- "fitting"
- adaptive MC integ'n
- quasi-optimal observables (FT, ACAT'02)

ACAT'03
KEK

POUZYRY:

MODELLING ③ A FUNCTION ① FROM A RANDOM SAMPLE ②

F. V. TKACHOV (MOSCOW)

POUZYR(Y) = BUBBLE(S)

French spelling
[Maison F.-J.]

Fundamental phenom. where's "physics of bubbles"

A recycling of an idea from
Optimal Jet Definition (F.T. '00)
and of an algorithm from
Optimal Jet Finder (see talk ...)

S Small random raindrops
Z can hardly hurt POUZYRY
S ... Oh! JOY OF RESEARCH!

Edmonton, 5 May 03

WHAT IS FUNCTION:

Conventional:

$f: x \to f(x)$ $\qquad f_n \to f \Leftrightarrow f_n(x) \to f(x)\ \forall x$

too limited: $\qquad \Theta(0) = ?$

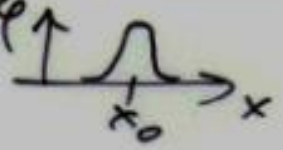
Modern:

$f: \varphi \to \langle f, \varphi \rangle = \text{“}\int f(x)\varphi(x)\,dx\text{”}$

$f_n \xrightarrow{W} f \Leftrightarrow \boxed{\langle f_n \varphi \rangle \to \langle f\varphi \rangle \quad \forall \varphi}$

better for many things... no time for philosophy

Equivalence for continuous:

φ around x_0: 

$\qquad \int \varphi = 1$

$\langle f\varphi \rangle \simeq f(x_0)$

$$\boxed{\{\langle f\varphi \rangle\}_\varphi \Leftrightarrow \{f(x)\}_x}$$

radius$(\varphi) \to 0$ to restore $f(x)$

PRINCIPLE OF ABSENCE OF MIRACLES

remember...

(Quasi) Random Sample:

$$\{x_n\}_{n=1}^{N} \overset{(?)}{\Longleftrightarrow} \mathcal{R}(x)$$

More generally

$$F_N \equiv \{x_n, f_n\}_{n=1}^{N} \overset{(?)}{\Longleftrightarrow} \mathcal{R}(x), f(x)$$

textbooks rarely discuss BUT ESSENTIAL STARTING POINT FOR ALL THINKING ...

$$F_N \equiv \frac{1}{N}\sum_n f_n \delta(x - x_n) \xrightarrow{W} f(x)\mathcal{R}(x)$$

Models "regularization"

$$\frac{1}{N}\sum_n f_n \delta(x - x_n) \xrightarrow[N\to\infty]{W} f(x)\mathcal{R}(x)$$

$$\frac{1}{N}\sum_n f_n \delta(x - x_n) \xrightarrow{W} M_{N,f}(x)$$

$\not W$? $\forall x$? L_2 etc.

(f>0)

A) $x \to M(x)$

B) $M \to$ random x

TYPES OF MODELS

1. Decompositional:

$$\mathcal{D}_{omain} = \bigcup_k \mathcal{D}_k \qquad \mathcal{D}_k \cap \mathcal{D}_{k'} = \emptyset$$

$$M(x)|_{\mathcal{D}_k} = \text{const}$$

adaptive MC

2. Galiorkin: $M(x) = \sum_k m_k P_k(x)$

histogramming: 1, 2

3. Parametric: $M(x, p)$, p found e.s. by fitting
must know in advance

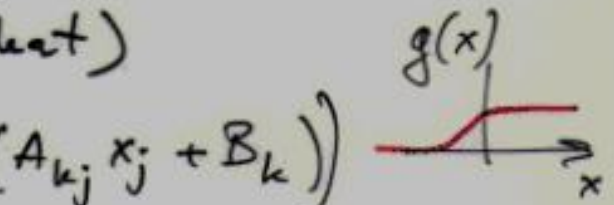
4. VEGAS: $M(x) = \prod_i M_i(x_i)$,

$M_i \in 1.$

5. Kernels: $\sum_n f_n K(x - x_n)$

6. NN (simplest; P. Bhat)

$$M(x) = g\left(\sum_k c_k g(A_{kj} x_j + B_k)\right)$$



sum of rotated and shifted step functions! (?)

POWHEG

$$F_N(x) = \frac{1}{N}\sum_n f_n \delta(x - x_n) \xrightarrow{W} f(x)\,\pi(x)$$

$$\tilde{F}_P(x) = \frac{1}{P}\sum_p \tilde{f}_p \delta(x - x_p) \xrightarrow{W} \frac{1}{P}\sum_p \tilde{f}_p\, h_p(x)$$

$P \ll N$

POWHEG = intermediate step + kernels.

$$h_p(x) = \frac{1}{R^{Dim}}\, H\left(\frac{\vec{x} - \vec{x}_p}{R}\right)$$

HAT-FN $\quad \int H(x)\,dx = 1$

- Obvious connection with jets
- Best for identifying narrow structures
- In combination with global smooth fits

Criterion to find $\tilde{f}_p, x_p$

$$\boxed{\min \left| \langle \tilde{F}_I \varphi \rangle - \langle F_N \varphi \rangle \right| \quad \forall \varphi}$$

All reasoning similar to Optimal Jet Definition FT'00

END RESULT:

recombination matrix: $\boxed{0 \le z_{u,p} \le 1}$, $\boxed{\sum_p z_{u,p} \le 1}$

$(x_u - x_p)^0$ $\quad \tilde{f}_p = \sum_u z_{up} f_p \qquad \bar{z}_u \equiv 1 - \sum_p z_{up}$

$(x_u - x_p)^1$ $\quad \tilde{f}_p \vec{x}_p = \sum_u z_{up} f_u \vec{x}_u \qquad \tilde{f}_p = 0$?

To find z_{up}:

$(x_u - x_p)^2$ $\quad \boxed{\min \left(\frac{1}{R^2} \sum_{u,p} z_{up} |f_p| (\vec{x}_u - \vec{x}_p)^2 + \sum_u \bar{z}_u |f_u| \right)}$

well-defined criterion → max rading of a powzyr.

REASONABLY FAST ALGORITHM EXISTS

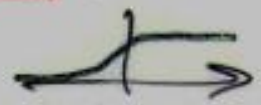
OJF → THURSDAY 10^{30}

$t \sim O(N \times P)$

pictures ...

Connection with NN (simplest)

$$g\left(\sum_k c_k\, g\left(\sum_i A_{ik} x_i + B_k\right)\right)$$

$g(x) \sim$ step function 

→ SUM OF POLYZRY OF $R=\infty$????

But to represent an arbitrary function one needs $R \to 0$!

MAKE YOUR OWN CONCLUSIONS ON HOW NN WOULD RULE THE WORLD IN 10 YEARS (see Summary at ACAT'02)

OJF — "verification version" [N function]

Fortran-77
highly optimized for jets

ideally: $P \sim \sqrt{N}$

POLYZRY
YET TO BE OPTIMIZED SPECIFICALLY FOR MODELLING

- FLEXIBILITY
  "LAYERS" $R_0 > R_1 > R_2$ .. foam
  Choice of hats & background
- POLYZRY overlaps "orthogonalization" .....