Isovector nucleon matrix elements with $N_f = 2+1$ dynamical DWF

Takeshi YAMAZAKI

Yukawa Institute for Theoretical Physics, Kyoto University

Y. Aoki, T. Blum, H-W. Lin, M. F. Lin, S. Ohta, S. Sasaki, R. J. Tweedie, and J. M. Zanotti

for RBC-UKQCD Collaborations

Refs. PoS(LAT2007):165(2007) and PRL100:171602(2008)

KEK workshop@ KEK

August 8, 2008

Outline

1. Introduction
2. Simulation parameters
3. Results
   - g_A/g_V
   - Axial vector and induced pseudoscalar form factors
   - Vector and induced tensor form factors
   - Moments of structure functions
4. Summary

1. Introduction

Motivation :

understand nucleon physics from first principle (lattice) QCD

We calculate matrix elements related to isovector form factors and moments of structure functions of nucleon on $N_f = 2 + 1$ domain wall fermion (DWF) configuration

(generated by RBC-UKQCD collaborations on QCDOC)

Isovector form factors (elastic scattering $Ne \to Ne$)

- Vector and induced tensor form factors

(elastic proton-electron scattering)

$$\langle N, p | V_\mu(q) | N, p' \rangle \;=\; \overline{u}_N(p) \left(\gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_N} \right) u_N(p')$$

$$q_\nu = p'_\nu - p_\nu$$

$$F_1(q^2), F_2(q^2) \to \quad \begin{aligned} & F_1(0) = F_1^p(0) - F_1^n(0) = 1 \\ & F_2(0) = \mu_p - \mu_n - 1 \ (\mu_i : \text{ magnetic moment}) \\ & \langle r_1^2 \rangle, \langle r_2^2 \rangle \text{ related to charge radii } \langle r_p^2 \rangle, \langle r_n^2 \rangle \end{aligned}$$

- Axial vector and induced pseudoscalar form factors

(β decay; muon capture on proton; neutrino-nucleon scattering; pion electroproduction)

$$\langle N, p | A_\mu(q) | N, p' \rangle \;=\; \overline{u}_N(p) \left(i\gamma_5 \gamma_\mu G_A(q^2) + i\gamma_5 q_\mu G_P(q^2) \right) u_N(p')$$

$$G_A(q^2), G_P(q^2) \to \quad \begin{aligned} & G_A(0) : \text{ axial charge, } \langle r_A^2 \rangle \\ & g_{\pi NN} : \text{ pion-nucleon coupling} \\ & g_P : \text{ pseudoscalar coupling for muon capture} \end{aligned}$$

Structure functions (deep inelastic scattering $Ne \to Xe$)

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{Q^4}\left(\frac{E'}{ME}\right) l_{\mu\nu} W^{\mu\nu}, \quad W^{\mu\nu} = W^{\{\mu\nu\}} + W^{[\mu\nu]}$$

$$W^{\{\mu\nu\}} = \left(-g^{\mu\nu} + \frac{q^\mu g^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{q\cdot P}{q^2} q^\mu\right)\left(P^\nu - \frac{q\cdot P}{q^2} q^\nu\right)\frac{F_2(x,Q^2)}{q\cdot P}$$

$$W^{[\mu\nu]} = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{q\cdot P}\left(g_1(x,Q^2) + g_2(x,Q^2)\right) - \frac{q\cdot S P_\sigma}{(q\cdot P)^2} g_2(x,Q^2)\right)$$

$$S^2 = -m_N^2, \ x = Q^2/2(q\cdot P)$$

Unpolarized structure function

$F_1(x,Q^2)$ and $F_2(x,Q^2)$

Polarized structure function

$g_1(x,Q^2)$ and $g_2(x,Q^2)$

Moments of structure functions (Operator Product Expansion)

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c^q_{1,n}(\mu^2/Q^2, g(\mu)) \langle x^{n-1}\rangle_q(\mu) + O(1/Q^2) \\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{q=u,d} c^q_{2,n}(\mu^2/Q^2, g(\mu)) \langle x^{n-1}\rangle_q(\mu) + O(1/Q^2) \\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e^q_{1,n}(\mu^2/Q^2, g(\mu)) \langle x^{n}\rangle_{\Delta q}(\mu) + O(1/Q^2) \\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{n}{2(n+1)} \sum_{q=u,d} \Big[e^q_{2,n}(\mu^2/Q^2, g(\mu)) d^q_n(\mu) \\
&\quad - 2 e^q_{1,n}(\mu^2/Q^2, g(\mu)) \langle x^{n}\rangle_{\Delta q}(\mu) \Big] + O(1/Q^2)
\end{aligned}
$$

Wilson coefficients: $c^q_{i,n}$, $e^q_{i,n}$

Our calculations (Isovector):

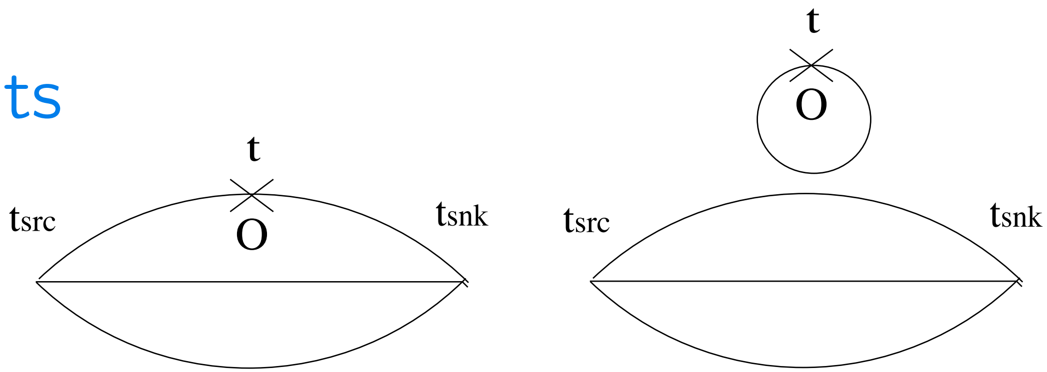
$$\langle x\rangle_q(\mu),\ \langle x\rangle_{\Delta q}(\mu),\ d^q_1(\mu),\ (\langle 1\rangle_{\delta q}(\mu))$$

Operators of moments of structure functions $\langle P, S|O|P, S\rangle$

$$
\begin{aligned}
\langle x^{n-1}\rangle_q \quad &: \quad \bar{q}\left[\left(\frac{i}{2}\right)^{n-1} \gamma_{\{\mu_1}\ \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \ - \ \text{trace}\right] q \\
\langle x^{n-1}\rangle_{\Delta q} \quad &: \quad \bar{q}\left[\left(\frac{i}{2}\right)^{n-1} \gamma_5\gamma_{\{\mu_1}\ \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \ - \ \text{trace}\right] q \\
d^q_{n-1} \quad &: \quad \bar{q}\left[\left(\frac{i}{2}\right)^{n-1} \gamma_5\gamma_{[\mu_1}\ \overleftrightarrow{D}_{\{\mu_2]} \cdots \overleftrightarrow{D}_{\mu_n\}} \ - \ \text{trace}\right] q \\
\langle x^{n}\rangle_{\delta q} \quad &: \quad \bar{q}\left[\left(\frac{i}{2}\right)^{n} \gamma_5\sigma_{\rho\nu}\ \overleftrightarrow{D}_{\{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n\}} \ - \ \text{trace}\right] q
\end{aligned}
$$

{}:symmetrize, []:antisymmetrize

Matrix elements



Disconnected diagram : very noisy and expensive $\sim 10\times$connected one
canceled in Isovector quantities when $m_u = m_d$

In this work only connected diagram $\rightarrow$ Isovector quantity

$$R_{\vec{p}}^{\mathcal{P}\mathcal{O}}(t, t_{snk}, t_{src}) = \frac{G_{\vec{p}}^{\mathcal{P}\mathcal{O}}(t)}{G_{\vec{0}}^{G}(t_{snk})} \left[\frac{G_{\vec{p}}^{L}(t_{snk} - t + t_{src}) G_{\vec{0}}^{G}(t) G_{\vec{0}}^{L}(t_{snk})}{G_{\vec{0}}^{L}(t_{snk} - t + t_{src}) G_{\vec{p}}^{G}(t) G_{\vec{p}}^{L}(t_{snk})} \right]^{1/2}$$

$$\propto \langle N(0) | \mathcal{O}(q) | N(p) \rangle \quad (t_{src} \ll t \ll t_{snk})$$

Normalization of nucleon operator is canceled.

$G_{\vec{p}}^{\mathcal{P}\mathcal{O}}$: 3-point function of $\mathcal{O}$ with $\vec{p}$ and projector $\mathcal{P}$
gauge invariant Gauss smearing source is employed.

$G_{\vec{p}}^{G,L}$: 2-point function with $\vec{p}$ and gauss smearing(G) or local(L) sink
gauss smearing source

(Isovector) Nucleon matrix elements

Recent works: Alexandrou *et al* PRD74:034508; PRD76:094511 ($N_f = 0, 2$ Wilson)
Göckeler *et al* PRD71:034508; PoS(LAT2007)161($N_f = 0, 2$ Wilson)
Hägler *et al* arXiv:0705.4295 ($N_f = 2 + 1$ Mixed action)
Sasaki and TY PRD78:014510 ($N_f = 0$ DWF)
Lin *et al* arXiv:0802.0863 ($N_f = 2$ DWF)

	valence	sea	N_f	L[fm]	$m_\pi >$[GeV]
This work	DWF	DWF	2+1	2.7(1.8)	0.33
Alexandrou	Wilson	—	0	3.0	0.41
Alexandrou	Wilson	Wilson	2	1.9	0.38
Göckeler	Clover	—	0	1.7	0.55
Göckeler	Clover	Clover	2	2.0	0.30
Hägler	DWF	Imp. staggered	2+1	2.5(3.5)	0.35
Lin	DWF	DWF	2	1.9	0.49
Sasaki	DWF	—	0	3.6	0.39

DWF has good chiral symmetry on lattice. It is advantage to calculate nucleon matrix element, especially axial vector current.

Our calculation is carried out with $N_f = 2 + 1$ dynamical quark effect on relatively larger volume at lighter pion mass.

Finite volume effect at lighter pion mass

2. Simulation parameters

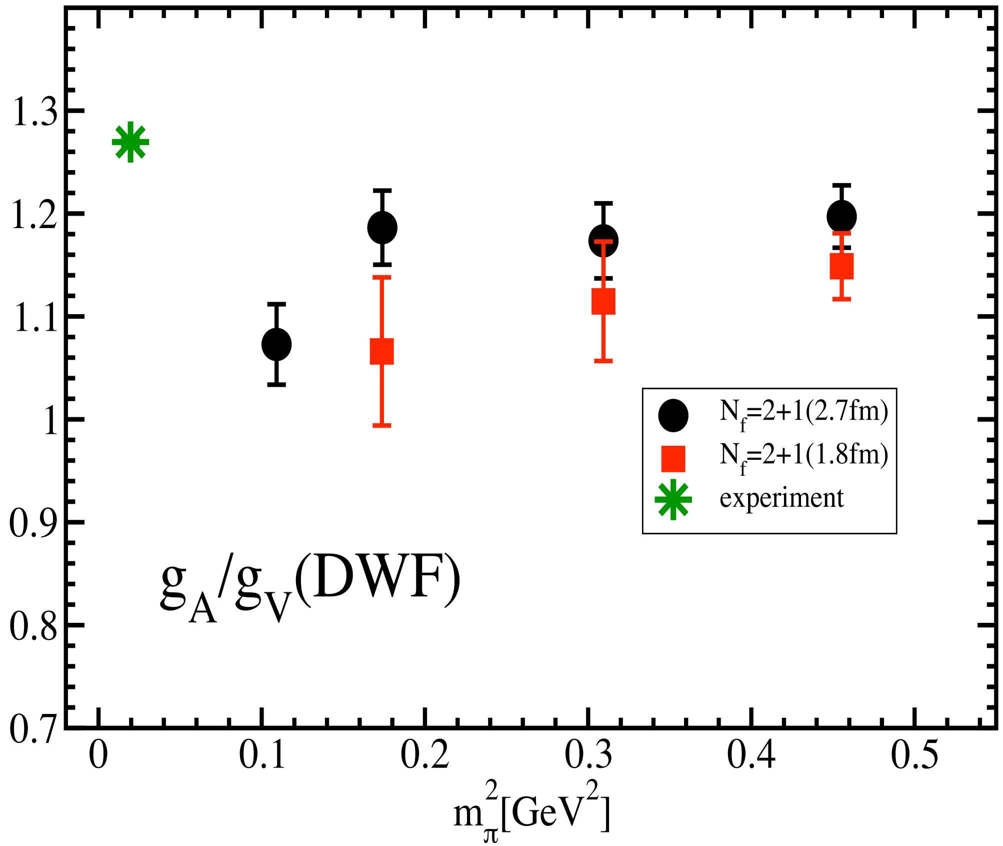
- $N_f = 2 + 1$ Iwasaki gauge + Domain Wall fermion actions
- $\beta = 2.13$ $a^{-1} = 1.73$ GeV $M_5 = 1.8$ $m_{\text{res}} \approx 0.003$
- Lattice size $24^3 \times 64 \times 16$ ($La \approx 2.7$ fm)
  $16^3 \times 32 \times 16$ ($La \approx 1.8$ fm) for g_A/g_V
- $m_s = 0.04$ fixed (close to m_s^{phys})
- quark masses $m_f = m_{sea} = m_{val}$ and confs.

m_f	m_π[MeV]	m_N[GeV]	# of confs.	N_{meas}
0.005	330	1.15	932	4
0.01	420	1.22	356	4
0.02	560	1.39	98	4
0.03	670	1.55	106	4

- We focus only on isovector quantities. (no disconnected diagram)
$$F_i(q^2) = F_i^p(q^2) - F_i^n(q^2)$$
- Four different non-zero $q^2 \approx 0.2, 0.4, 0.6, 0.75$[GeV^2]
- Matrix elements are evaluated by ratio of 3- and 2-point functions.
- $t_{snk} - t_{src} = 12 \approx 1.37$ fm

3. Results

3.1. Axial charge $g_A/g_V = G_A(0)/F_1(0)$

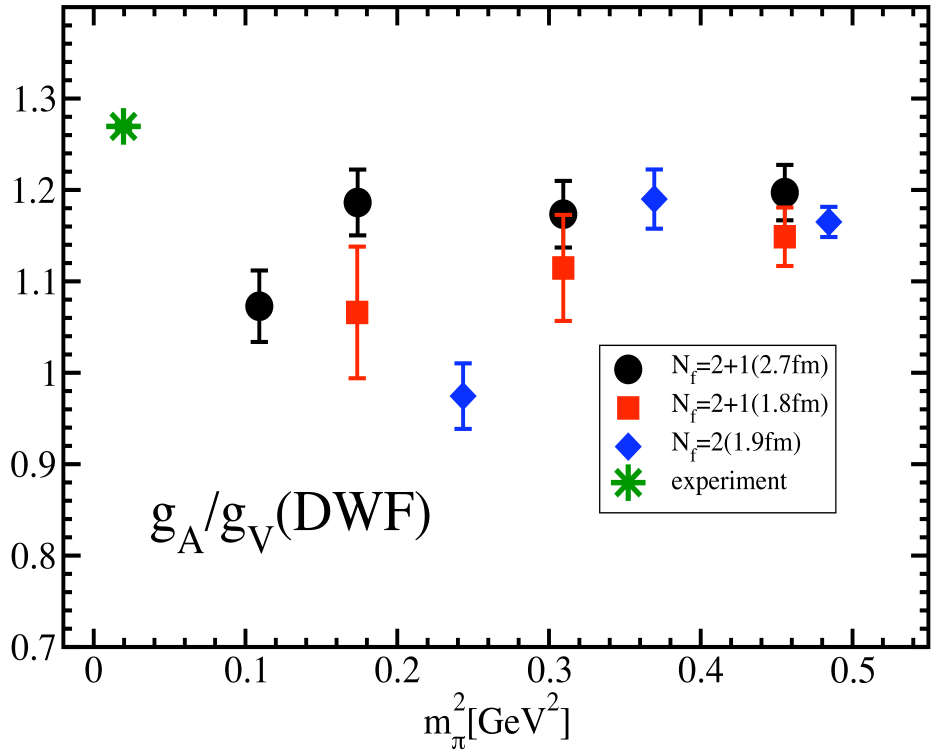


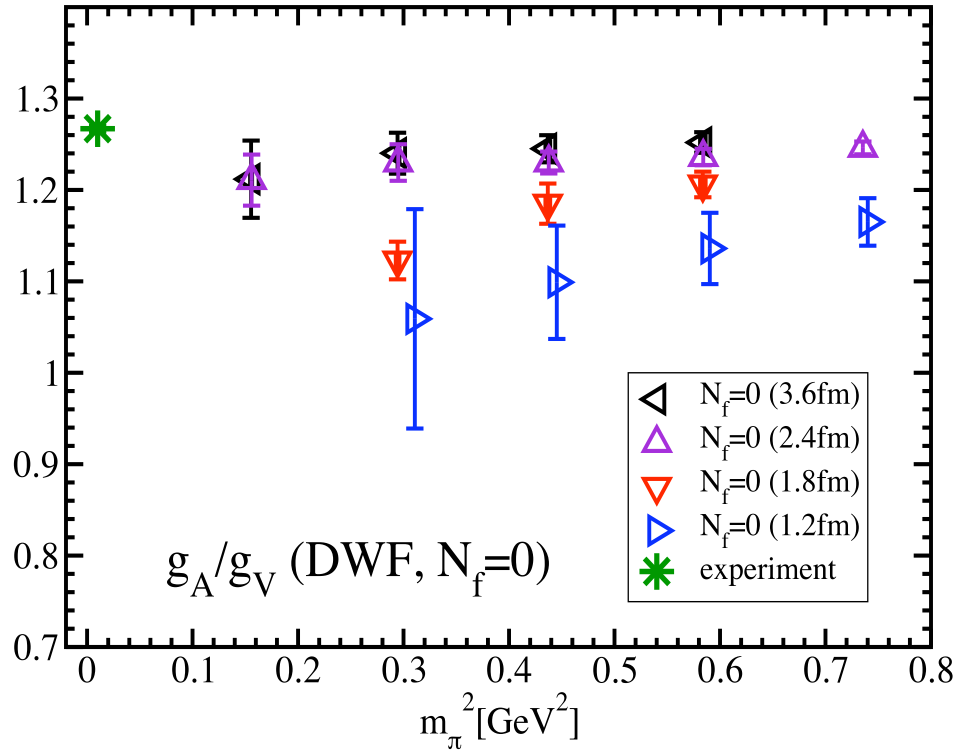
Heavier three data are almost independent of m_π^2, while lightest data is 9% smaller than other masses.

Smaller volume data are systematically below larger volume data.

(1.8 fm) data are calculated on $16^3 \times 32 \times 16$ with heavier three quark masses

g_A/g_V in DWF

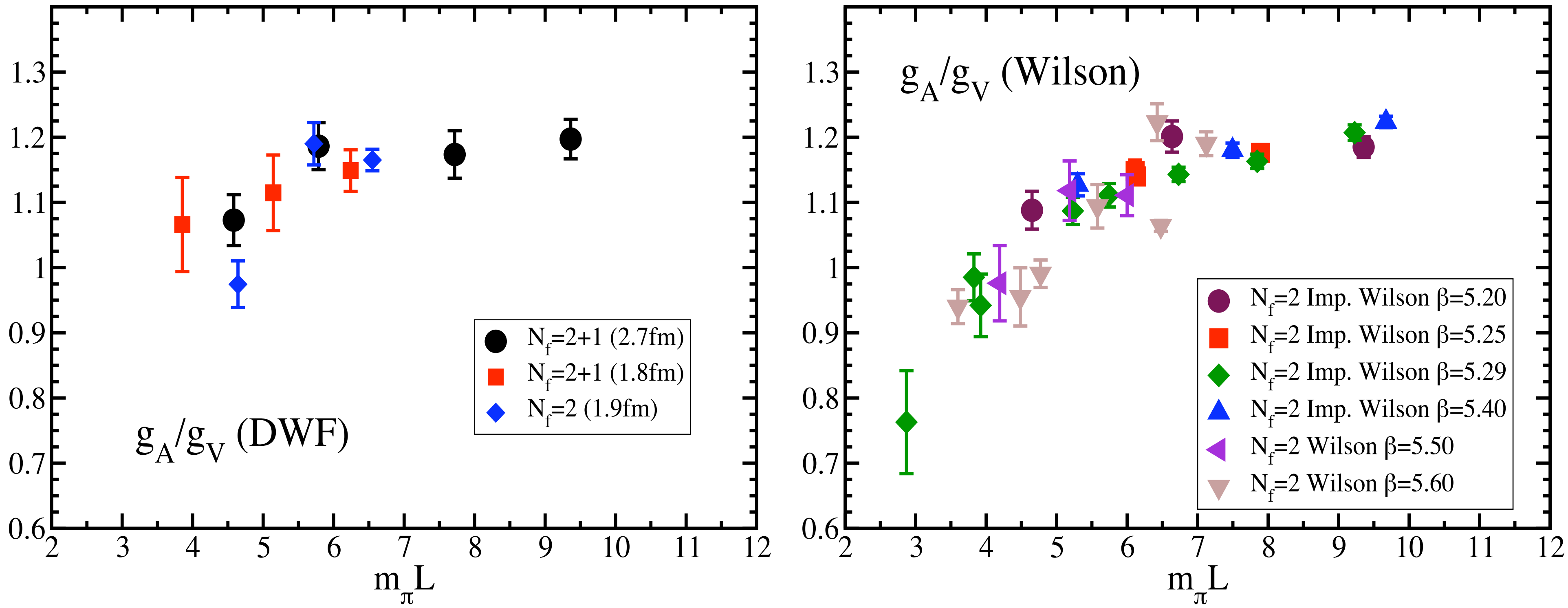




Refs. $N_f = 0$ DWF PRD68:054509(RBCK), PRD78:014510;
$N_f = 2$ DWF arXiv:0802.0863(RBC)

Similar behavior was seen in $N_f = 2$, but it sets in at heavier pion mass.
We suspect that downward behavior is caused by finite volume effect.
m_π dependence of $N_f = 2+1$ is similar to $N_f = 0$ on $L < 2.4$ fm, which is caused by finite volume.
In $N_f = 0$ such a dependence disappears when $L > 2.4$ fm. Large finite volume effect is not expected on 2.7 fm.

$m_\pi L$ scaling of g_A/g_V in dynamical calculations



Refs. Wilson PRD66:034506(LHPC and SESAM); Wilson PRD76:094511;

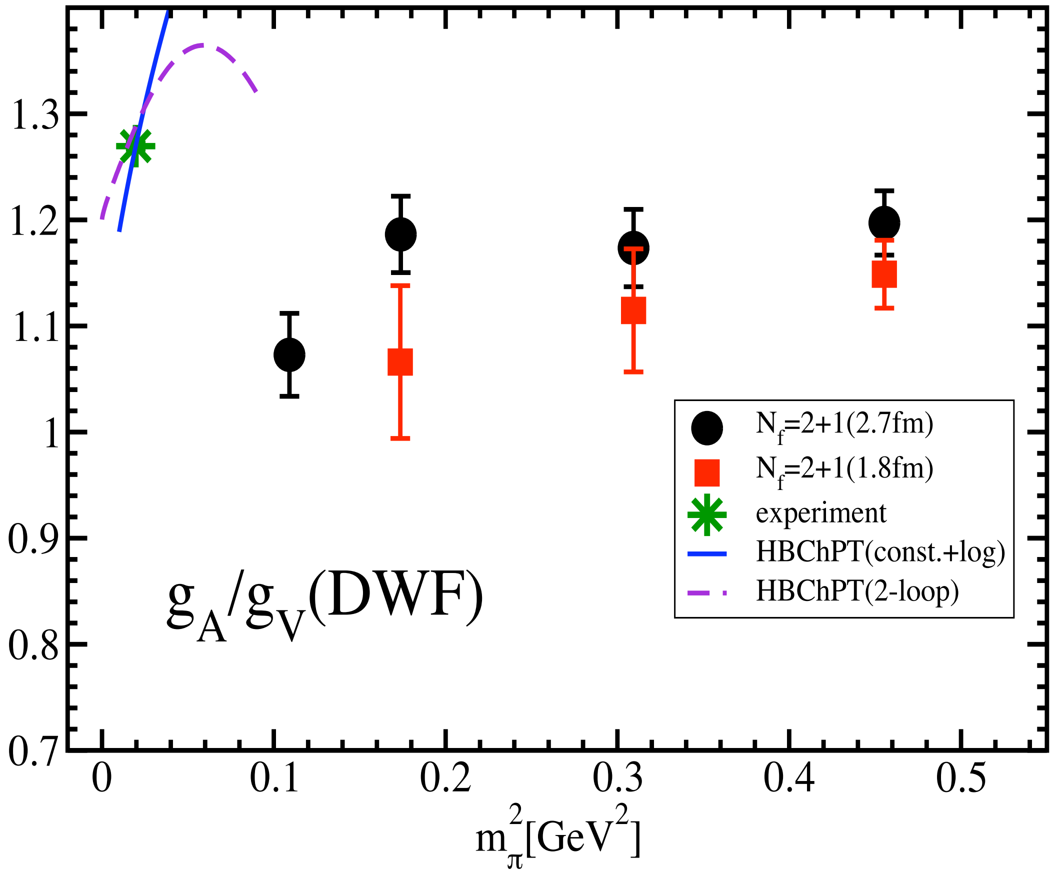
Imp. Wilson PRD74:094508(QCDSF)

$N_f = 2+1$ data on two volumes scale in $m_\pi L$.

Similar scaling is seen in two-flavor (Imp.) Wilson fermion calculations with various $m_\pi = 0.38$–1.18 GeV, $V = (0.95$–$2.0\text{ fm})^3$, and β.

This observation is used for chiral extrapolation of g_A/g_V.

Chiral extrapolations of g_A/g_V



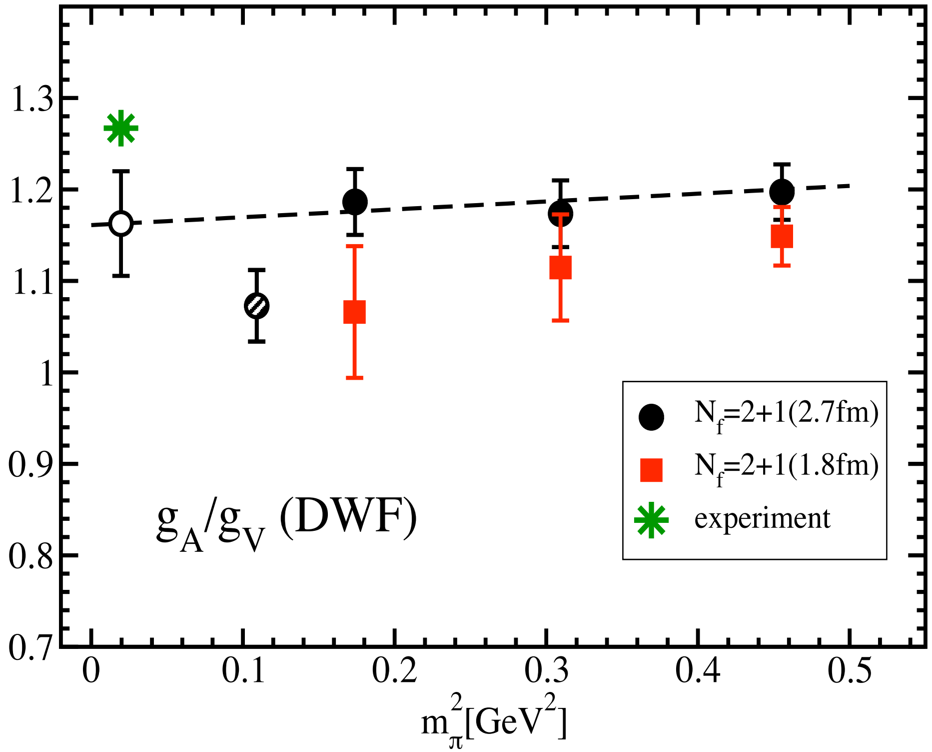
We did not use heavy baryon chiral perturbation theory(HBChPT) formula for chiral extrapolation.

Our m_π is beyond the region where HBChPT is valid even at two-loop order, $m_\pi < 300$ MeV. Bernard and Meißner PLB639:278

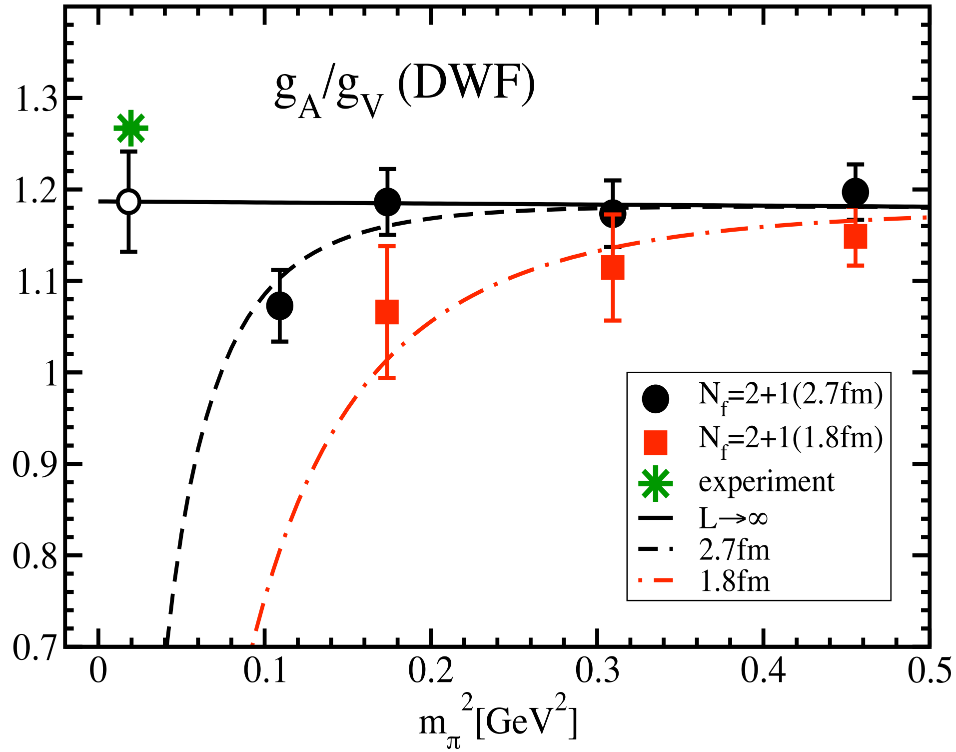
Most works employed HBChPT with Δ baryon for chiral extrapolation, but estimated finite volume effect is less than 1% at lightest point.

Chiral extrapolations of g_A/g_V (cont'd)

linear



linear+$f_V(m_\pi L)(= e^{-m_\pi L})$



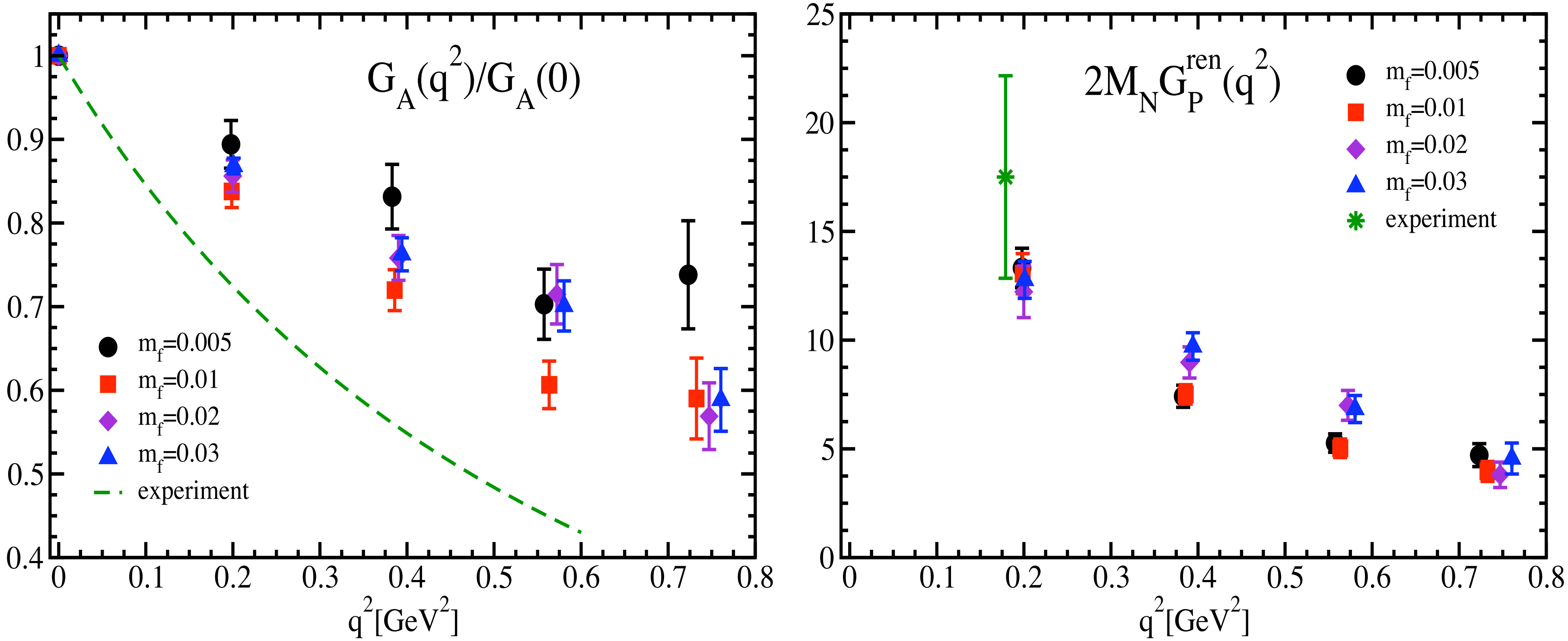
1.17(6) at $m_\pi = 135$ MeV

1.20(6)(4) at $m_\pi = 135$ MeV

Second error is systematic determined from different choice of f_V, such as x^{-3}, $x^{1/2}e^{-x}$, and $m_\pi^2 e^{-x}/x^{1/2}$ with $x = m_\pi L$.

From fit with f_V, we estimate that one needs $L \approx$ 3.5–4.5 fm ($m_\pi L \approx$ 6–8) to aim finite volume effect being below 1% at $m_\pi = 330$ MeV.

3.2. Axial vector and induced pseudoscalar form factors



$G_P(q^2)$ is renormalized by $Z_V = 1/F_1(0) \approx Z_A$.

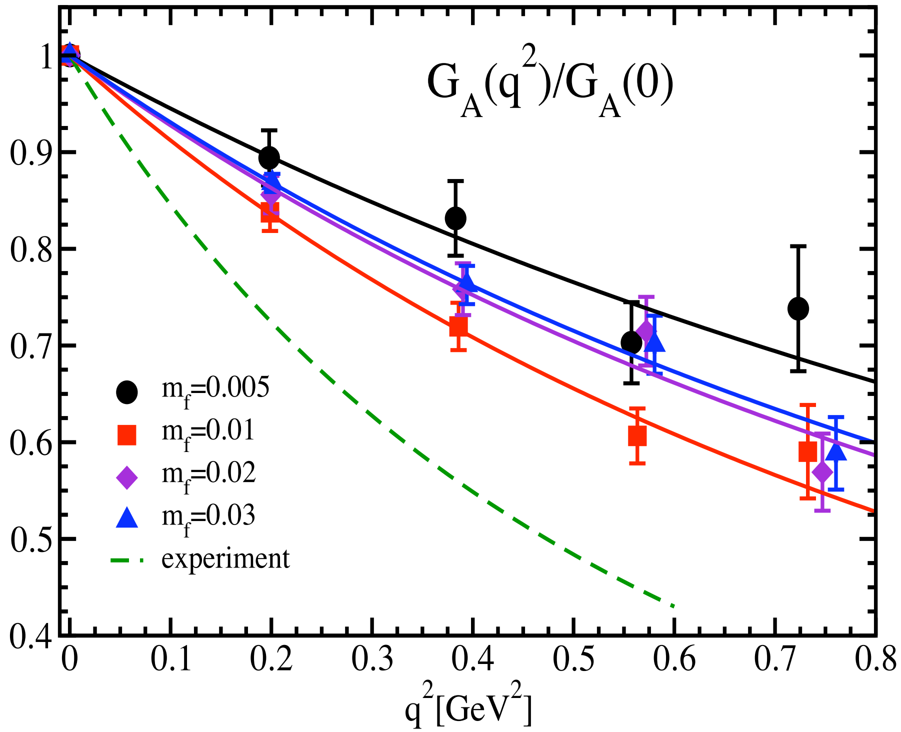
m_f dependence of G_A is strange, and not monotonic function of m_f.
Lightest data is larger than heavier mass data.

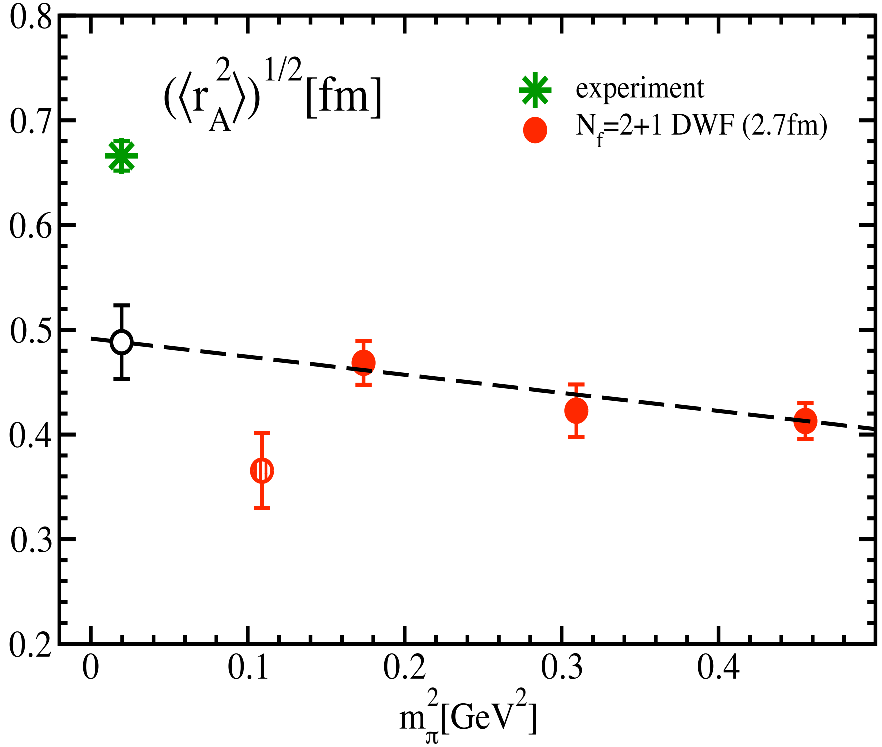
Dipole fit of $G_A(q^2)$ and Axial charge rms radius $\sqrt{\langle r_A^2 \rangle}$

1 parameter fit $\langle r_A^2 \rangle$

$$\frac{G_A(q^2)}{G_A(0)} = \frac{1}{(1 + \langle r_A^2 \rangle q^2/12)^2}$$

$\sqrt{\langle r_A^2 \rangle}$

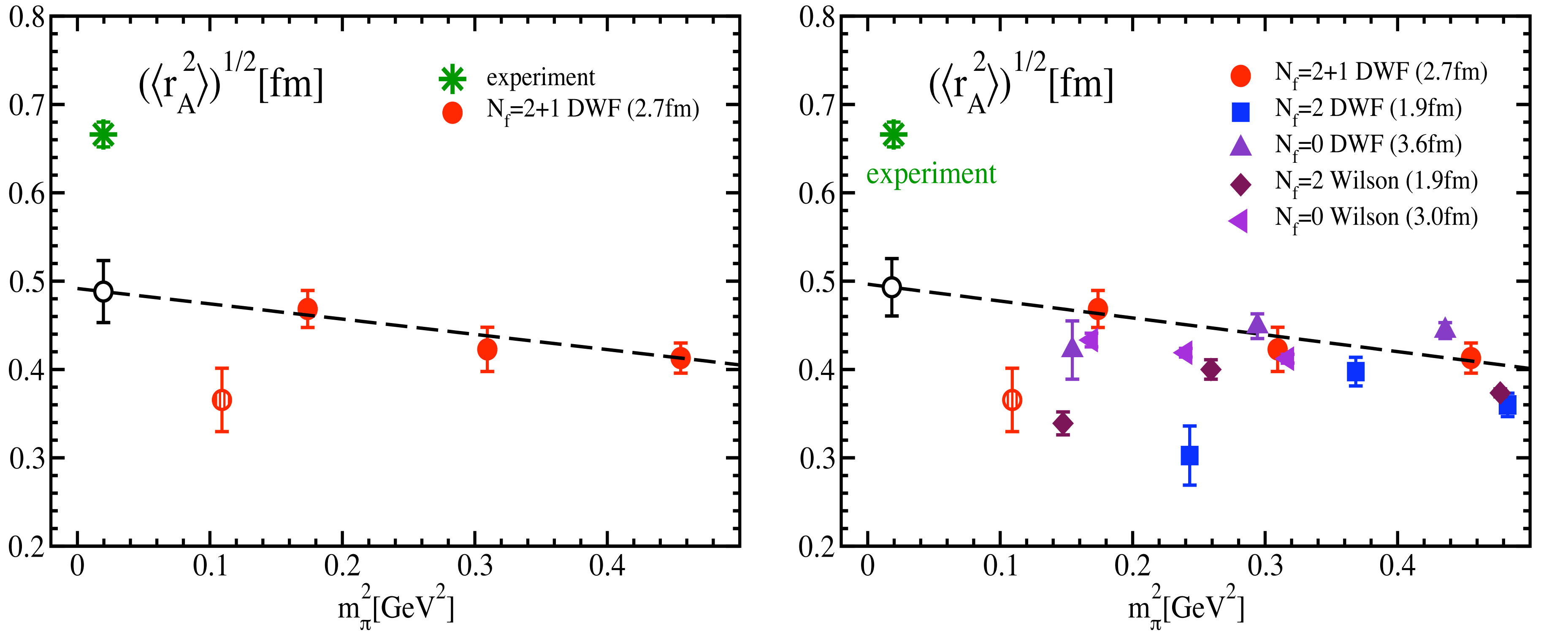




Lightest result of $\sqrt{\langle r_A^2 \rangle}$ is smaller than other mass points, and goes away from experiment.

This m_π^2 dependence is similar to one in g_A/g_V

Axial charge rms radius $\sqrt{\langle r_A^2 \rangle}$



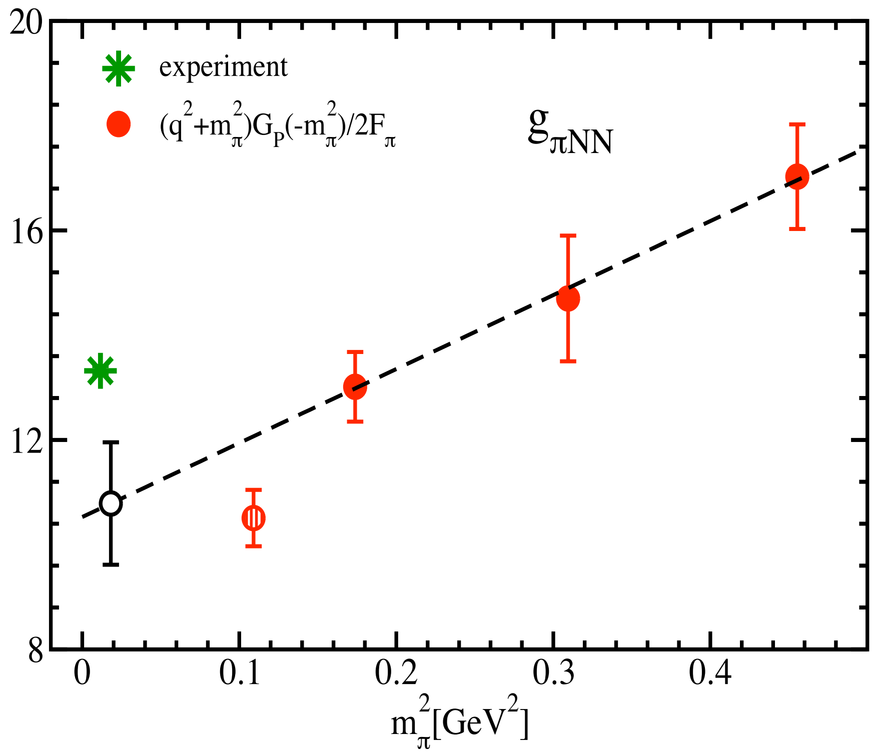
$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 2$ DWF arXiv:0802.0863;

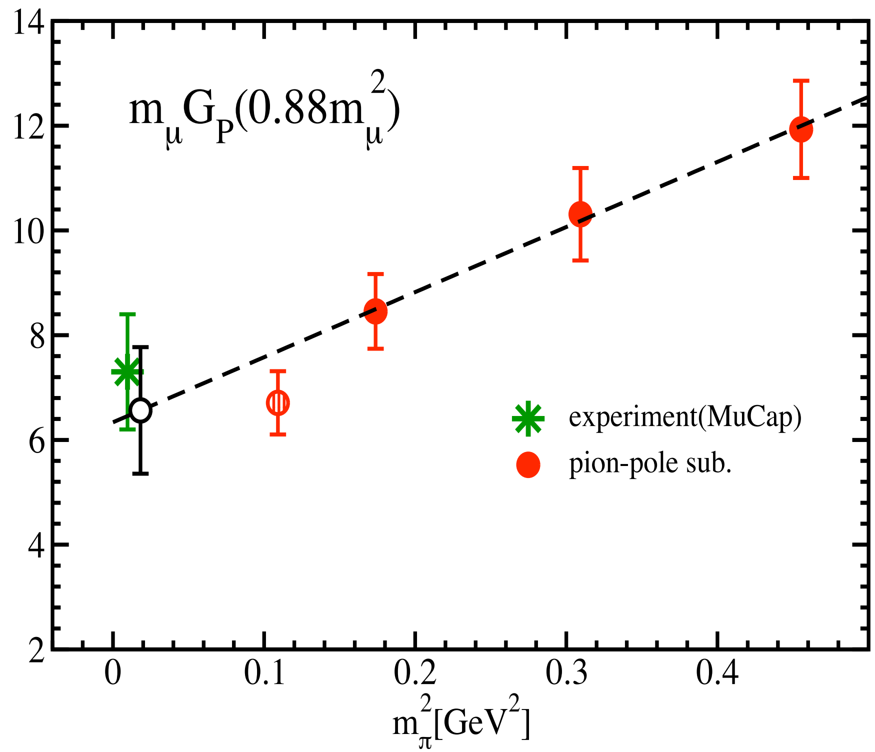
$N_f = 0$ DWF PRD78:014510

Similar trend is seen in $N_f = 2$ data on (1.9 fm)^3 volume, but the downward behavior sets in at heavier m_π as in g_A/g_V.

This would indicate large finite volume effect at lightest point.

$g_{\pi NN}$ coupling and g_P for muon capture





$g_{\pi NN}$ is evaluated by definition of $g_{\pi NN}$ with $G_P(q^2)$.

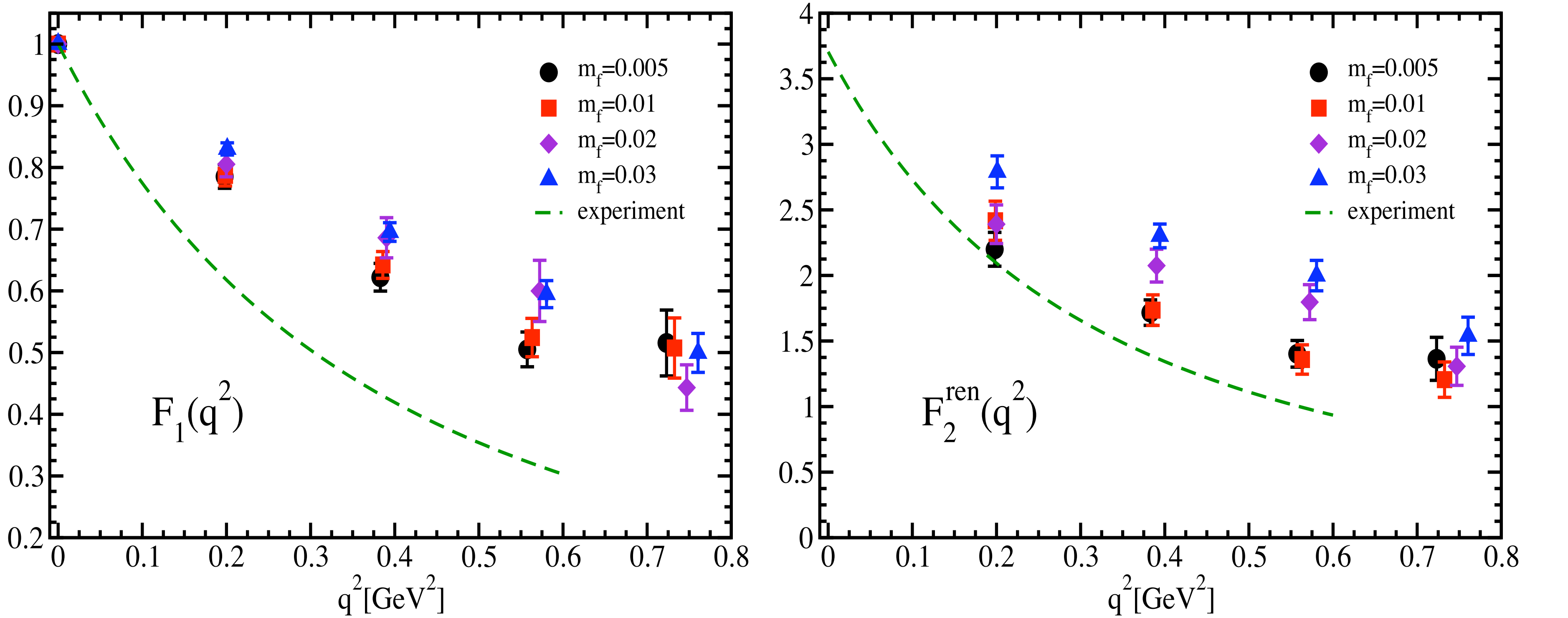
g_P is determined with assumption that $G_P(q^2)$ has pion-pole.

Results at m_π^{phys} reasonably agree with experiments.

Lightest data are smaller than linear fit line using heavier data.

G_A and G_P are sensitive to finite volume.

3.3 Vector and induced tensor form factors



renormalized by $Z_V = 1/F_1(0)$

There is no strange, non-monotonic behavior in the form factors.

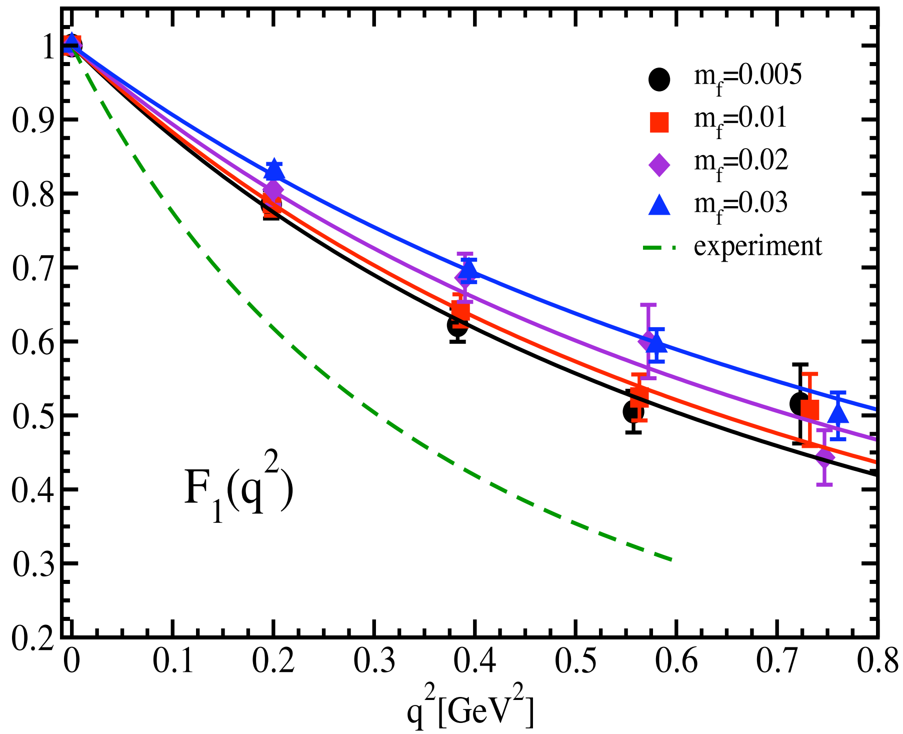
Dipole fit of form factors

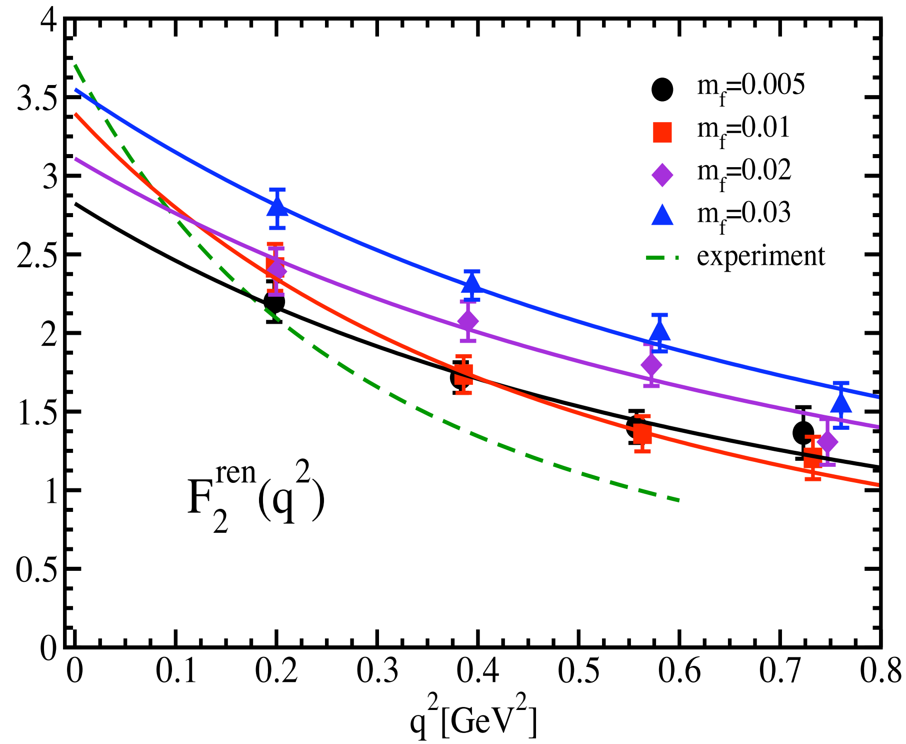
1 parameter fit $\langle r_1^2 \rangle$

$$\frac{1}{(1+\langle r_1^2 \rangle q^2/12)^2}$$

2 parameters fit $F_2(0)$ and $\langle r_2^2 \rangle$

$$\frac{F_2(0)}{(1+\langle r_2^2 \rangle q^2/12)^2}$$

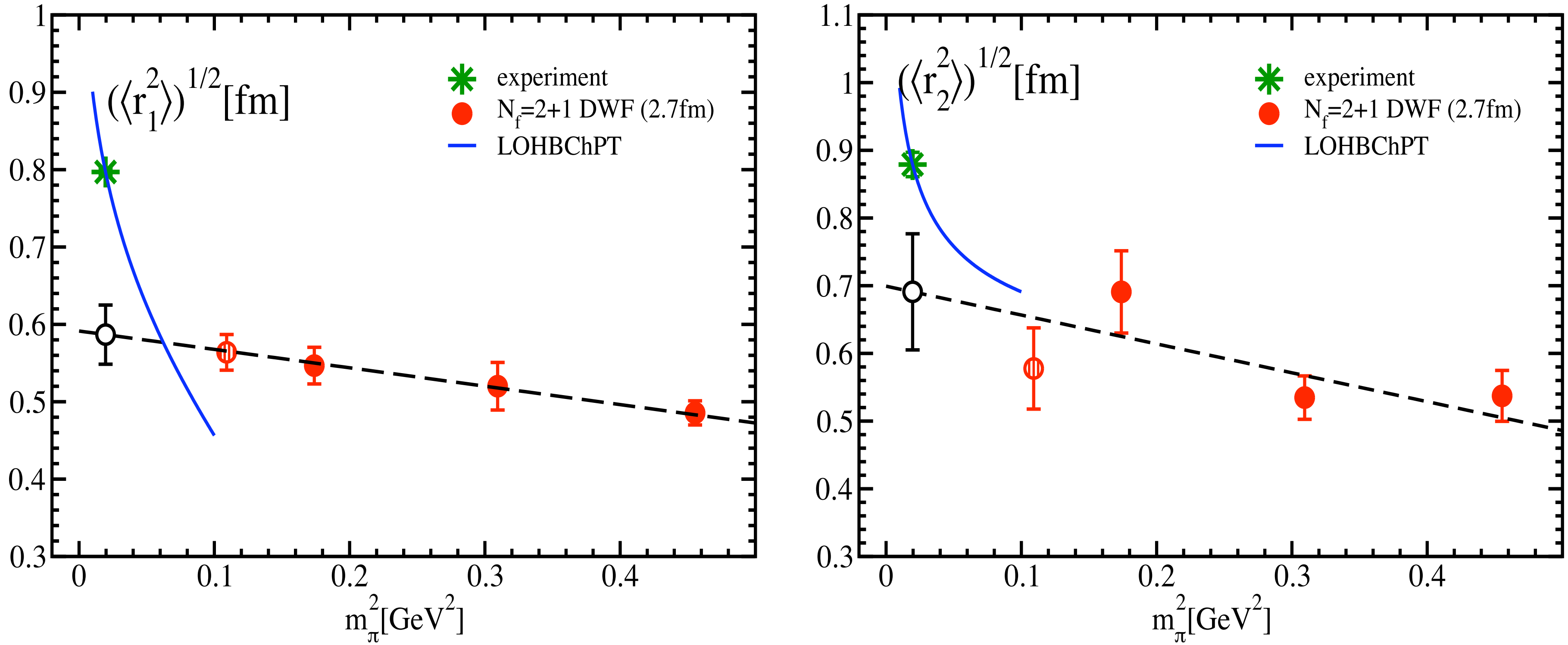




$$F_2(0) = \mu_p - \mu_n - 1$$

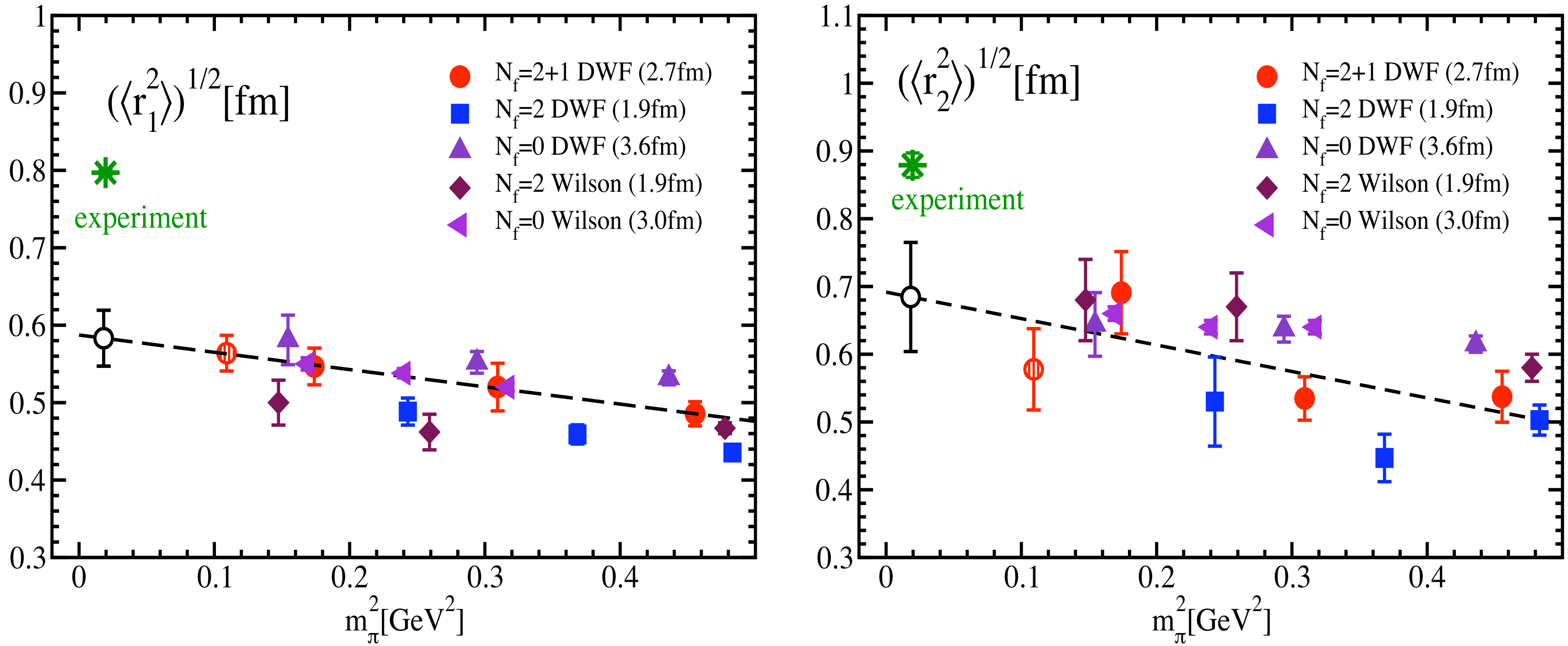
$F_2(0)$ cannot be calculated directly, so that $F_2(0)$ is a free parameter in $F_2(q^2)$ fit.

Dirac and Pauli rms radius $\sqrt{\langle r_1^2 \rangle}$, $\sqrt{\langle r_2^2 \rangle}$



Result increases as m_π decreases, but are smaller than experiments.
Lightest results are consistent with linear extrapolations with other data.
In HBChPT both radii diverge at chiral limit, while such a behavior is not seen.
Lighter quark mass calculation, *e.g.*, $m_\pi \ll$ 200 MeV, would be necessary to observe divergent behavior.

Dirac and Pauli rms radius $\sqrt{\langle r_1^2 \rangle}$, $\sqrt{\langle r_2^2 \rangle}$



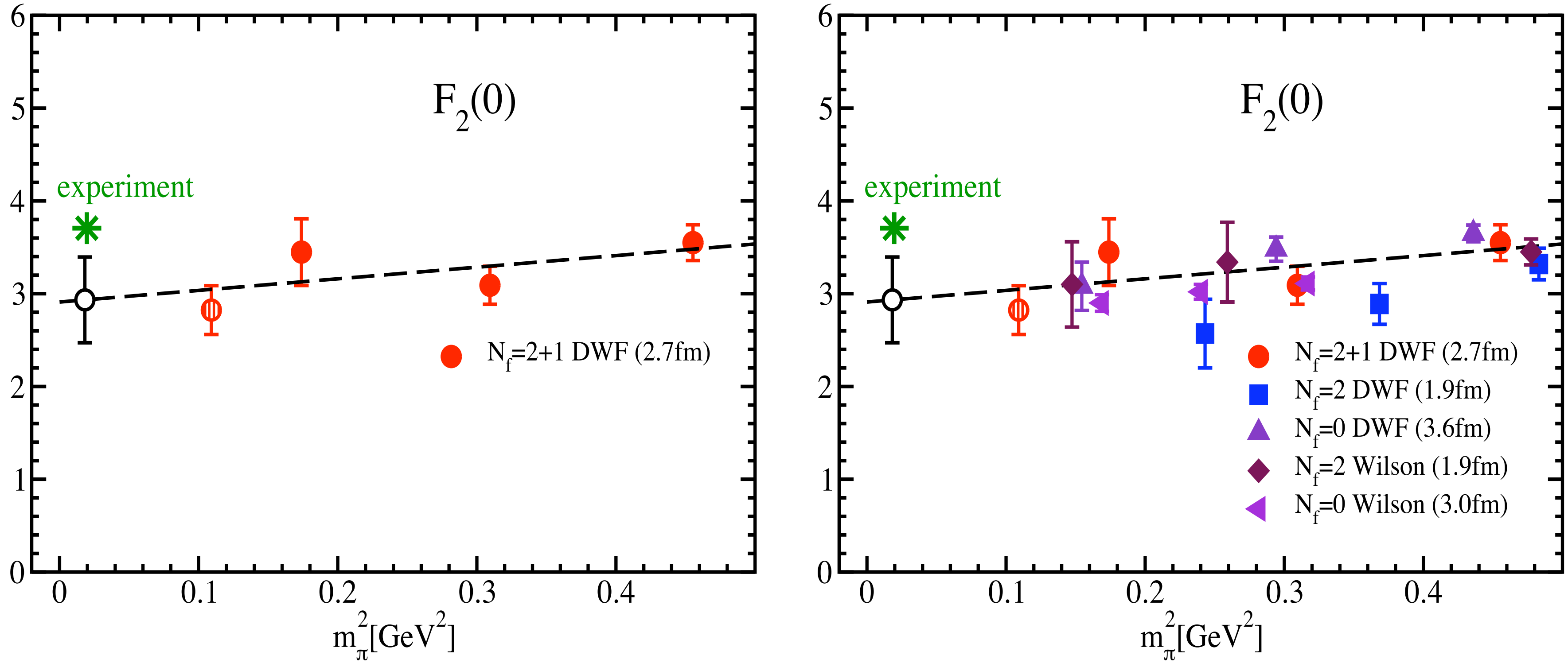
$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 0$ DWF PRD78:014510;

Lin *et al* arXiv:0802.0863

Linear m_π^2 dependences are consistent with previous results in quenched and dynamical calculations.

All the results do not have divergent behavior.

$F_2(0) = \mu_p - \mu_n + 1$

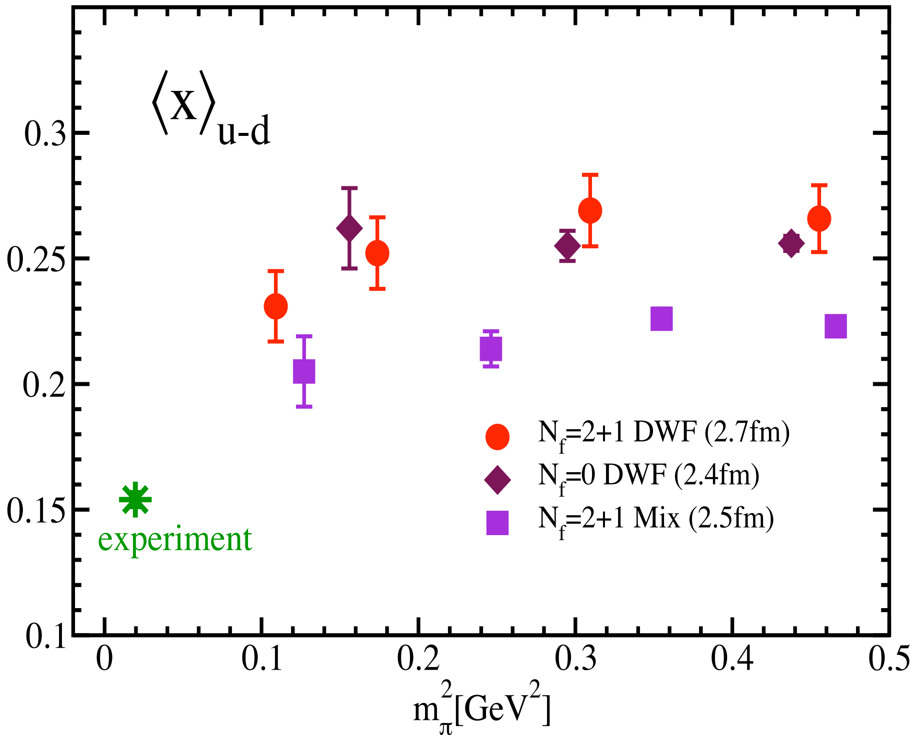


$N_f = 0, 2$ Wilson PRD74:034508; $N_f = 2$ DWF arXiv:0802.0863;
$N_f = 0$ DWF PRD78:014510

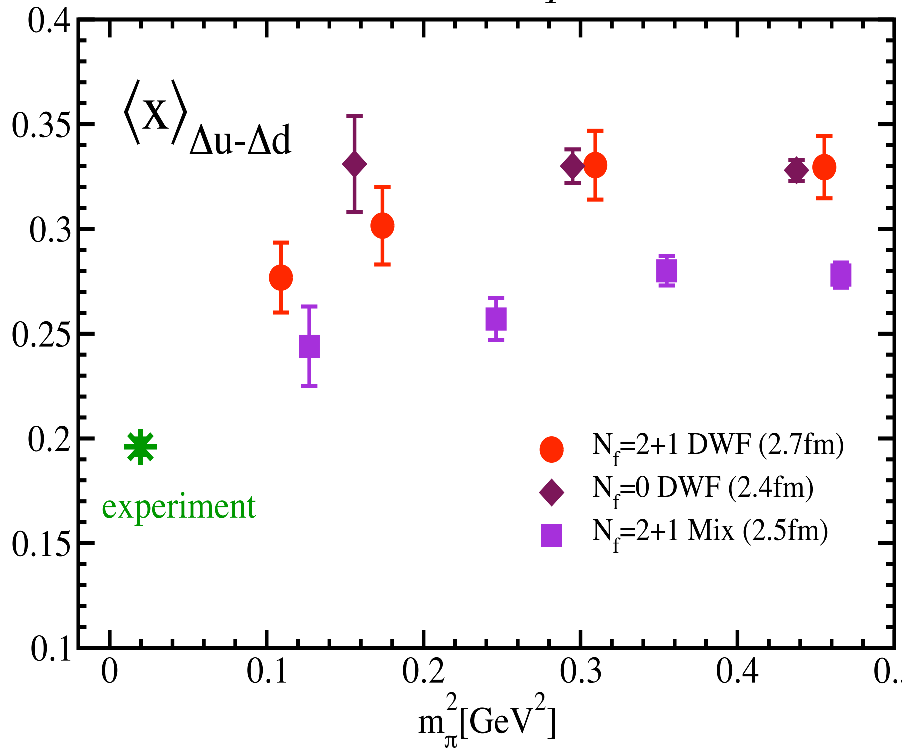
$F_2(0)$ has mild m_π dependence, and reasonably agrees with experiment in physical pion mass. m_π dependence is similar to previous results. Results obtained from $F_1(q^2)$ and $F_2(q^2)$ have no strange m_π dependence in contrast to $G_A(q^2)$ and $G_P(q^2)$. (less sensitive to finite volume effect)

3.4. Moments of structure functions

unpolarized $\langle x \rangle_q(2\text{GeV})$



polarized $\langle x \rangle_{\Delta q}(2\text{GeV})$



$N_f = 2+1$ Mixed action arXiv:0705.4295; $N_f = 2$ DWF arXiv:0802.0863;

$N_f = 0$ DWF PRD73:094503

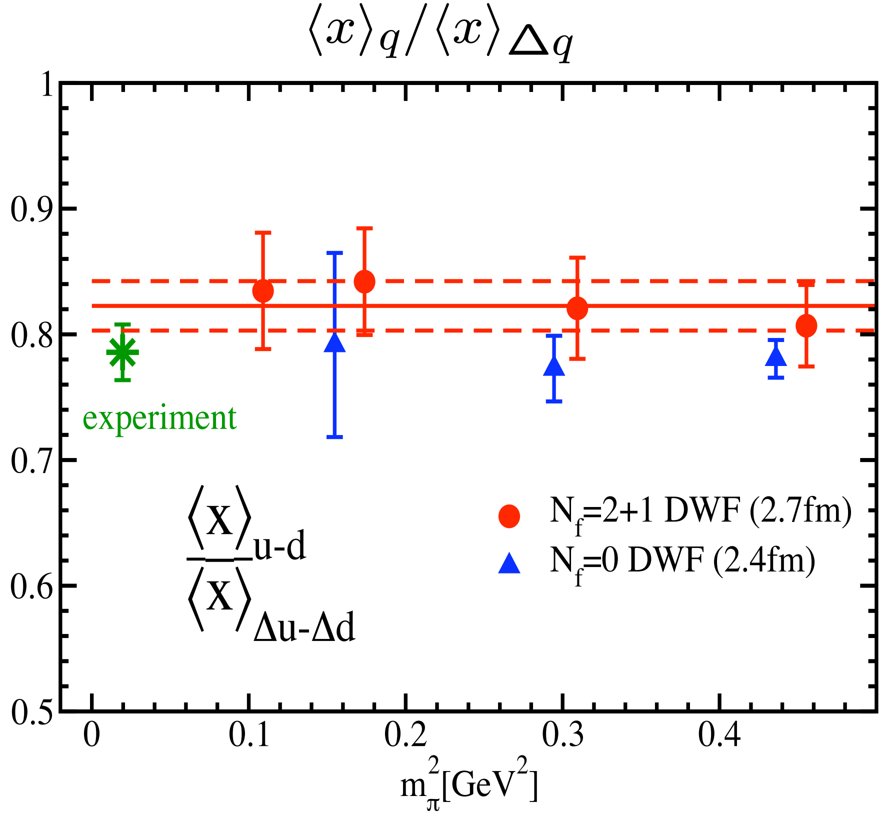
Renormalized at $\mu = 2$GeV with non-perturbative method

Tendency to approach experiment

Similar behavior in mixed action with perturbative renormalization

Need finite volume study to confirm the curvature

Ratio of moments

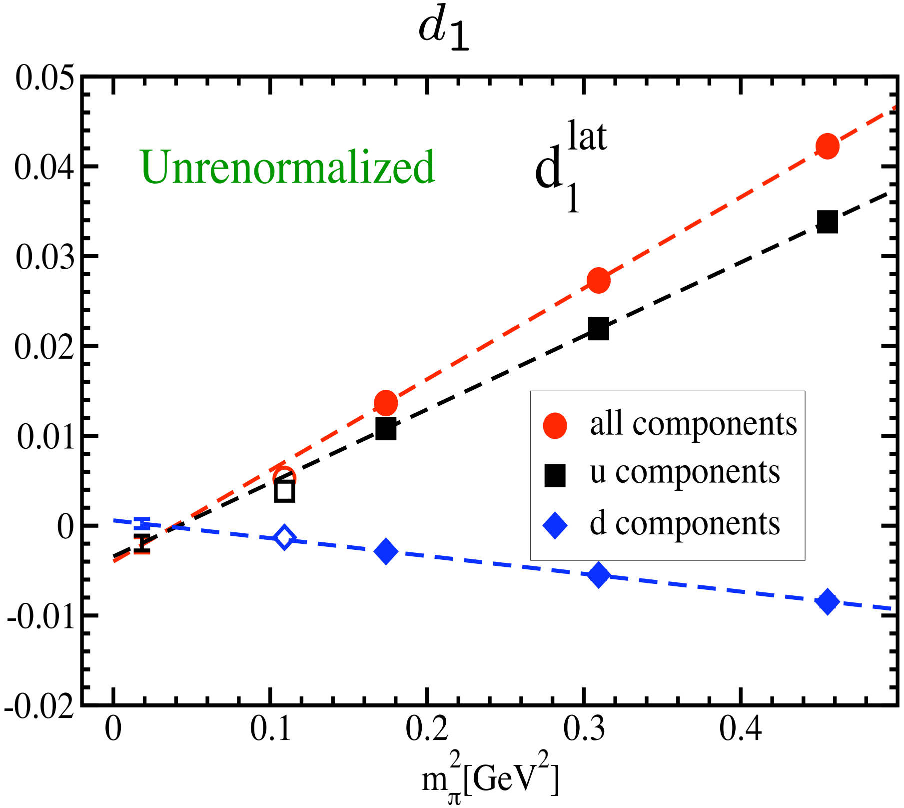


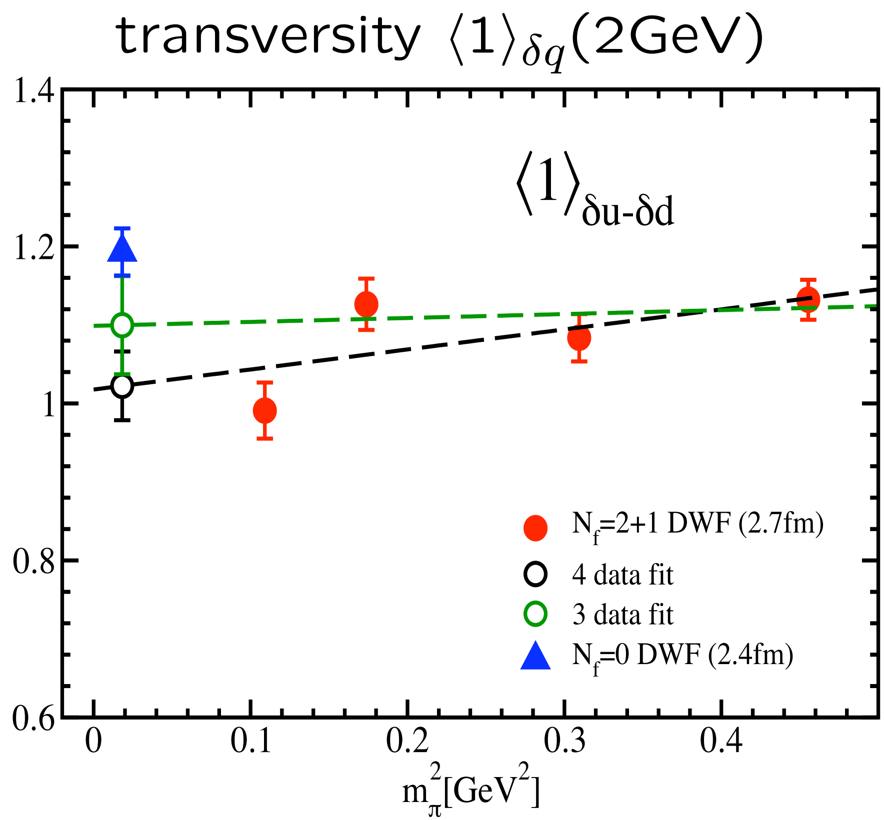
$N_f = 0$ DWF PRD73:094503

Renormalization factors are cancelled (due to good chiral symmetry).

Almost flat, and consistent with experiment as in $N_f = 0$ DWF

d_1 and transversity $\langle 1 \rangle_{\delta u - \delta d}$





$N_f = 2+1$ Mixed action arXiv:0705.4295; $N_f = 2$ DWF arXiv:0802.0863; $N_f = 0$ DWF PRD73:094503

Renormalized at $\mu = 2$GeV with non-perturbative method in $\langle 1 \rangle_{\delta q}$

d_1 decreases as $m_\pi \to 0$.

Finite volume effect in lightest point?

4. Summary

We calculated nucleon form factors and moments of structure functions with $N_f = 2+1$ dynamical domain wall fermions.

- Axial charge $g_A/g_V = G_A(0)/F_1(0)$
  - Large finite volume effect at $m_\pi = 330$ MeV on (2.7 fm)^3
  - g_A/g_V scaling in DWF and (Imp.) Wilson
  - $g_A/g_V = 1.20(6)(4)$ at physical pion mass
- Form factors
  - large finite volume effect in $G_A(q^2)$ and $G_P(q^2)$
  - less sensitive to finite volume in $F_1(q^2)$ and $F_2(q^2)$
- Moments of structure functions
  - $\langle x \rangle_q$, $\langle x \rangle_{\Delta q}$, d_1^q, $\langle 1 \rangle_{\delta q}$
  - Need finite volume effect study

Future work

- Large volume and lighter pion mass
- Finite volume study (difference $\langle N|V_\mu|N\rangle$ and $\langle N|A_\mu|N\rangle$)