量子ゲーム理論について

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Plan

I. classical game theory

strategy of players, payoff, Nash equilibrium (NE),
Prisoners Dilemma, Battle of the Sexes

II. quantum game theory

strategy in Hilbert space, entangled strategy,
classical limit, quantum NE, phase structure

III. quantum vs classical

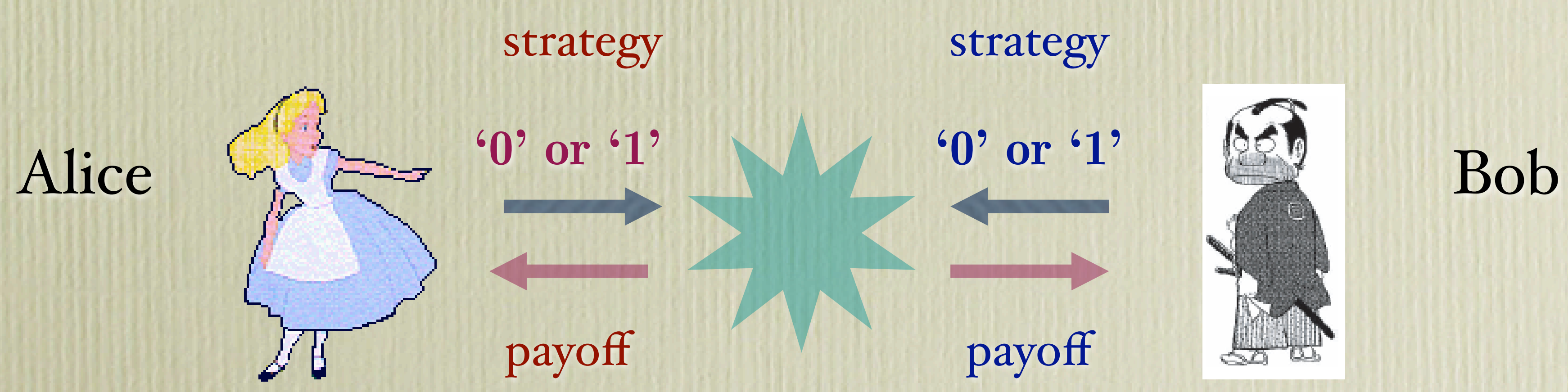
resolution of dilemmas? quantization schemes,
roles of referee, further development

I. classical game theory

m players try to optimize their payoffs by making their choices in n strategies properly

von Neumann - Morgenstein (1947)

for m = 2, n = 2 case



Prisoner's Dilemma

Alice \ Bob	Deny	Confess
Deny	(3, 3)	(0, 5)
Confess	(5, 0)	(1, 1)

Pareto optimal

Nash equilibrium (NE)

NE is not the best choice (Pareto Optimal)
for the two players

→ dilemma

Battle of the Sexes

Bob Alice	Opera	Football
Opera	(2, 1)	(0, 0)
Football	(0, 0)	(1, 2)

two Nash equilibria

players cannot decide on the choice Opera/Football

$\longrightarrow$ dilemma

Stag Hunt

Alice \ Bob	Stag	Hare
Stag	(4, 4)	(0, 3)
Hare	(3, 0)	(3, 3)

Pareto optimal NE

risk dominant NE

players cannot decide on the choice Stag/Hare

→ dilemma

general payoff table

Alice \ Bob	S_1	S_2
S_1	(a, b)	(e, f)
S_2	(c, d)	(g, h)

strategy

payoff matrices

Alice: $A = \begin{pmatrix} a & e \\ c & g \end{pmatrix}$

Bob: $B = \begin{pmatrix} b & f \\ d & h \end{pmatrix}$

classical game theory is based on

- bit ('0' or '1') or their mixture (mixed strategies)
- payoffs are specified by matrices $A_{ij},\ B_{ij}$

useful for analyzing decision making processes
in economics, sociology, ...

what if the players can resort to qubit strategies?

$\longrightarrow$ 'quantization' of game

Eisert-Wilkens-Lowenstein (1999), Meyer (1999)

resolution of the Prisoner's Dilemma!

II. quantum game theory

strategic space : Hilbert space $\quad \mathcal{H} = \mathcal{H}_A \times \mathcal{H}_B$

Alice's strategy $\quad |\alpha\rangle_A \in \mathcal{H}_A$

Bob's strategy $\quad |\beta\rangle_B \in \mathcal{H}_B$

total 'strategy' $\quad |\alpha, \beta; \gamma\rangle = J(\gamma)\, |\alpha\rangle_A \, |\beta\rangle_B$

Cheon-Tsutsui (2005)

quantum correlation (entanglement) by referee

payoff: self-adjoint operators A and B

Alice's payoff	$\Pi_A(\alpha,\beta;\gamma) = \langle\alpha,\beta;\gamma\|A\|\alpha,\beta;\gamma\rangle$
Bob's payoff	$\Pi_B(\alpha,\beta;\gamma) = \langle\alpha,\beta;\gamma\|B\|\alpha,\beta;\gamma\rangle$

expectation values

quantum Nash equilibrium (QNE) $|\alpha^\star,\beta^\star\rangle$

$$\Pi_A(\alpha^\star,\beta^\star;\gamma) \geq \Pi_A(\alpha,\beta^\star;\gamma)$$

$$\Pi_B(\alpha^\star,\beta^\star;\gamma) \geq \Pi_B(\alpha^\star,\beta;\gamma)$$

each player cannot improve their payoff by deviating from the QNE unilaterally

2 player 2 strategy quantum games

$$\dim \mathcal{H}_A = \dim \mathcal{H}_B = n = 2$$

strategy as qubit

$$|\alpha\rangle_A = \alpha_0|0\rangle_A + \alpha_1|1\rangle_A \qquad |\alpha_0|^2 + |\alpha_1|^2 = 1$$

$$|\beta\rangle_B = \beta_0|0\rangle_B + \beta_1|1\rangle_B \qquad |\beta_0|^2 + |\beta_1|^2 = 1$$

probabilistic distribution and interference is now possible!

quantum correlation

$$\boxed{J(\gamma) = e^{i\frac{\gamma_1}{2}S}\, e^{i\frac{\gamma_2}{2}T}}$$

three convenient operations

Swap $S\,|i,j\rangle = |j,i\rangle$ $\quad |i,j\rangle = |i\rangle_A\,|j\rangle_B$

swap of strategies between two players

$$S\,|\alpha,\beta\rangle = |\beta,\alpha\rangle$$
$$|\alpha,\beta\rangle = |\alpha\rangle_A\,|\beta\rangle_B$$

Conversion $C\,|i,j\rangle = |\bar{i},\bar{j}\rangle$ $\quad \bar{i} = 1-i, \quad i = 0,\,1$

conversion of strategies of each player

$$|\bar{\alpha}\rangle_A = \sum_i \alpha_i\,|1-i\rangle_A\,, \quad |\bar{\beta}\rangle_B = \sum_j \beta_j\,|1-j\rangle_B$$

Twist $T\,|i,j\rangle = |\bar{j},\bar{i}\rangle$

combination $T = SC$

algebra $S^2 = C^2 = T^2 = I \qquad S = CT$ and $C = TS$

$$[S,T] = ST - TS = 0$$

classical limit

if $[A, B] = 0$ choose the basis so that

$$\langle i', j' | A | i, j \rangle = A_{ij} \delta_{i'i} \delta_{j'j},$$
$$\langle i', j' | B | i, j \rangle = B_{ij} \delta_{i'i} \delta_{j'j}.$$

then at $\gamma = 0$ (classical limit) we have

$$\Pi_A(\alpha, \beta; 0) = \langle \alpha, \beta; 0 | A | \alpha, \beta; 0 \rangle = \sum_{i,j} x_i A_{ij} y_j$$
$$\Pi_B(\alpha, \beta; 0) = \langle \alpha, \beta; 0 | B | \alpha, \beta; 0 \rangle = \sum_{i,j} x_i B_{ij} y_j$$

reproduce payoff under classical mixed strategies

$x_i = |\alpha_i|^2$ represents probability to choose $|i\rangle_A$

$y_j = |\beta_j|^2$ represents probability to choose $|j\rangle_B$

classical game is contained in quantum game

symmetric games

- S-symmetric $\Pi_B(\beta, \alpha; \gamma) = \Pi_A(\alpha, \beta; \gamma)$

ensured if $B = S\,A\,S.$

game is ‘fair’

examples: Prisoner’s Dilemma, Stag Hunt

- T-symmetric $\Pi_B(\bar{\beta}, \bar{\alpha}; \gamma) = \Pi_A(\alpha, \beta; \gamma)$

ensured if $B = T\,A\,T.$

game is ‘anti-fair’

example: Battle of the Sexes

correlated (effective) payoff operator

$$\mathcal{A}(\gamma) = J^\dagger(\gamma) A J(\gamma) \rightarrow \Pi_A(\alpha, \beta; \gamma) = \langle \alpha, \beta | \mathcal{A}(\gamma) | \alpha, \beta \rangle$$

meanings of quantum correlation

$$\mathcal{A}(\gamma) = \mathcal{A}^{\mathrm{pc}}(\gamma) + \mathcal{A}^{\mathrm{in}}(\gamma)$$

altruistic

converted

$$\mathcal{A}^{\mathrm{pc}}(\gamma) = \cos^2\frac{\gamma_1}{2} A + \left(\cos^2\frac{\gamma_2}{2} - \cos^2\frac{\gamma_1}{2}\right) SAS + \sin^2\frac{\gamma_2}{2} CAC,$$ pseudo-classical

$$\mathcal{A}^{\mathrm{in}}(\gamma) = \frac{i}{2}\sin\gamma_1 (AS - SA) + \frac{i}{2}\sin\gamma_2 (AT - TA).$$ interference

quantum Nash equilibria

'edge' solutions $|\alpha^\star, \beta^\star\rangle = |0,0\rangle\,,\ |1,1\rangle\,,\ |0,1\rangle\,,\ |1,0\rangle$

$\|\alpha^\star, \beta^\star\rangle$	$\|0,0\rangle$	$\|1,1\rangle$	$\|0,1\rangle$	$\|1,0\rangle$
Condition	$H_+ > 0$	$H_- > 0$	$H_+ < 0$	$H_+ < 0$
			$H_- < 0$	$H_- < 0$
$\mathrm{Max}(\Pi_A^\star)$	A_{00}	A_{00}	A_{11}	$A_{01}+A_{10}-A_{11}$
$\mathrm{Max}(\Pi_B^\star)$	A_{00}	A_{00}	$A_{01}+A_{10}-A_{11}$	A_{11}

$$H_\pm(\gamma) = \tau(A) \pm \left[G'_+(\gamma) + G'_-(\gamma)\right]$$

$$\tau(A) = A_{00} - A_{01} - A_{10} + A_{11}$$

$$G'_+(\gamma) = (A_{00} - A_{11})\cos\gamma_2 \qquad G'_-(\gamma) = (A_{01} - A_{10})\cos\gamma_1$$

quantum Prisoner's Dilemma

$A_{00} = 3, \quad A_{01} = 0$

$A_{10} = 5 \quad A_{11} = 1$

$(\gamma_1^\star, \gamma_2^\star) = (2 \arcsin \sqrt{\lambda}, 0)$

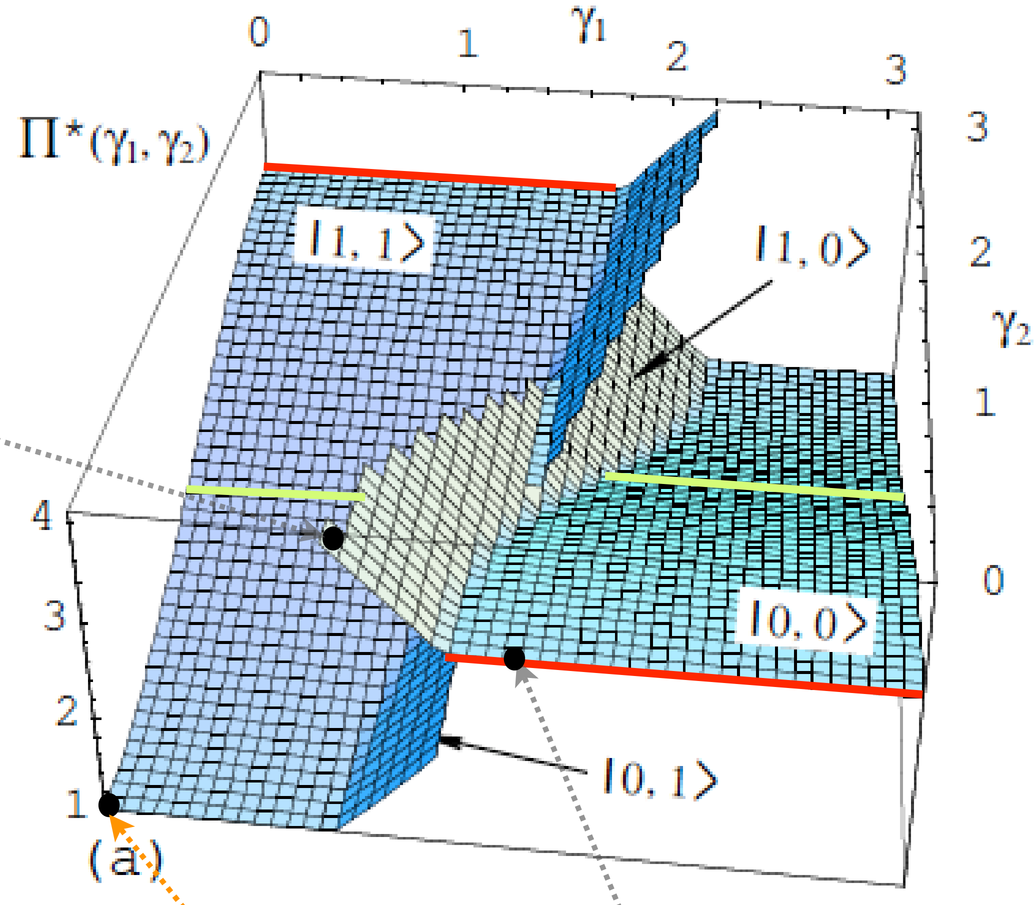
maximal payoff point

$\lambda = (A_{11} - A_{01})/(A_{10} - A_{01})$

but in another dilemma

entropy of entanglement

$S(\rho_{\rm red}) = -\lambda \log \lambda - (1 - \lambda) \log (1 - \lambda)$



$\gamma = 0$

classical PD

$(\gamma_1, \gamma_2) = (\pi/2, 0)$

QNE by Eisert et.al.

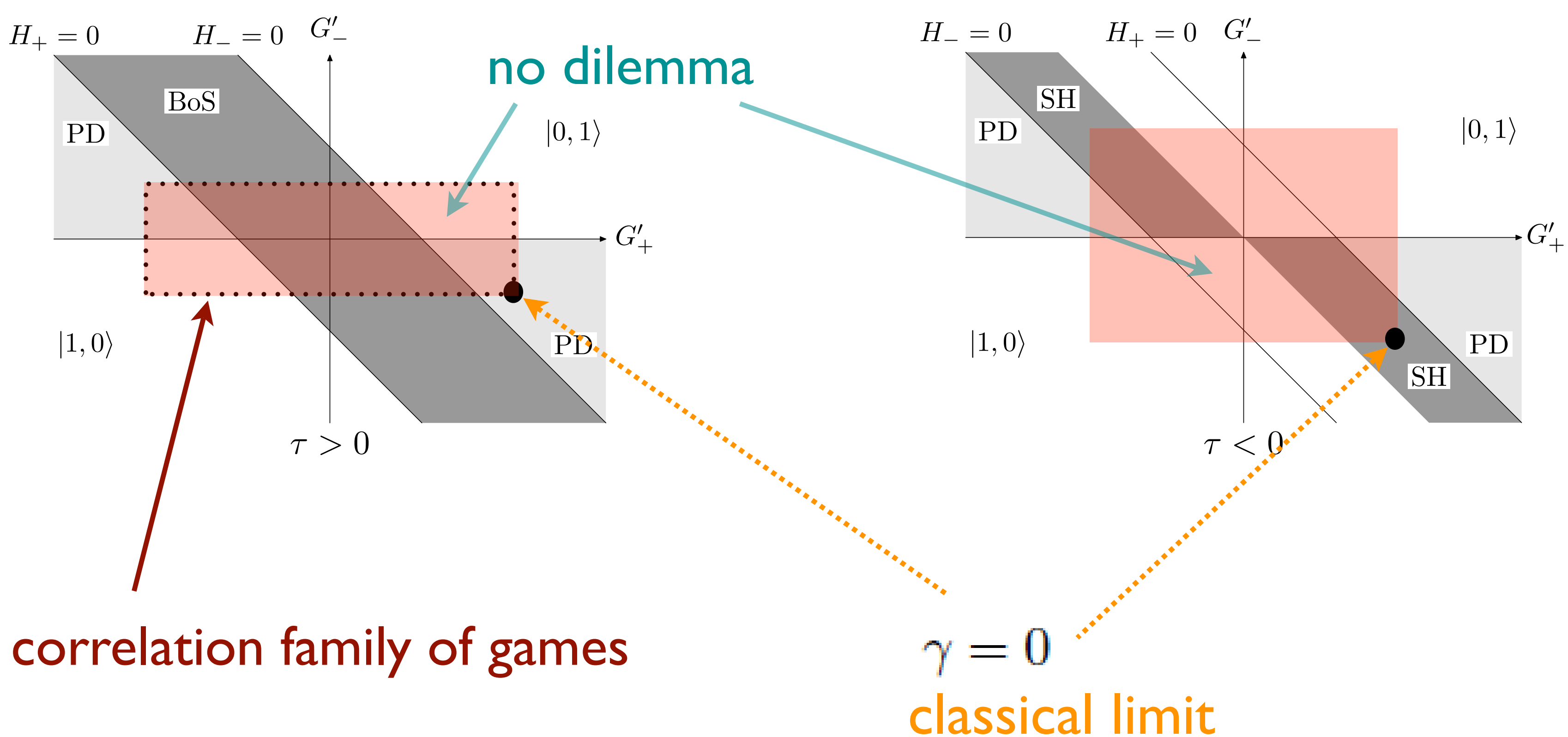
'phase structure' of quantum PD

phase structure of symmetric quantum games

Ichikawa-Tsutsui (2006)

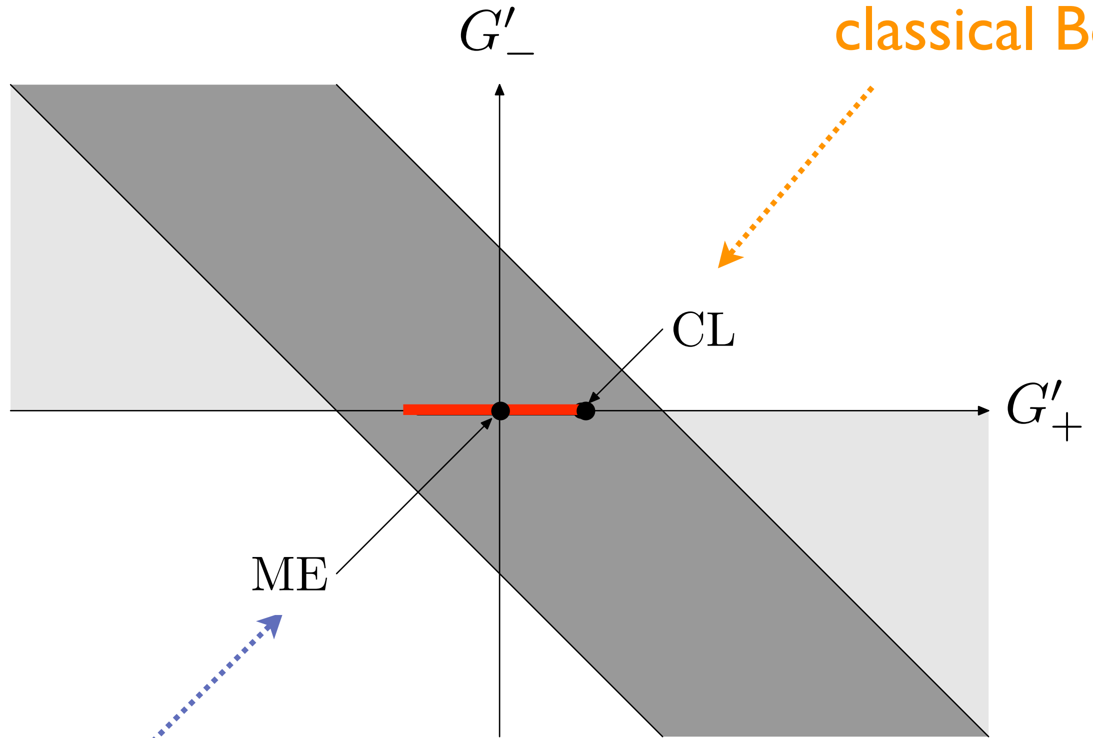
quantum PD

quantum SH



quantum BoS

classical BoS $\gamma = 0$



best possible improvement (Alice and Bob have the same payoff at QNE) at maximally entangled strategy

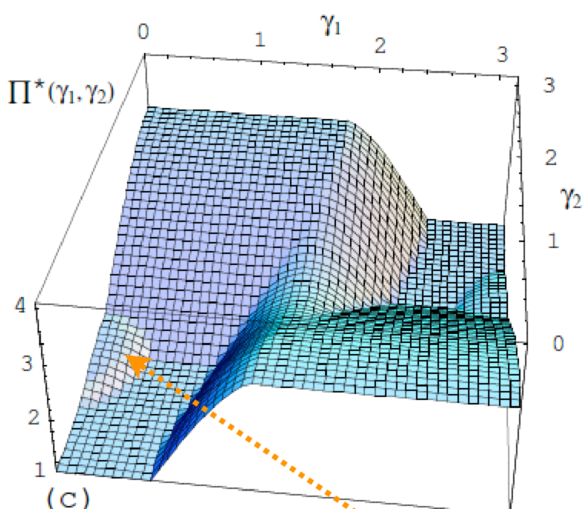
... but the dilemma remains

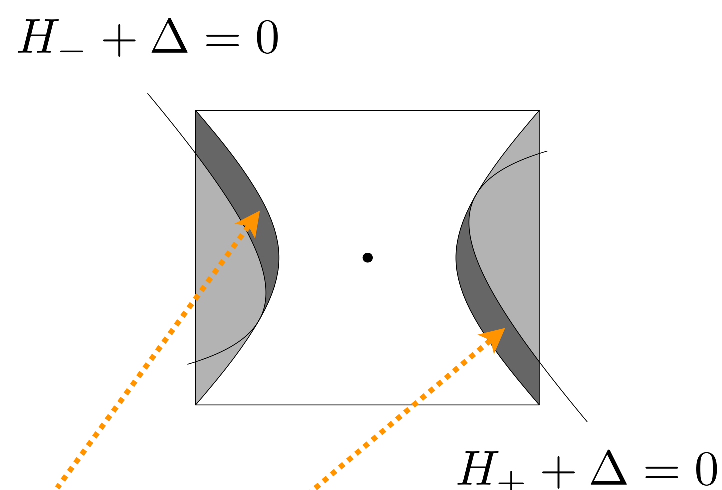
‘non-edge’ solutions $|\alpha^\star, \beta^\star\rangle$

exist depending on correlation γ and payoff A, B

(modified) quantum PD

quantum SH

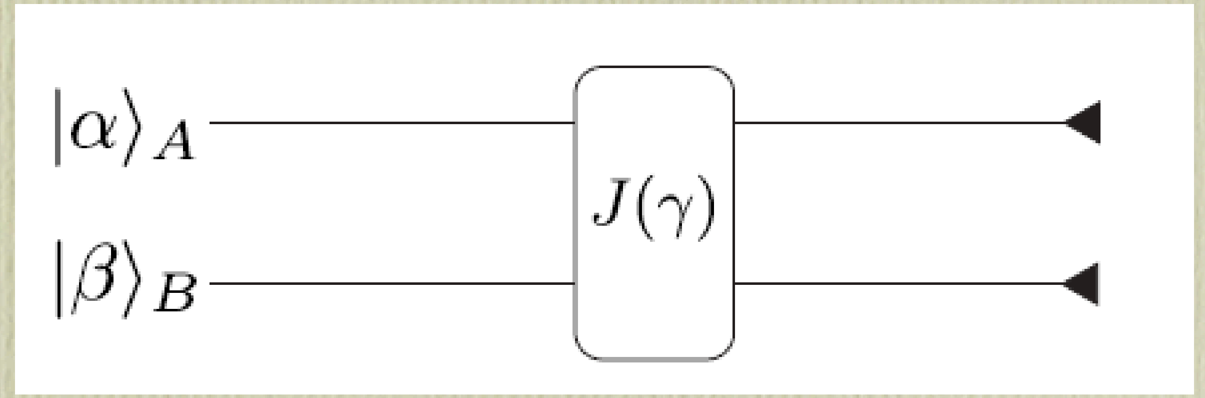




non-edge QNE

purely quantum

III. quantum vs classical



Cheon-Tsutsui (2005)

provides full deformation of CG

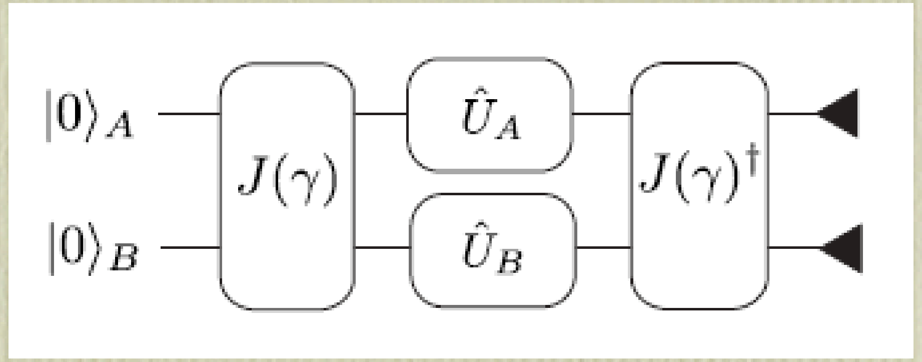
initial state　　correlation　　measurement

players　　referee

novelty of quantum game: defines a family of games

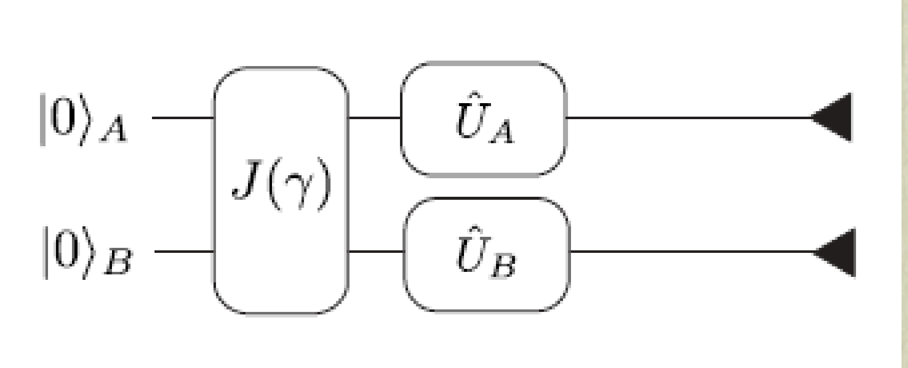
roles of referee: collaborative or probabilistic (noise)?

various different schemes



Eisert et.al. (1999)

provides partial extension of CG

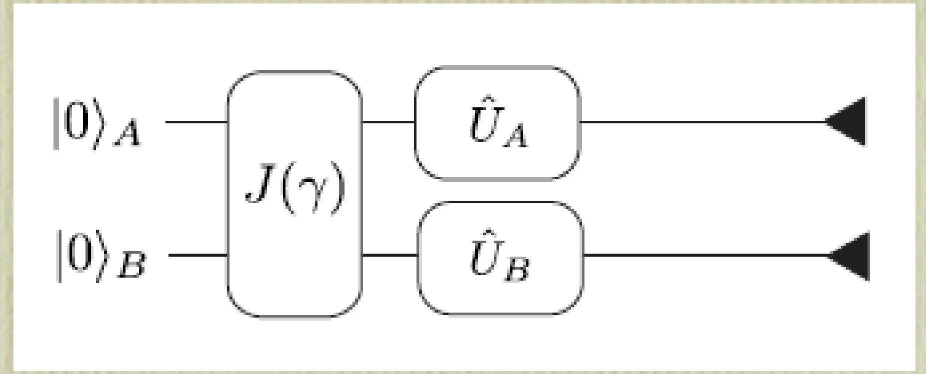


Marinatto-Weber (2000)

provides partial deformation of CG

- resolution of dilemmas: scheme-dependent
- phase structure of QNE: (basically) scheme-independent

Schmidt scheme



provides full deformation of CG

$$|\Psi\rangle = \sqrt{p}\,|\alpha\rangle_A|\beta\rangle_B + \sqrt{1-p}\,|\alpha'\rangle_A|\beta'\rangle_B,$$

$$= U_A \otimes U_B \underbrace{\left(\cos\gamma\,|0,0\rangle + \sin\gamma\,|1,1\rangle\right)}_{J(\gamma)\,|0,0\rangle}$$

Schmidt decomposition

- γ entanglement parameter
- analysis becomes extremely simple

→ Ichikawa's poster

implementation of quantum strategies

$$|\Psi\rangle = J(\delta) \cdot U_A(\alpha) \otimes U_B(\beta) \cdot J(\gamma)|\Psi_0\rangle \qquad |\Psi_0\rangle = |0\rangle_A|0\rangle_B$$

Scheme	**Correlation Op.**	**Players' Strategy**	**Merit**	**Demerit**
EWL	$J(\delta) = J^{\dagger}(\gamma)$	restricted unitary op.	-extended family	-unnatural $\mathcal{S}$
('99)	$J(\gamma) = e^{i\frac{\gamma}{2}D_2}$	$U_A, U_B \in \mathcal{S} \subset SU(2)$		-singular at $\gamma = \frac{\pi}{2}$
MW	$J(\delta) = I$	restricted mixed states	-clear entanglement	-small strategic space
('00)	$J(\gamma) = e^{i\frac{\gamma}{2}D_2}$	$(p, q) \in [0,1] \times [0,1]$		-deformed family
NT	$J(\delta) = I$	restricted mixed states	-clear/general entg.	-small strategic space
('04-a)	$J(\gamma) \in U(4)$	$(p, q) \in [0,1] \times [0,1]$		-deformed family
NT	$J(\delta) = e^{i\frac{\delta}{2}D_2}$	restricted unitary op.	-general: contains	-unnatural $\mathcal{S}$
('04-b)	$J(\gamma) = e^{i\frac{\gamma}{2}D_2}$	$U_A, U_B \in \mathcal{S} \subset SU(2)$	EWL and MW	-deformed family
CT	$J(\delta) = e^{i\frac{\delta_1}{2}S}e^{i\frac{\delta_2}{2}T}$	pure states	-full strategic space	-involved entanglement
('05)	$J(\gamma) = I$	$U_A, U_B \in SU(2)$		-deformed family
CIT	$J(\delta) = I$	pure states	-full strategic space	-deformed family
('06)	$J(\gamma) = e^{i\frac{\gamma_1}{2}X}e^{i\frac{\gamma_2}{2}D_2}$	$U_A, U_B \in SU(2)$	-clear/general entg.	

future directions

- firmer foundation: furnish a standard framework for QG based on some guiding principle (which is missing at present)
- generalization: study non-commutative case $[A, B] \neq 0$, establish m-player, n-strategy QG, evolutionary QG theory (introduction of dynamics), ..., as done in CG
- explore quantum mechanics: interpret Bell (CHSH) inequality, Kochen-Specker, etc. in terms of QG
- application: find systems where QG may be relevant (quantum information/communication, molecular evolution of species?)