Nuclear parton distributions and structure functions

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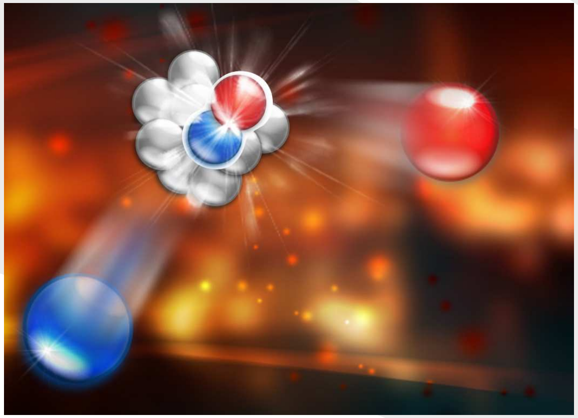
Theme

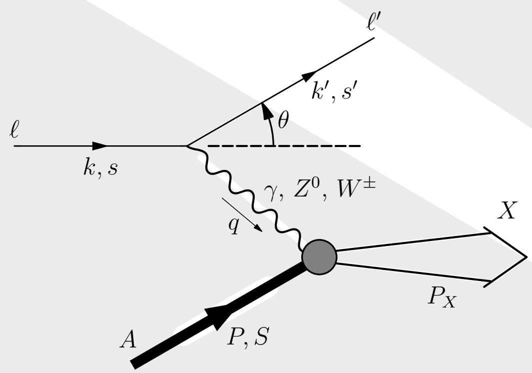
- **Are nucleon properties modified when they are bound inside a nucleus?**

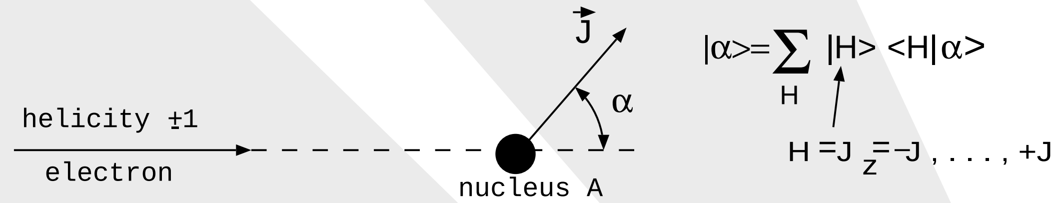


Points of this talk:

- **Unpolarized EMC effect**
- **Parity violation in DIS, NuTeV anomaly**
- **Polarized EMC effect**
- Prospect: **Semi-inclusive DIS on nuclear targets**

Deep inelastic scattering





Photon cross section (B.L.) for electron helicity $\pm$, target helicity H ($x_A = \frac{Q^2}{2\overline{M}_N \nu}$ with $\overline{M}_N = M_A/A$, $y = \frac{\nu}{E}$):

$$\frac{\mathrm{d}\sigma_\pm^H}{\mathrm{d}x_A \mathrm{d}y} = \frac{2\pi\alpha^2}{Q^2}\left[\frac{2-2y+y^2}{x_A y} F_{2A}^H(x_A) \pm 2\,(2-y)\, g_{1A}^{(H)}(x_A)\right]$$

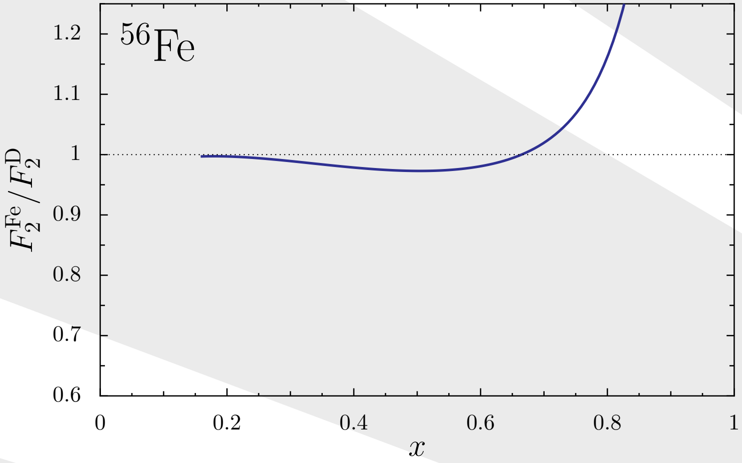
$$F_{2A}^H(x_A) = x_A \sum_q e_q^2\, q_A^H(x_A) = x_A \sum_q e_q^2 \left(q_{A\uparrow}^H(x_A) + q_{A\downarrow}^H(x_A)\right)$$

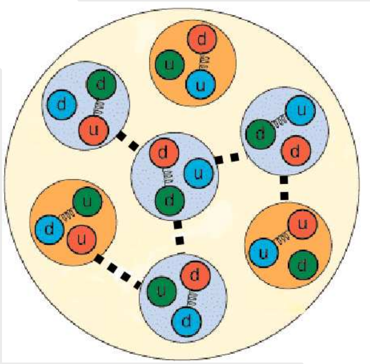
$$g_{1A}^H(x_A) = \frac{1}{2}\sum_q e_q^2\, \Delta q_A^H(x_A) = \frac{1}{2}\sum_q e_q^2 \left(q_{A\uparrow}^H(x_A) - q_{A\downarrow}^H(x_A)\right)$$

Note: There are $2J+1$ independent distributions (or structure functions) for spin J target.

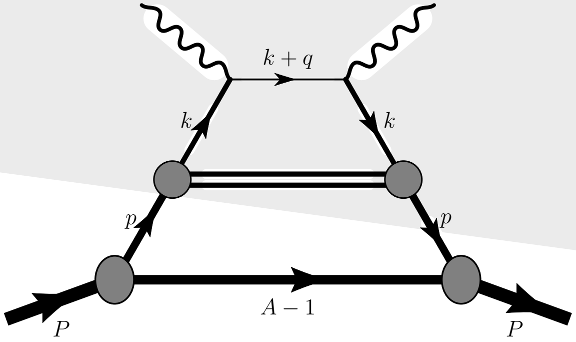
Prior to discovery of EMC effect

Basic picture: Nuclear mean fields couple to nucleons like elementary particles.





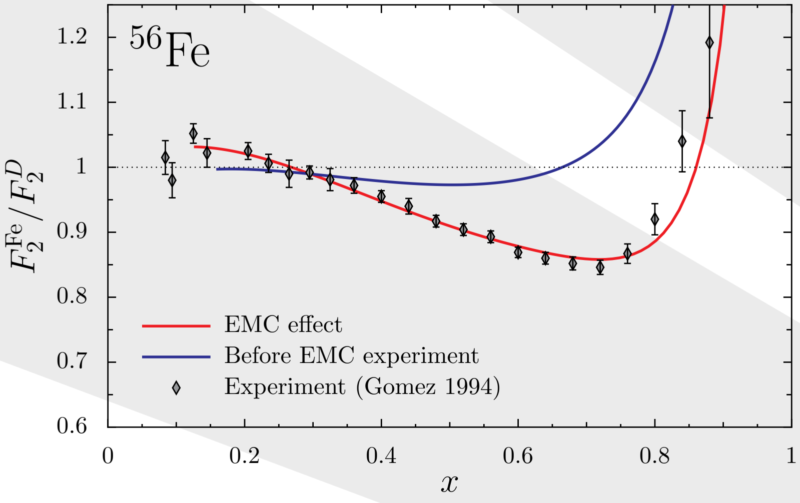
[Nucleon distribution in nucleus] $\otimes$ [quark distribution in free nucleon]: **Binding effects on nucleon level.**

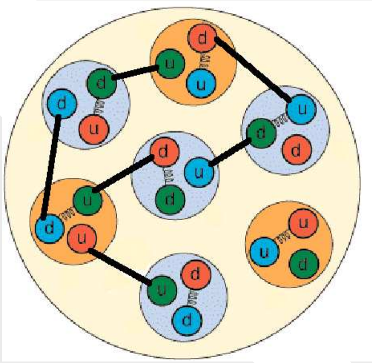


Constituent quark and diquark **do not experience** nuclear mean fields.

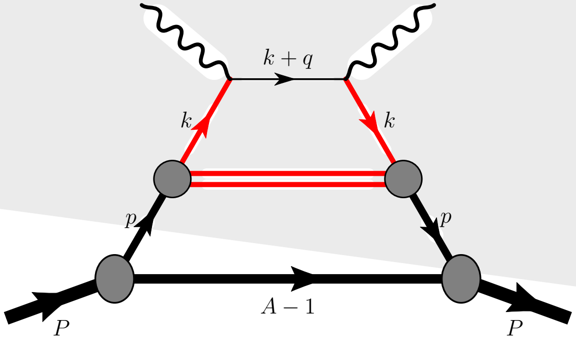
After discovery of EMC effect

Better picture: Nuclear mean fields couple to quarks in the nucleons!





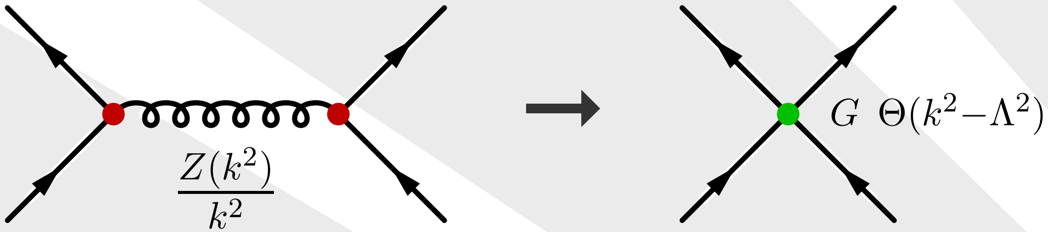
[Nucleon distribution in nucleus] $\otimes$ [quark distribution in bound nucleon]:
Binding effects on quark level.



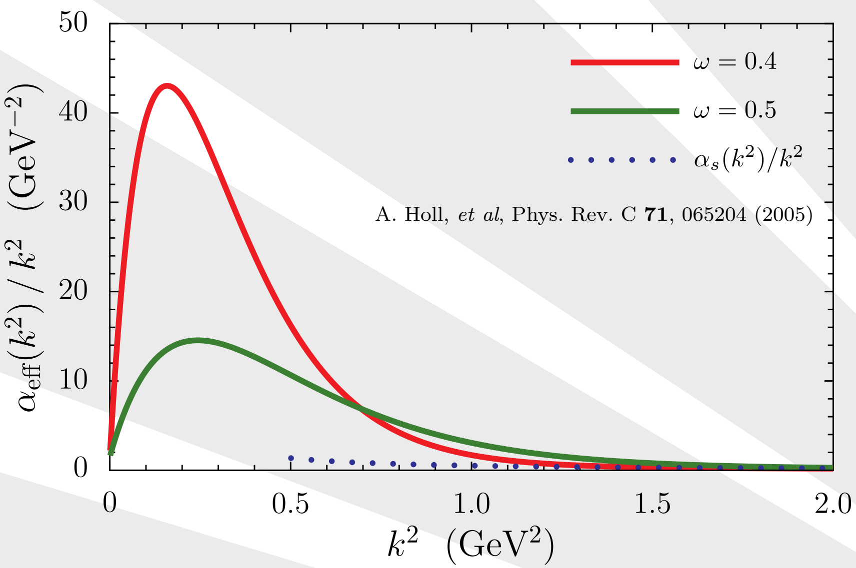
Constituent quark and diquark **experience** the nuclear mean fields.

Nambu–Jona-Lasinio Model

- Interpreted as low energy chiral effective theory of QCD



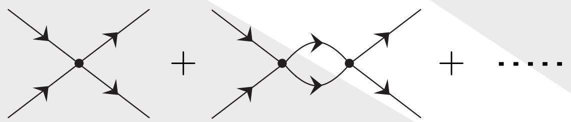
- Motivated by infrared enhancement of gluon propagator
  - Dyson-Schwinger equations
  - lattice QCD

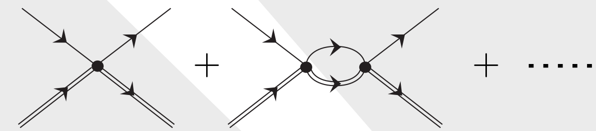


- Lagrangian: $\mathcal{L}_{\rm NJL} = \overline{\psi}\,(i\,\not\nabla - m)\,\psi + G\left(\overline{\psi}\,\Gamma\,\psi\right)^2$
- Spontaneous breakdown of chiral symmetry
- Easily extended to finite density and temperature

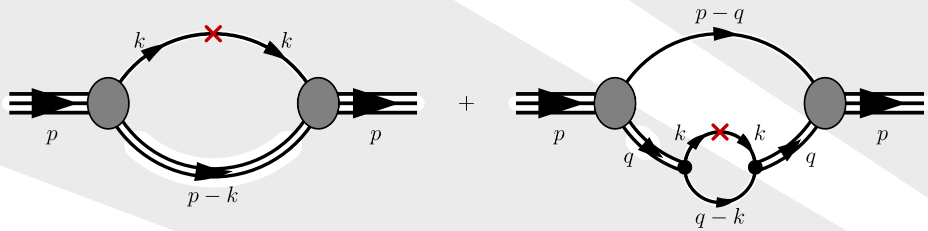
Free nucleon

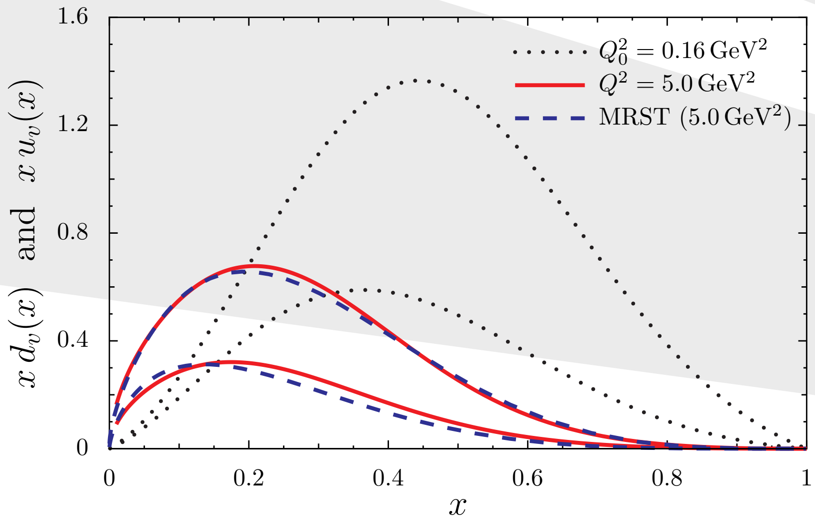
- Quark-diquark description based on the Faddeev method.
  We include scalar (0^+) and axial vector (1^+) diquarks.

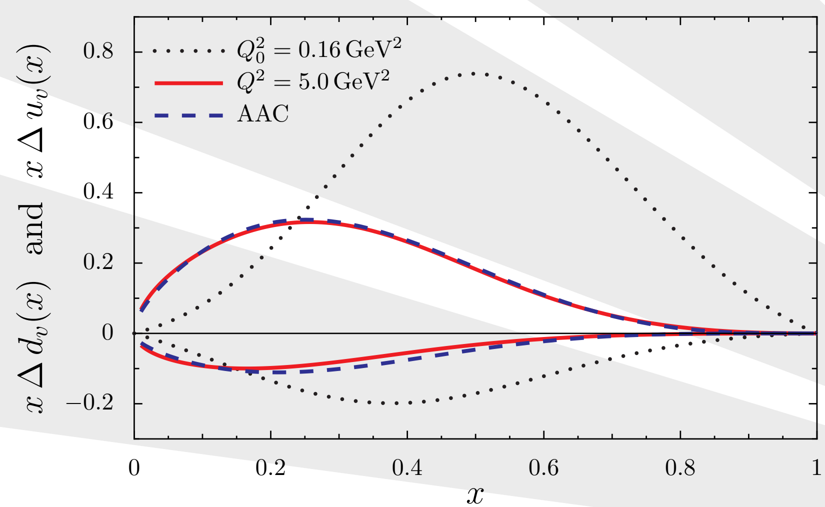
diquark: 

nucleon: 

$\Rightarrow$ Calculate **parton distributions in the free nucleon**.



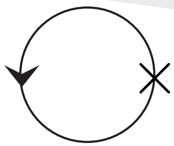




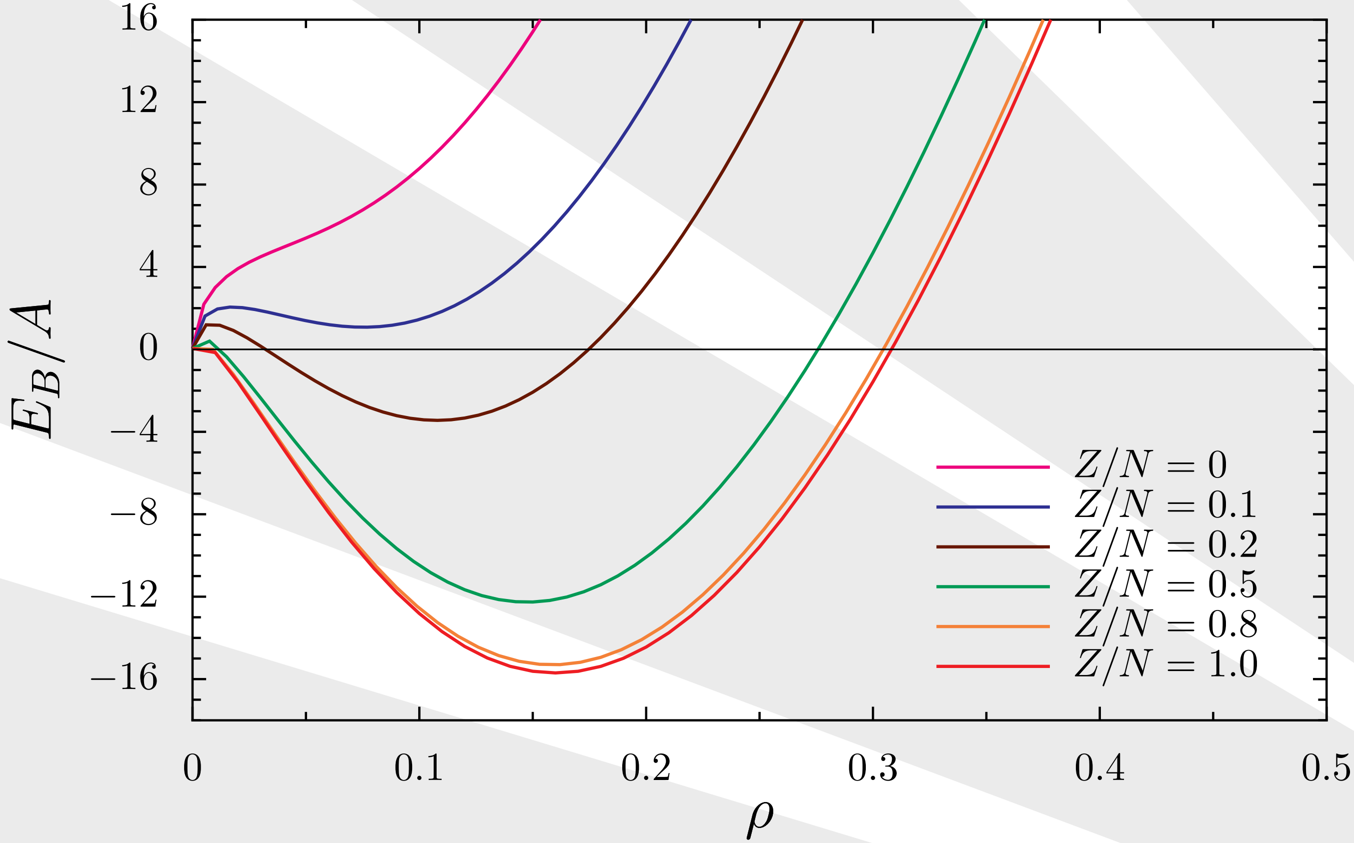
Nuclear matter

- Nuclear matter described in mean field approximation:
  Self consistent mean scalar and vector fields couple to the quarks in the nucleons.
- Mean fields in effective quark Lagrangian
  $\mathcal{L} = \overline{\psi}\left[i \not\nabla - M - \gamma^0\left(\omega_0 + \tau_z \rho_0\right)\right]\psi - \frac{(M-m)^2}{4G_\pi} + \frac{\omega_0{}^2}{4G_\omega} + \frac{\rho_0{}^2}{4G_\rho}$
  are calculated by minimizing the energy density. Here
  $M = m - 2G_\pi \langle \overline{\psi}\psi \rangle$,
  $\omega_0 = 2G_\omega \langle \overline{\psi}\gamma_0\psi \rangle$, $\rho_0 = 2G_\rho \langle \overline{\psi}\gamma_0\tau_3\psi \rangle$.
- These mean fields are incorporated in the quark propagators to calculate parton distributions in the bound nucleon.
- Convolution to get the parton distributions in nuclear matter:

$$q_A(x_A) = \sum_{N=p,n} f_N(y) \otimes q_N(z) \,, \qquad \text{where}$$

$f_N(y) = \; p$  $(\gamma^+, \gamma^+\gamma_5)\, \delta\left(y - \frac{p^+}{M_A/\sqrt{2}}\right)$

Binding energy of nuclear matter

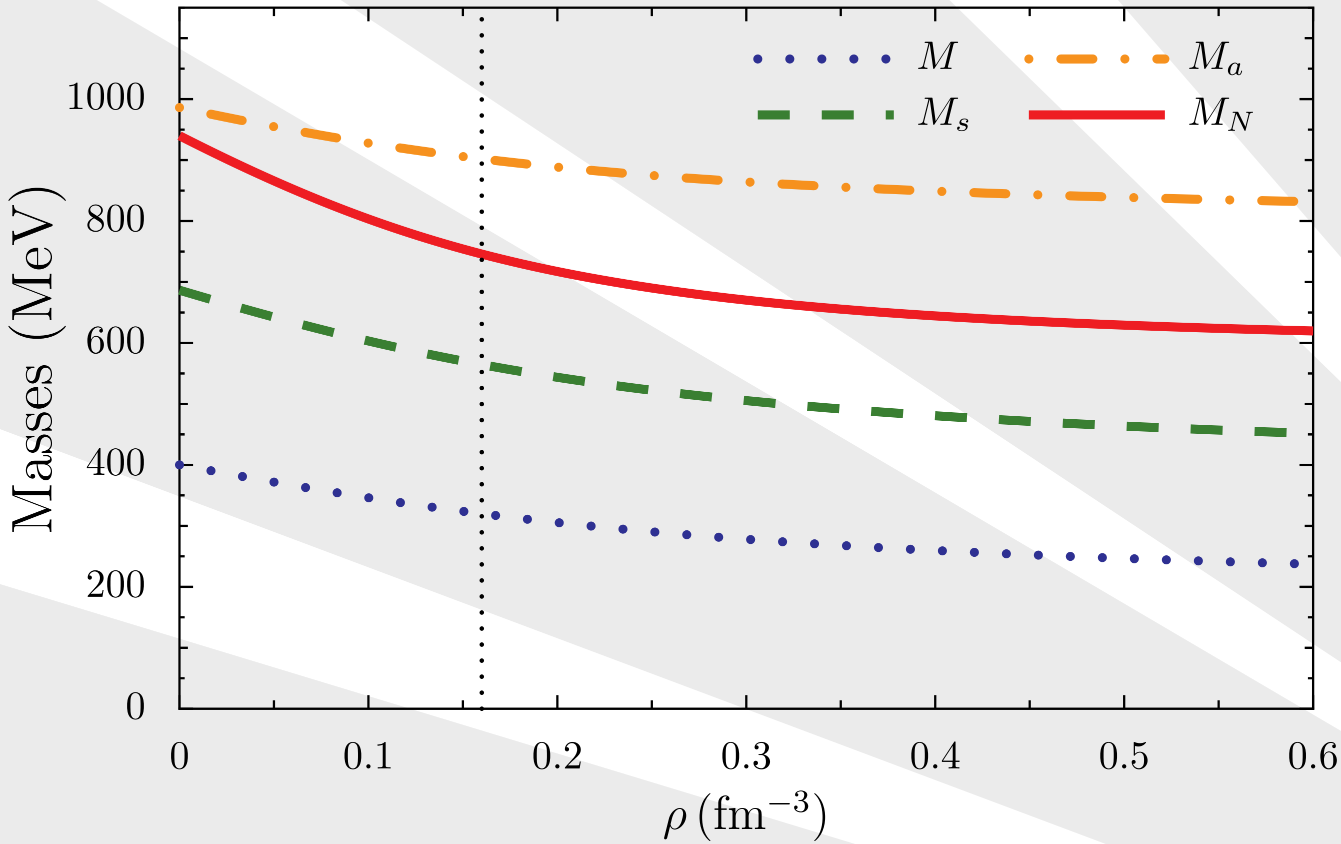


Mean vector fields: $\omega^0 = 2G_\omega \langle \psi^\dagger \psi \rangle$, $\rho^0 = 2G_\rho \langle \psi^\dagger \tau_3 \psi \rangle$

Vector potentials: $V_{p(n)} = 3\omega^0 \pm \rho^0$, $V_{u(d)} = \omega^0 \pm \rho^0$

$G_\omega \Leftrightarrow$ saturation point for $Z = N$, $G_\rho \Leftrightarrow$ symmetry energy.

Effective masses in symmetric nuclear matter

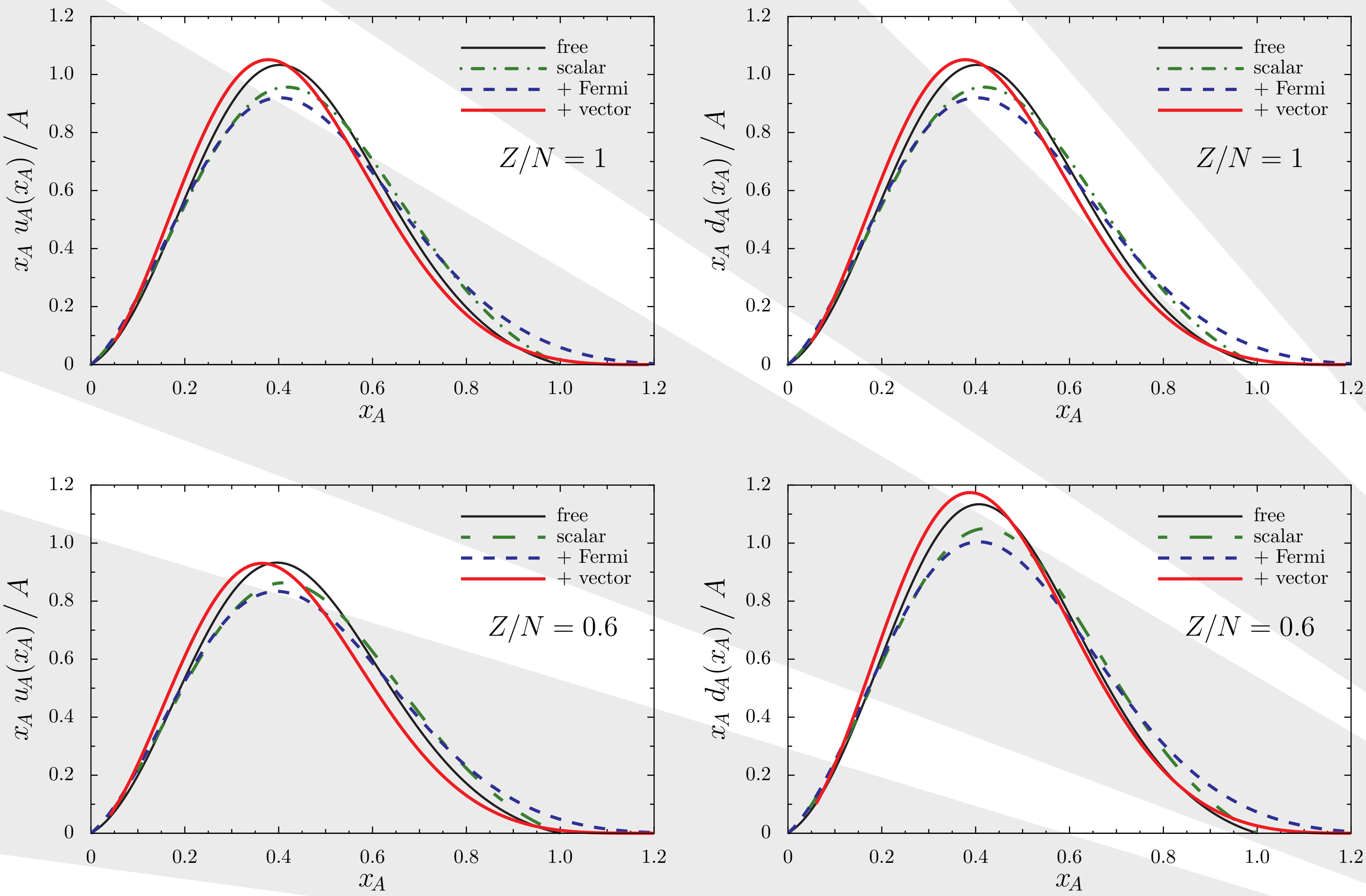


$M \ldots$ constituent quark mass $(M = m - 2G_\pi \langle \bar{\psi}\psi \rangle)$

$M_{s(a)} \ldots$ scalar (axial vector) diquark mass (pole of qq t-matrix)

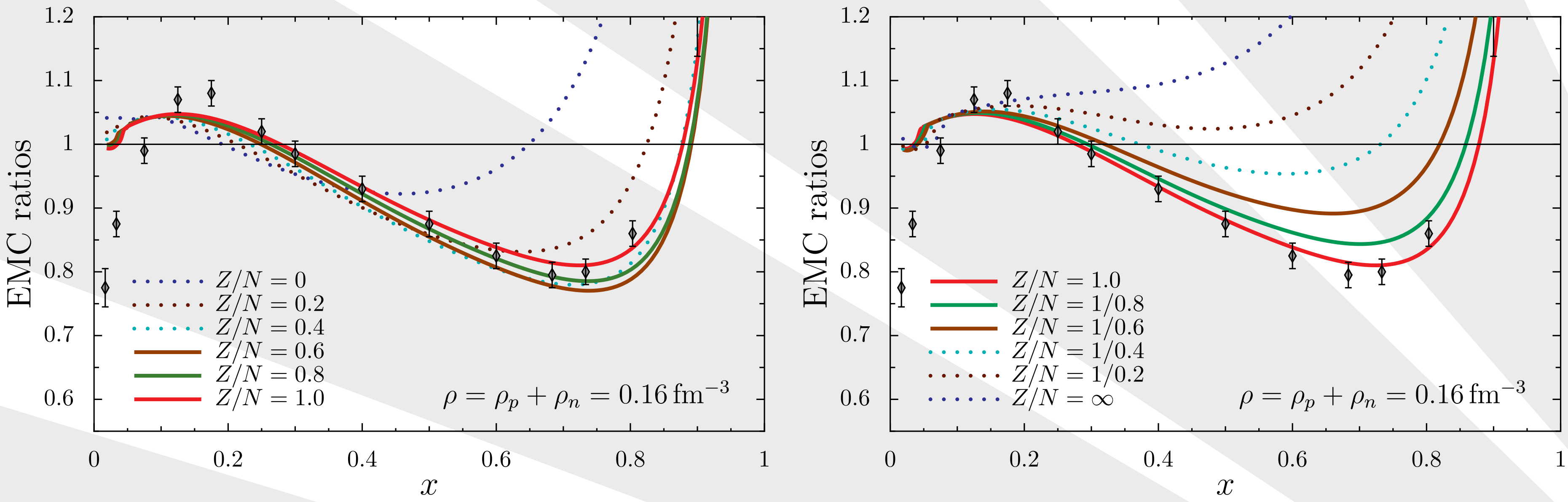
$M_N \ldots$ nucleon mass (pole of q-diquark t-matrix).

In-medium distributions: Flavor dependence



- In-medium distributions softer than free ones: Binding effect on quark level.
- For $N > Z$, u-quarks feel additional binding (**symmetry energy!**) $\Rightarrow$ **larger medium effects for u-quarks in neutron rich matter.** (This effect is caused mainly by the ρ^0 field.)

EMC effect: Isospin dependence



EMC ratio = $\frac{F_{2A}}{F_{2A,\mathrm{naive}}} \simeq \frac{4u_A + d_A}{4u_{A\mathrm{f}} + d_{A\mathrm{f}}}$, where $q_{A\mathrm{f}} = Z q_{p\mathrm{f}} + N q_{n\mathrm{f}}$.

- Case $N > Z$: When matter becomes neutron-rich, medium-modification of u-quarks **increases**, but their number **decreases** $\Rightarrow$ EMC effect becomes more pronounced as Z/N decreases from 1 to 0.6, but for $Z/N < 0.6$ the EMC effect becomes smaller because d-quarks begin to dominate.
- Case $N < Z$: When matter becomes proton-rich, medium modification of u-quarks **decreases** and their number **increases** $\Rightarrow$ EMC effect becomes smaller.

Applications

This flavor dependence should show up in many places, e.g.,
$e + A \to e' + \pi^\pm + X, \quad \pi^\pm + A \to (\ell^+\ell^-) + X$.

Here: Consider some physical quantity R, which is a ratio of nuclear parton distributions:

$$\begin{aligned} R &= \frac{c_1 u_A + c_2 d_A}{c_3 u_A + c_4 d_A} \simeq A + B\,\frac{d_A - u_A}{d_A + u_A} \\ &\equiv R_0 + \delta_{\mathrm{naive}} R + \delta_{\mathrm{med}} R \end{aligned}$$

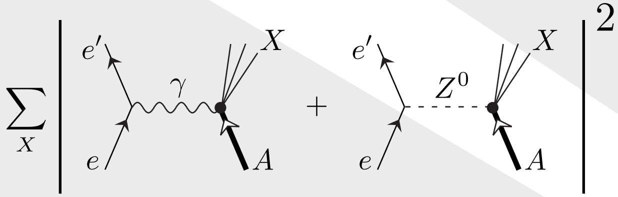
$A,\ B$ = known constants,
$R_0 = A =$ value for $N = Z$,
$\delta_{\mathrm{naive}} R =$ **neutron excess correction** obtained from **free** (no-medium) parton distributions.

- If R could be measured, any deviation from the "**naive value**" $R_0 + \delta_{\mathrm{naive}} R$ would be an indication for the in-medium flavor dependence $\delta_{\mathrm{med}} R$.

Note: Effects of **charge symmetry breaking** ($m_d > m_u$) should also be considered.

Application 1: Parity-violating DIS

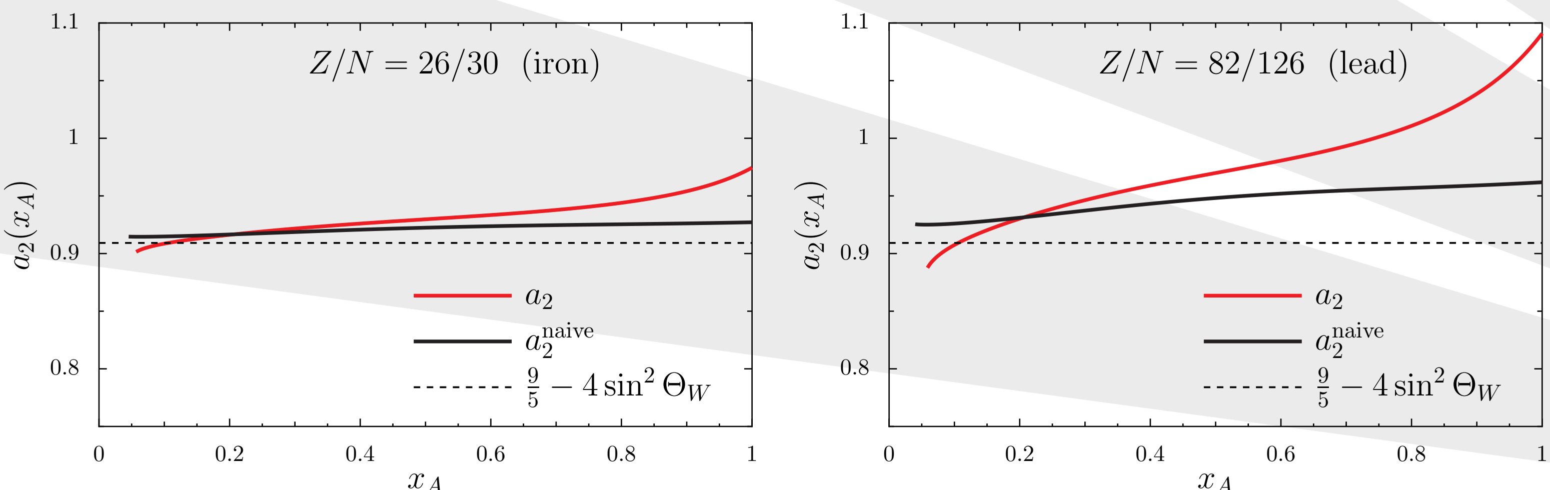
Parity violation from $\gamma - Z^0$ **interference**:



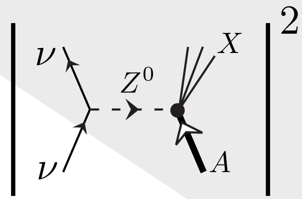
$$W^{\mu\nu} = \frac{1}{4\pi}\sum_X < X|J^\mu|N >^* < X|J^\nu|N >$$
$$\times\ (2\pi)^4\delta(p_X - p - q)$$

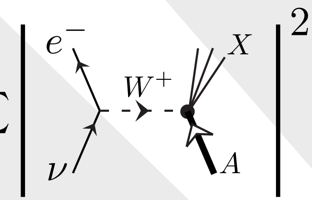
leads to electron spin asymmetry $A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$ for unpolarized targets:

$$A_{PV} = \frac{G_F Q^2}{4\pi\alpha\sqrt{2}}\left[a_2(x_A) + \text{small corrections}\right]$$
$$a_2 \simeq \left(\frac{9}{5} - 4\sin^2\Theta_W\right) + \frac{12}{25}\frac{d_A - u_A}{d_A + u_A}$$



Application 2: DIS of neutrinos

NC: $\sum_X$ 

CC: $\sum_X$ 

In 2002, the NuTeV collaboration measured the following Paschos-Wolfenstein ratio (all cross sections integrated over x_A and y):

$$
\begin{aligned}
R &= \frac{\sigma(\nu\mathrm{Fe} \to \nu\mathrm{X}) - \sigma(\overline{\nu}\mathrm{Fe} \to \overline{\nu}\mathrm{X})}{\sigma(\nu\mathrm{Fe} \to \mu^-\mathrm{X}) - \sigma(\overline{\nu}\mathrm{Fe} \to \mu^+\mathrm{X})} \\
&\simeq \left(\frac{1}{2} - \sin^2\Theta_W\right) - \left(1 - \frac{7}{3}\sin^2\Theta_W\right)\frac{\langle x_A d_A - x_A u_A\rangle}{\langle x_A d_A + x_A u_A\rangle} \\
&\equiv R_0 + \delta_{\mathrm{naive}} R + \delta_{\mathrm{med}} R
\end{aligned}
$$

- If the Standard Model value of $\sin^2\Theta_W$ is used: Measured R deviates from the “naive value” $R_0 + \delta_{\mathrm{naive}} R$ ($\Rightarrow$ “NuTeV anomaly”).
- However: Including medium effects, and also charge symmetry breaking effects ($m_d > m_u$), the measured value of R is reproduced with the Standard Model value of $\sin^2\Theta_W$: **There is no anomaly!**

Finite nuclei

Self consistent calculation in progress . . .
For the present, we use the following method (for $N \simeq Z$ nuclei):

- Assume Woods-Saxon scalar and vector potentials for nucleons: Depth from self consistent nuclear matter calculation ($V_s = -194$ MeV, $V_v = 133$ MeV), text book values for $r_0 = 1.2$ fm, $a = 0.65$ fm.
- Calculate average scalar and vector fields for each nucleon orbit, and translate to average fields for quarks by using the quark-diquark equation. Use these fields in the quark propagators to calculate quark distributions in nucleons.
- Calculate nucleon momentum distributions for each orbit, and use convolution to get the nuclear quark distributions.

Application: Polarized EMC effect

- Definition of **spin dependent EMC ratio**

$$R_s^H(x) = \frac{g_{1A}^H(x_A)}{P_p^H g_{1p}(x) + P_n^H g_{1n}(x)} \xrightarrow{\text{NR,no-medium}} 1$$

where the $P's$ are the proton and neutron polarization factors: $P_\alpha^H = \langle J, H | 2S_z^\alpha | J, H \rangle \quad (\alpha = p, n)$.

- Only few (valence) nucleons contribute to nuclear polarization $\Rightarrow$ $g_{1A}^H \propto 1/A$ (relative to F_{2A}.)
Possible candidates for measurement: Stable nuclei with polarization dominated by protons; A not too large (^{11}B, ^{15}N, etc).

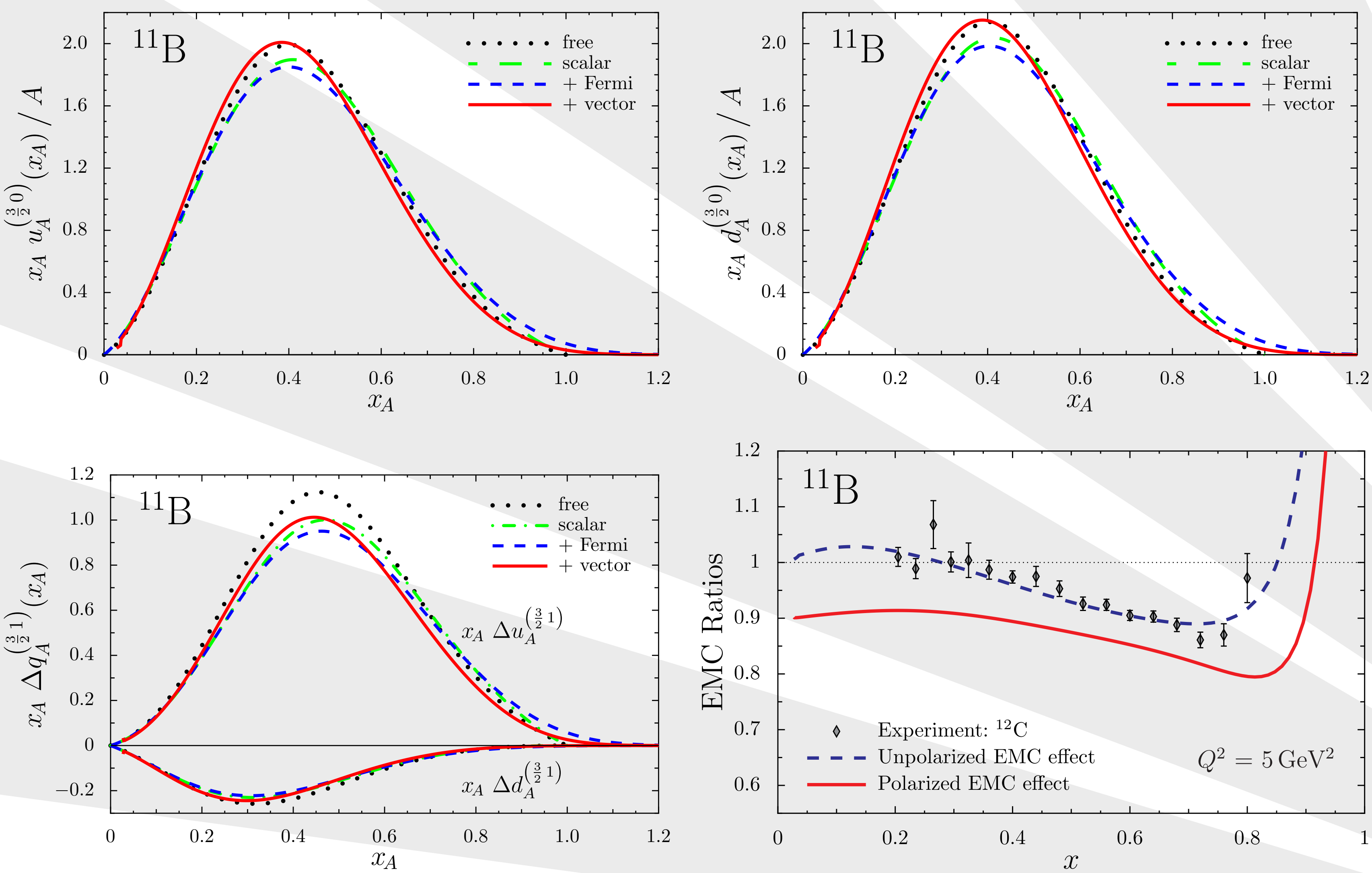
Nuclear spin sums

- **Nuclear spin sums**:

$$\int \mathrm{d}x_A \, \Delta q_A^H(x_A)$$
$$= \left(N_{q\uparrow/N\uparrow} - N_{q\downarrow/N\uparrow}\right) \times \left(N_{N\uparrow/A} - N_{N\downarrow/A}\right)$$
$$= (\text{spin sum for nucleon}) \times (\text{nuclear polarization factor})$$

Interesting connections to other spin phenomena in nuclei, like Gamow-Teller matrix elements.

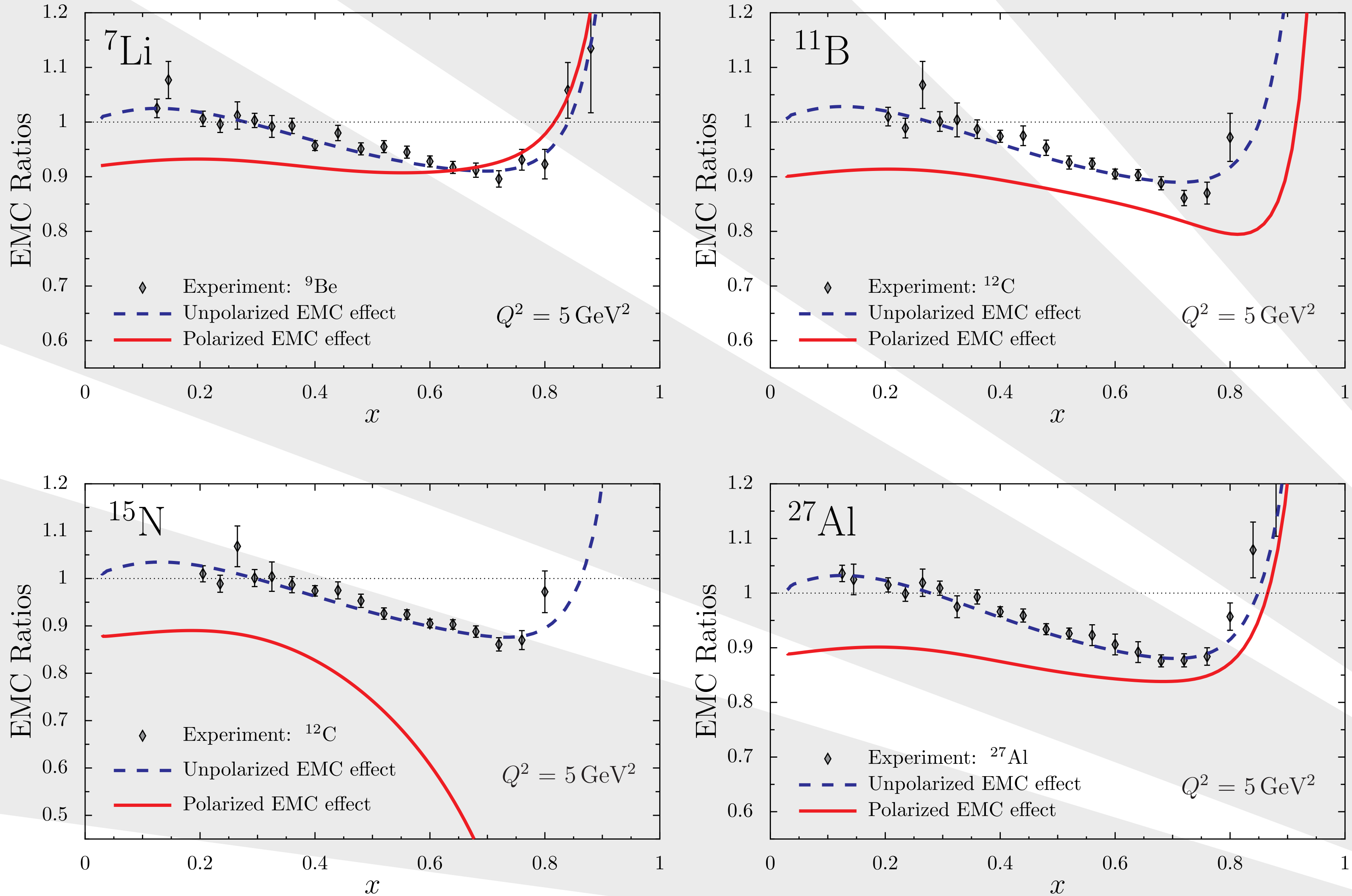
Case of ^{11}B



$(J, K) = (3/2, 0)$ is the lowest multipole for unpolarized case,

$(J, K) = (3/2, 1)$ is the lowest multipole for polarized case.

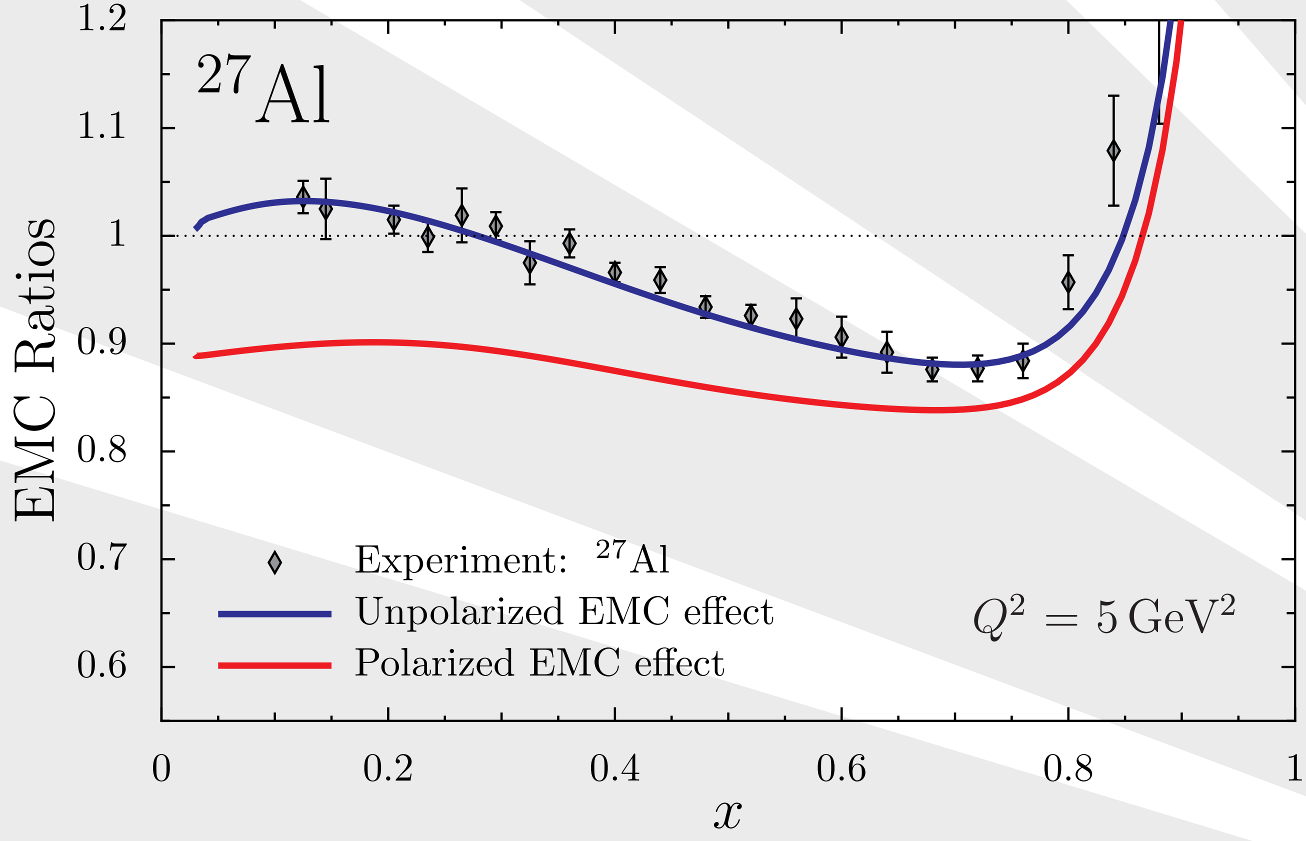
EMC in finite nuclei



- **Unpolarized case**: Nuclear vector potential leads to rescaling of Bjorken x, and plays the essential role!
- **Polarized case**: Nuclear scalar potential (smaller quark mass) leads to quenching of quark spin sum and enhancement of quark orbital angular momentum!

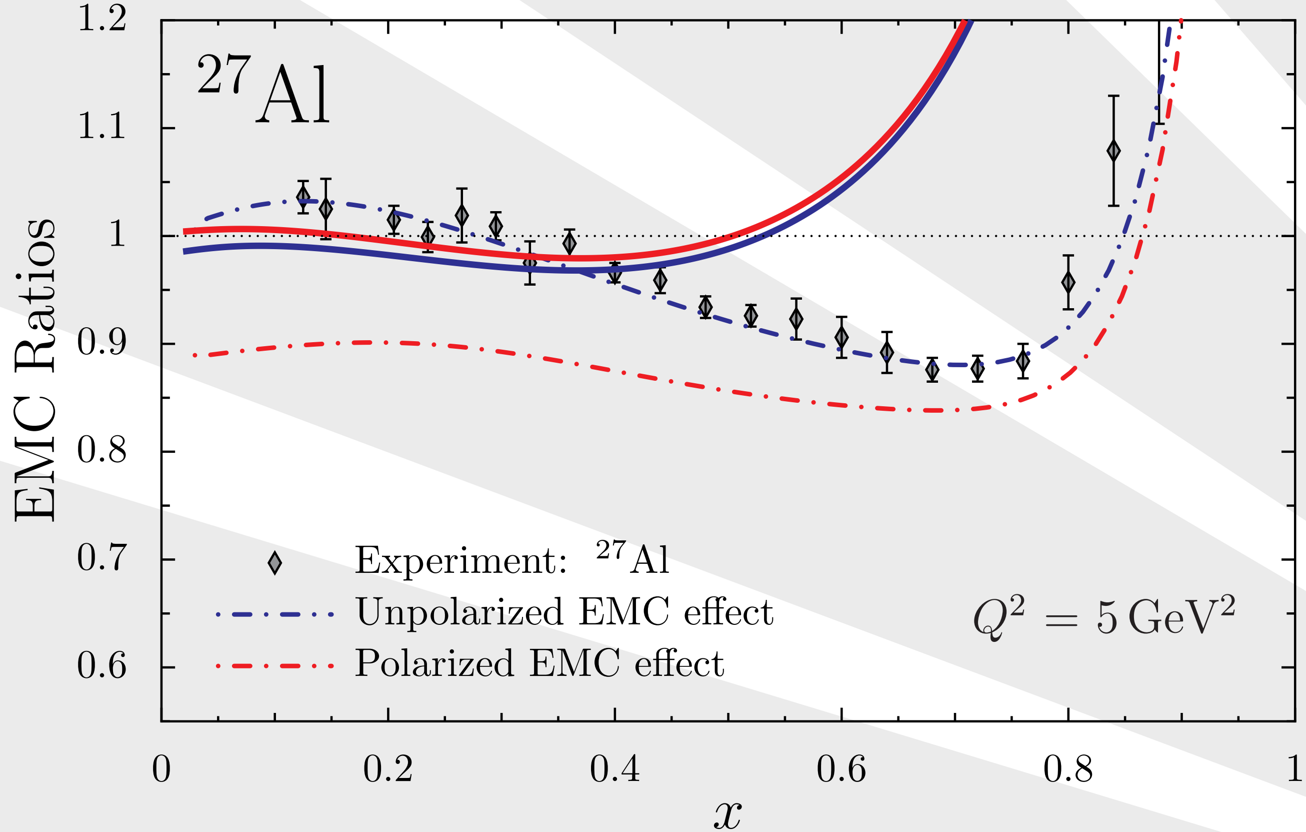
Quark effects in finite nuclei

EMC ratios with medium modified quark distributions:



Quark effects in finite nuclei

EMC ratios with free nucleon quark distributions:



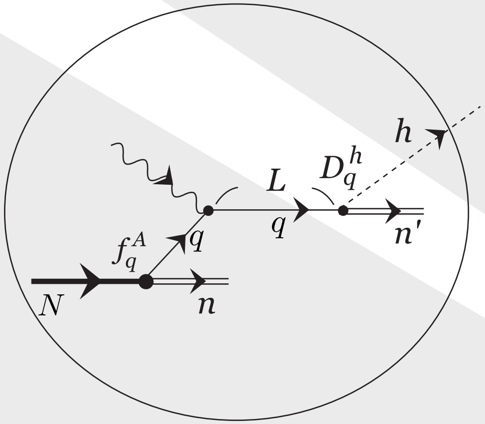
Results for spin sums:

	Δu	Δd	Σ	g_A
p	0.97	-0.30	0.67	1.27
^{7}Li	0.91	-0.29	0.62	1.19
^{11}B	0.88	-0.28	0.60	1.16
^{15}N	0.87	-0.28	0.59	1.15
^{27}Al	0.87	-0.28	0.59	1.15
nucl. matt.	0.74	-0.25	0.49	0.99

- Isoscalar spin sum: $\Delta u_A + \Delta d_A \equiv \Sigma \cdot (P_p + P_n)$, where $\Sigma \equiv \Delta u + \Delta d$ is the **isoscalar spin sum for a nucleon bound in the valence level**.
- Isovector spin sum: $\Delta u_A - \Delta d_A \equiv g_A \cdot (P_p - P_n)$, where $g_A \equiv \Delta u - \Delta d$ is the **isovector (Bjorken) spin sum for a nucleon bound in the valence level**.

Outlook: SIDIS on nuclear targets (EIC)

Recent data for nuclear targets (HERMES, JLab) indicate medium modification of SIDIS process:



Nucleus A

- **Distribution function $f_q^A \Leftrightarrow$ EMC effect.**
- **Average hadron formation length L.**
- **Quark energy loss in medium.**
- **Fragmentation function D_q^h.**
- **Interaction of hadron h with medium.**

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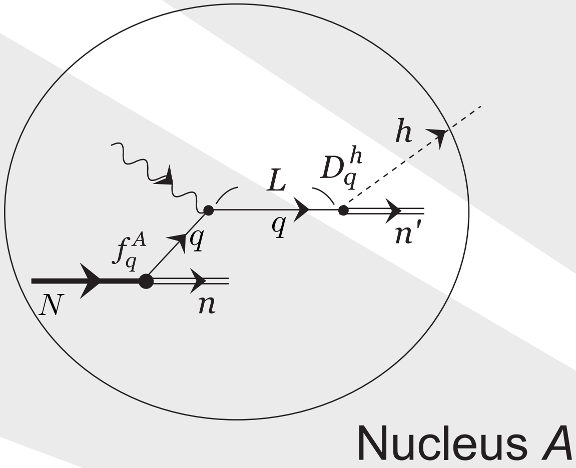
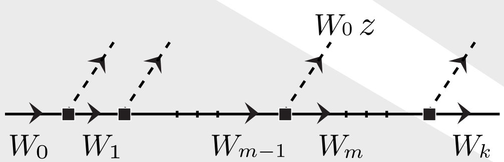


- **Distribution function $f_q^A \Leftrightarrow$ EMC effect. (Done!)**
- **Average hadron formation length L. (Lund model)**
- **Quark energy loss in medium. (Modified evolution)**
- **Fragmentation function D_q^h. (Done in vacuum!)**
- **Interaction of hadron h with medium. (Glauber)**

Fragmentation functions: NJL-jet model

We calculated the fragmentation functions $D_q^h(z)$ in vacuum, including multifragmentation process. ($\Leftrightarrow$ Product ansatz of Field and Feynman.)
For the case of pion channel only:

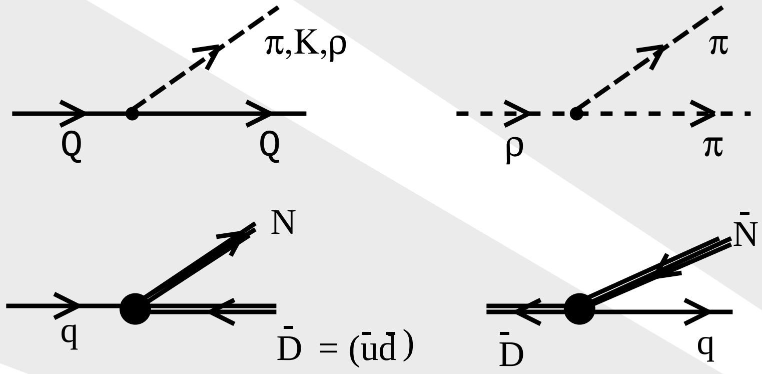
$$D_q^\pi(z) = \sum_{k=1}^{N} P(k) \int_0^1 \mathrm{d}\eta_1 \ldots \int_0^1 \mathrm{d}\eta_k \, F(\eta_1) \ldots F(\eta_k) \left(\sum_{m=1}^{k} \delta(z - z_m) \right)$$

$$D_q^\pi(z) = \sum_{k=1}^{N} P(k) \left(\sum_{m=1}^{k} \right.$$  $\left. \right)$

- N ... Maximum number of produced pions
- $P(k)$ ... Probability that k pions are produced.
  (For $N \to \infty$, $P(k)$ becomes a **normal distribution**.)
- $F(\eta)$ ... Probability that momentum fraction η (of incoming quark) is left to outgoing quark, in each elementary step.
- For $N \to \infty$, **100% of quark momentum is converted to pions.**
  (**The limit $N \to \infty$ corresponds to the Bjorken (infinite energy) limit.**)

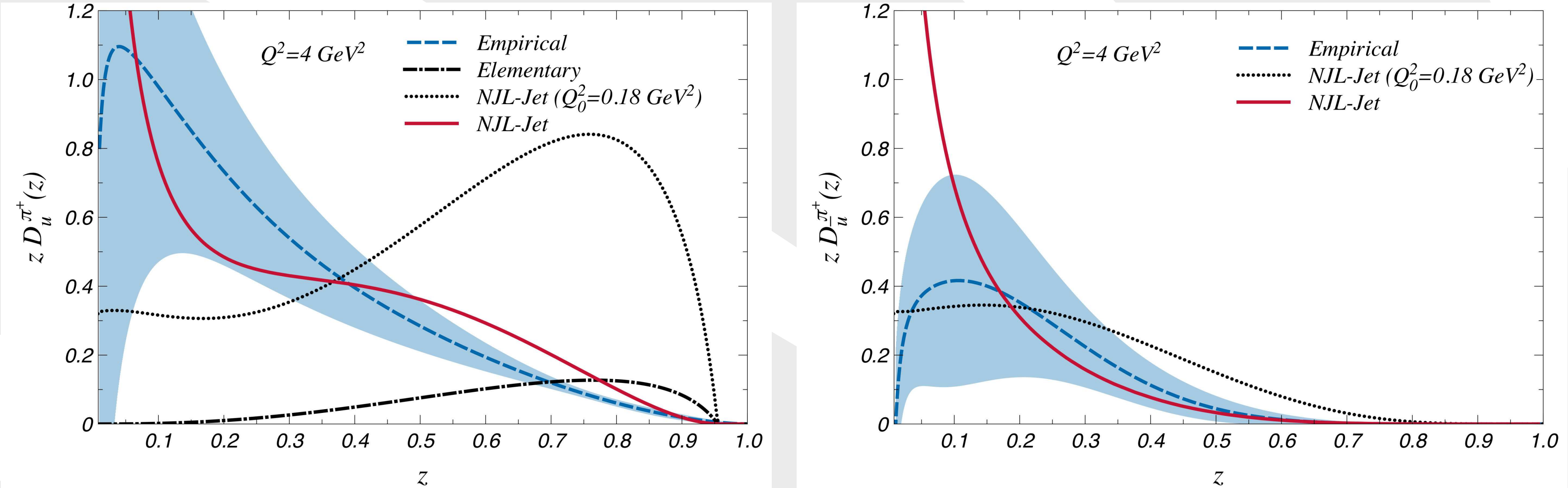
More about NJL-jet

- **Generalization**: Fragmentation to other hadrons (K, N) included via the elementary splitting functions:



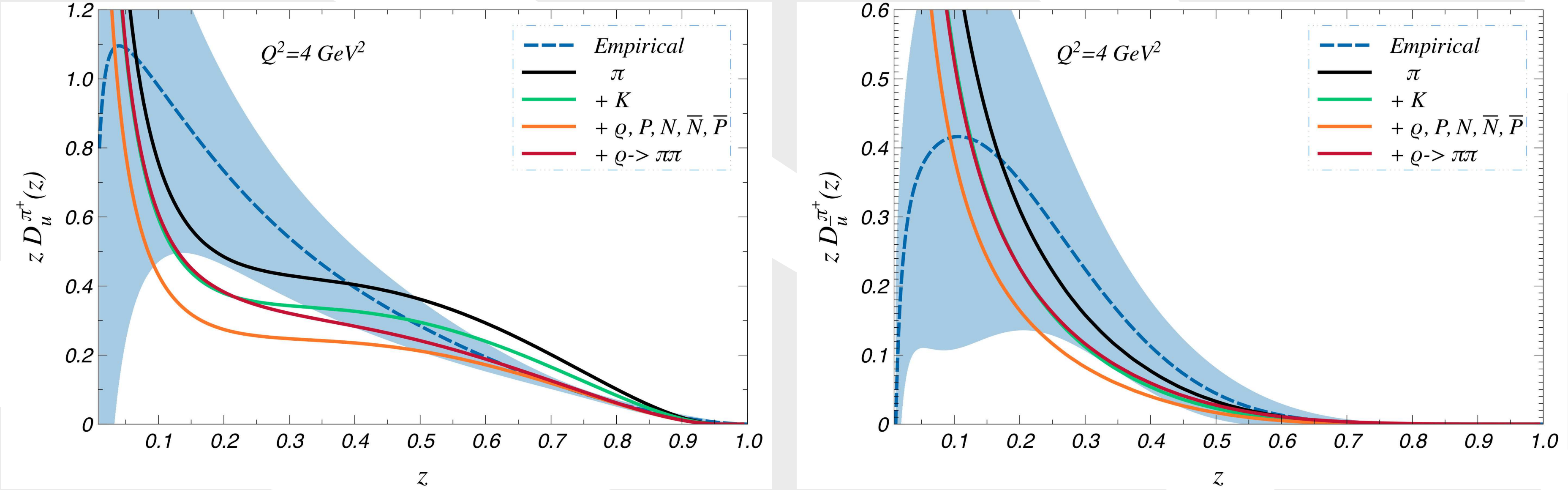
Here $Q = (u, d, s)$, $q = (u, d)$.
Nucleon described as bound state of **scalar diquark** (D) and quark (q).

Fragmentation to pions: π channel only



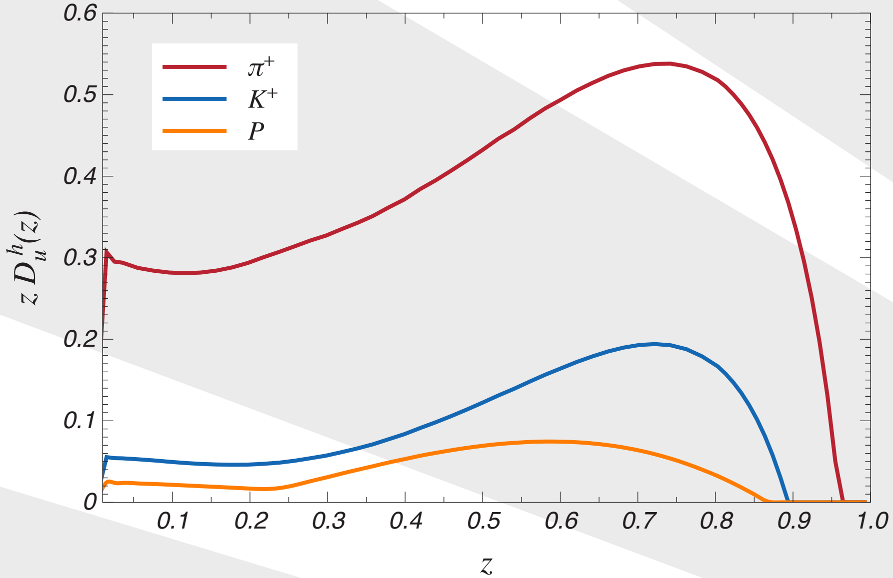
- **Cascade-like processes enhance the fragmentation functions tremendously!**
- Low energy (NJL) scale $Q_0^2 = 0.18$ GeV^2 determined by reproducing empirical **distribution** functions.
- Evolution to $Q^2 = 4$ GeV^2 in NLO performed using **QCDNUM-17** (M. Botje et al, 2010).
- Empirical NLO parametrizations and uncertainties from **M. Hirai** et al (PR **D 75** (2007) 094009).

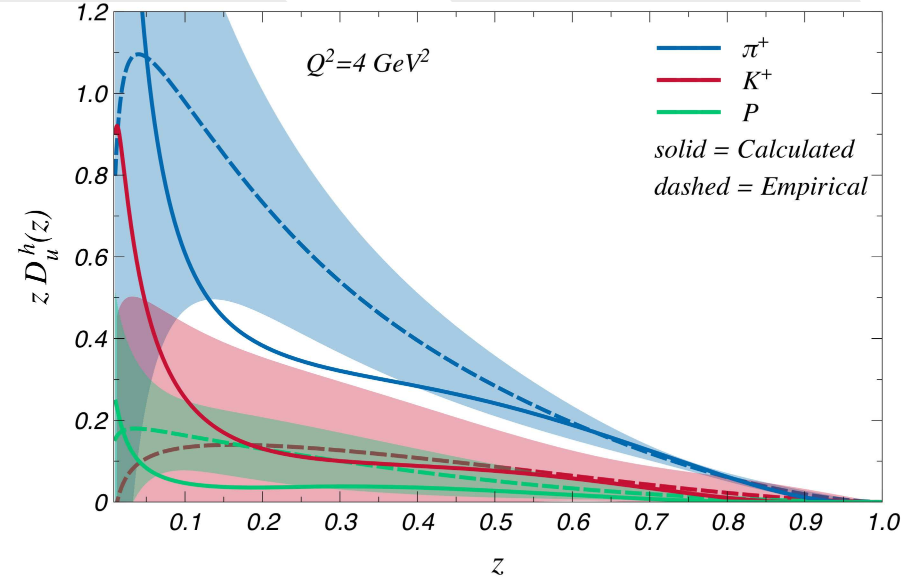
Fragmentation to pions, incl. other channels



- Inclusion of other channels makes the pion momentum softer.
- Inclusion of $\rho \rightarrow \pi\pi$ gives back part of the strength.

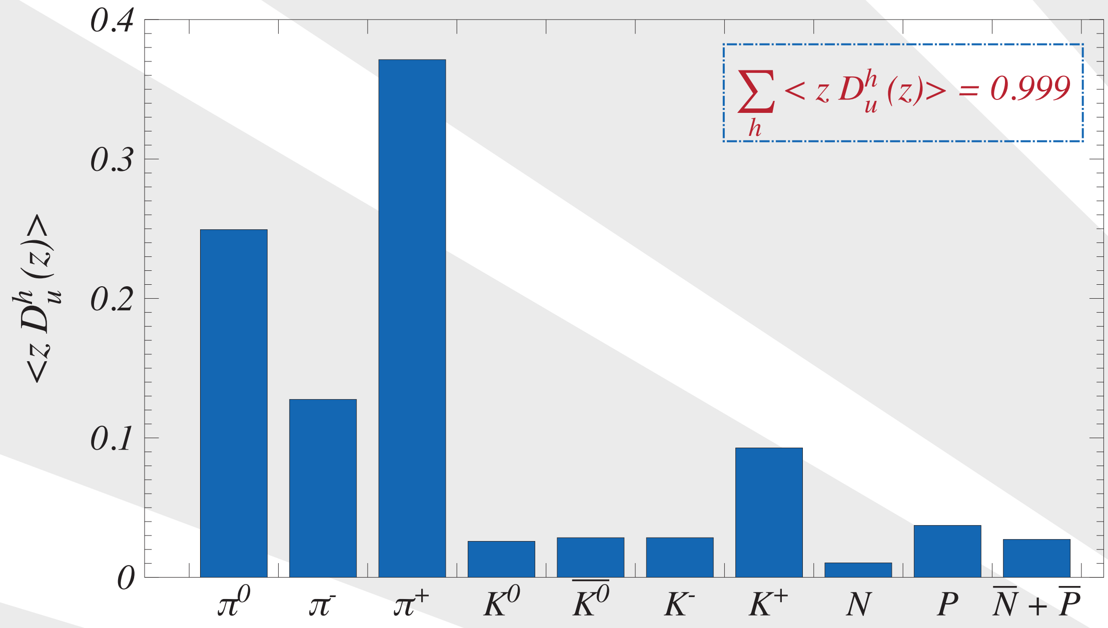
Fragmentation $u \to \pi, K, N$





- Left figure: NJL model scale (Q_0^2 = 0.18 GeV^2).
- Right figure: Evolved to Q^2=4 GeV^2 using QCDNUM-17, empirical curves with uncertainties from M. Hirai et al.

Momentum sum for u quark



- Momentum of u quark goes largely to π, followed by K.
- In this NJL calculation, we used the **invariant mass cut-off** (Lepage and Brodsky). We fixed $M_u = M_d = 300$ MeV, and fitted $M_s = 537$ MeV to m_K.

Summary

- Medium modifications at the quark level can explain the EMC effect.
- Isovector EMC effect has impact on NuTeV analysis and spin asymmetry in parity violating DIS.
- In the medium, part of quark spin is converted to orbital angular momentum, which leads to the polarized EMC effect.
- The recently developed NJL-jet model qualitatively describes the empirical fragmentation functions for SIDIS in vacuum.
- Very interesting project: SIDIS for nuclear targets!!

References used for NLO evolution

- Unpolarized distributions: M. Miyama, S. Kumano, Comput. Phys. Commun. **94** (1996) 185.
- Polarized distributions: M. Hirai, S. Kumano, M. Miyama, Comput. Phys. Commun. **108** (1998) 38.
- Unpolarized fragmentations: M. Botje (QCDNUM), Comput. Phys. Commun. **182** (2011) 490.

Role of nuclear vector field

Role of the mean vector field V_q acting on quark $q = u, d$: The following rescaling relation holds:

$$q_A(x) = \frac{M_A}{M_{A0}} q_{A0} \left(x_0 = \frac{M_A}{M_{A0}} x - \frac{V_q}{M_{A0}} \right),$$

where M_A = mass per nucleon, and $'0'$ means: 'without vector field'.
The can be understood from the relations

$$x = \frac{k}{M_A}, \qquad x_0 = \frac{k_0}{M_{A0}} = \frac{k - V_q}{M_{A0}} = x \frac{M_A}{M_{A0}} - \frac{V_q}{M_{A0}}.$$

(Here $k \equiv k^+ \cdot \sqrt{2}$, $k_0 \equiv k_0^+ \cdot \sqrt{2}$.)

Isovector corrections to M_A or M_{A0} are $\propto \left(\frac{N-Z}{A}\right)^2$, but those to V_q are $\propto \mp \frac{N-Z}{A}$ (for $q = u, d$).

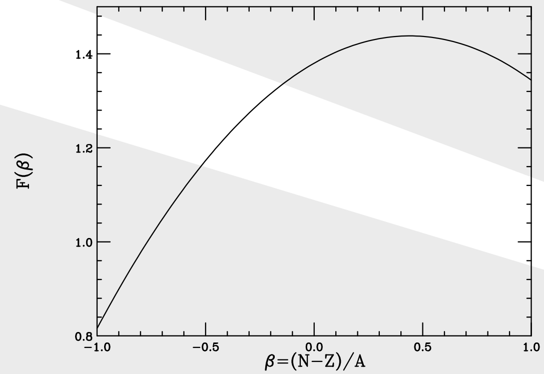
$\Rightarrow$ If neutron excess increases: Shift to left (from $M_A/M_{A0} > 1$, important for large x, same for u, d) does not change much,
but shift to right (from V_q) becomes weaker for u and stronger for d.

$\Rightarrow$ **Cancellation of shifts for the d quark at large x,**
Increasing left-shift for the u quark at large x.

Simple estimate of the isospin dependence

Isospin dependence of EMC effect:

- Assume: EMC effect $\propto$ binding energy of quarks (E_u, E_d) weighted by their numbers and squared charges:
  $F(\beta) \equiv \mathrm{const} \times (4N_u E_u + N_d E_d)$, where $\beta = (N - Z)/A$.
- Use $E_q = M_0 - \mu_q$, where $M_0 = 400$ MeV, and the chemical potentials follow from energy density as:
  $\mu_{d(u)} = \frac{1}{3}\overline{M}_N \pm 2\beta a_4$, where $\overline{M}_N = (940 - 15)$ MeV, $a_4 = 30$ MeV.



Note: Maximum at $\beta = 0.44$ ($Z/N = 0.4$).