時間における情報理論的Bell不等式と量子計算

Information-theoretic temporal Bell inequality and quantum computation

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#### **Motivation**

Separation between classical and quantum phenomena ~ crucial for understanding the weirdness of quantum physics

Rigid distinction between them by Bell-type inequalities

Bell inequality: for spatial correlation

J. S. Bell, *Physics* **1**,195 (1964).

Leggett-Garg inequality: for temporal correlation

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985).

*Information-theoretic temporal Bell inequality* ~ discrimination between classical and quantum computations

Another motivation:

Explore temporal correlations in quantum information processing

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# Outline

- 1. Information-theoretic (spatial) Bell inequalities
- 2. "Database" search problem
- 3. Information-theoretic temporal Bell inequality
- 4. Violation of the inequality
- 5. Temporal non-locality in quantum computation
- 6. Summary

#### Information-theoretic (spatial) Bell inequalities

S. L. Braunstein and C. M. Caves, Phys. Rev. Lett. 61, 662 (1988).



<u>Classically, there exists a joint probability distribution of</u> <u>measurement outcomes in all possible measurement settings.</u>

 $H(A \mid B) \leq H(A \mid B') + H(B' \mid A') + H(A' \mid B)$ ~ Information-theoretic Bell inequality

However, this inequality can be violated in quantum theory! H(A | B) > H(A | B') + H(B' | A') + H(A' | B)

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#### "Database" search problem

Given a black box (oracle) that calculates the function f such that

$$f(x) = \begin{cases} 0 & (x \neq s) \\ 1 & (x = s) \end{cases} \qquad \begin{pmatrix} x = 0, \dots, 2^n - 1 \\ s : \text{unknown} \end{pmatrix},$$

find the unknown number s by asking the oracle as few times as possible.



# Information-theoretic temporal Bell inequality for classical computation



Consider an algorithm solving the problem in L steps with certainty.

The amo

information

$$n \text{ bits} \leq H(A_0, \dots, A_L) = \sum_{i=1}^{L} H(A_i \mid A_{i-1}, \dots, A_0) + \frac{H(A_0)}{= 0}$$
  
The amount of  
information obtained  
by solving the problem  
$$\leq H(A_L \mid A_{L-1}) + \dots + H(A_1 \mid A_0)$$

Information-theoretic temporal Bell inequality  $n \leq H(A_{I} | A_{I-1}) + \dots + H(A_{I} | A_{0})$ 

#### Some properties of conditional entropy

(i) Chain rule for conditional entropy:

 $H(A_1, A_2) = H(A_2 | A_1) + H(A_1)$  $H(A_1, A_2, A_3) = H(A_3 | A_2, A_1) + H(A_2 | A_1) + H(A_1)$ 

(ii) Conditioning never increases entropy:



# "Experiment" to show the violation of the inequality

1. Prepare many quantum computers and run Grover's algorithm independently.



2. Randomly choose two successive oracle queries (Grover iterations) for each computer and measure only the output registers to calculate  $H(A_{k+1} | A_k)$ .



(N.B.) Classically, the same "experiment" does not violate the inequality.

# Violation of the inequality in quantum computation



#### In the k-th Grover iteration:



#### **Temporal non-locality in quantum computation**

#### **Conventional view**

Parallel computation due to the superposition of all possibilities ~ Quantum parallelism



# Spatial non-locality due to entanglement is crucial.

#### Another view

A quantum computer seems to select promising candidates within  $O(\sqrt{2^n})$  steps as if it referred to *unperformed* queries!



"Temporal non-locality" in quantum theory is essential.

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# Summary

- Information-theoretic temporal Bell inequality for classical algorithms solving the "database" search problem
- Violation of the inequality in Grover's algorithm for quantum computation

- Temporal non-locality in quantum computation
  - ~ Quantum computer looks as if it exploited unperformed computation as well as performed one.