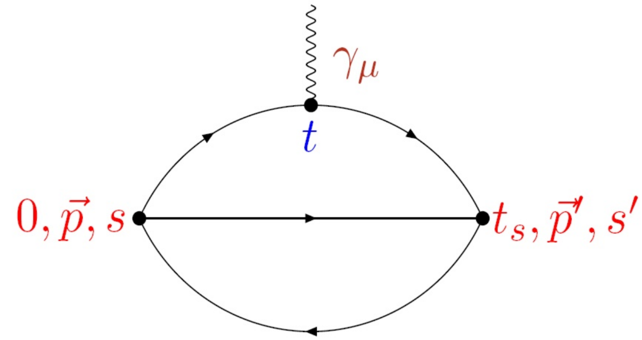

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- $\beta = 5.3, \quad 32^3 \cdot 64, \quad a = 0.069\,\mathrm{fm}$
- Iso-vector axial charge at $m_\pi = 550\,\mathrm{MeV}$, $t_s = 12, 15$
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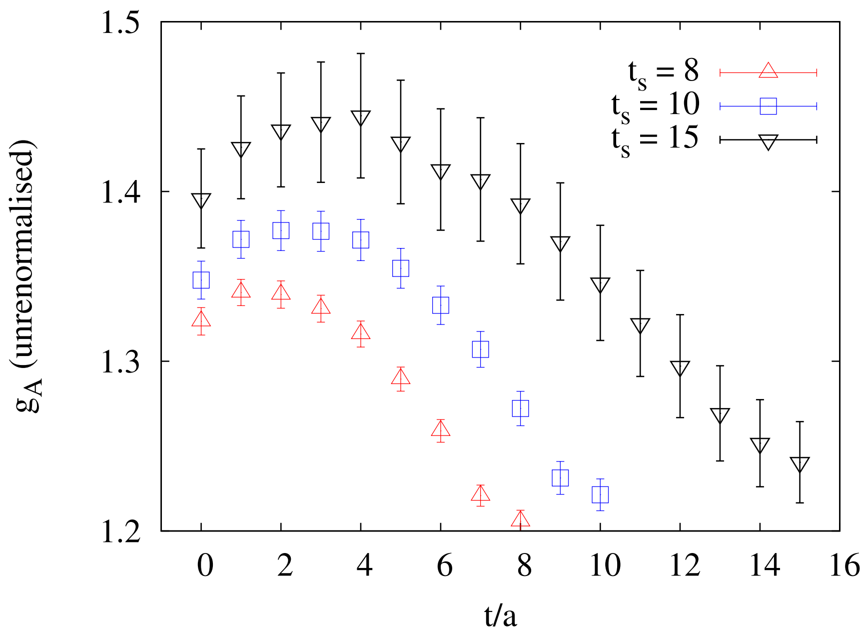

Summed insertions

[Maiani et al. 1987; B. Knippschild @ Lattice2010]

- Standard method:

$$R(\vec{q},t,t_s) = R_{\mathrm{G}}(\vec{q}) + \mathrm{O}(\mathrm{e}^{-\Delta t}) + \mathrm{O}(\mathrm{e}^{-\Delta'(t_s-t)})$$

- Summed insertion:

$$\sum_{t=0}^{t_s} R(\vec{q},t,t_s) = R_{\mathrm{G}}(\vec{q}) \cdot t_s + K(\Delta,\Delta') + \mathrm{O}(\mathrm{e}^{-\Delta t_s}) + \mathrm{O}(\mathrm{e}^{-\Delta' t_s})$$

- Excited state contributions more strongly suppressed

- Determine $R_{\mathrm{G}}(\vec{q})$ from linear slope of summed ratio

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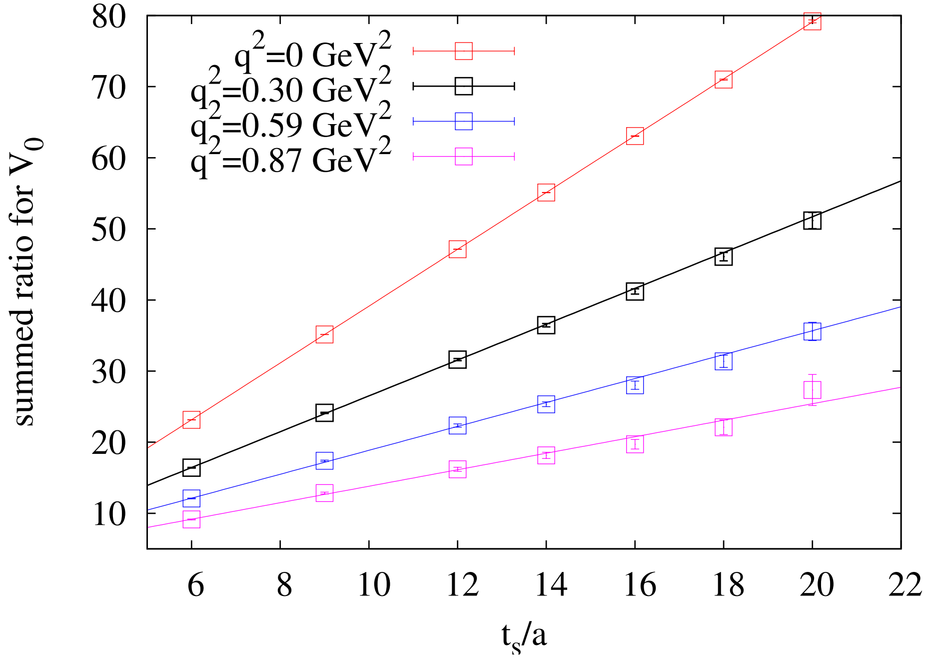
- Connected Iso-scalar form factor

- Smeared-local correlators



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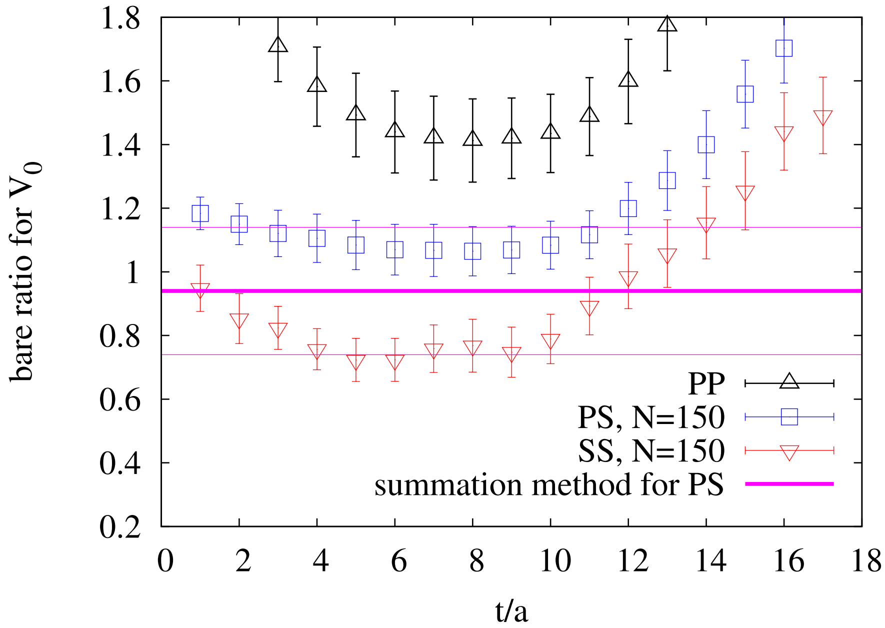

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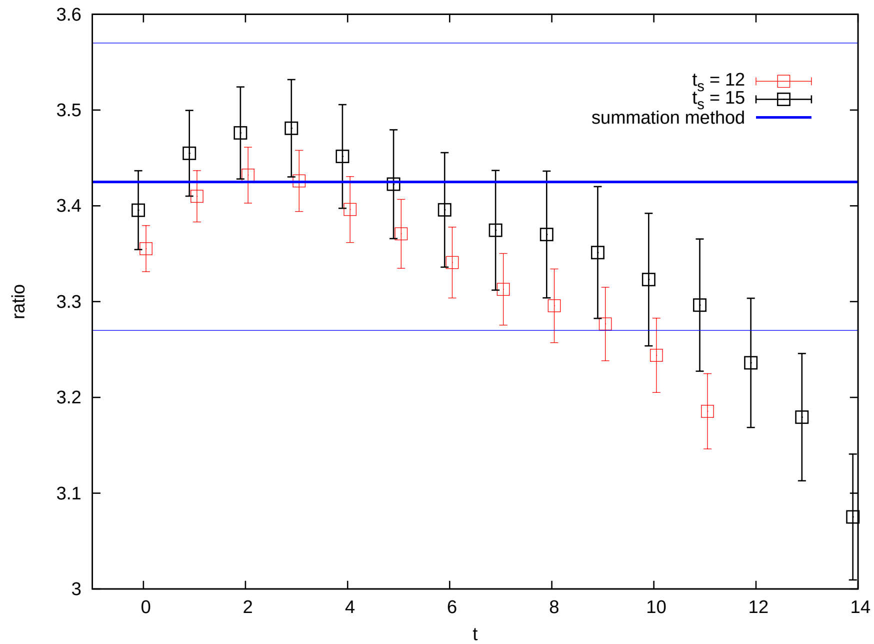

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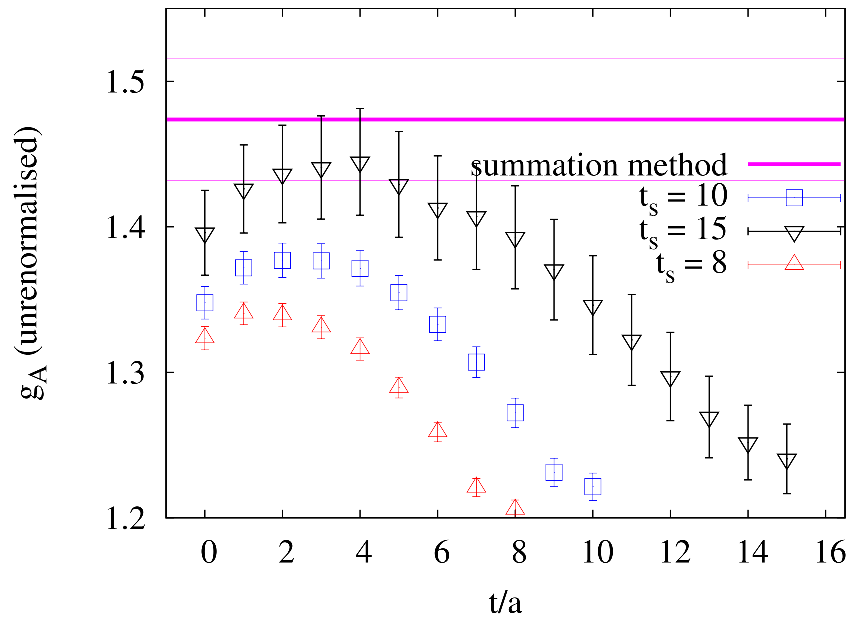

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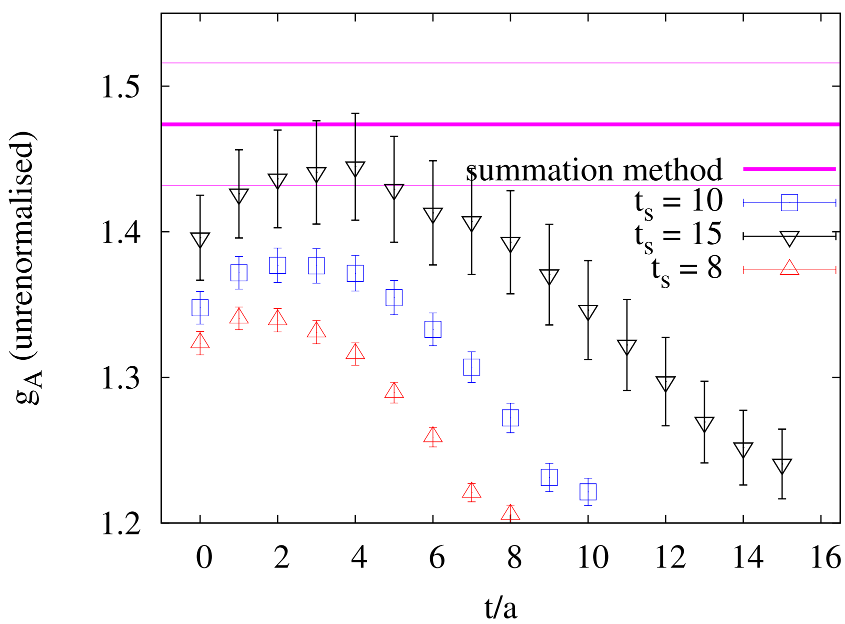
  $m_\pi = 550\,\mathrm{MeV}$, $t_s = 12, 15$

- Smeared-local correlator



- Better control over excited state contamination

- Larger statistical errors

- Requires more values of t_s

Results

[Capitani, Della Morte, Jüttner, Knippschild, Meyer, H.W.]

- Ensembles at $\beta = 5.3, \quad a = 0.069\,\mathrm{fm}, \quad 32^3{\cdot}64$ and $48^3{\cdot}96$

  $m_\pi = 290 - 590\,\mathrm{MeV}, \quad L = 2.2$ and $3.3\,\mathrm{fm}, \quad Lm_\pi \gtrsim 5$

  Employ summed insertions for ≈ 6 values of t_s

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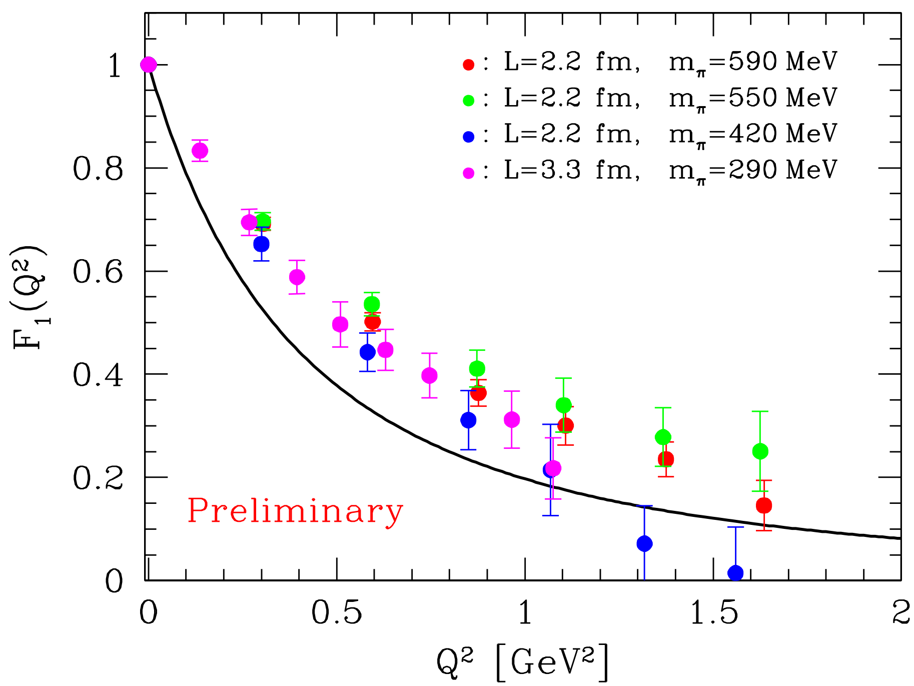
  Employ summed insertions for ≈ 6 values of t_s

- Dirac form factor: *[Knippschild @ Lattice 2010]*



Results

[Capitani, Della Morte, Jüttner, Knippschild, Meyer, H.W.]

- Ensembles at $\beta = 5.3$, $a = 0.069\,\mathrm{fm}$, $32^3{\cdot}64$ and $48^3{\cdot}96$

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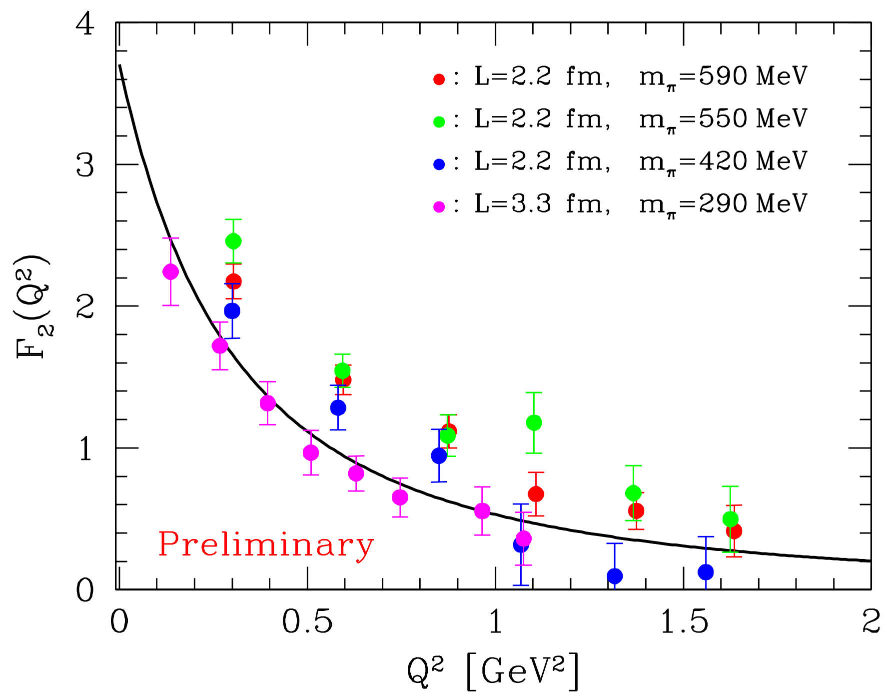
  Employ summed insertions for ≈ 6 values of t_s

- Pauli form factor: *[Knippschild @ Lattice 2010]*



Results

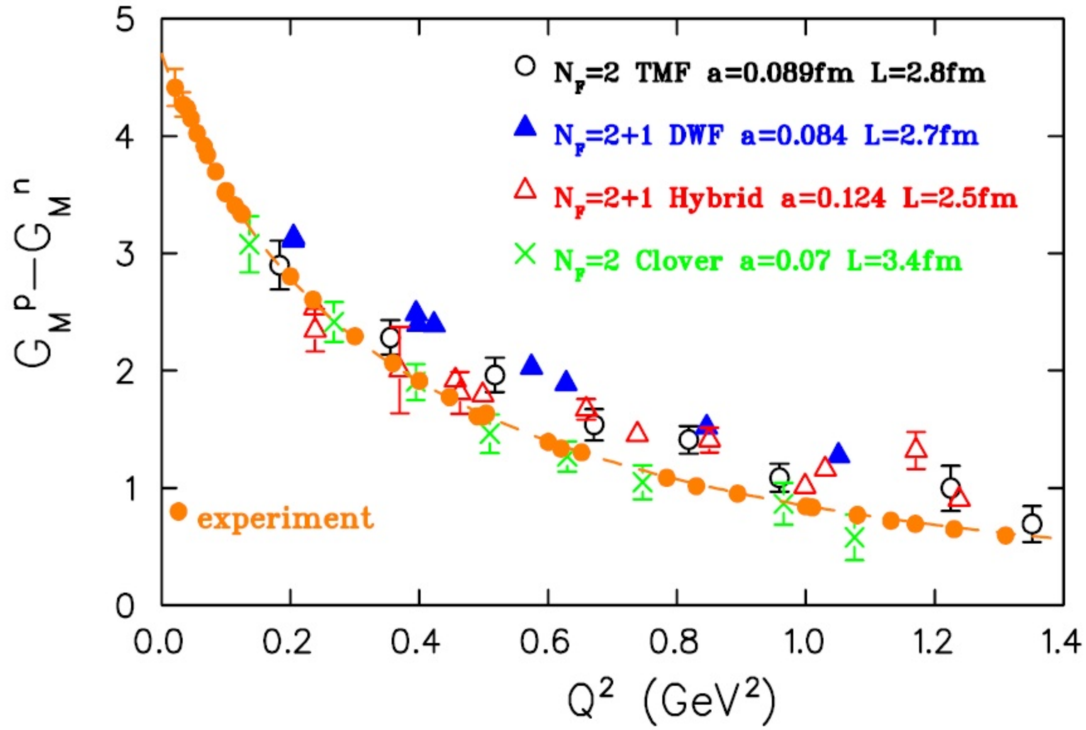
[Capitani, Della Morte, Jüttner, Knippschild, Meyer, H.W.]

- Ensembles at $\beta = 5.3,\quad a = 0.069\,\mathrm{fm},\quad 32^3{\cdot}64$ and $48^3{\cdot}96$

  $m_\pi = 290 - 590\,\mathrm{MeV},\quad L = 2.2$ and $3.3\,\mathrm{fm},\quad Lm_\pi \gtrsim 5$

  Employ summed insertions for ≈ 6 values of t_s

- Comparison: magnetic form factor @ $m_\pi \simeq 300\,\mathrm{MeV}$ *[D. Alexandrou @ Lattice 2010]*



Results

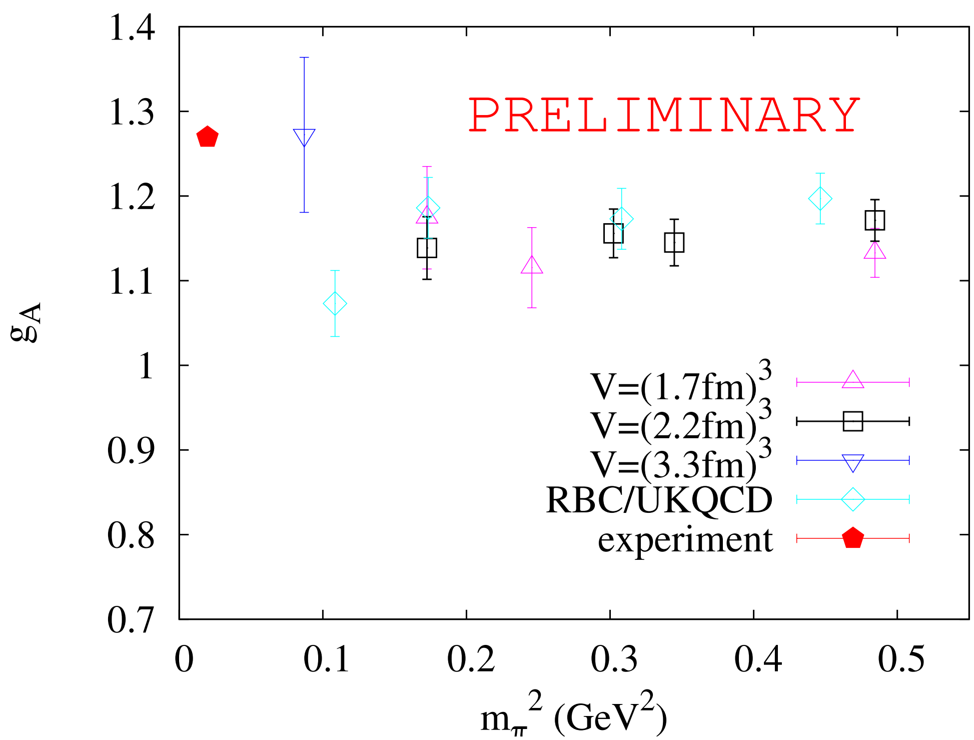
[Capitani, Della Morte, Jüttner, Knippschild, Meyer, H.W.]

- Ensembles at $\beta = 5.3,\quad a = 0.069\,\mathrm{fm},\quad 32^3{\cdot}64$ and $48^3{\cdot}96$

  $m_\pi = 290 - 590\,\mathrm{MeV},\quad L = 2.2$ and $3.3\,\mathrm{fm},\quad Lm_\pi \gtrsim 5$

  Employ summed insertions for ≈ 6 values of t_s

- Axial charge: *[Knippschild @ Lattice 2010]*



Results

[Capitani, Della Morte, Jüttner, Knippschild, Meyer, H.W.]

- Ensembles at $\beta = 5.3, \quad a = 0.069\,\mathrm{fm}, \quad 32^3{\cdot}64$ and $48^3{\cdot}96$

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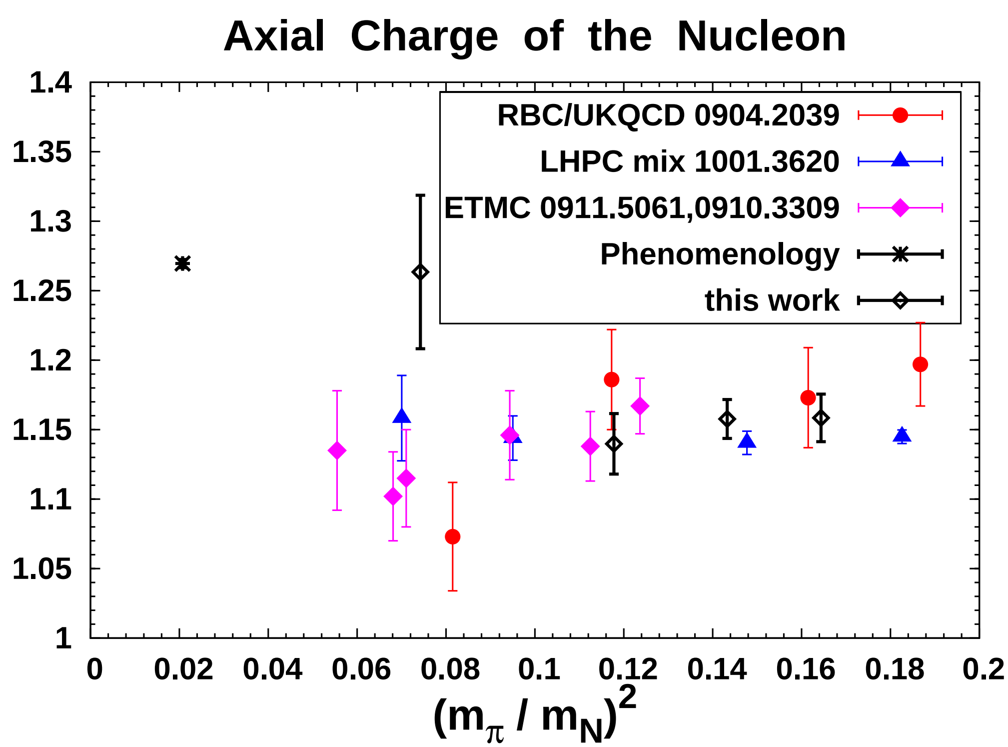
  Employ summed insertions for ≈ 6 values of t_s

- Axial charge: *[Knippschild @ Lattice 2010]*



Still to come. . .

- Process CLS ensembles at $\beta = 5.2,\ a = 0.08\,\mathrm{fm}$ and $\beta = 5.5,\ a = 0.05\,\mathrm{fm}$
  $\longrightarrow$ Lattice artefacts

- Comparison of $L \simeq 2.6\,\mathrm{fm}$ and $L \simeq 3.8\,\mathrm{fm}$ at $\beta = 5.2$

Summary

- Nucleon form factors and axial charge:
  "2^{nd} Generation Benchmark" for lattice QCD

- Progress in controlling systematic uncertainties in form factor calculations using CLS ensembles

- Pion form factor:
  - precise, model-independent estimates of $\langle r_\pi^2 \rangle$ via twisted boundary conditions

- Nucleon form factors and g_{A} :
  - summed insertions help control excited state contamination
  - situation still not settled (pion masses, volumes, discretisation effects)