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２重荷電交換反応によるハイパー核の生成とハイペロン混合

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3. **The (K$^-$,K$^+$) reactions**

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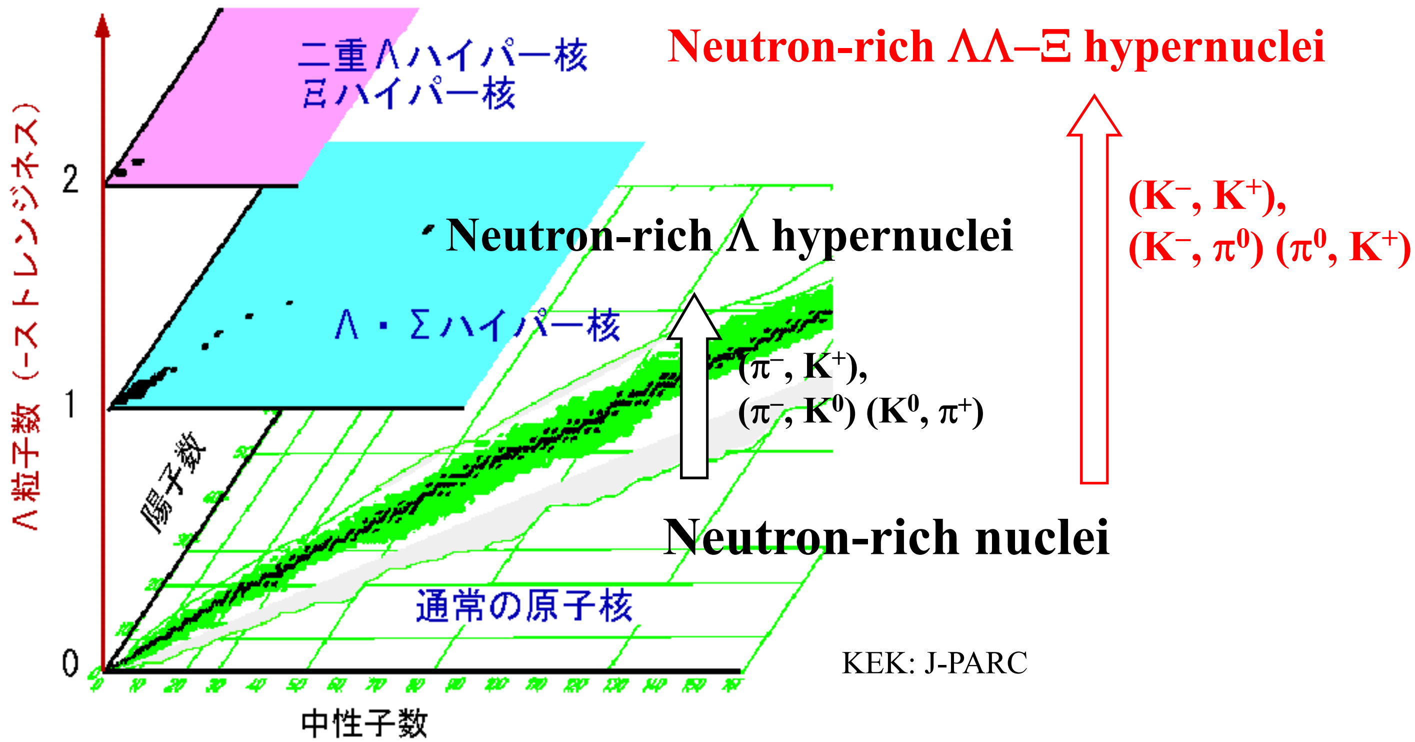
2重荷電交換反応

(π^-,K^+), (K^-,π^+) S=−1 Σ⁻ハイパー核の研究
中性子過剰Λハイパー核の研究

(K^-,K^+) S=−2 Ξ⁻ハイパー核の研究
（中性子過剰）ダブルΛハイパー核の研究

@J-PARC

バリオン相互作用の解明 → 中性子星の構造



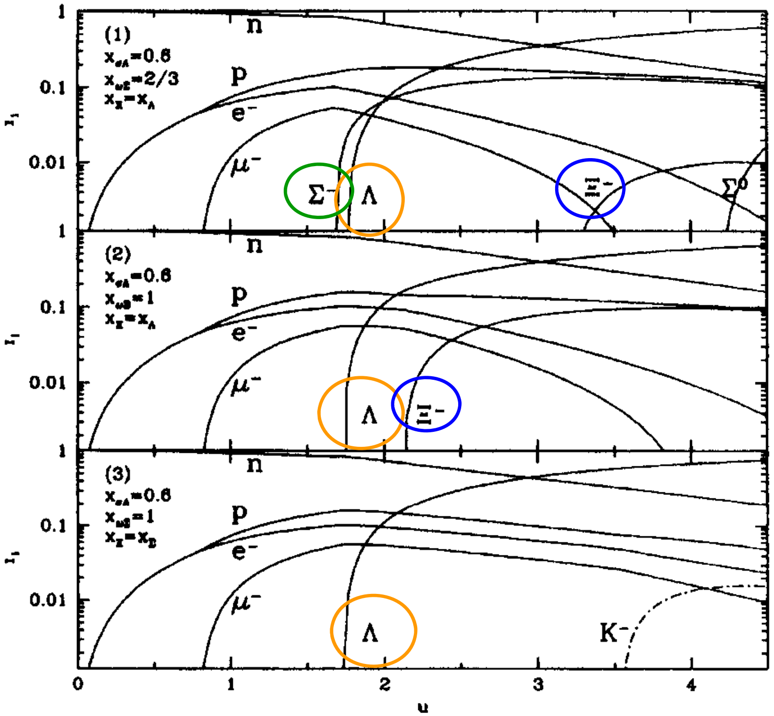
KEK: J-PARC

Neutron star core

"An interesting neutron-rich hypernuclear system"

Hyperon-mixing

Coupling constant ratio; $x_{iY} = g_{iY} / g_{iN}$ $(i = \sigma, \omega, \rho)$

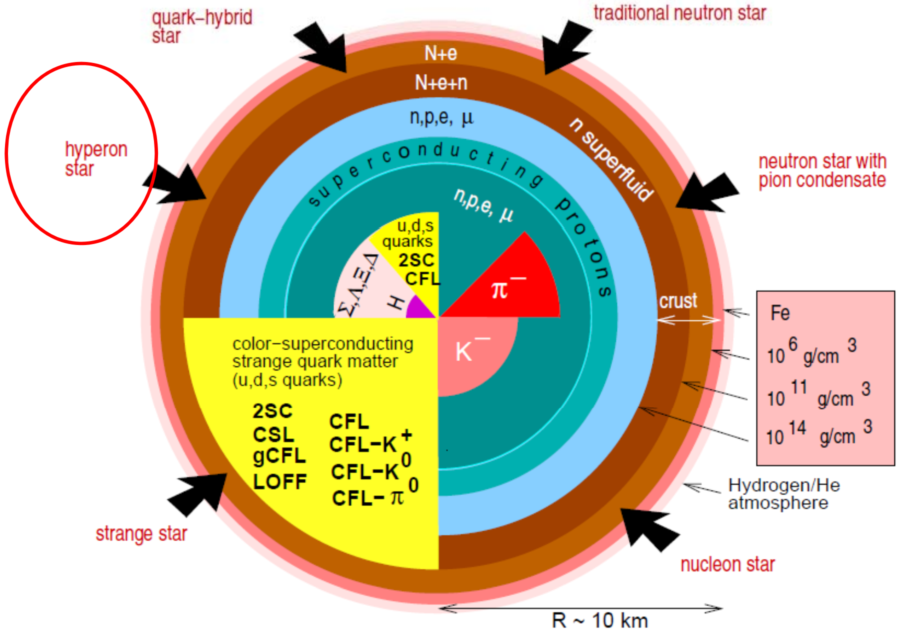


$U_\Sigma < 0$, $U_\Xi < 0$

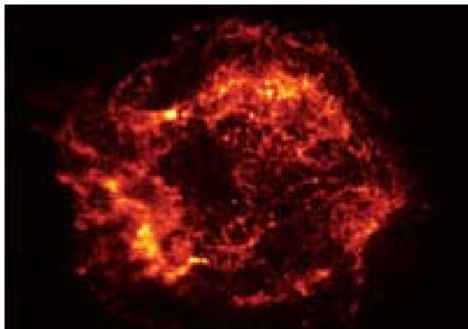
$U_\Sigma > 0$, $U_\Xi < 0$

$U_\Sigma > 0$, $U_\Xi > 0$

R. Knorren, M. Prakash, P.J.Ellis, PRC52(1995)3470



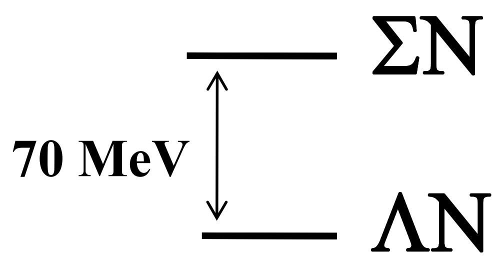
F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193



Cassiopeia A nebula
NASA/CXC/SAO.

The (π^-,K^+) reactions

Production of neutron-rich Λ hypernuclei
via Σ^- doorways



"Feasibility of extracting a Σ^- admixture probability in the neutron-rich hypernucleus"

$^{10}_{\Lambda}$**Li**

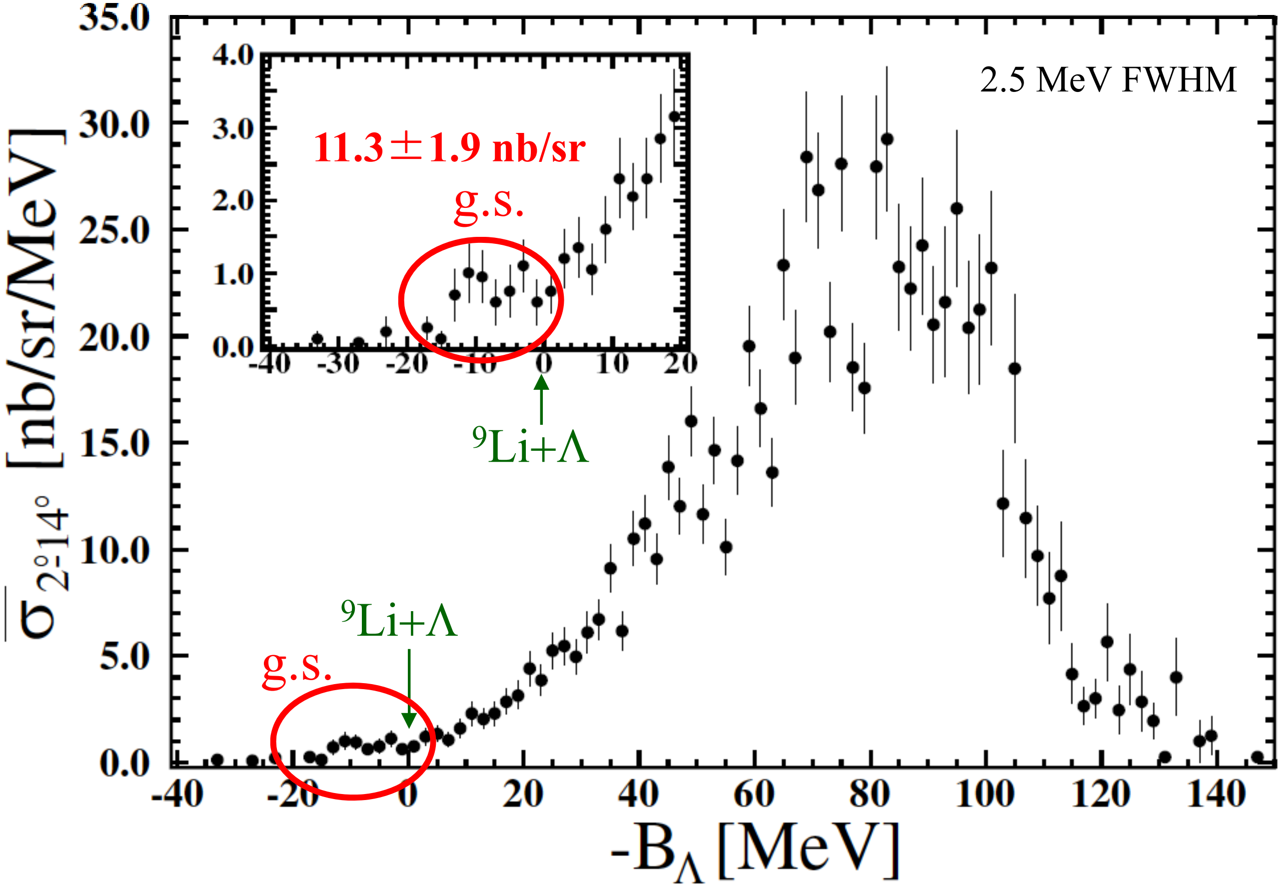
T. Harada, A. Umeya, Y. Hirabayashi, PRC79 (2009) 014603

First production of neutron-rich Λ hypernuclei

$^{10}B(\pi^-, K^+)^{10}_{\Lambda}Li$

Λ spectrum by DCX (π⁻,K⁺) reaction at 1.2GeV/c

KEK-PS-E521 P. K. Saha, et al., PRL94(2005)052502



Cross sections

- p_π=1.20 GeV/c

$$\frac{d\sigma}{d\Omega_L} \approx 11.3 \pm 1.9 \text{ nb/sr}$$

- p_π=1.05 GeV/c

$$\frac{d\sigma}{d\Omega_L} \approx 5.8 \pm 2.2 \text{ nb/sr}$$

～1/1000

$^{12}C(\pi^+, K^+)^{12}_{\Lambda}C$ (1.2 GeV/c)

17.5±0.6 μb/sr

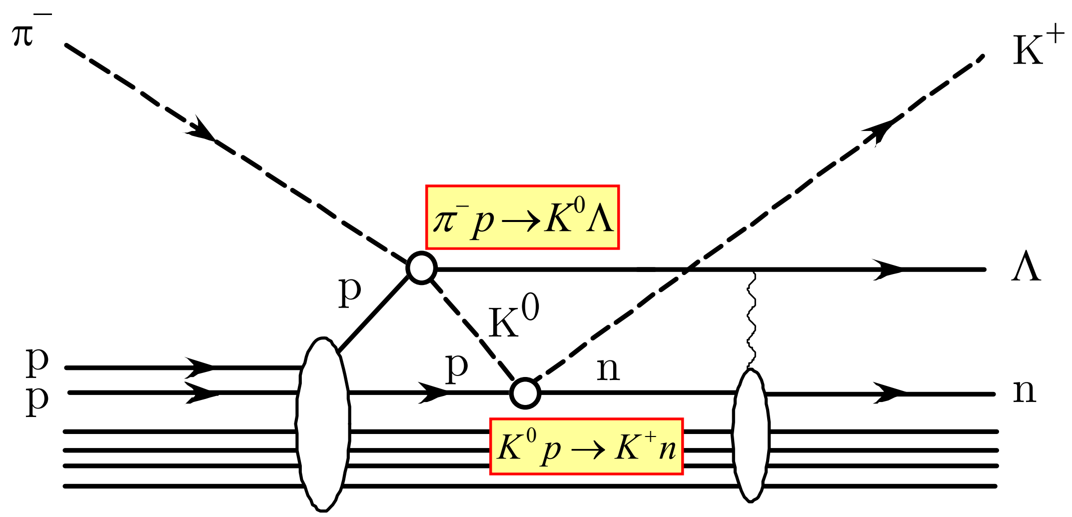
The DCX (π⁻,K⁺) *reaction at 1.2GeV/c can produce the neutron-rich Λ hypernucler states, whereas the cross section is as small as 1/1000 of the* (π⁺,K⁺) *reaction.*

(π^-, K^+) – Double Charge Exchange (DCX) Reaction

***Two-step process*:**

$$\pi^- p \rightarrow K^0 \Lambda$$
$$K^0 p \rightarrow K^+ n$$

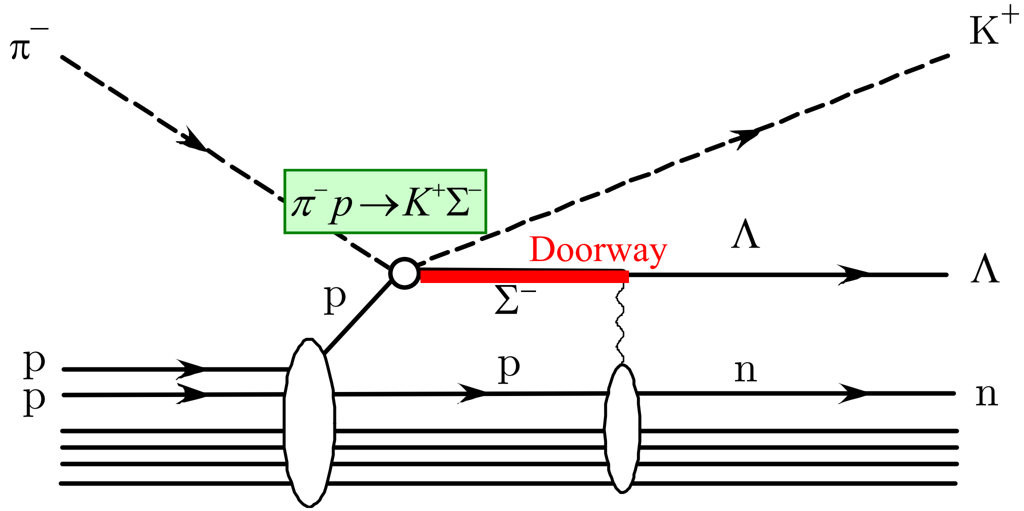
$$\pi^- p \rightarrow \pi^0 n$$
$$\pi^0 p \rightarrow K^+ \Lambda$$



***One-step process*:**

$$\pi^- p \rightarrow K^+ \Sigma^-$$
$$\Sigma^- p \leftrightarrow \Lambda n$$

via Σ^- doorways caused by ΛN-ΣN coupling

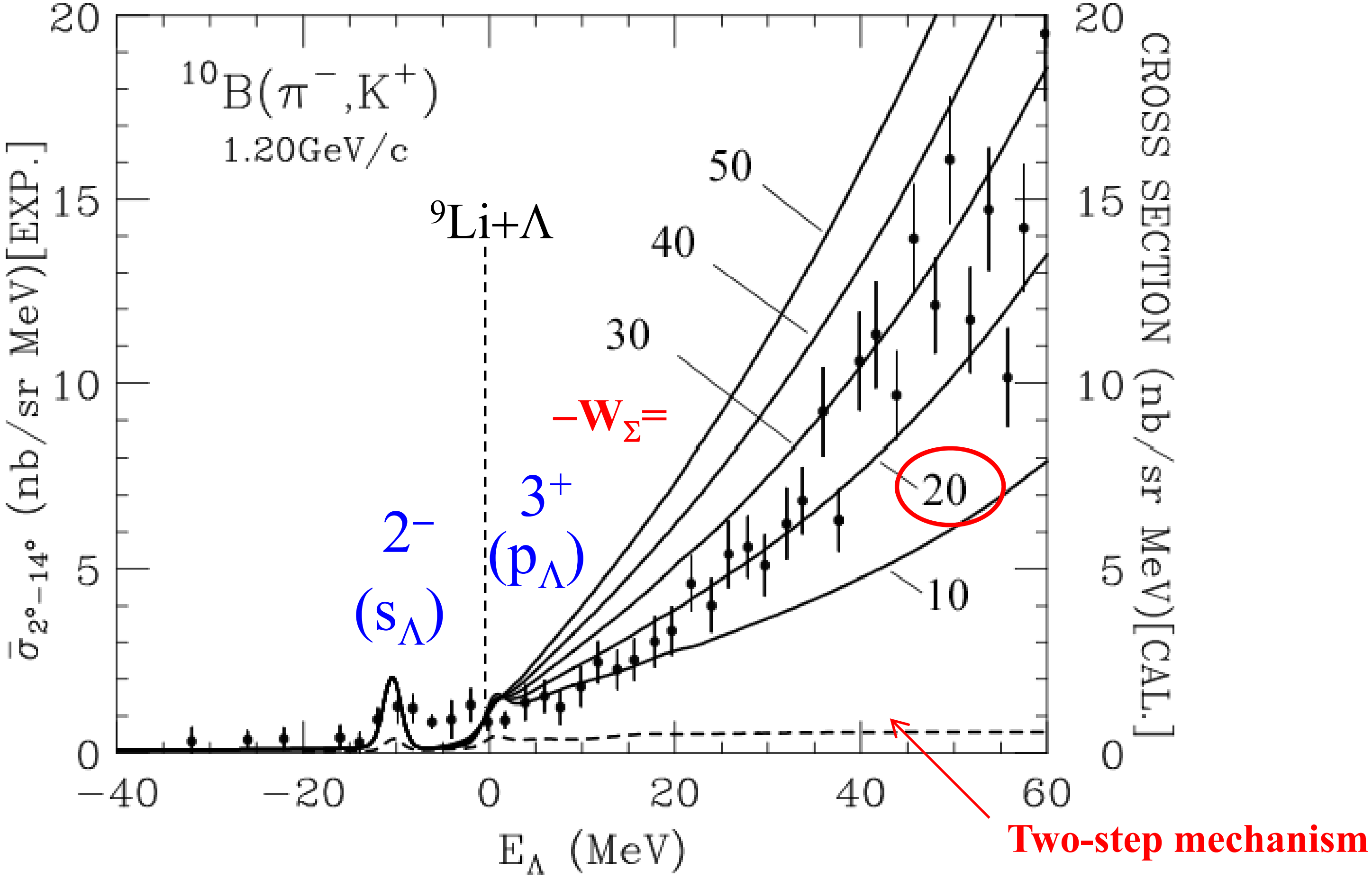


Results (1) Λ spectrum by DCX (π^-,K^+) reaction at 1.2GeV/c

Spreading potential dep. W_Σ

$U_X = 11$ MeV is fixed. P_{Σ^-}=0.57%

^{10}B



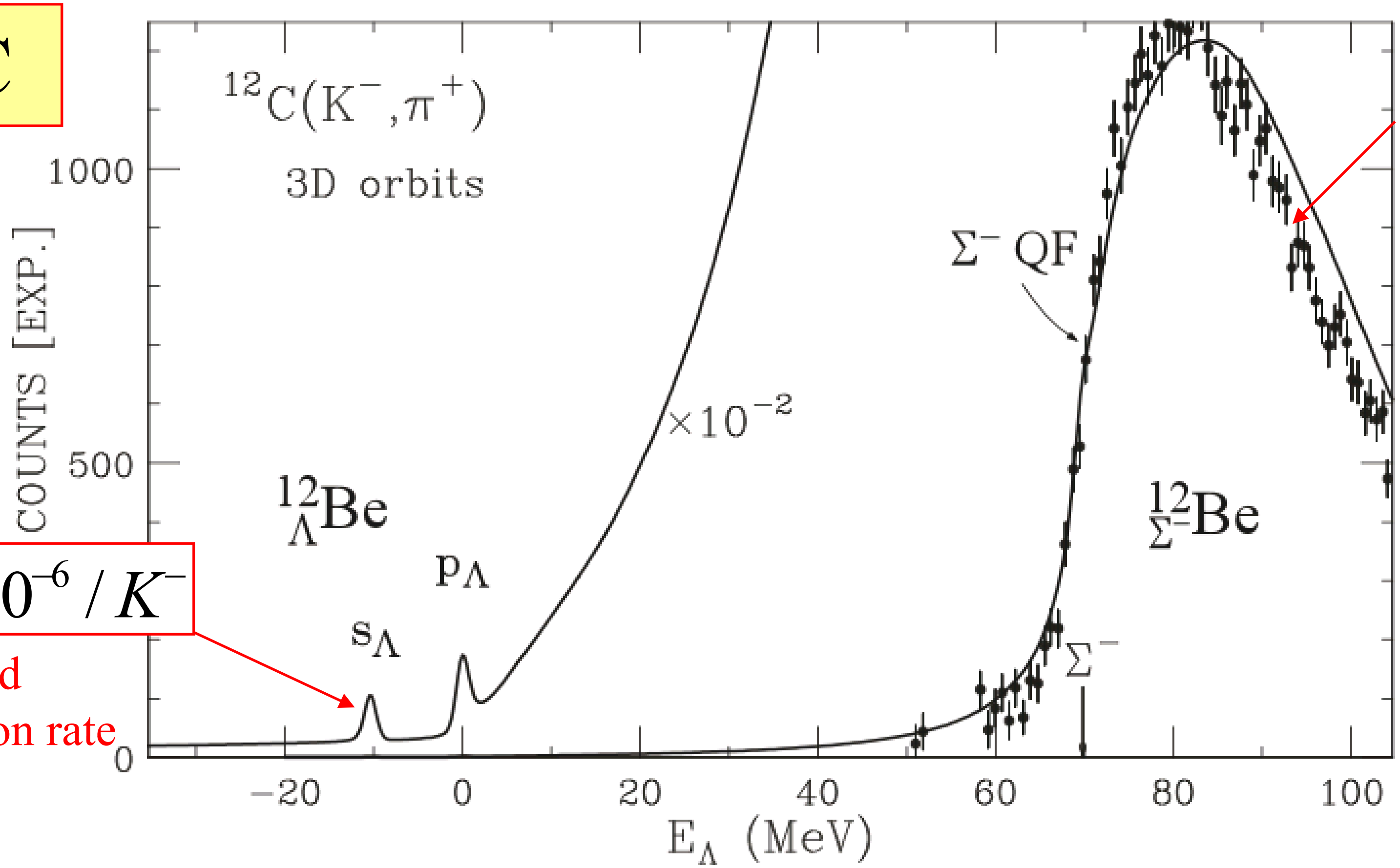
Harada, Umeya, Hirabayashi, PRC79(2009)014603

The calculated spectrum with $-W_\Sigma$= 20-30 MeV can reproduce the shape of the data in the continuum region, and these values of $-W_\Sigma$ are consistent with the analysis of Σ^- QF production by the (π^-, K^+) reactions.

Calculation for DCX (stopped K^-, π^+) reactions

If the Σ^- admixture probability of ~0.6 % is <u>assumed</u> in $^{12}_{\Lambda}$Be, we demonstrate the (stopped K^-,π^+) spectrum on a ^{12}C target.

^{12}C



Early KEK data

$\sim 4\times10^{-6} / K^-$

Integrated production rate

This result is consistent with recent data form DAΦNE.

The DAΦNE data: $U.L. \sim (2.0\pm0.4)\times10^{-5} / K^-$ M.Agnello, et al.,PLB640(2006)145.

Remarks

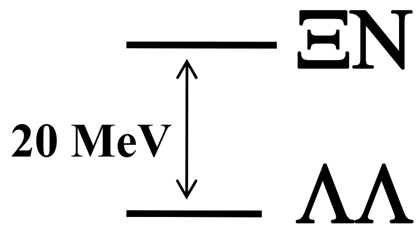
The calculated spectrum by the one-step mechanism fully explains the $^{10}B(\pi^-, K^+)$ data.

The Σ^- admixture probability is on the order of 10^{-1} % for $^{10}_{\Lambda}Li$ due to Λ-Σ couplings.

The (π^-, K^+) reactions can provide the ability to extract a production mechanism and **Σ^- admixture probabilities** of neutron-rich Λ hypernuclei from experimental data.

The (K^-, K^+) reactions

Production of S = -2 hypernuclei
via Ξ^- doorways



“Production of doubly strange hypernuclei via Ξ^- doorways in the $^{16}O(K^-, K^+)$ reaction at 1.8GeV/c”

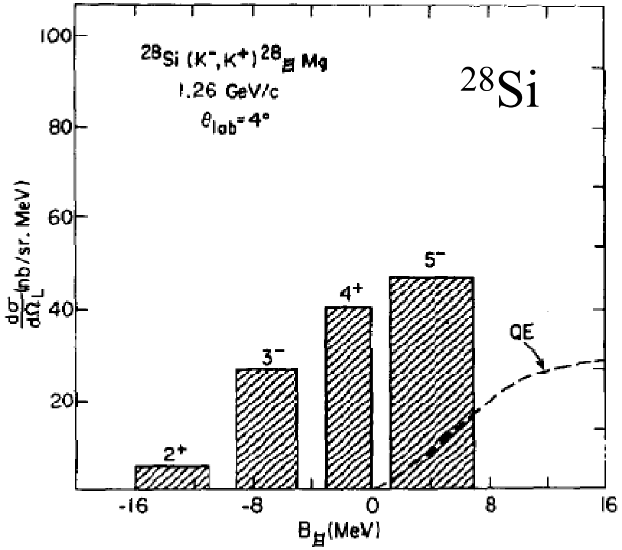
T.Harada, Y.Hirabayashi, A.Umeya, Phys. Lett. B690(2010)363.

$$^{16}_{\Lambda\Lambda}C - {}^{16}_{\Xi^-}C$$

Studies of interaction of Ξ⁻ hyperon with the nucleus

V_{Ξ} ?

Studies of Ξ-hypernuclei via (K–,K+) reactions



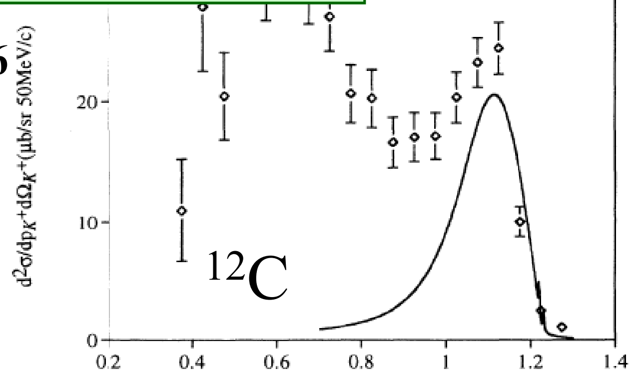
C.B. Dover, A.Gal, Ann. Phys. 146 (1989) 309.

Analysis of the nuclear K⁻p → K⁺Ξ⁻ reaction
The potential depth parameters is obtained by

$$V_{\Xi}^{0} = -24 \pm 4 \text{ MeV} \quad \text{for} \quad r_0 = 1.1 \text{ fm} \quad (W_{\Xi}^{0} \simeq -1 \text{ MeV})$$

Data: T.Iijima, et al.,NPA546(1992)588.

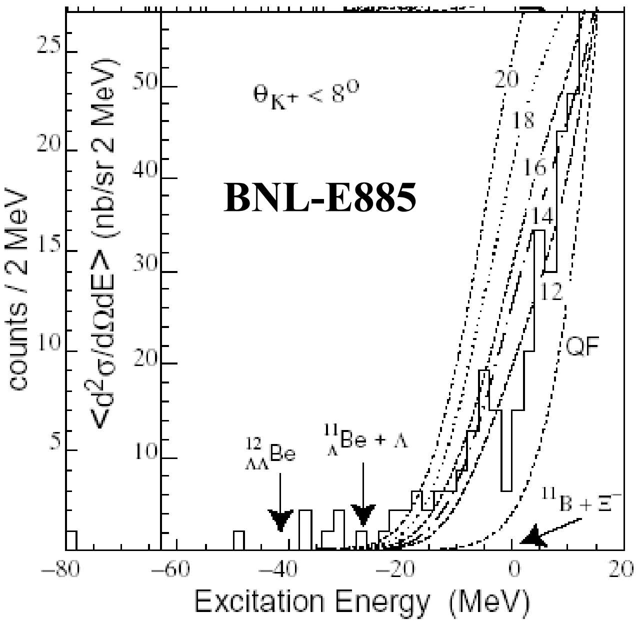
Coulomb-assisted Ξ⁻ hybrid bound states



Tadokoro et al.,PRC51(1995)2656

Analysis of 12C(K-,K+) spectrum suggests

$$V_{\Xi}^{0} \simeq -16 \text{ MeV}$$



DWIA analysis of ¹²C(K–,K+) data at 1.8GeV/c

P.Khaustov et al., PRC61(2000)054603

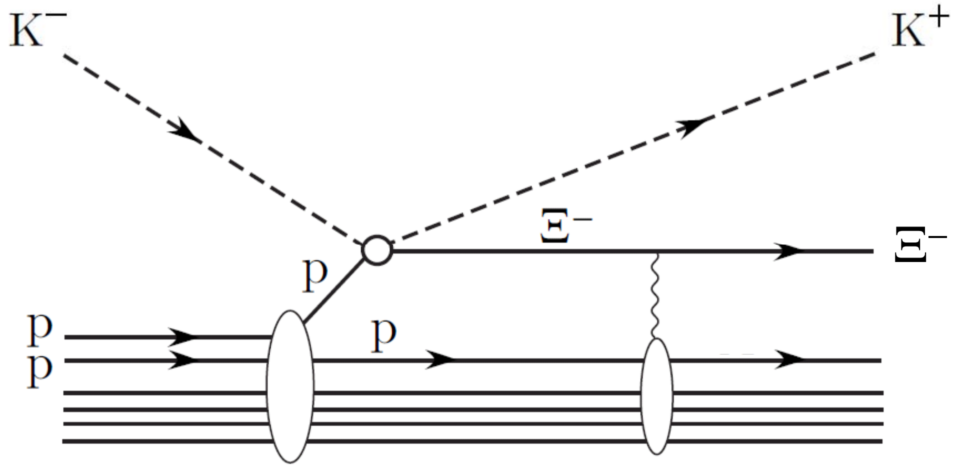
Comparison with the data in the Ξ bound region

$$V_{\Xi}^{0} \simeq (-12)\text{-}(-14) \text{ MeV}$$ ⟷

Kohno-Fujiwara,
PRC79(2009)054318
" V_{Ξ}∼0 MeV"

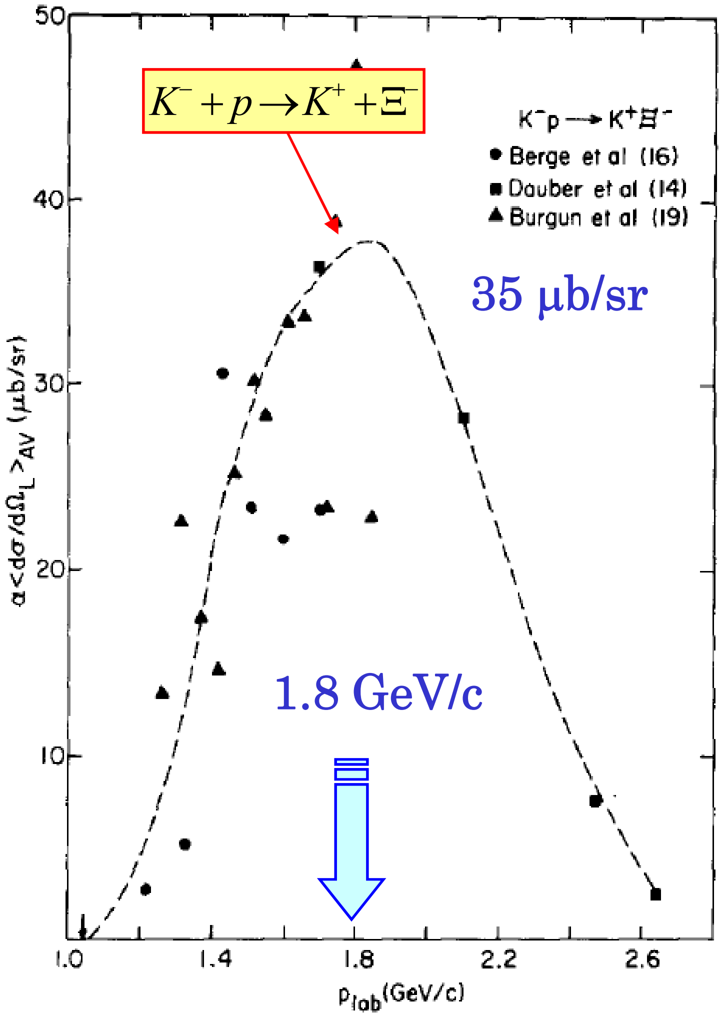
(K $^-$,K$^+$) －Double Charge Exchange (DCX) Reactions

$$K^- + p \to K^+ + \Xi^-$$



Elementary cross sections

Dover and Gal, Ann. Phys, 146 (1983) 309.



Green's function method

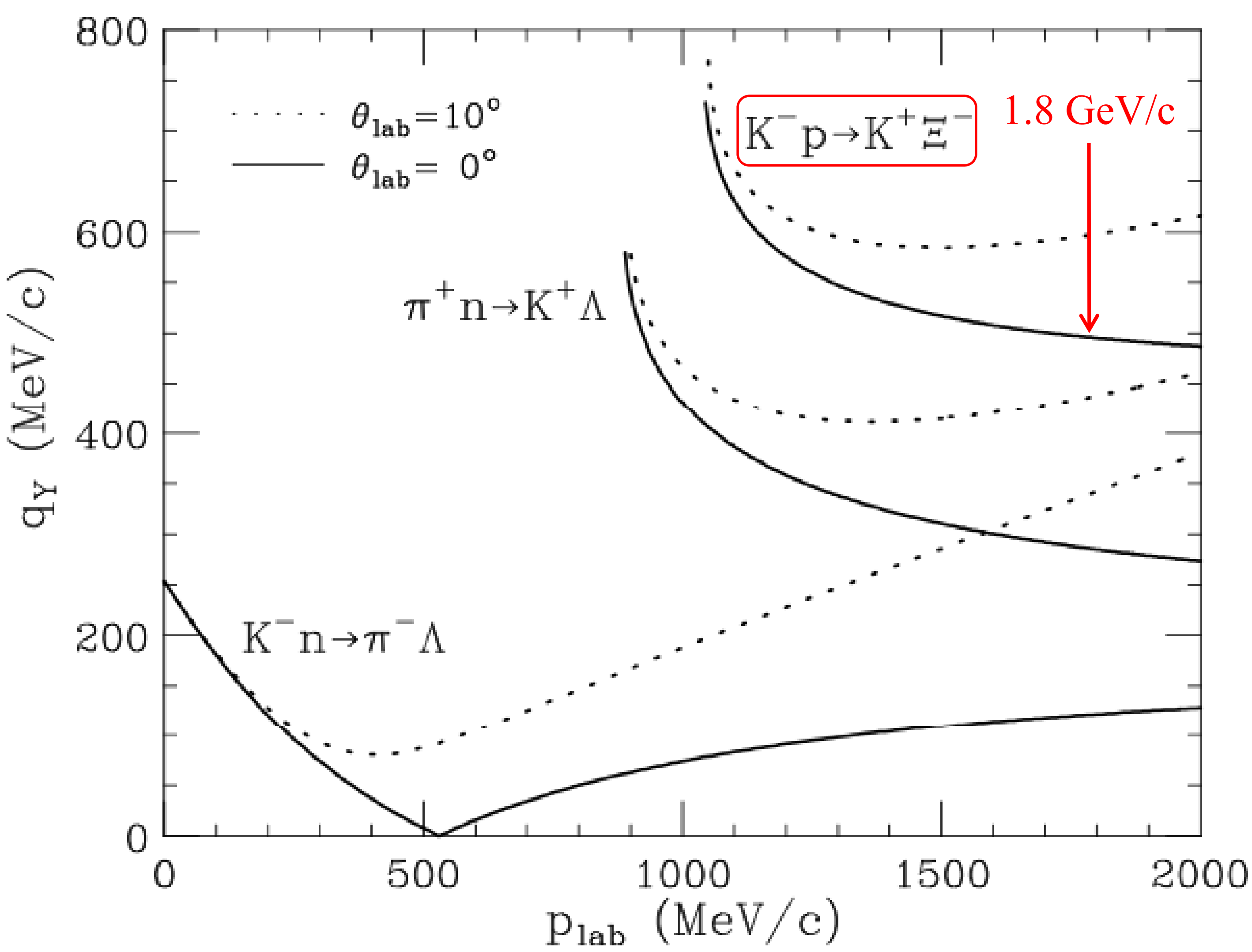
Morimatsu, Yazaki, NPA483(1988)493

$$S(\omega) = \sum_f |< f \,|\, \hat{O} \,|\, i >|^2 \; \delta(\omega + E_K - E_\pi)$$

$$= -\frac{1}{\pi} \mathrm{Im} \int d\mathbf{r} d\mathbf{r}' F^\dagger(\mathbf{r}) G(\omega + i\varepsilon; \mathbf{r}, \mathbf{r}') F(\mathbf{r}')$$

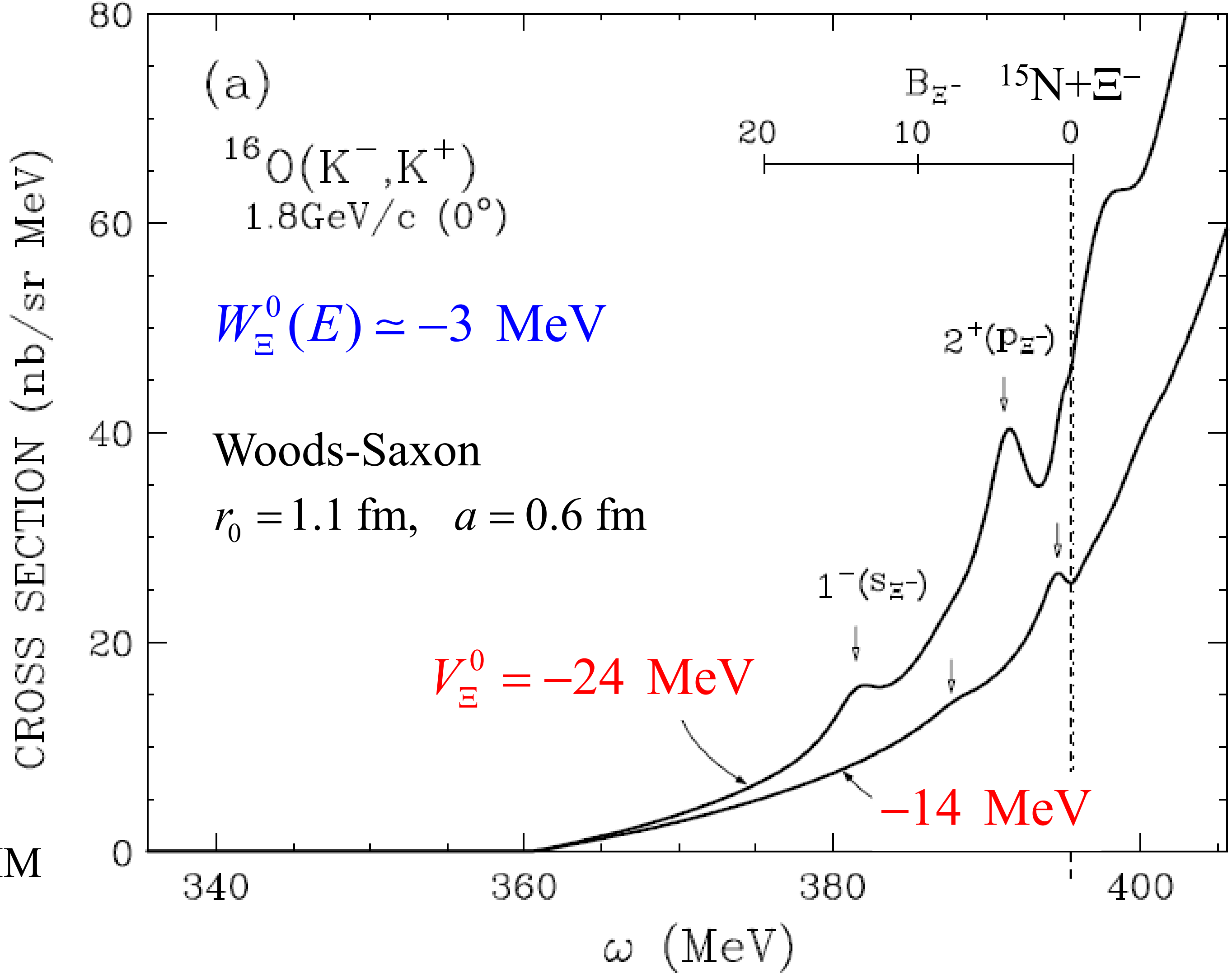
Green's function

Momentum transfer to the Λ, Ξ⁻ hyperon



Ξ^- spectrum in DCX (K^-,K^+) reactions at 1.8GeV/c

^{16}O

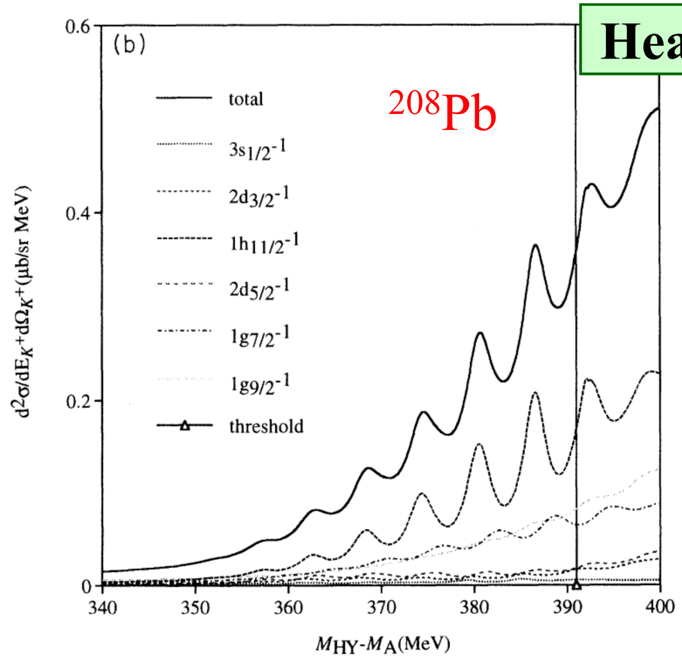


1.5MeV FWHM

- ***Spin-stretched Ξ^- states*** ***can be populated due to*** ***the high momentum transfer.***

$d\sigma/d\Omega[^{15}N(1/2^-)\otimes s_\Xi](1-) = 6$ nb/sr, $d\sigma/d\Omega[^{15}N(1/2^-)\otimes p_\Xi](2+) = 9$ nb/sr for $V_\Xi = -14$ MeV.

Recent Theoretical calculations



Heavy Ξ^--hypernuclei via (K–,K+) reaction

Tadokoro et al.,PRC51(1995)2656

Analysis of ^{12}C(K-,K+) spectrum suggests

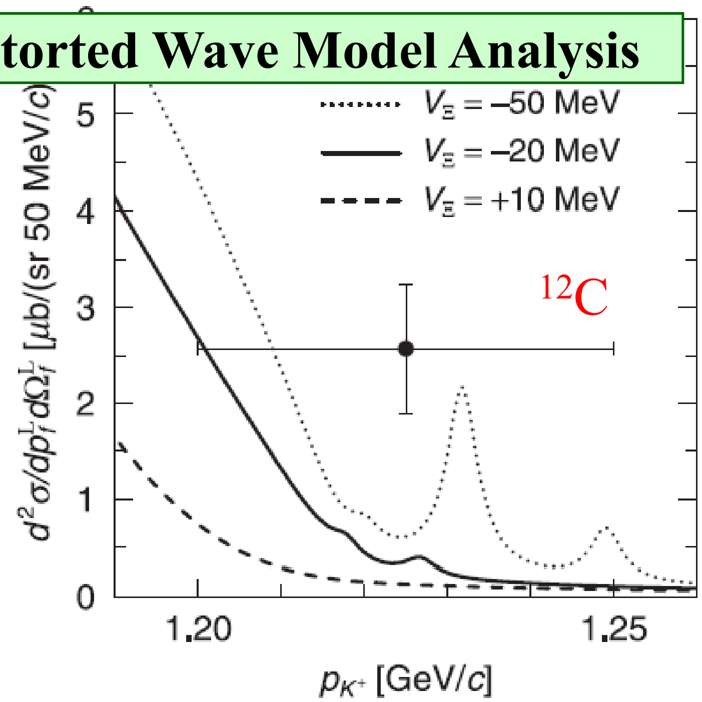
$$V_\Xi^0 = -16 \text{ MeV} \quad \text{for} \quad r_0 = 1.1 \text{ fm}$$

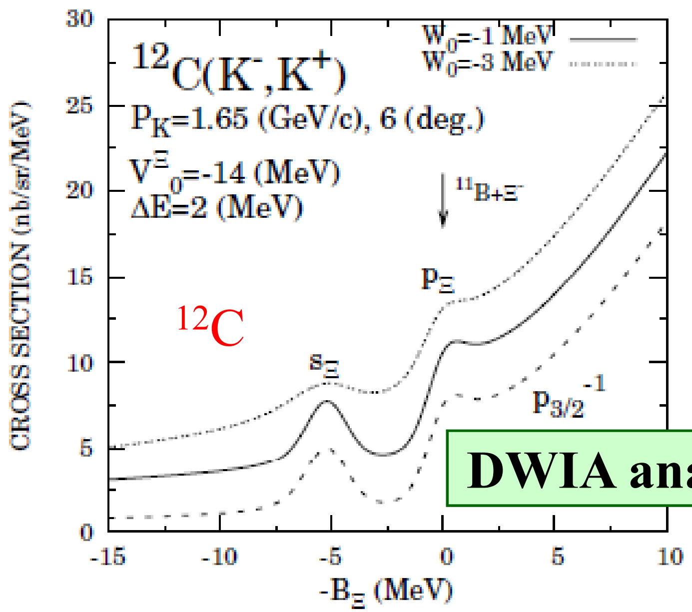
Coulomb-assisted hybrid bound state

Semi-Classical Distorted Wave Model Analysis

S. Hashimoto, et al., PTP119 (2008)1005

$$V_\Xi^0 = -50, -20, +20 \text{ MeV}$$



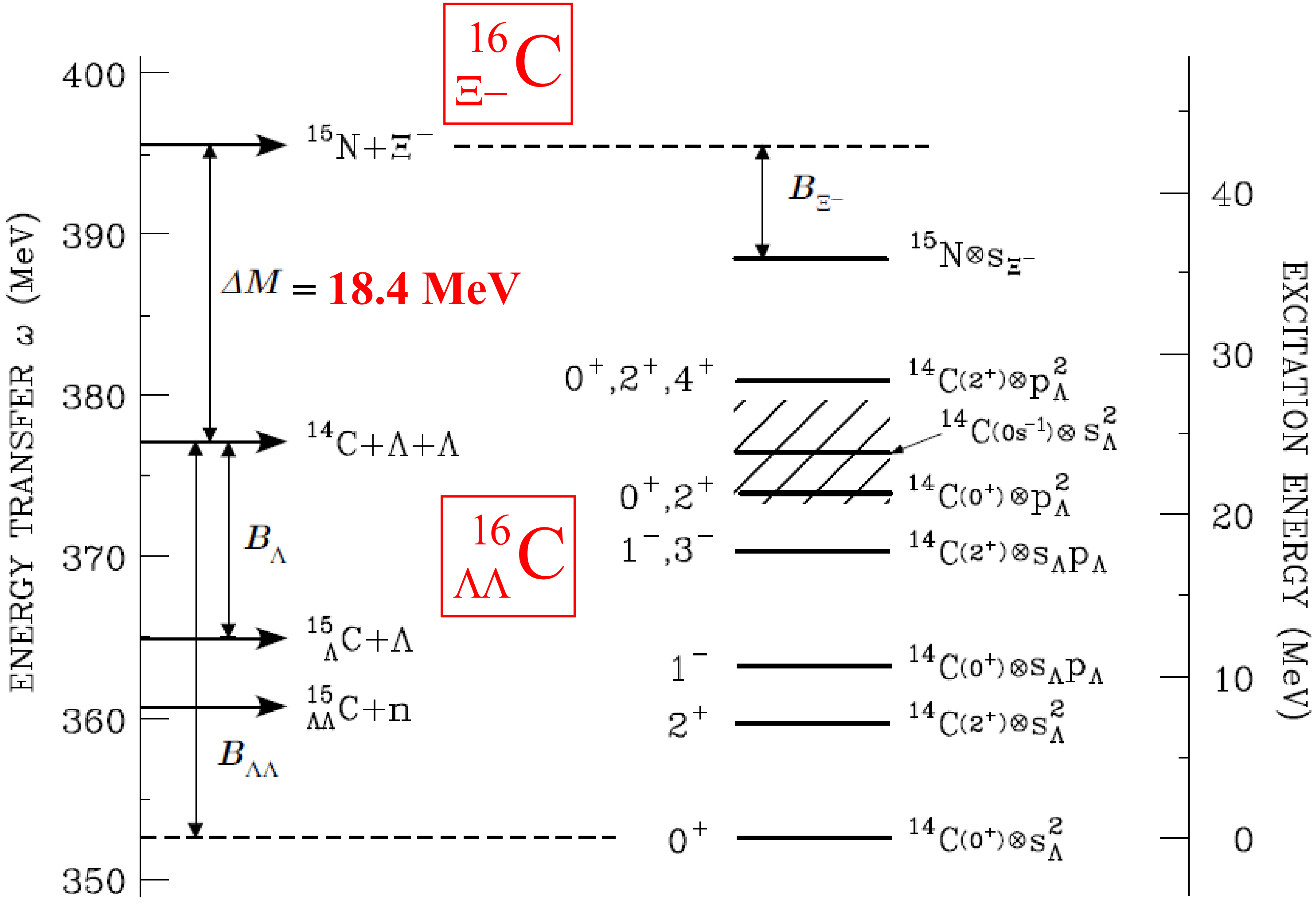


$$V_\Xi^0 \simeq -14 \text{ MeV}$$

DWIA analysis of the nuclear (K–,K+) reaction at 1.65GeV/c

H. Maekawa, et al., arXiv:0704.3929

Energy spectrum of Ξ^- and $\Lambda\Lambda$ nuclei on a ^{16}O target



. *The energy shifts $\Delta B_{\Lambda\Lambda}$ are not taken into account.*

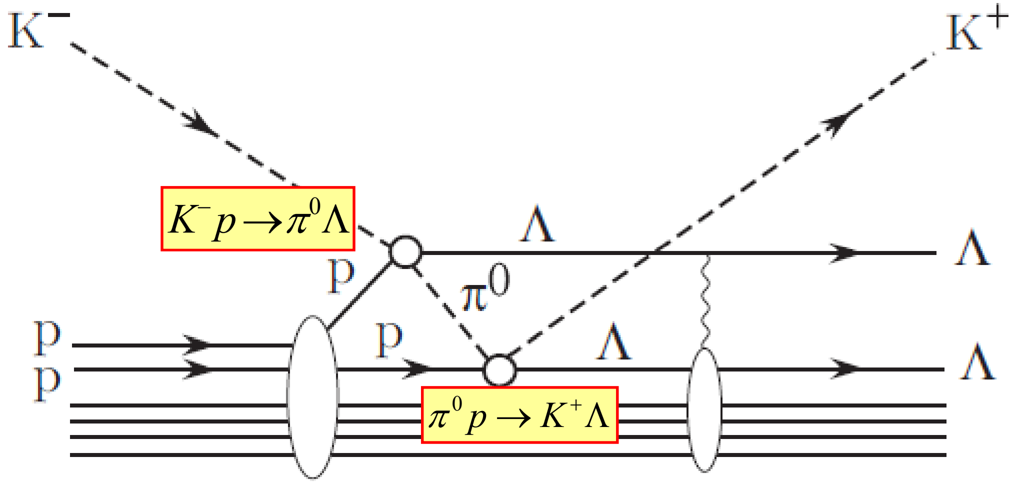
See also Dover, Gal and Millener, NPA572(1994) 85.

(K⁻,K⁺) －Double Charge Exchange (DCX) Reactions

· ***Two-step process:***

$$K^- p \to \pi^0 \Lambda$$

$$\pi^0 p \to K^+ \Lambda$$



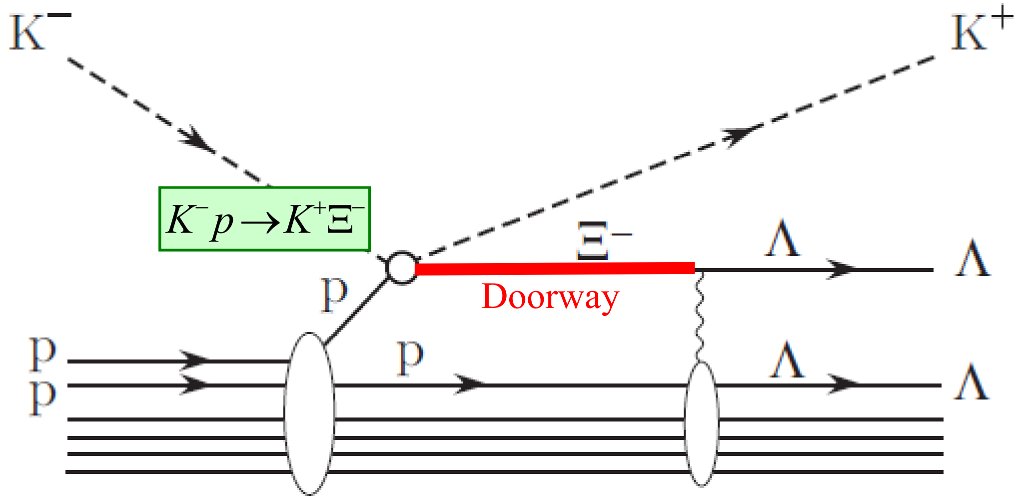
(a)

· ***One-step process:***

$$K^- p \to K^+ \Xi^-$$

$$\Xi^- p \leftrightarrow \Lambda\Lambda$$

ΞN-ΛΛ coupling



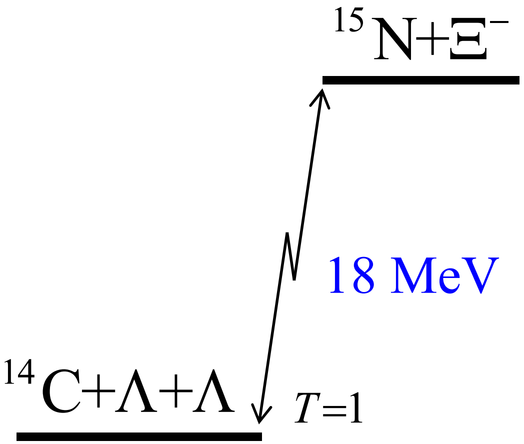
$$\boxed{t^*_{K^- p \to K^+ \Xi^-} G_{\Xi^-} t_{K^- p \to K^+ \Xi^-}} \simeq t^*_{K^- p \to K^+ \Xi^-} G^{(0)\dagger}_{\Xi^-} U^{\dagger}_{\Xi - \Lambda\Lambda} G^{(0)}_{\Lambda\Lambda} U_{\Lambda\Lambda - \Xi} G^{(0)}_{\Xi^-} t_{K^- p \to K^+ \Xi^-} + \cdots$$

Models for calculations

Single-particle shell model wf.

$$\left|{}^{16}_{\Lambda\Lambda\text{-}\Xi}\mathrm{C}\right\rangle = \sum_{JJ''j_2}\left[[\Phi_J({}^{14}\mathrm{C})\phi^{(\Lambda)}_{j_1}(\mathbf{r}_{\Lambda_1})]_{J''}\,\boxed{\phi^{(C\Lambda)}_{j_2}(\mathbf{r}'_{\Lambda_2})}\right]$$

$$+\sum_{JJ'j_3}\left[[\Phi_J({}^{14}\mathrm{C})\phi^{N}_{j_p}(\mathbf{r}_p)]^{({}^{15}\mathrm{N})}_{J'}\,\boxed{\phi^{(C\Xi-)}_{j_3}(\mathbf{r}'_{\Xi})}\right]$$



Hyperon-nucleus potentials

Woods-Saxon + derivative form $R = r_0(A-1)^{1/3}\,\mathrm{fm},\quad r_0 = 1.080 + 0.395A^{-2/3}\ \mathrm{fm},\quad a = 0.6\ \mathrm{fm}$

$$U_{Y=\Lambda,\Xi} = V_Y f(r,R,a) + iW_Y(r,R',a') + iW^{(D)}_Y g(r,R',a')$$

-29.34 MeV for Λ

Spreading potential: energy-dependent g(E) = excited states

$$U_X(r) = \left\langle [\Phi_{J'}({}^{15}\mathrm{N})\otimes \mathcal{Y}^{(\Xi^-)}_{j'\ell's'}(\hat{\boldsymbol{r}})]_{J_B}\right| \sum v_{\Xi N,\Lambda\Lambda}(\boldsymbol{r}'_i,\boldsymbol{r}) \times \left| [[\Phi_J({}^{14}\mathrm{C}),\varphi^{(\Lambda)}_{j_1}]_{J''}\otimes \mathcal{Y}^{(\Lambda)}_{j\ell s}(\hat{\boldsymbol{r}})]_{J_B}\right\rangle$$

coupling Λ-Ξ pot.

zero-range interaction: $v_{\Lambda\Lambda\text{-}\Xi N} = v^0_{\Lambda\Lambda\text{-}\Xi N}\,\delta(\mathbf{r}-\mathbf{r}')$

volume integral: $v^0_{\Lambda\Lambda\text{-}\Xi N} = \int v_{\Lambda\Lambda\text{-}\Xi N}(\mathbf{r})\,d\mathbf{r}$

Coupled-channel Green's function

Green's function method

Morimatsu, Yazaki, NPA483(1988)493

$$S(\omega)=\sum_f |<f|\hat{O}|i>|^2\,\delta(\omega+E_K-E_\pi)=-\frac{1}{\pi}\mathrm{Im}\int d\mathbf{r}d\mathbf{r}'F^\dagger(\mathbf{r})\mathrm{G}(\omega+i\varepsilon;\mathbf{r},\mathbf{r}')F(\mathbf{r}')$$

Green's function

The completeness relation including the intermediate states

$$G_\ell(\omega;r',r)=\sum_n\frac{\varphi_{n\ell}(r')(\tilde{\varphi}_{n\ell}(r))^*}{\omega-E_{n\ell}+i\epsilon}+\frac{2}{\pi}\int_0^\infty dk\frac{k^2S_\ell(k)u_\ell(k,r')(\tilde{u}_\ell(k,r))^*}{\omega-E_k+i\epsilon}$$

Coupled-channel Green's function

T.Harada, NPA672(2000)181

$$\hat{\mathbf{G}}(\omega)=\hat{\mathbf{G}}^{(0)}(\omega)+\hat{\mathbf{G}}^{(0)}(\omega)\hat{\mathbf{U}}\hat{\mathbf{G}}(\omega)$$

$$\hat{\mathbf{G}}^{(0)}(\omega)=\begin{bmatrix}G^{(0)}_{\Lambda\Lambda} & \\ & G^{(0)}_{\Xi}\end{bmatrix}\qquad \hat{\mathbf{U}}=\begin{bmatrix}U_{\Lambda\Lambda} & U_{\Lambda\Lambda-\Xi}\\ U_{\Xi-\Lambda\Lambda} & U_{\Xi}\end{bmatrix}$$

$$\mathrm{Im}\,\hat{G}=\underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}^{(0)}_{\Lambda\Lambda}\}\hat{\Omega}^{(-)}}_{\Lambda\Lambda\text{ escape}}+\underbrace{\hat{\Omega}^{(-)\dagger}\{\mathrm{Im}\,\hat{G}^{(0)}_{\Xi}\}\hat{\Omega}^{(-)}}_{\Xi^-\text{ escape}}+\underbrace{\hat{G}^\dagger\{W_{Y,T}\}\hat{G}}_{\text{Spreading}}$$

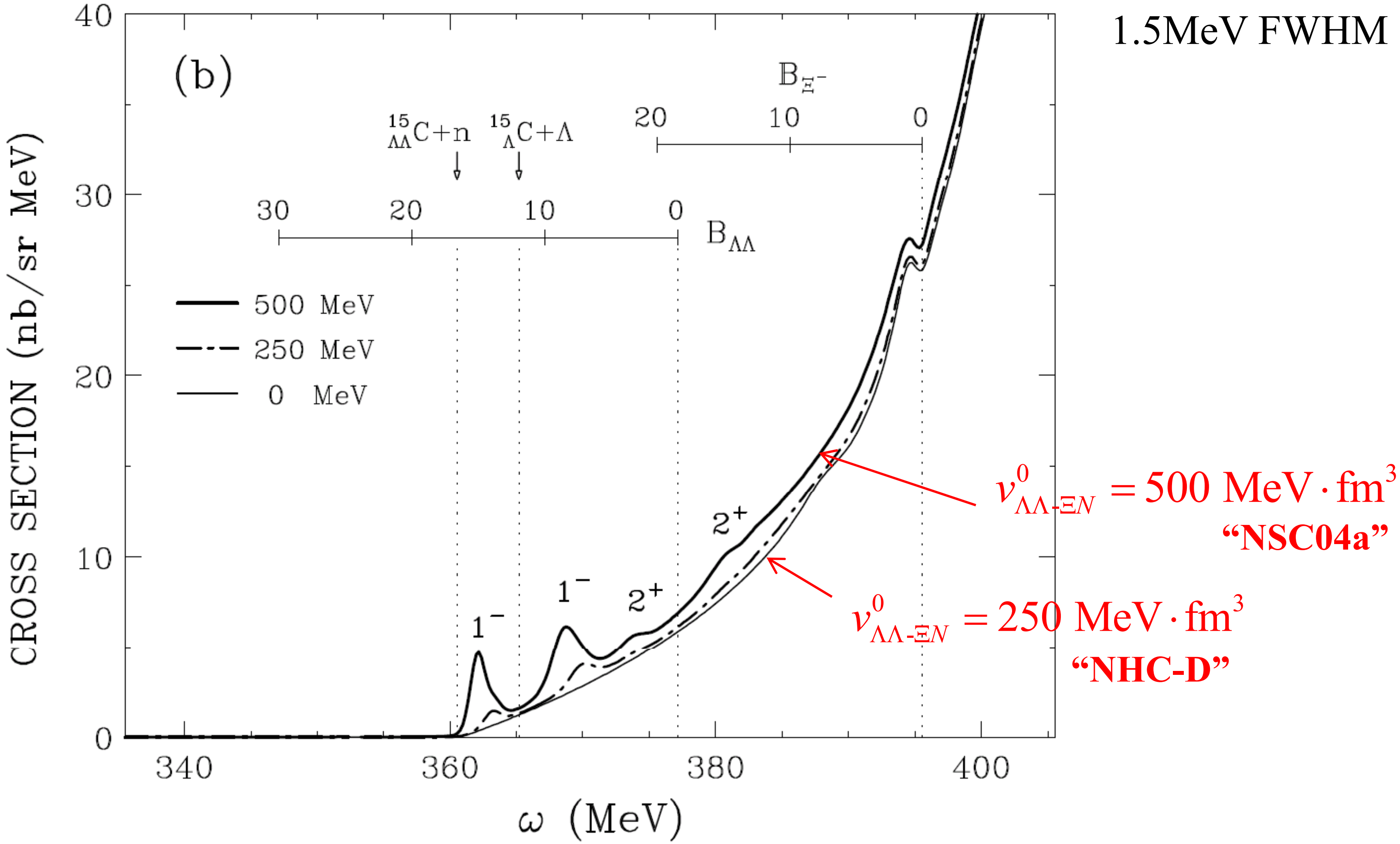
ΛΛ escape

Ξ^- escape

Spreading (nuclear-core breakup)
= Complicated excited states

Ξ^- spectrum in DCX (K^-,K^+) reactions at 1.8GeV/c

^{16}O



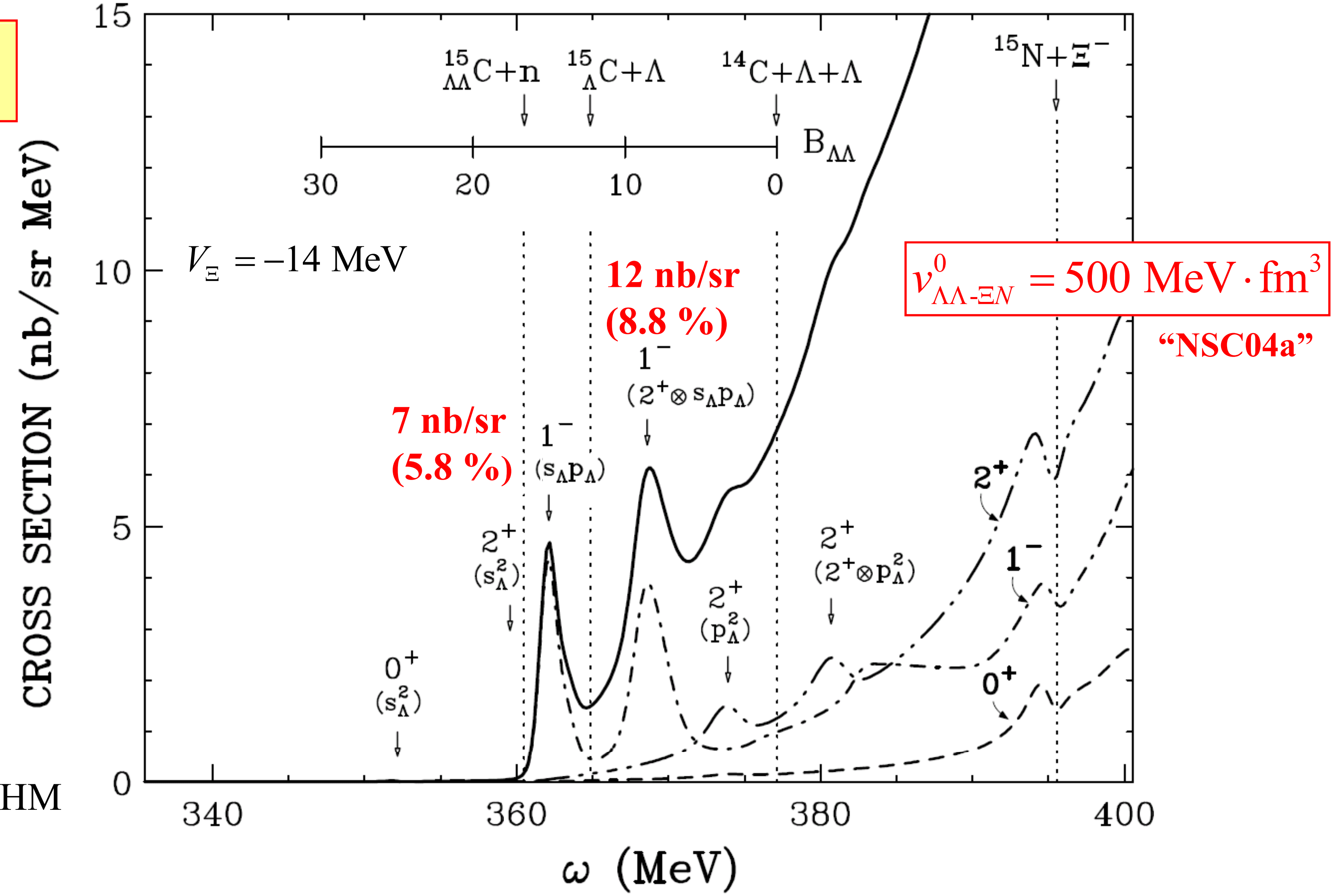
- *The shape of the calculated spectrum is quite sensitive to the value of* $v^0_{\Xi N,\Lambda\Lambda}$.
- *Significant peaks of the 1− excited states with*

  $^{14}C(0^+)\otimes s_\Lambda p_\Lambda$ *at* ω = 362.1 MeV ($B_{\Lambda\Lambda}$ = 15.1 MeV)

  $^{14}C^*(2^+)\otimes s_\Lambda p_\Lambda$ *at* ω =368.5 MeV ($B_{\Lambda\Lambda}$ = 8.7 MeV)

Ξ^- spectrum in DCX (K^-,K^+) reactions at 1.8GeV/c

^{16}O



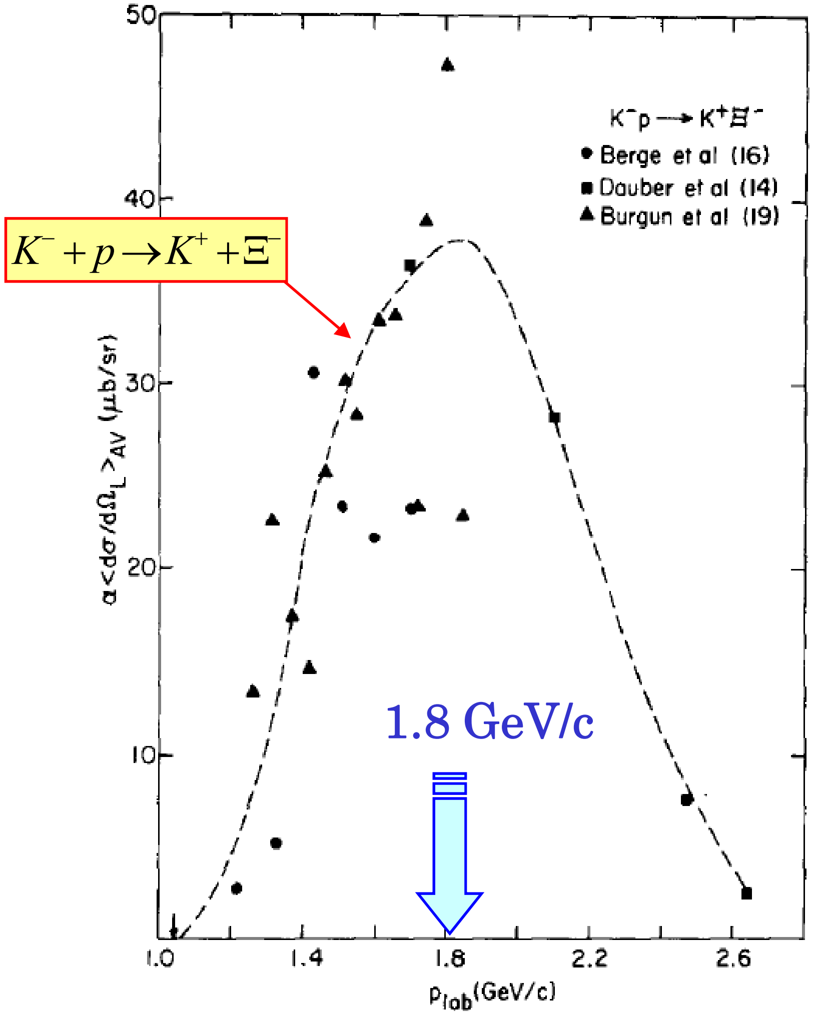
1.5MeV FWHM

The large momentum transfer $q_{\Xi^-} \simeq 400$ MeV/c leads to *the spin-stretched* Ξ^- *doorways states* followed by $[^{15}N(1/2^-, 3/2^-) \otimes s_{\Xi^-}]1^- \rightarrow [^{14}C(0^+, 2^+) \otimes s_\Lambda p_\Lambda]1^-$

Elementary Cross sections for (K^-,K^+) reactions

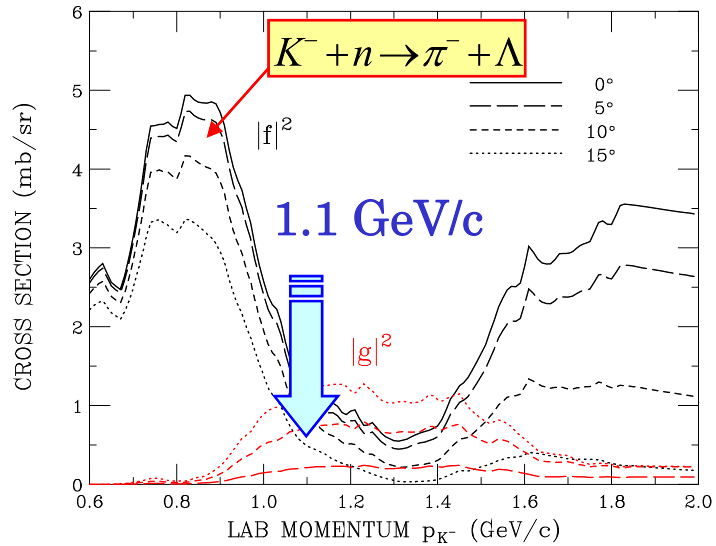
$$K^- + p \rightarrow K^+ + \Xi^-$$

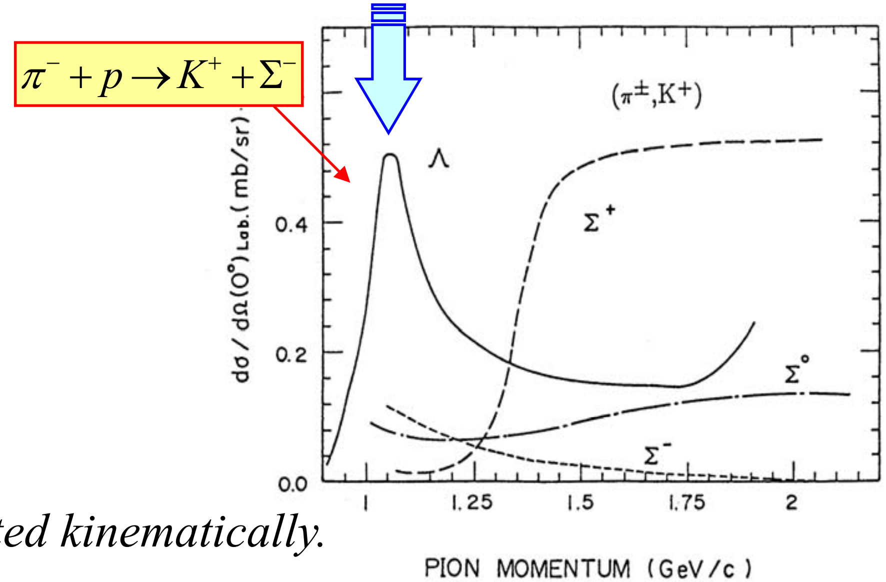
Dover and Gal, Ann. Phys, 146 (1983) 309.



$$K^- + p \rightarrow \pi^0 + \Lambda,\ \pi^0 + p \rightarrow K^+ + \Lambda$$

Bando et al., Int.J.Mod.Phys. A5(1990)4021



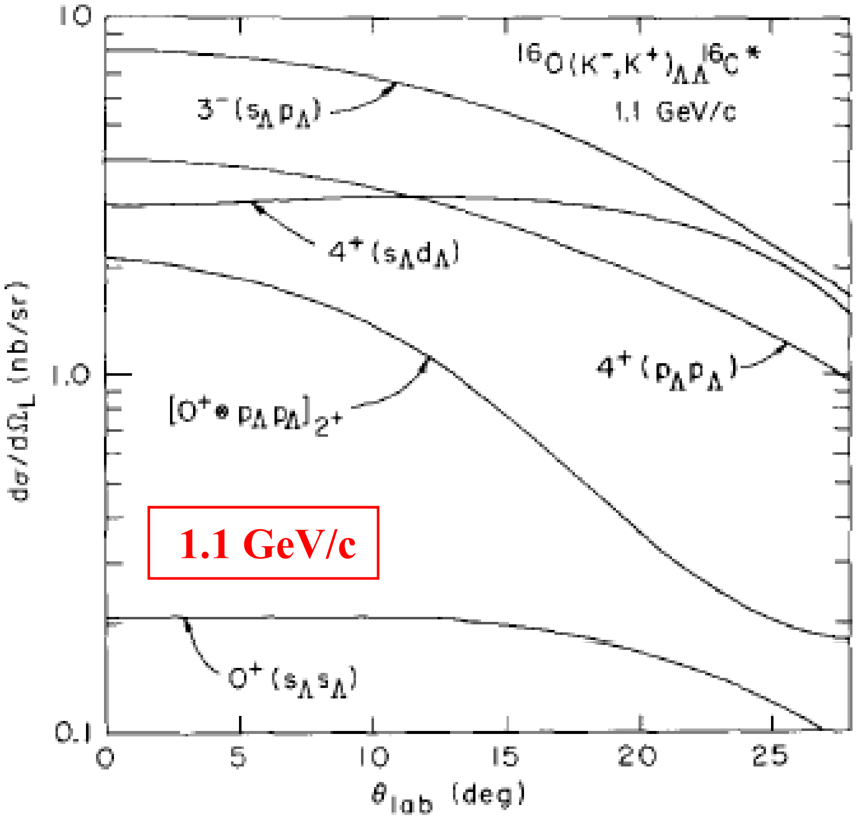


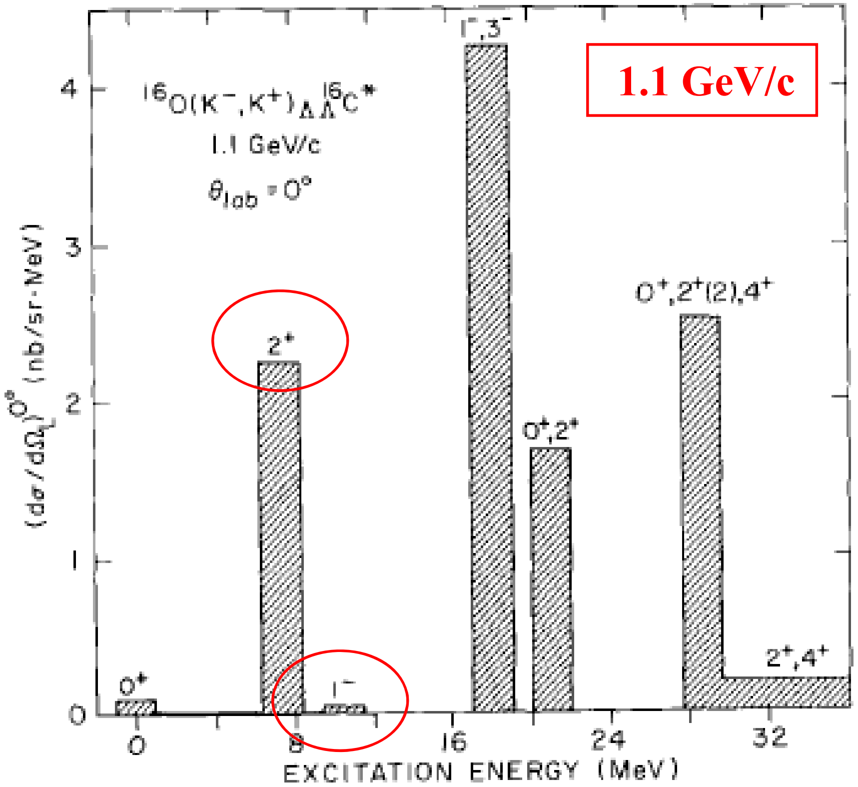
These different mechanisms are well separated kinematically.

Compared with the two-step mechanism

"On the formation and spectroscopy of ΛΛ hypernuclei"

A.J. Baltz, C.B. Dover and D.J. Millener, PLB123(1981)12.





The ΛΛ states populated by the two-step processes are quite different from those by the one-step processes.

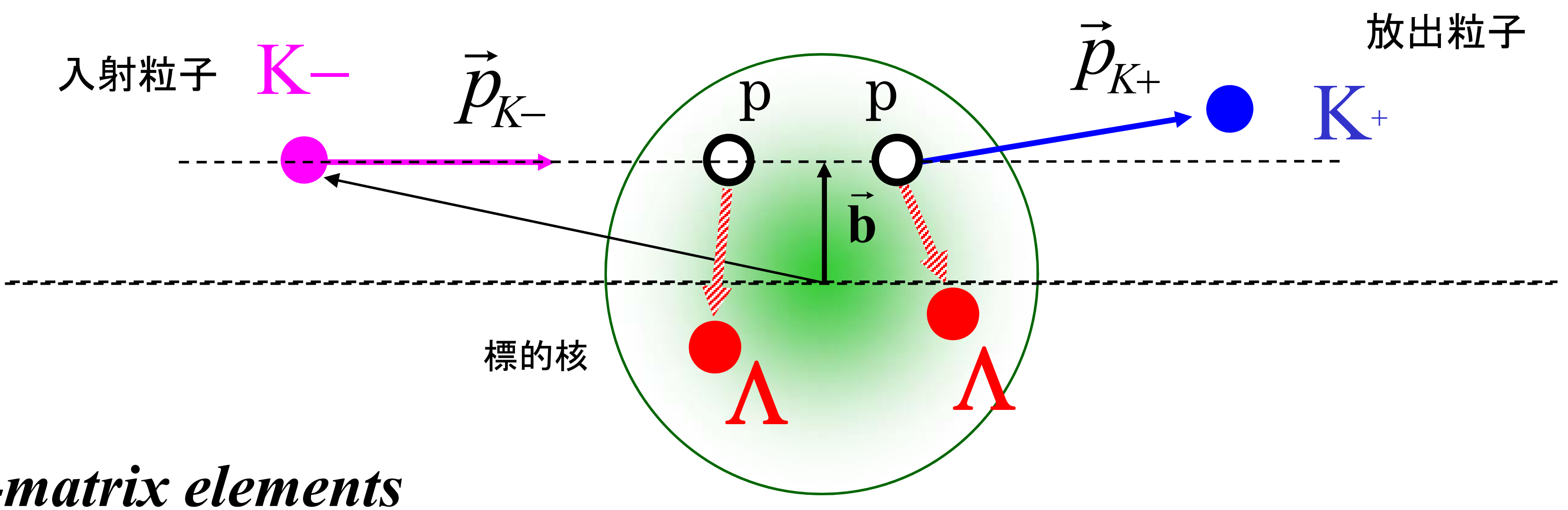
$$^{16}\mathrm{O} \xrightarrow[\Delta L_1]{K^- p \to \pi^0 \Lambda} {}^{16}_{\Lambda}\mathrm{N} \xrightarrow[\Delta L_2]{\pi^0 p \to K^+ \Lambda} {}^{16}_{\Lambda\Lambda}\mathrm{C}$$

$$\underset{2+}{\Delta L} = \underset{1-}{\Delta L_1} \otimes \underset{1-}{\Delta L_2}$$

Production of ΛΛ-hypernuclei via 2-step reactions

Eikonal approximation

C.B. Dover, Nukleonica 25 (1980) 521



T-matrix elements

$$T_{fi} = -\frac{i}{v_\pi}\int d^{(2)}\mathbf{b}\int dz'\int dz\ \theta(z-z')e^{-i\mathbf{p}_{\pi\perp}\cdot\mathbf{b}}e^{-i(p_{K-}-p_\pi)z'}e^{-i(p_\pi-p_{K^+})z}$$

$$\times\ \exp\left(-i\frac{1}{v_{K^-}}\int_z^\infty U_{K^-}(\mathbf{b},z)dz\right)\exp\left(-i\frac{1}{v_\pi}\int_{z'}^{z}U_\pi(\mathbf{b},z)dz\right)\exp\left(-i\frac{1}{v_{K^+}}\int_{-\infty}^{z'}U_{K^+}(\mathbf{b},z)dz\right)$$

Eikonal distortion

$$\times\ t_1(\mathbf{b})t_2(\mathbf{b})\left\langle f\left|\sum_{l\neq j}V_-(l)\delta^{(3)}(\mathbf{r}-\mathbf{r}_l)V_-(j)\delta^{(3)}(\mathbf{r}-\mathbf{r}_j)\right|i\right\rangle$$

A crude estimation for the two-step contributions

Eikonal approximation in a harmonic oscillator model

C.B. Dover, Nukleonica 25 (1980) 521
T. Iijima et al., NPA546(1992)588.

Summed lab cross section at 0°

$$\sum_f \left(\frac{d\sigma_{f_i}^{(2)}}{d\Omega} \right)_{0^\circ} = \frac{2\pi\xi}{p_{\pi^0}^2} \left\langle \frac{1}{r^2} \right\rangle \left[\alpha_1 \frac{d\sigma}{d\Omega} \right]_{0^\circ}^{K^- p \to \pi^0 \Lambda} \left[\alpha_2 \frac{d\sigma}{d\Omega} \right]_{0^\circ}^{\pi^0 p \to K^+ \Lambda} N_{\text{eff}}^{PP}$$

$$\alpha_1 = \left(1 - Q_0^{(1)} / v_\pi \varepsilon_N\right) \qquad \alpha_2 = \left(1 - Q_0^{(2)} / v_K \varepsilon_\Lambda\right)$$

$$\xi = \int \frac{d\mathbf{Q}_\perp}{(2\pi)^2} \left| \frac{t_1(\mathbf{Q}_\perp)}{t_1(0)} \right|^2 \left| \frac{t_2(-\mathbf{Q}_\perp)}{t_2(0)} \right|^2 = 0.022 - 0.019\,\text{fm}$$

Shadowing effects

$$\left\langle 1/r^2 \right\rangle \simeq 0.028\,\text{mb}^{-1}$$

$$\left[\alpha_1 (d\sigma/d\Omega)\right]_{0^\circ}^{K^- p \to \pi^0 \Lambda} \simeq 1.57 - 1.26 \text{ mb/sr} \qquad \left[\alpha_2 (d\sigma/d\Omega)\right]_{0^\circ}^{\pi^0 p \to K^+ \Lambda} \simeq 0.070 - 0.067 \text{ mb/sr}$$

$N_{\text{eff}}^{pp} \simeq 1$ including the nuclear distortion effects.

Thus,

$$\sum_f \left(\frac{d\sigma_{fi}^{(2)}}{d\Omega} \right)_{0^\circ} \simeq 0.06 - 0.04\ \mu\text{b/sr}$$

Bound state production due to q = 400 MeV/c

$\times \sim 1\% \approx 0.6 - 0.4\,\text{nb/sr}$

Doverらの計算値から１桁以上小さくすべき !!

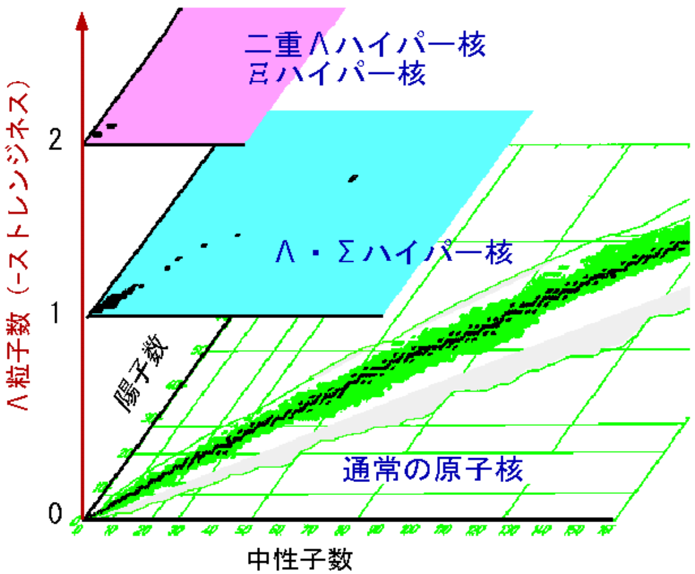
Remarks

One-step mechanism via **Ξ^- doorways**
predicts promising peaks of
the $\Lambda\Lambda$ bound and excited states
in the $^{16}O(K^-, K^+)$ reactions at **1.8 GeV/c** (0°).

The (K^-,K^+) reactions can provide the ability to
extract properties of the ΞN-$\Lambda\Lambda$ potentials
and **Ξ^- admixture probabilities**
in doubly strange hypernuclei.

Summary

Studies of the double-charge exchange reactions (DCX) for hypernuclear productions are
very important and promising
at J-PARC.



Future subjects:

More microscopic calculations based on YN, YY potentials are needed to compare them with the forthcoming experimental data at J-PARC.