

# 核子構造の基礎と 格子QCDによる核子構造研究の現状

大谷 宗久 (杏林大学)

## 1. 核子構造の基礎

- パートン分布と深非弾性散乱
- 形状因子と弾性散乱
- 一般化パートン分布と深部仮想コンプトン散乱

## 2. 格子 QCD による解析

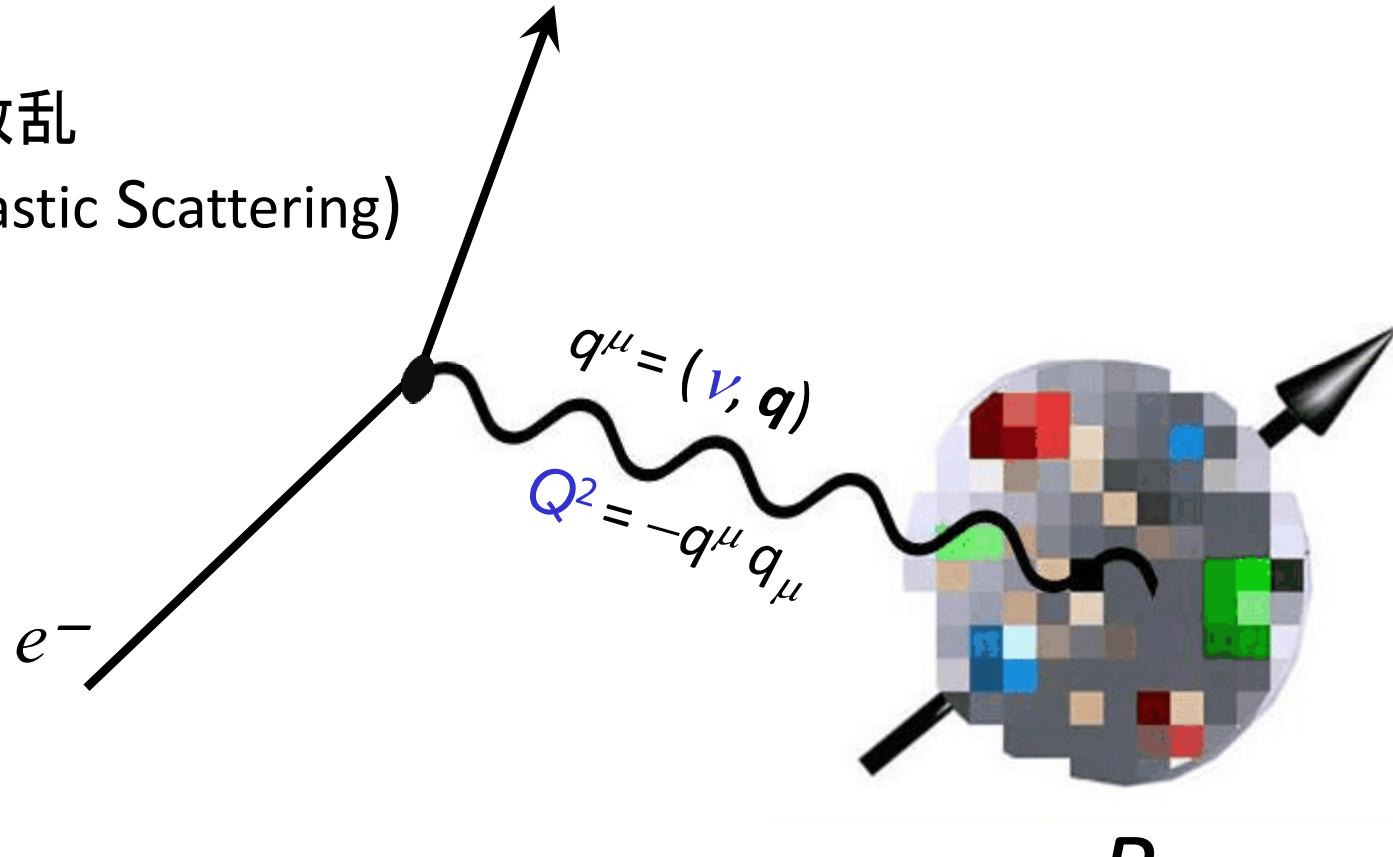
- 異常磁気能率, 荷電半径
- 軸性結合定数, 電磁分極率
- クオークの全角運動量, グルーオンからの寄与



# 核子の構成要素

- 核子の構造：核子の構成要素  $\otimes$  構成要素の状態分布

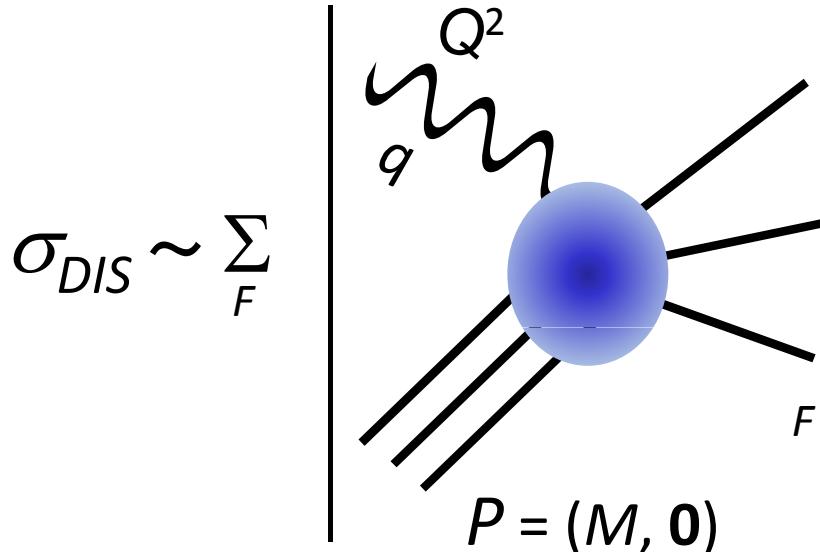
- 深非弾性散乱  
(Deep Inelastic Scattering)



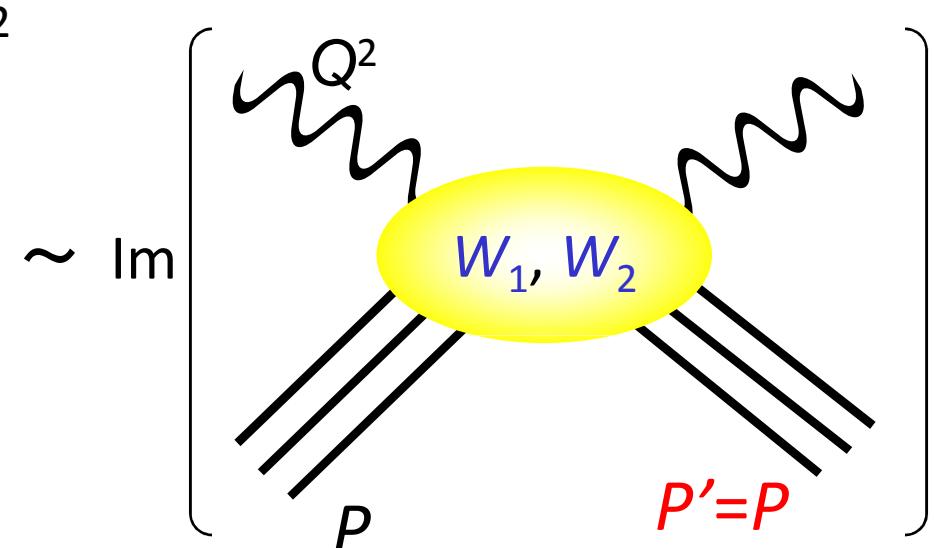
cf. 核構造: 低エネルギー  
集団運動  $\xrightarrow{\hspace{1cm}}$  中間エネルギー  
核子KO反応, pick up反応

# 構造関数

- 光学定理



前方散乱振幅

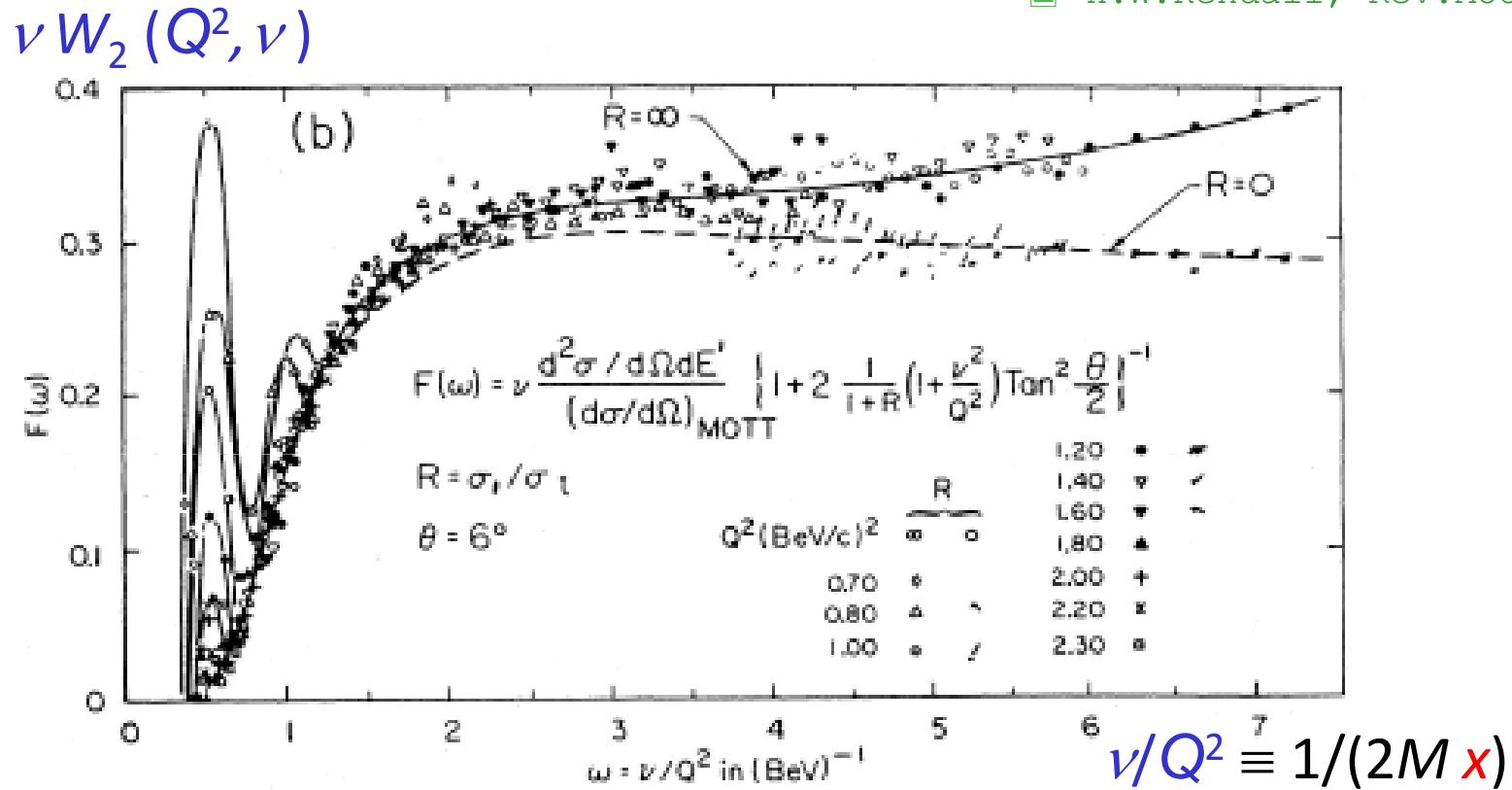


$$\sim j_\mu j_\nu [ \# (g^{\mu\nu} + \dots) W_1(Q^2, \cancel{q} \cdot \cancel{P}) + \# (P^\mu P^\nu + \dots) W_2(Q^2, \cancel{q} \cdot \cancel{P}) ]$$

$\cancel{M} \cancel{v}$  @標的静止系

# Bjorken scaling

■ H.W.Kendall, Rev.Mod.Phys. 63 (1991)

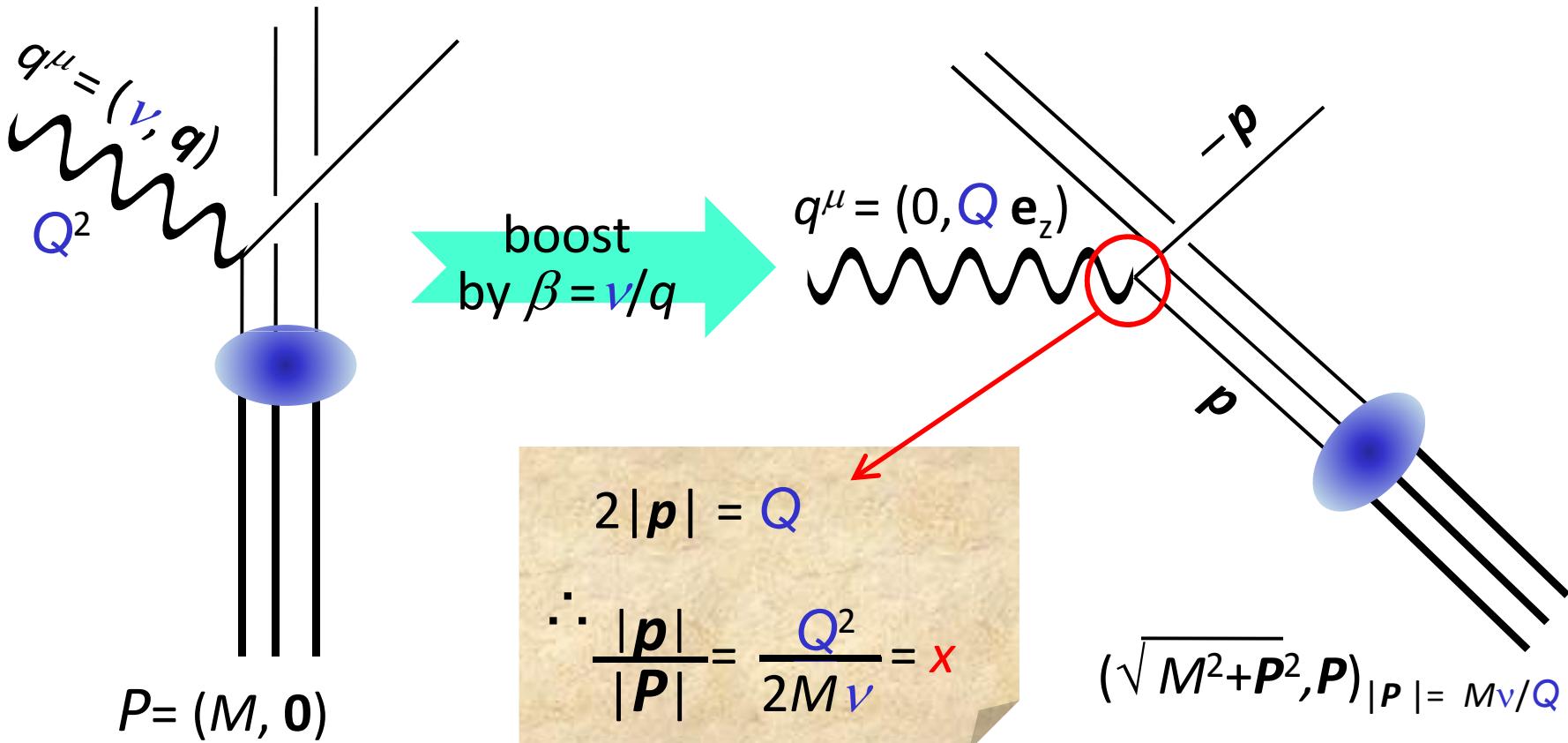


$$\nu W_2(Q^2, \nu) \xrightarrow[\nu, Q^2 \rightarrow \infty]{} F_2(x = Q^2/2M\nu)$$

解像度を上げても不变

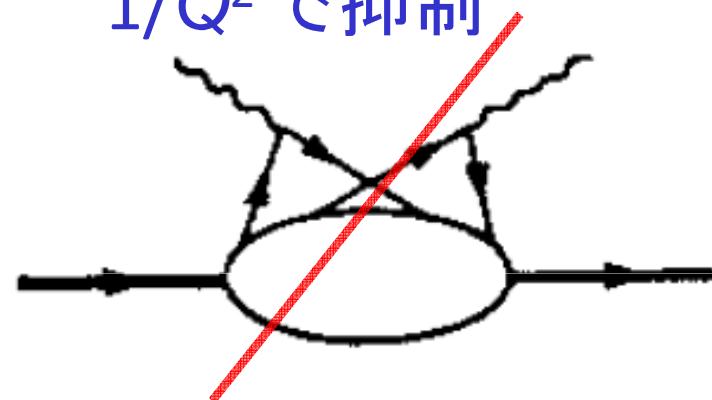
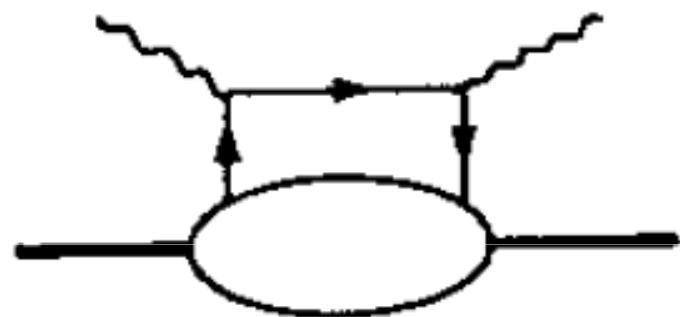
← Dirac Fermionからの  
incoherentな散乱

# 運動量比としての Scaling 変数

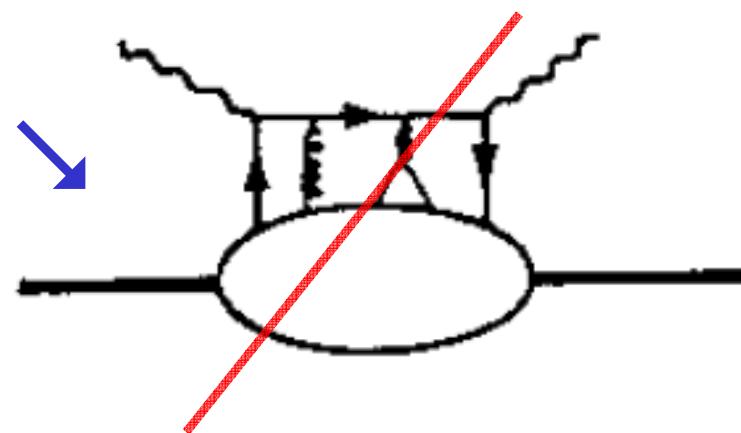
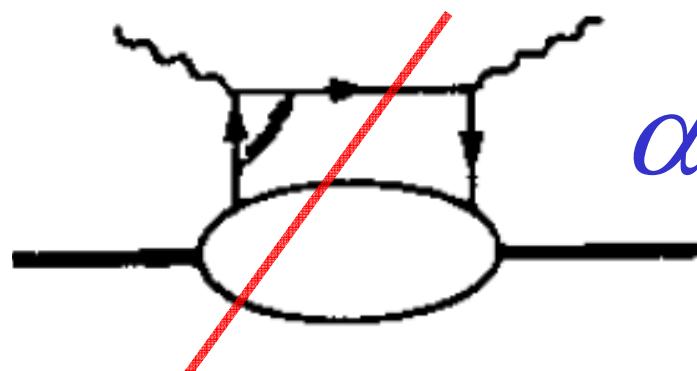


## 素朴なクオークパートン模型

光円錐特異性なし  
 $1/Q^2$  で抑制



$\alpha_s(Q^2) \downarrow$



# 演算子積展開とTwist

- 近距離展開:  $z^\mu \sim 0$

$$\hat{O}_A(z/2) \hat{O}_B(-z/2) \sim \sum_i \# (z)^{\textcolor{red}{d_i} - d_A - d_B} \hat{O}_i(0)$$

低次元の演算子が支配的

- 光円錐展開:  $z^\mu z_\mu \sim 0$

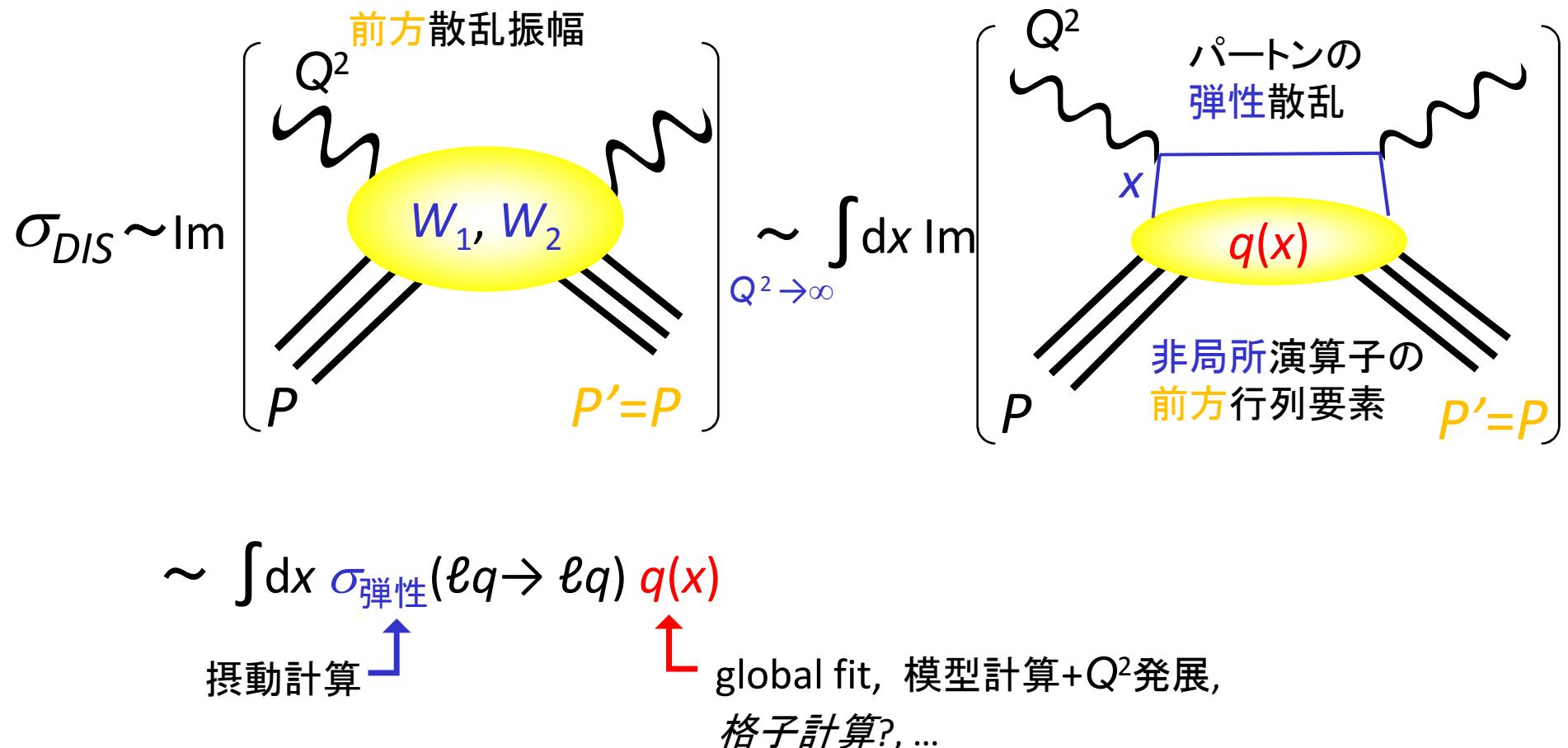
$$\int e^{iq \cdot z} \hat{O}_A(z/2) \hat{O}_B(-z/2) \sim \int \underbrace{e^{iq \cdot z} \sum_i \# (\sqrt{z^2})^{\textcolor{red}{d_i - s_i} - d_A - d_B}}_{\text{as } Q^2 \rightarrow \infty} z^{\mu_1} \cdots z^{\mu_s} \hat{O}_{i\mu_1 \cdots \mu_s}(0)$$



as  $Q^2 \rightarrow \infty$

光円錐特異性が高い  $\Leftrightarrow$  低 twist ( $\equiv$  次元-スピン)  
演算子が支配的

# 因子化



cf. Wigner-Eckert の定理

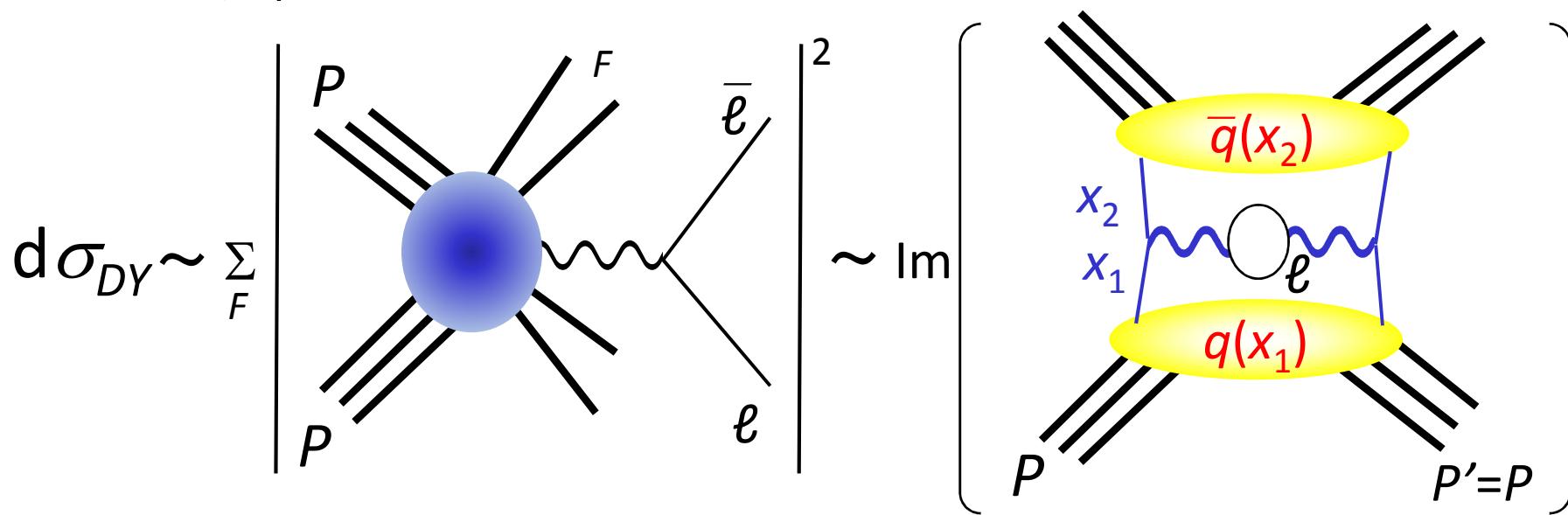
$$\langle j_1 m_1 | T_{LM} | j_2 m_2 \rangle = \frac{1}{\sqrt{2j_1 + 1}} \langle j_1 m_1 LM | j_2 m_2 \rangle \langle j_1 \| T_L \| j_2 \rangle$$

# パートン分布関数 (PDF)

- Callan Grossの関係式

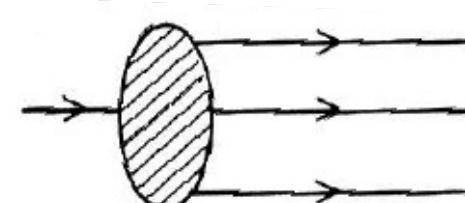
$$\begin{cases} MW_1(Q^2, \nu) \xrightarrow{\nu, Q^2 \rightarrow \infty} F_1(x = Q^2/2M\nu) = \sum_f \frac{1}{2} e_f^2 q_f(x) \\ \nu W_2(Q^2, \nu) \rightarrow F_2(x) = \sum_f e_f^2 x q_f(x) \end{cases} \quad \therefore 2x F_1(x) = F_2(x)$$

- Drell Yan 過程

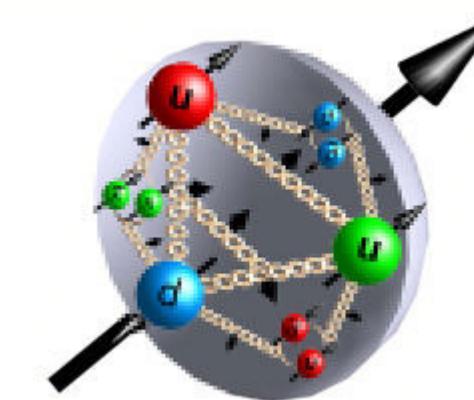
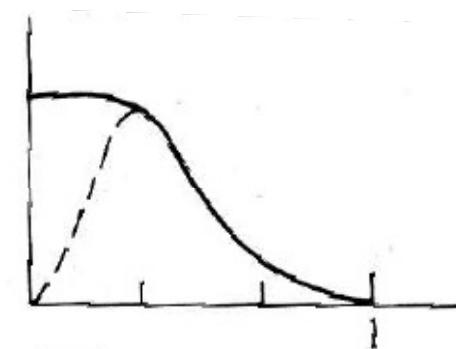
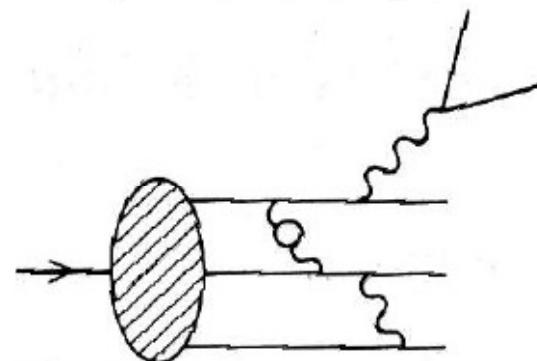
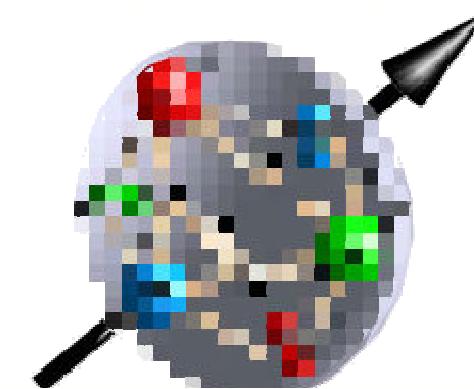
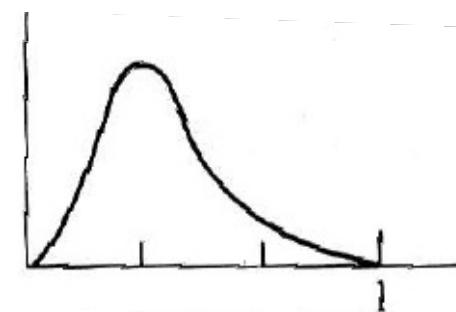
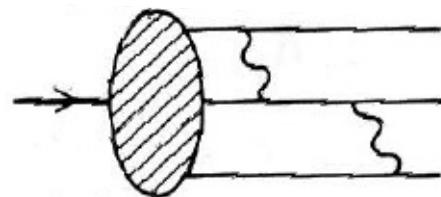
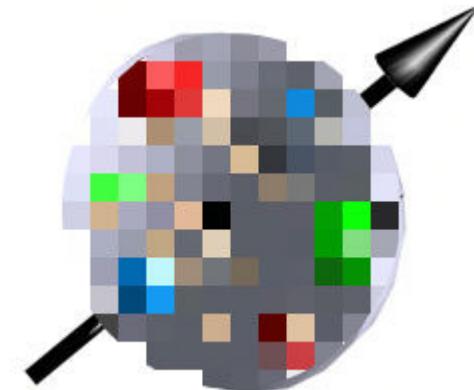
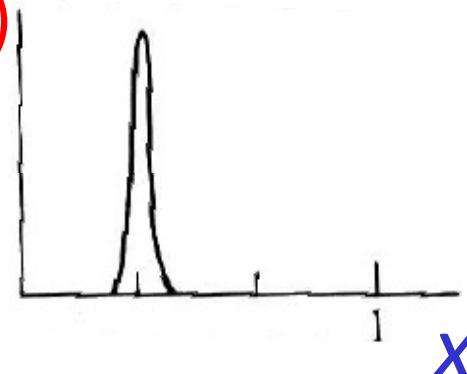


$$\sim \# \sum_f e_f^2 [q_f(x_1) \bar{q}_f(x_2) + \bar{q}_f(x_1) q_f(x_2)]$$

# PDFの概形

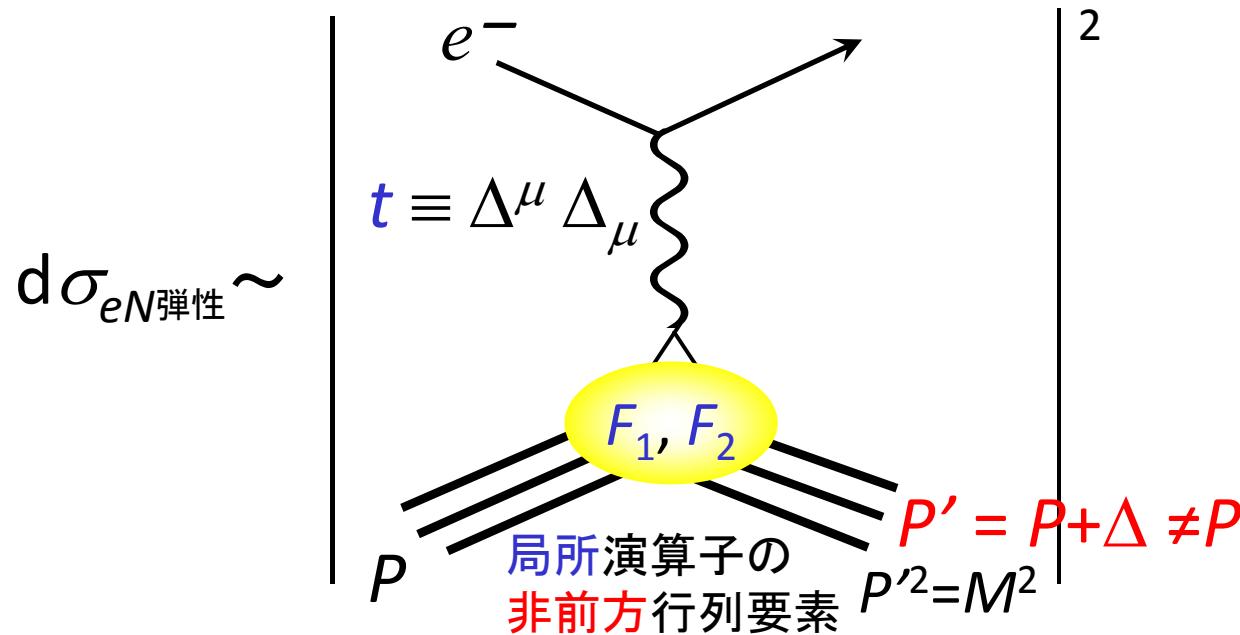


$q(x)$



# 形状因子

- 電子-陽子弹性散乱

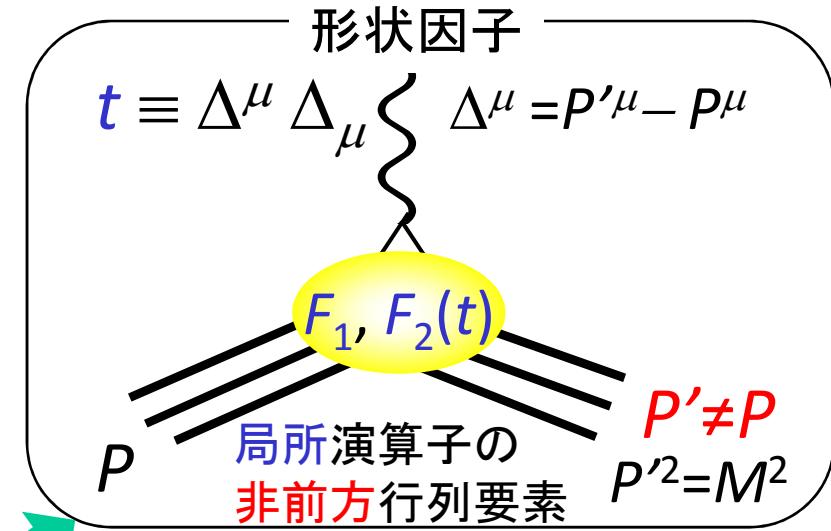
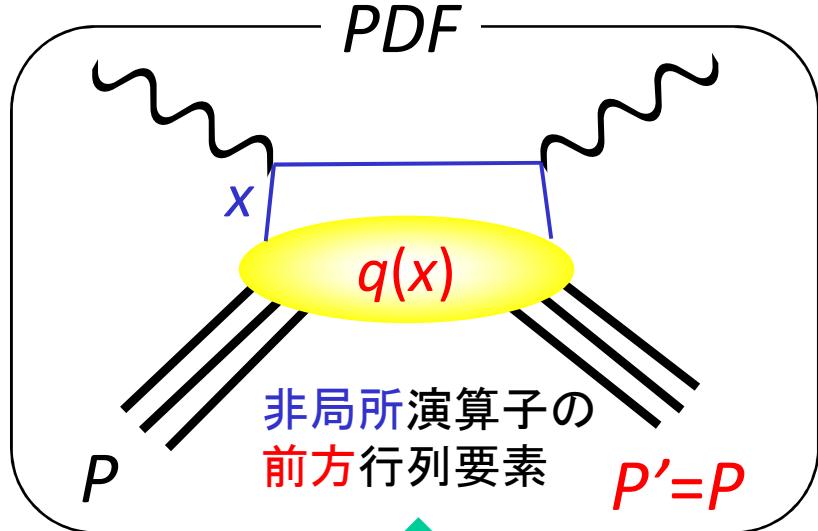


↓ 異常磁気能率の分布

$$\sim \# F_1(t)^2 + \# F_2(t)^2 + \# F_1(t) F_2(t)$$

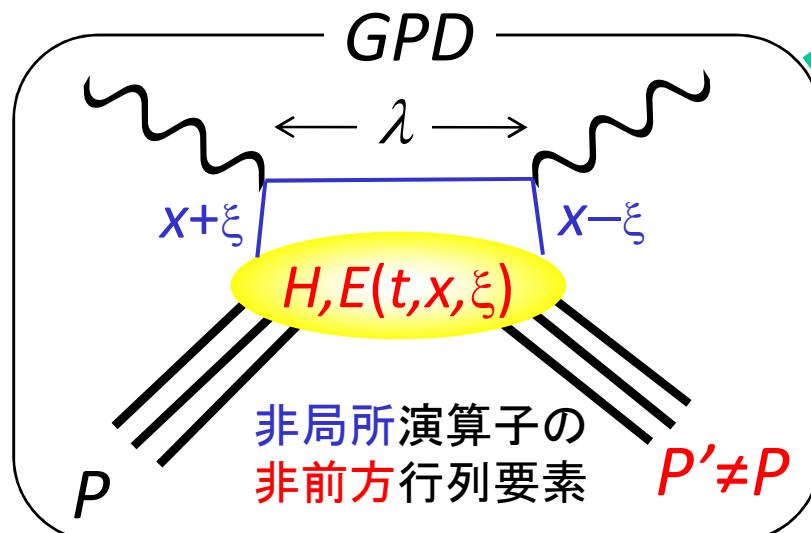
↑ 電荷分布

# 一般化パートン分布(Generalized Parton Dist.)

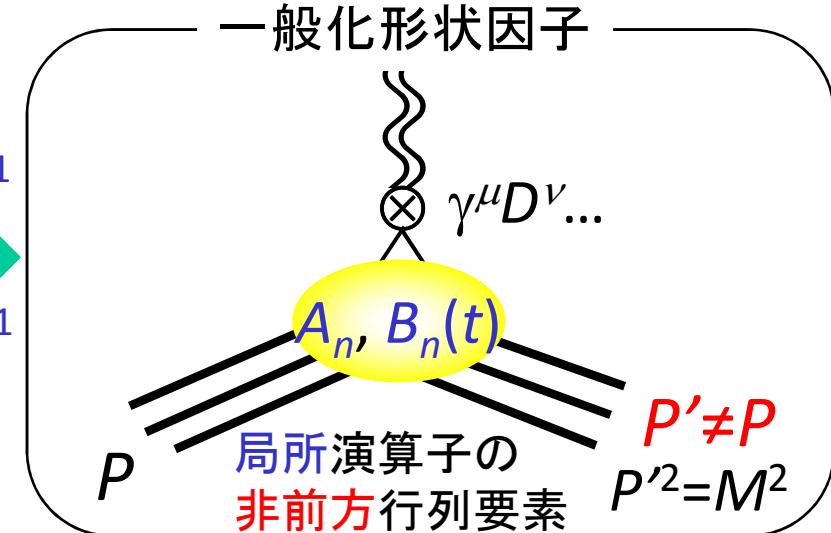


$P' \rightarrow P$  i.e.  $t \rightarrow 0$

$\lambda \rightarrow 0$  i.e.  $\delta(\lambda) = \int dx e^{ix\lambda}$



$$\int dx x^{n-1} \quad \text{II} \quad \int dx \partial_\lambda^{n-1}$$



# エネルギー運動量テンソルと角運動量

- 正準テンソルと軌道角運動量

$$T^{\mu\nu} = \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \partial^\nu \phi - g^{\mu\nu} \mathcal{L}$$

$$\textcolor{blue}{L} = \int d^3x (\mathbf{x} \times T^{0i}) \quad \Rightarrow \psi^\dagger \mathbf{x} \times (-i\nabla) \psi$$

$$cf. H = p \frac{dq}{dt} - L \Big|_{p=\partial L/\partial q}$$

- 対称テンソルと全角運動量

$$\Theta^{\mu\nu} = \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} \quad \Rightarrow \bar{\psi} \gamma^{\{\mu} i \partial^{\nu\}} \psi$$

$$cf. j^\mu = \frac{\delta \mathcal{L}}{\delta A_\mu} \quad \Rightarrow \bar{\psi} e \gamma^\mu \psi$$

$$\begin{cases} \langle x \rangle_q = \textcolor{red}{A}_{20}(t=0); \\ \textcolor{red}{J} = \int d^3x (\mathbf{x} \times \Theta^{0i}) \quad \Rightarrow \psi^\dagger [\mathbf{x} \times (-i\nabla) + \gamma^0 \sigma/2] \psi \\ \langle J \rangle \sim \#(\textcolor{red}{A}_{20} + B_{20})|_{t=0} \bar{N} \sigma N \end{cases}$$

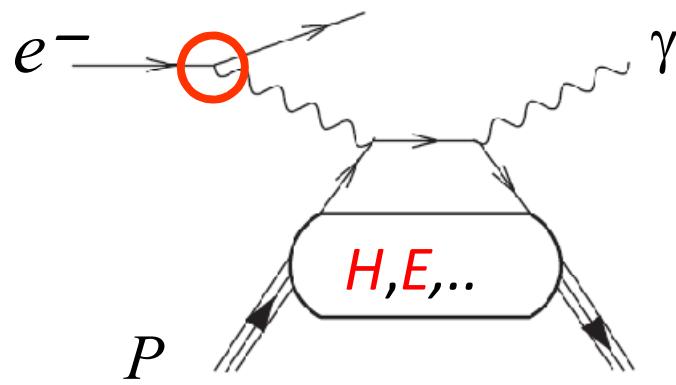
[ X.Ji, PRL 78 (1997) ]

$$\begin{cases} e_p = \textcolor{red}{F}_1(t=0); \\ K_p = \textcolor{red}{F}_2(t=0) \end{cases}$$

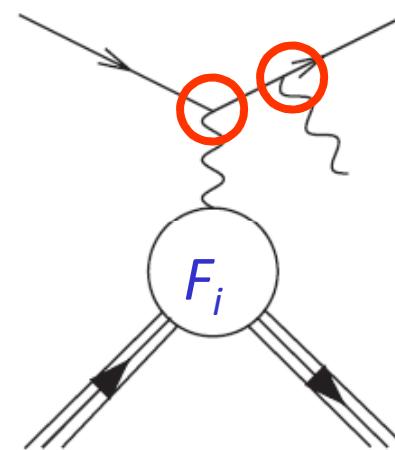
※ グルーオンのスピン・軌道分離 ← ゲージ不变性 vs. 局所演算子

# 深部仮想コンプトン散乱

Deeply Virtual Compton Scattering



Bethe-Heitler



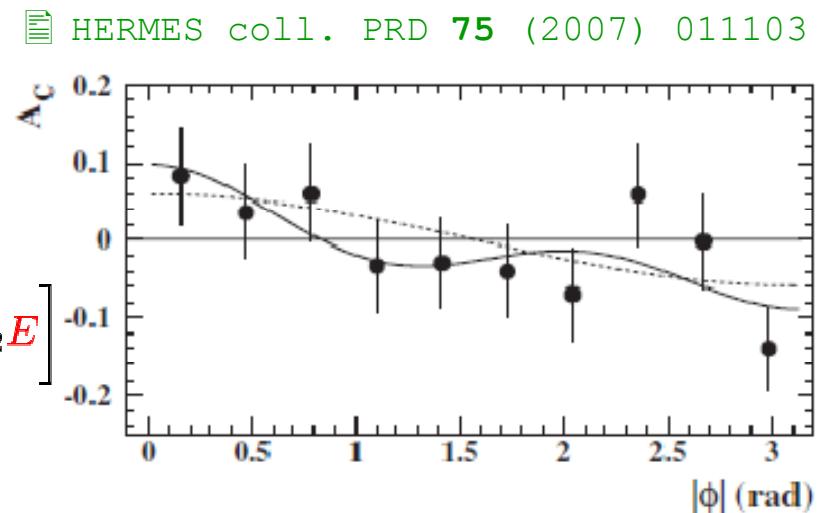
$$|\tau|^2 = |\tau_{\text{BH}}|^2 + |\tau_{\text{DVCS}}|^2 + \underbrace{\tau_{\text{DVCS}} \tau_{\text{BH}}^* + \tau_{\text{DVCS}}^* \tau_{\text{BH}}}_{F_1 \times H, F_2 \times E, \dots}$$

# 非対称度

- beam charge asymmetry

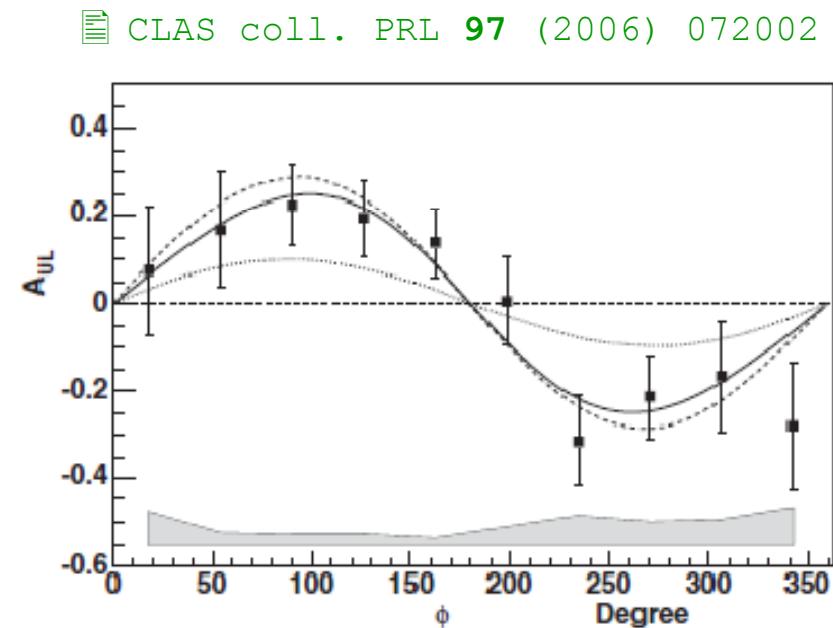
$$A_C(\phi) = \frac{N^+(\phi) - N^-(\phi)}{N^+(\phi) + N^-(\phi)}$$

$$\propto \cos \phi \operatorname{Re} \left[ F_1 \textcolor{red}{H} + \frac{x}{2-x} (F_1 + F_2) \tilde{\textcolor{red}{H}} - \frac{t}{4M^2} F_2 \textcolor{red}{E} \right]$$



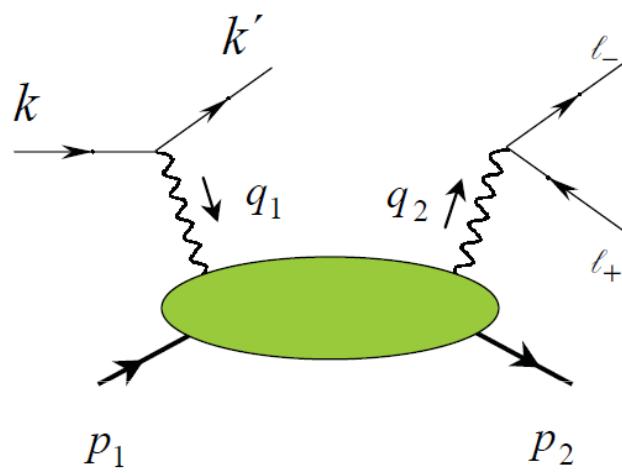
- Longitudinal target spin asymmetry

$$A_{UL}(\phi) = \frac{d\sigma^{\uparrow\uparrow}(\phi) - d\sigma^{\downarrow\downarrow}(\phi)}{d\sigma^{\uparrow\uparrow}(\phi) + d\sigma^{\downarrow\downarrow}(\phi)}$$

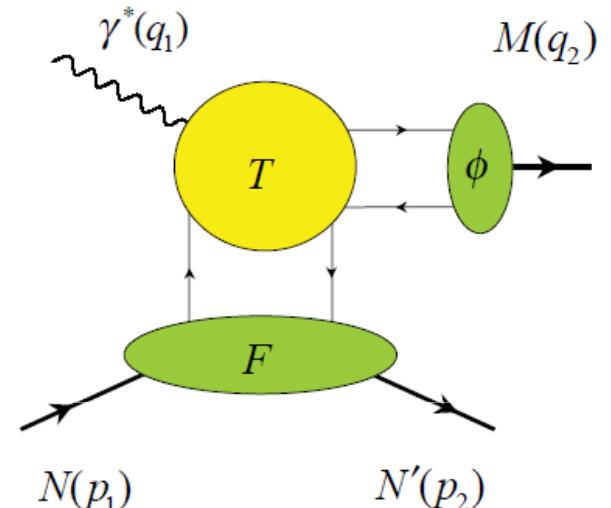


# DVCS以外の実験とGPD

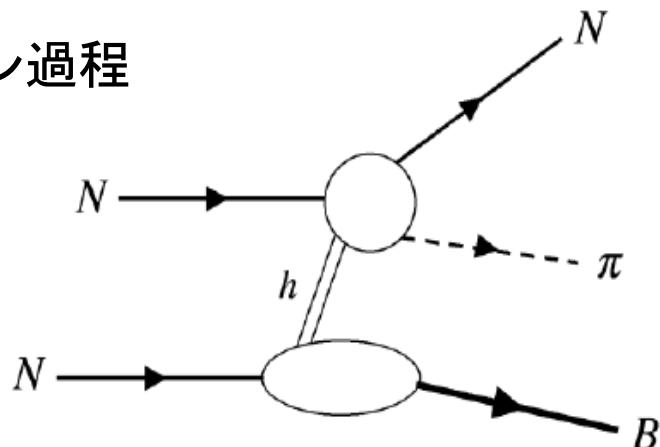
- レプトン対生成



- 中間子生成

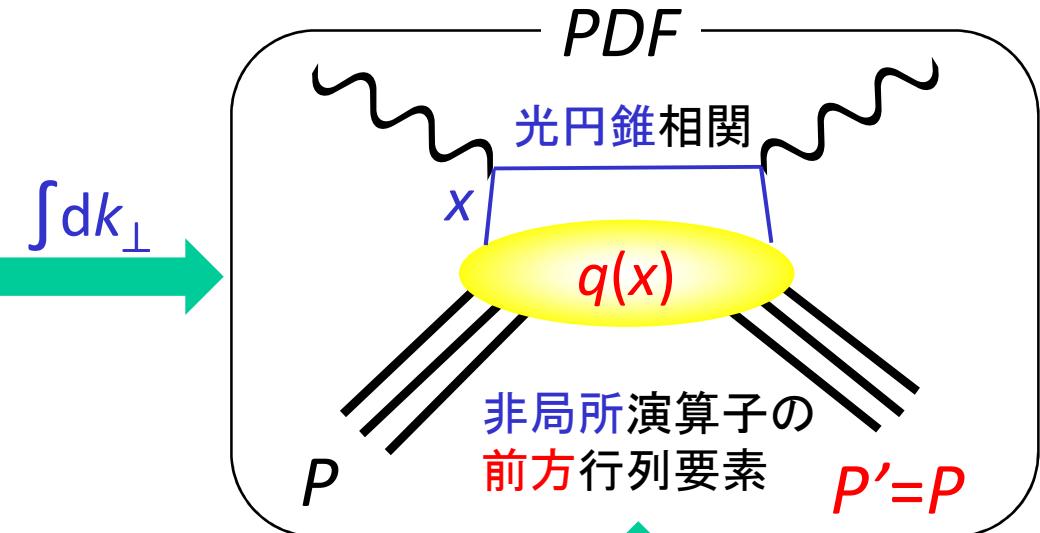
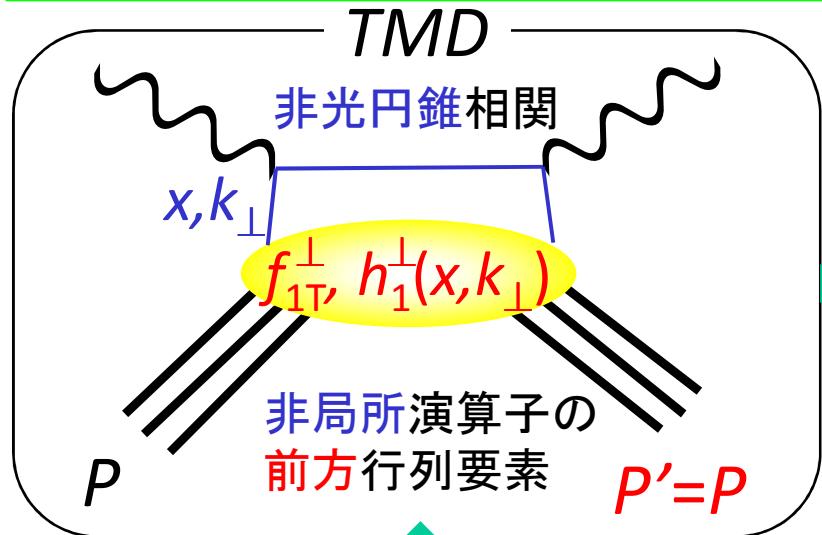


- ハドロン過程

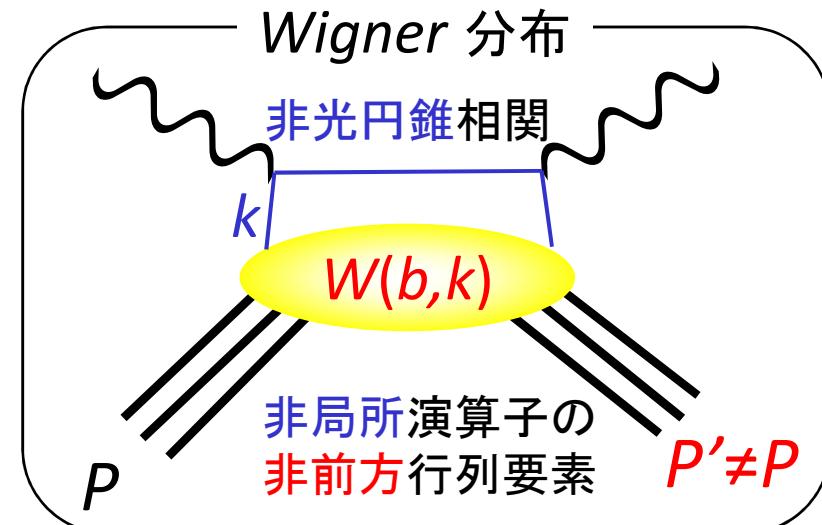


Accessible @ J-PARC, GSI..

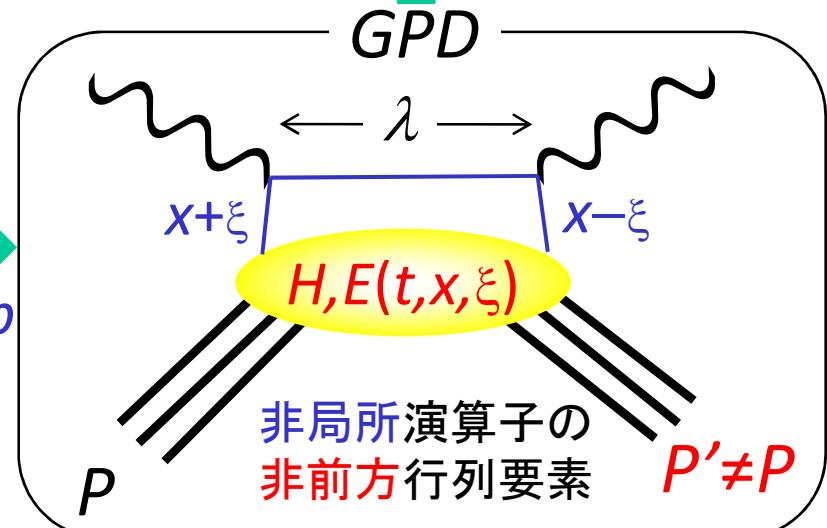
# GPD & TMD(*Transverse Momentum Dependent parton dist.*)



$P' \rightarrow P$  i.e.  $\int db$



$\int dk_\perp$   
F.T. w.r.t  $b$



# 実験との対応と前方極限値

	構造関数 (スケーリング関数)		PDF	形状因子 (一般化形状因子)		GPD
非偏極	$W_1, W_2(Q^2, x)$	$F_1, F_2; f_1, f_2(x)$	$q; f_q(x)$	$F_1, F_2; G_E, G_M; A_{10}, B_{10}(t)$	$A_{nk}, B_{nk}, C_n(t)$	$E, H(t, x, \xi)$
	DIS		DIS, DY	Elastic $\ell N$ ; $\xrightarrow{t \rightarrow 0} e, \kappa$	$\xrightarrow{t \rightarrow 0} \langle x^{n-1} \rangle_q, J_q$	DVCS, 中間子生成 $\xrightarrow{t \rightarrow 0} q(x)$
縦偏極	$G_1, G_2(Q^2, x)$	$g_1, g_2(x)$	$\Delta q(x)$	$G_A, G_P; g_A, h_A(t)$	$\tilde{A}_{nk}, \tilde{B}_{nk}, \tilde{C}_n(t)$	$\tilde{E}, \tilde{H}(t, x, \xi)$
	偏極DIS		偏極DIS, W生成, 偏極DY	$\nu N, \pi$ 生成; $\xrightarrow{t \rightarrow 0} g_A, s_q$	$\xrightarrow{t \rightarrow 0} \langle x^{n-1} \rangle_{\Delta q}$	DVCS, 中間子生成 $\xrightarrow{t \rightarrow 0} \Delta q(x)$
横偏極	$F_{UT}^{\sin(\phi)}, \dots$	$h_1(x)$	$\delta q; \Delta_T q(x)$	$G_T(t); A_{T10}, B_{T10} \dots$	$A_{Tnk}, B_{Tnk}(t) \dots$	$E_T, H_T(t, x, \xi) \dots$
	$A_{UT}$ in SIDIS, $A_{TT}$ in $\bar{p}p$ DY, ..			$\xrightarrow{t \rightarrow 0} g_T$	$\xrightarrow{t \rightarrow 0} \langle x^{n-1} \rangle_{\delta q}$	$\xrightarrow{t \rightarrow 0} \delta q(x)$

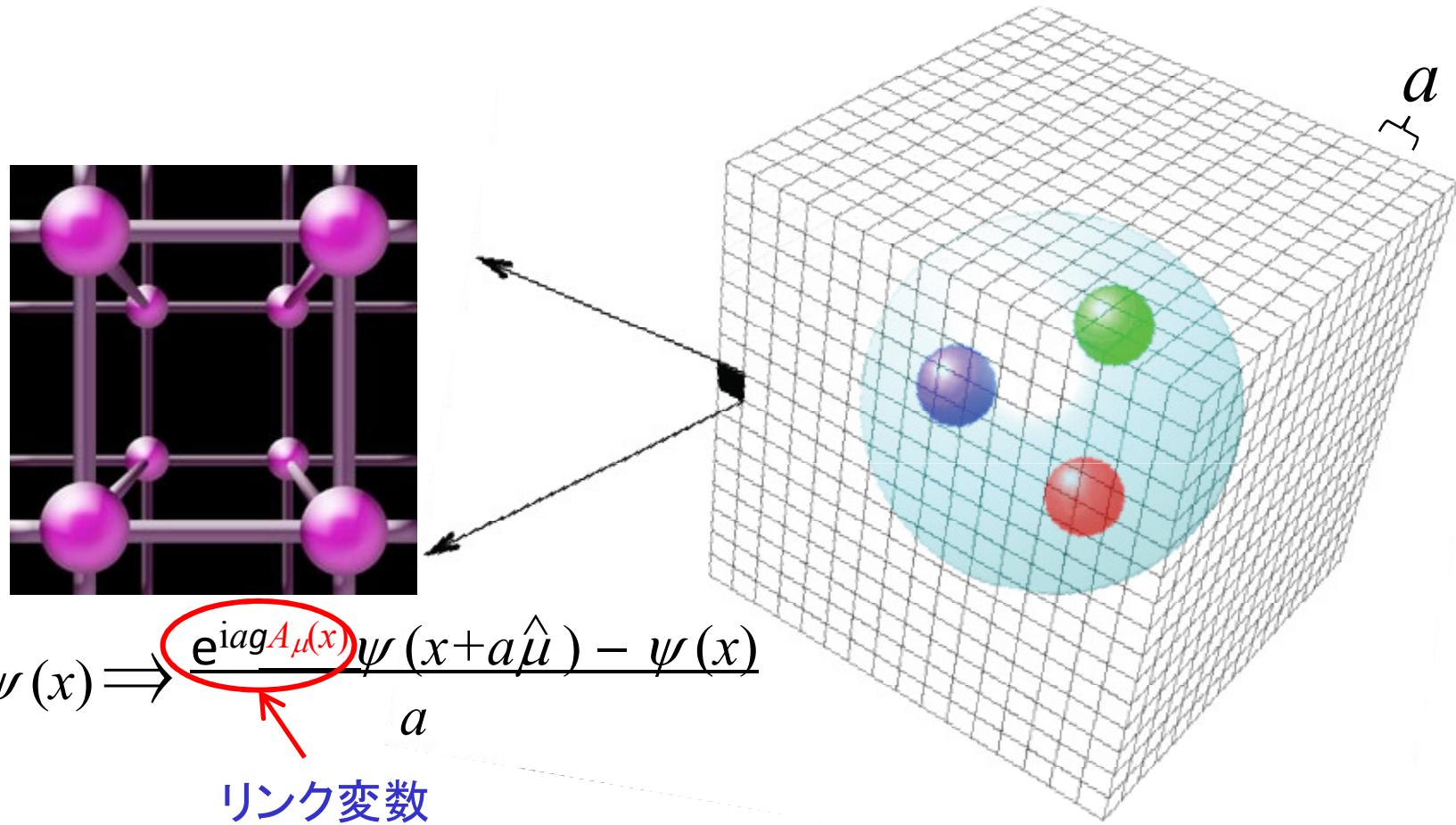
## 1. 核子構造の基礎

- パートン分布と深非弾性散乱
- 形状因子と弾性散乱
- 一般化パートン分布と深部仮想コンプトン散乱

## 2. 格子 QCD による解析

- 異常磁気能率と荷電半径
- 軸性結合定数, 電磁分極率
- クオークの全角運動量, グルーオンからの寄与

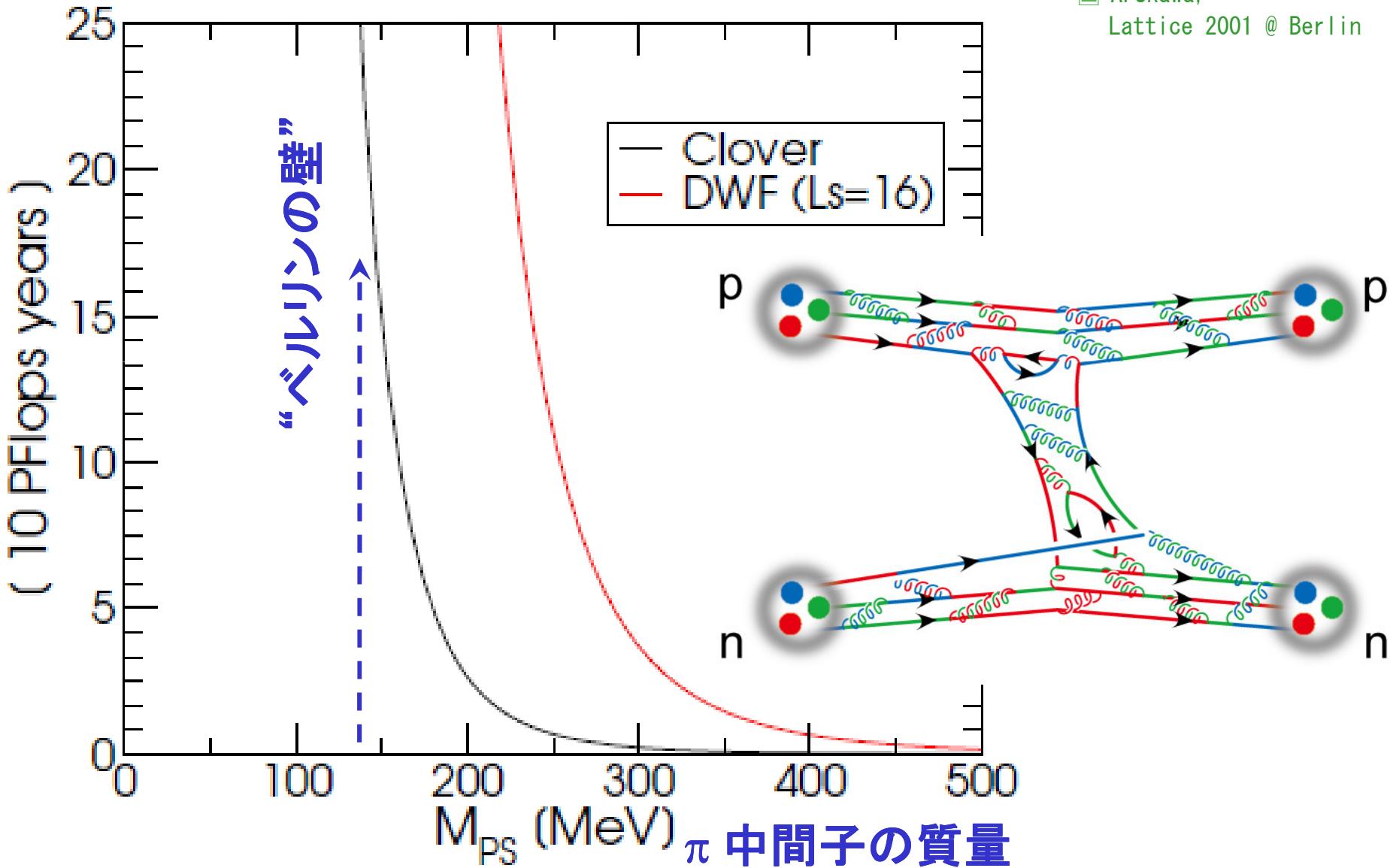
# 格子状に分割した時空と場



- フェルミオンの格子定式化: staggered, Wilson, Domain Wall, Overlap, ..  
計算コスト vs. カイラル対称性

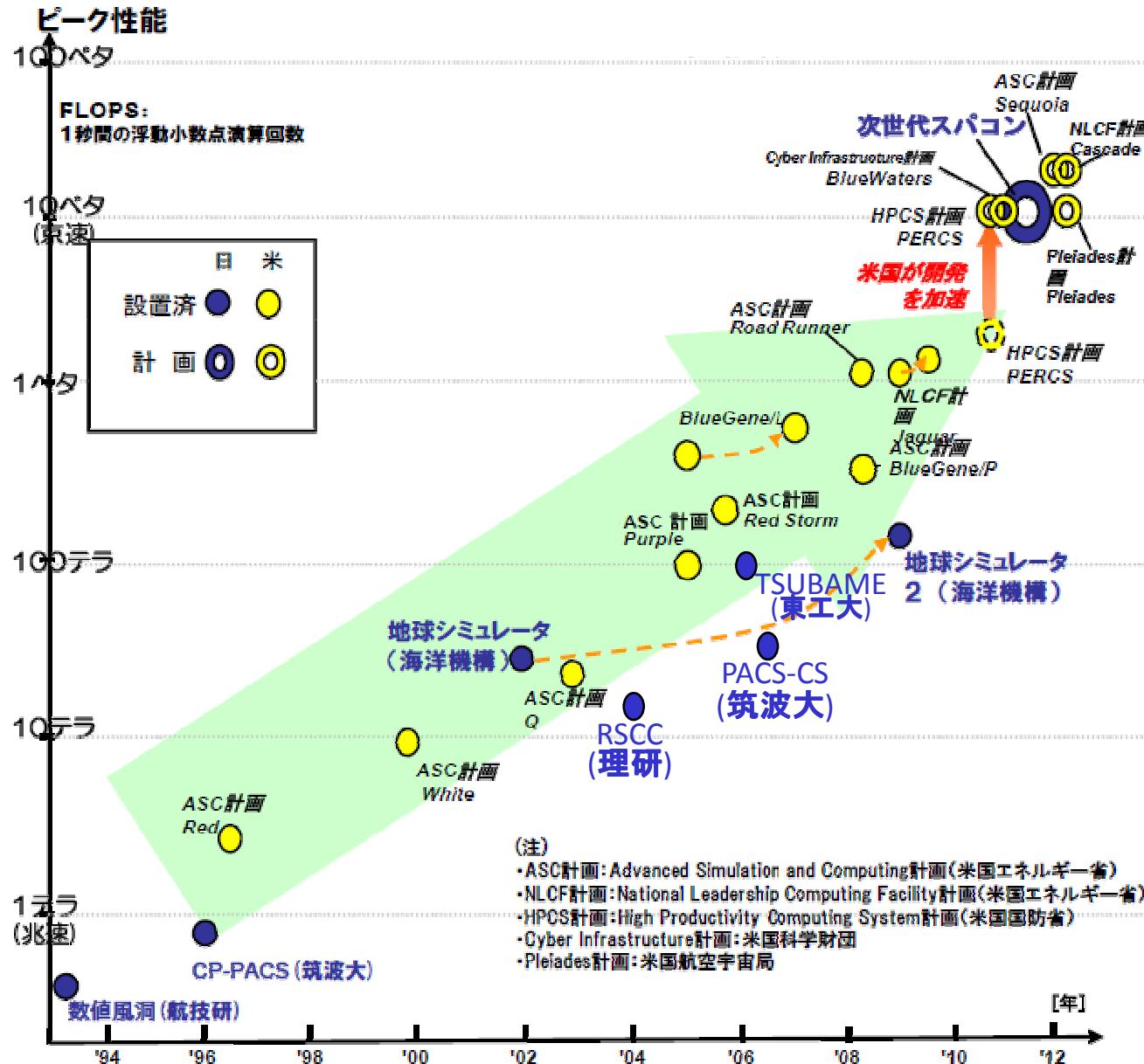
# 計算性能の限界

A. Ukawa,  
Lattice 2001 @ Berlin



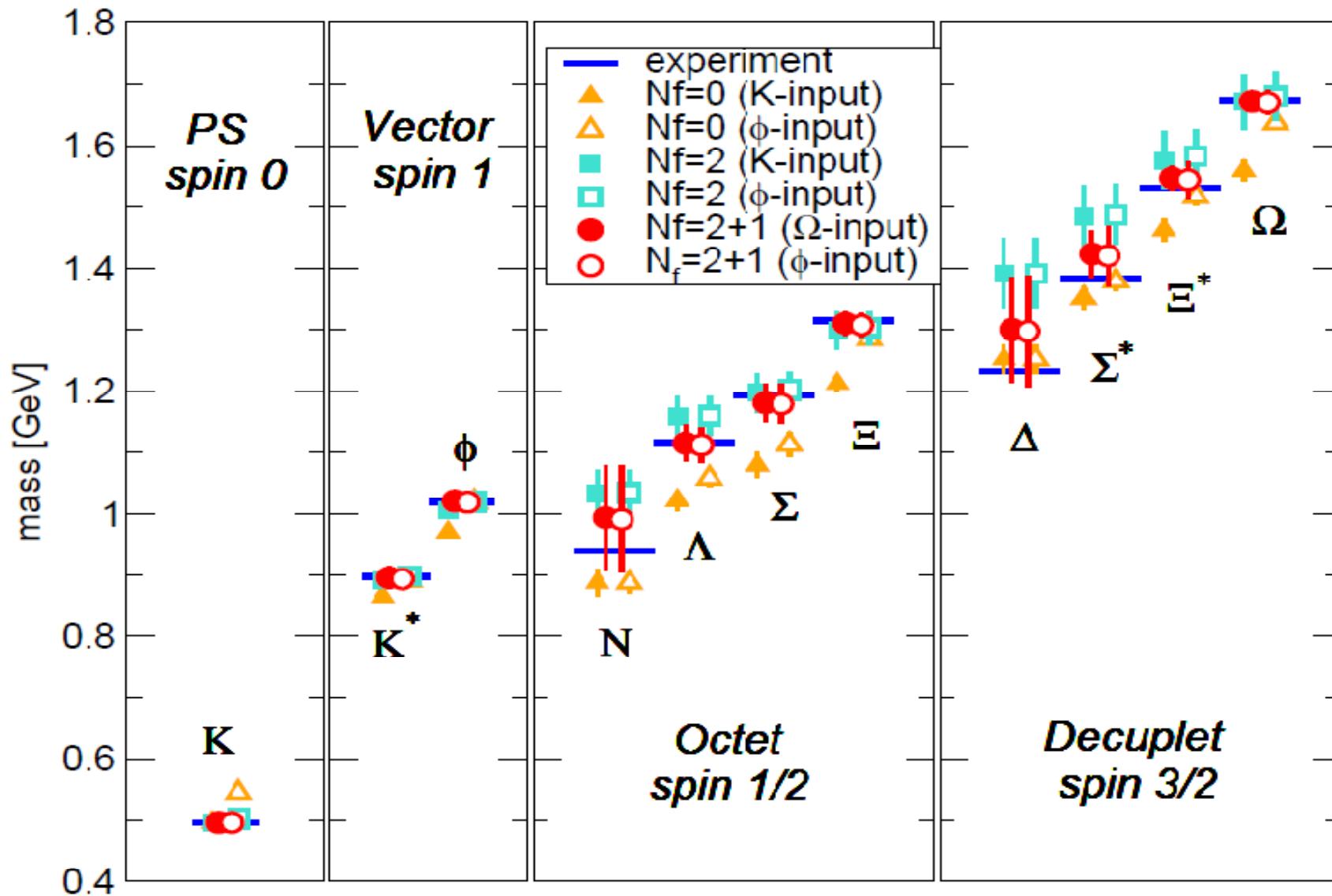
# 国内外のスパコンの変遷

文部科学省研究振興局、  
講演資料(2010)



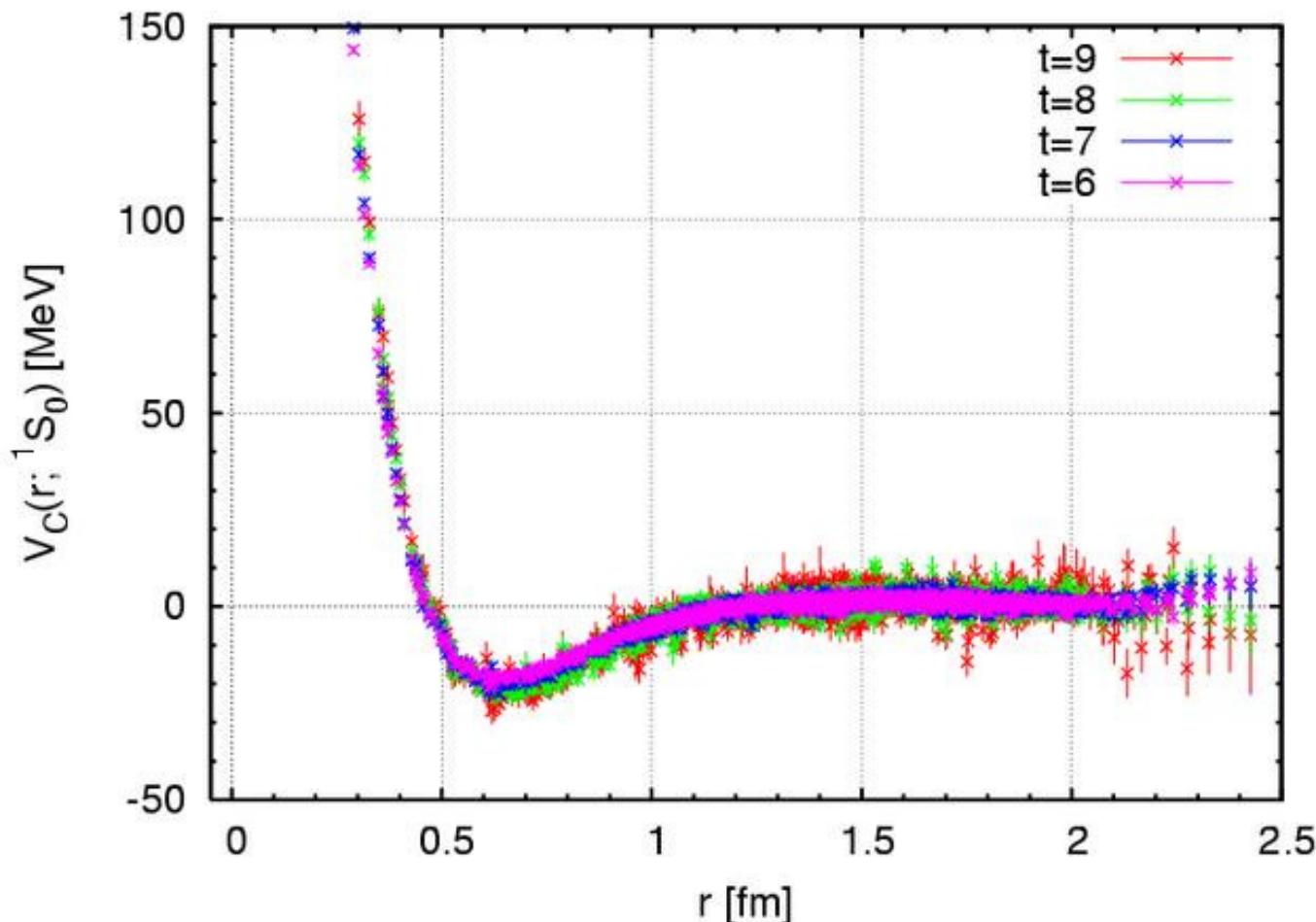
# 粒子質量の実験値と計算値

Y. Kuramashi, PoS Lat2007 (2007) 017



# 格子計算による重粒子間ポテンシャル

■ N. Ishii, S. Aoki & T. Hatsuda, PRL 99 (2007)



# Moments of GPD: Generalized Form Factors

- Polynomiality

X. Ji, J. Phys. **G24**(1998) 1181

$$\int dx x^{n-1} \begin{bmatrix} H(x, \xi, t) \\ E(x, \xi, t) \end{bmatrix} = \sum_{k=0}^{[n-1/2]} (2\xi)^{2k} \begin{bmatrix} A_{n,2k}(t) \\ B_{n,2k}(t) \end{bmatrix} \pm \delta_{n,\text{even}} (2\xi)^n C_n(t)$$



$A_{n,2k}, B_{n,2k}, C_n$  are related to  $\langle P | \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \dots D^{\mu_n\}} \psi | P' \rangle$

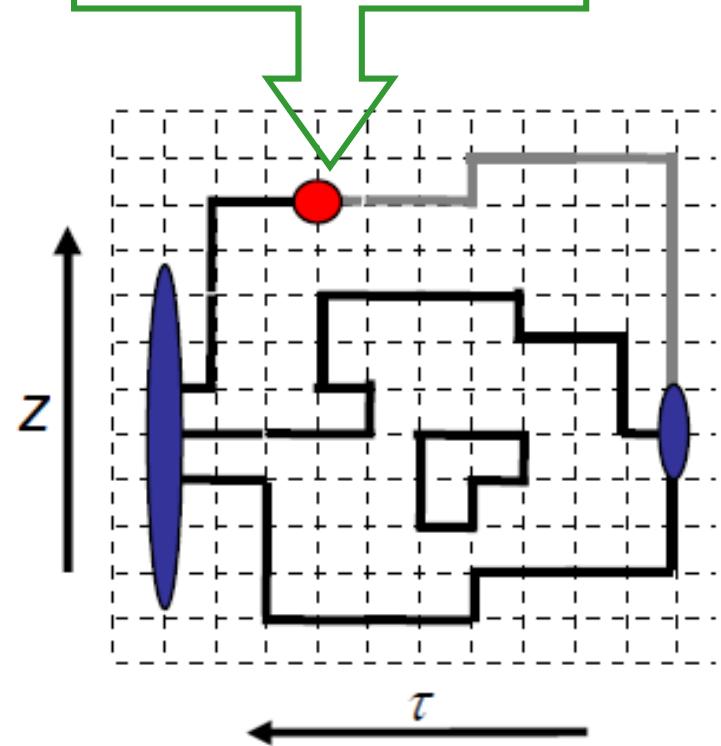
Calculate ratio of 2pt & 3pt correlation functions on lattice



Extract GFF

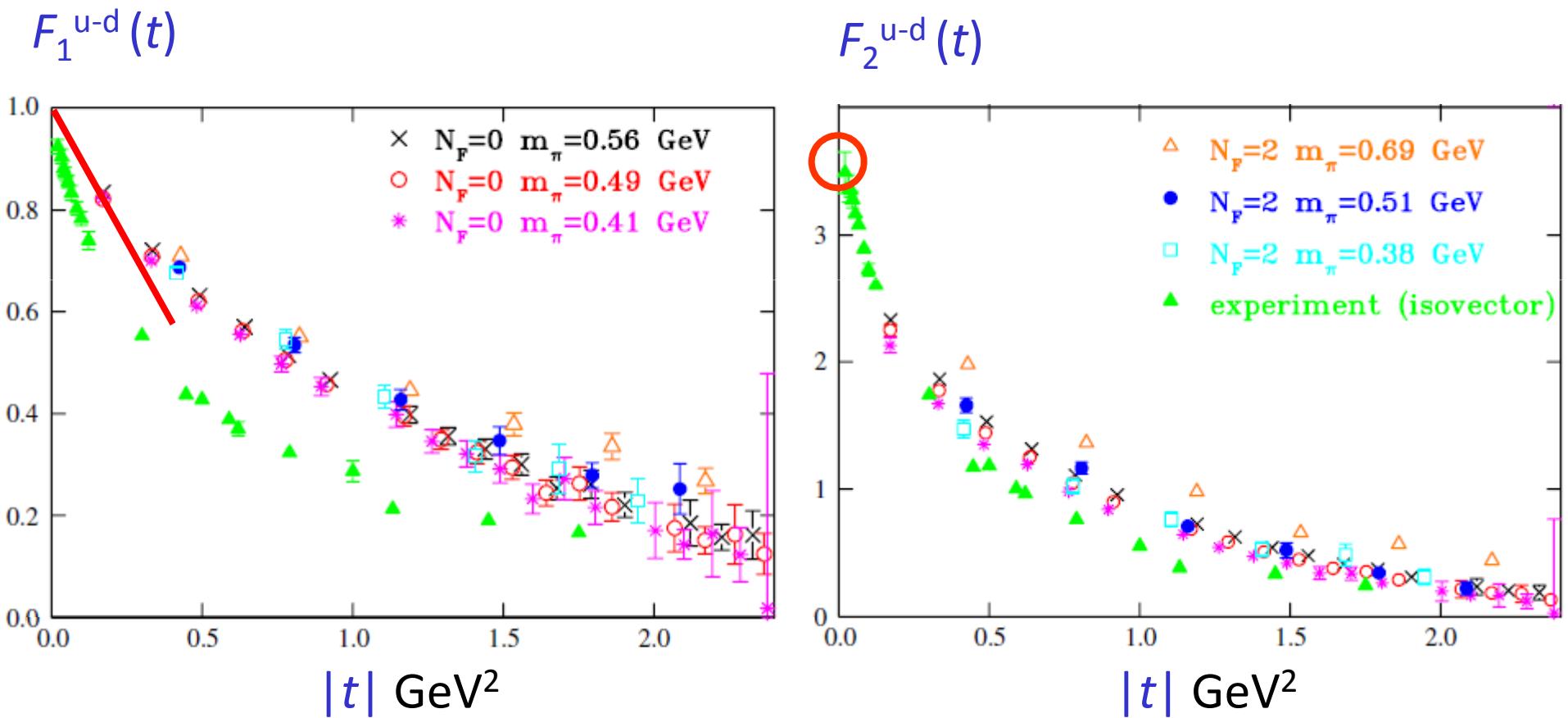
LHPC, PRD**68**(2003) 034505

$$\langle P | \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \dots D^{\mu_n\}} \psi | P' \rangle$$



# 核子の形状因子

■ C. Alexandrou, et al., PRD74 (2006)

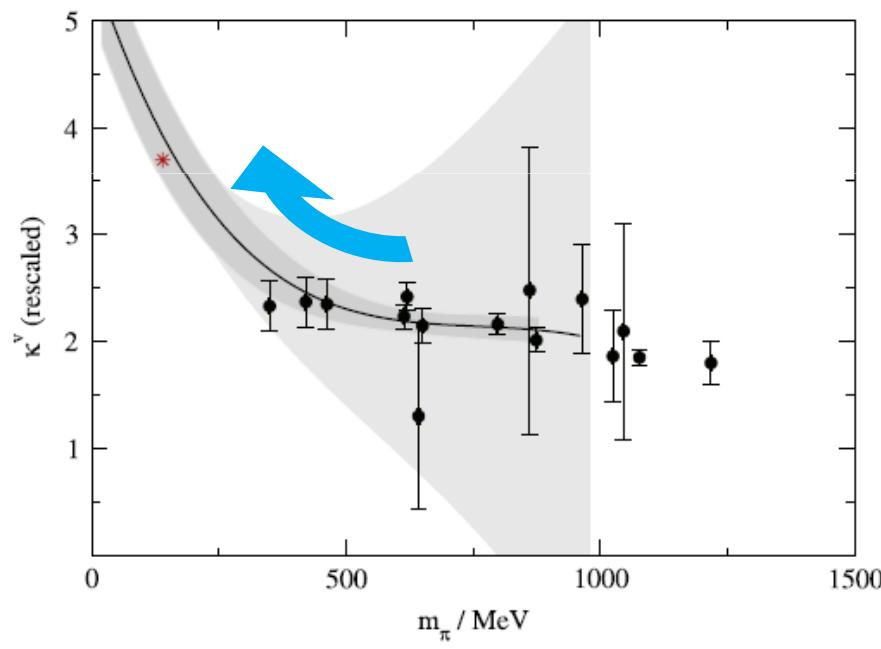


# 異常磁気能率

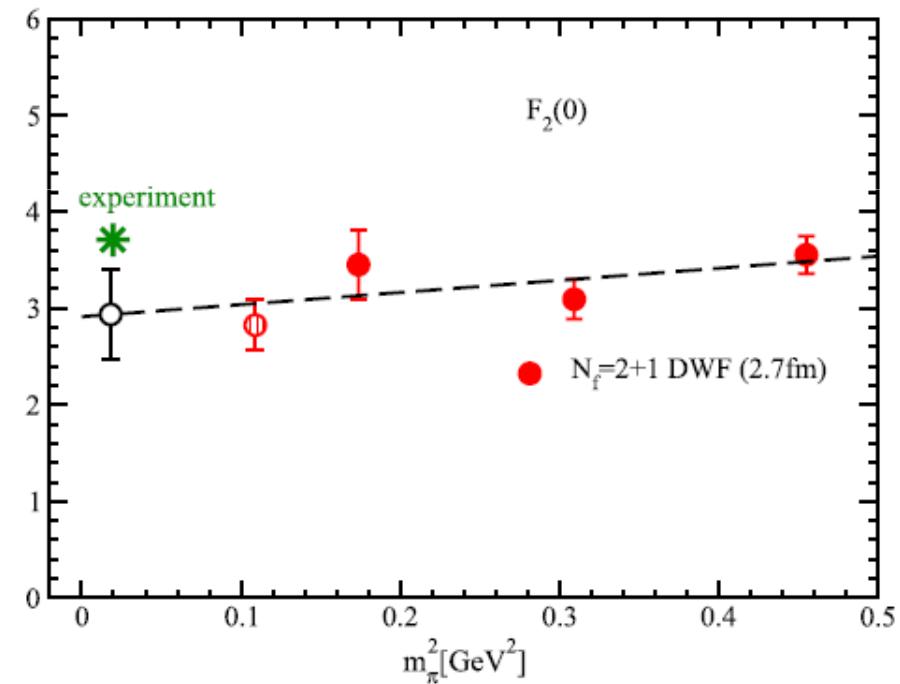
QCDSF, PoS Lattice2007, 161

RBC-UKQCD, PoS Lattice2008

## カイラル摂動論による外挿



$N_F=2$ , Wilson-clover

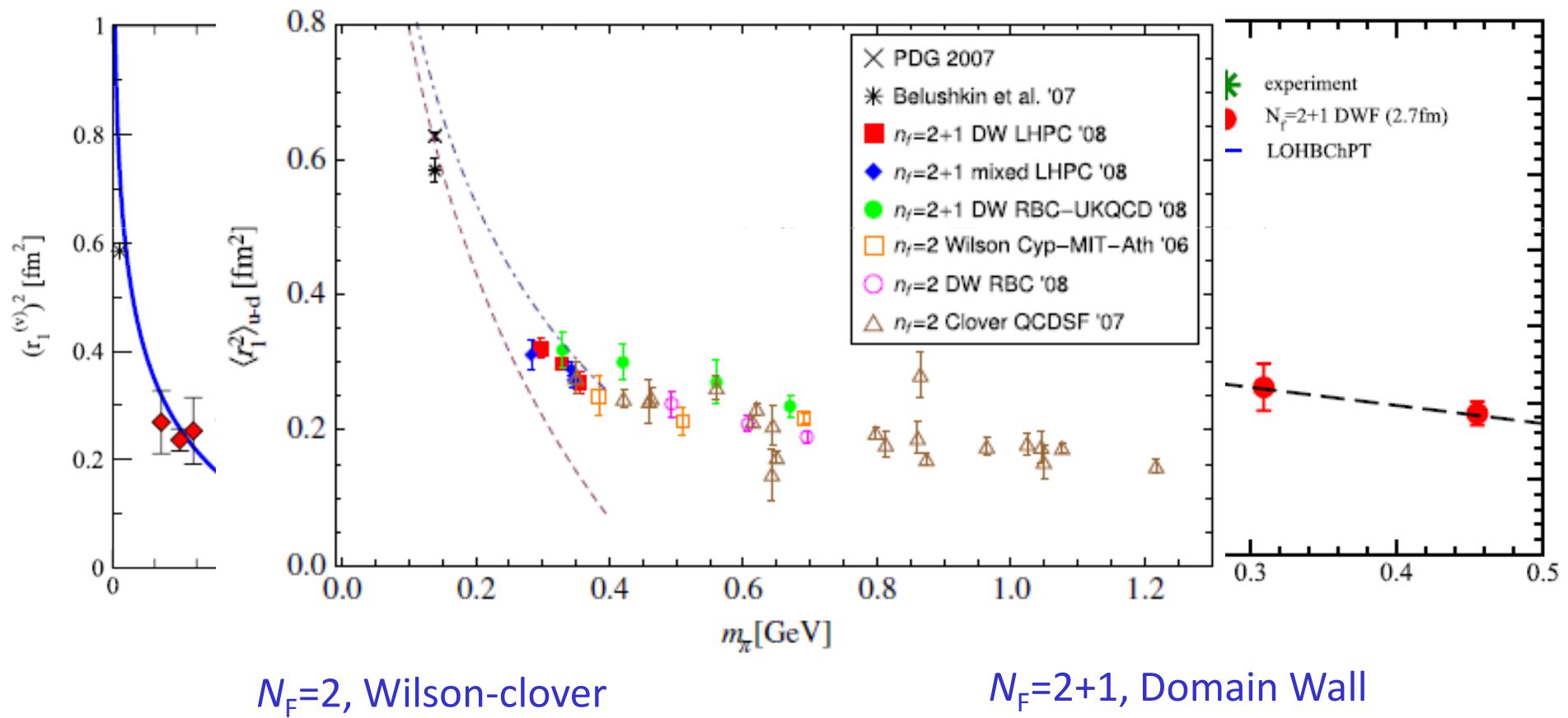


$N_F=2+1$ , Domain Wall

# 荷電半径

QCDSF, PoS Lattice2006, 120  
QCDSF, PoS Lattice2007, 161

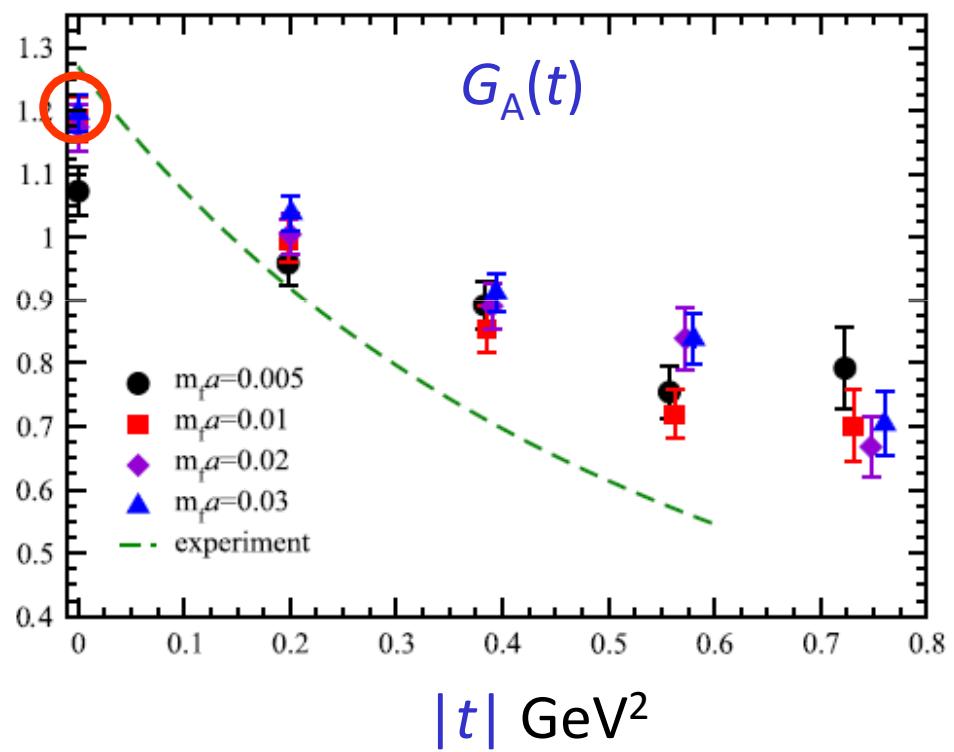
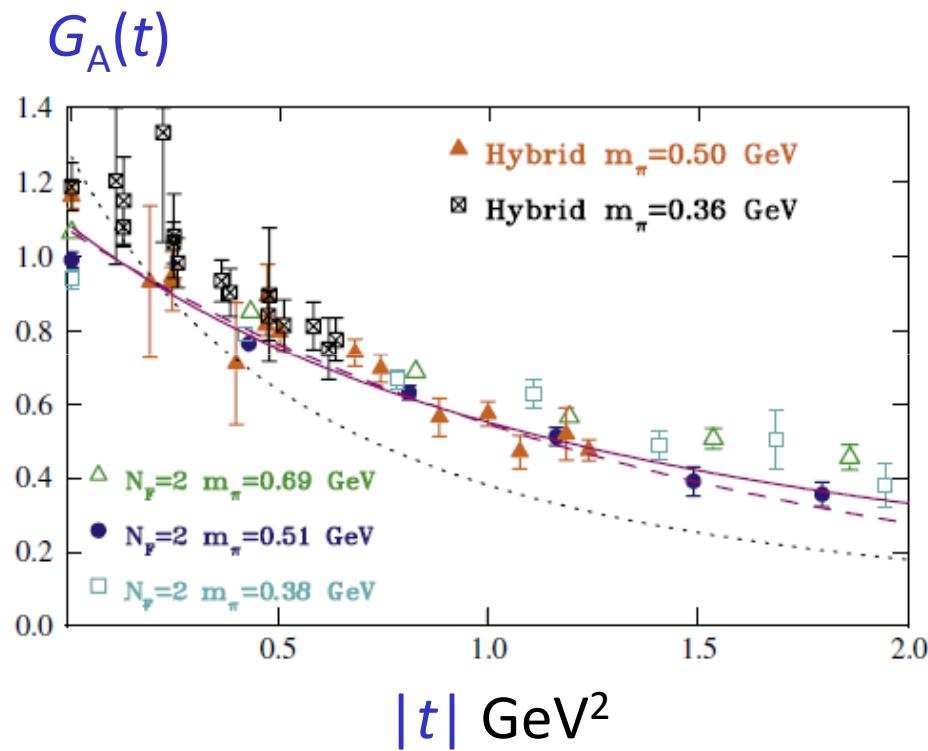
RBC-UKQCD, PoS Lattice2008



# 軸性形状因子

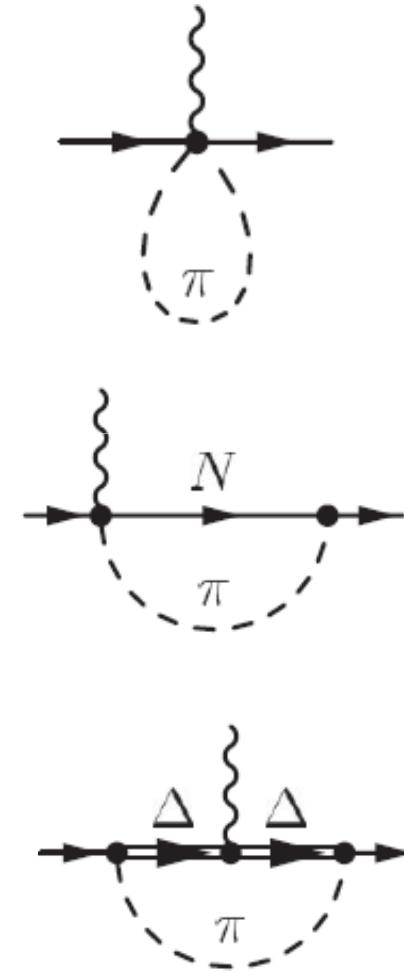
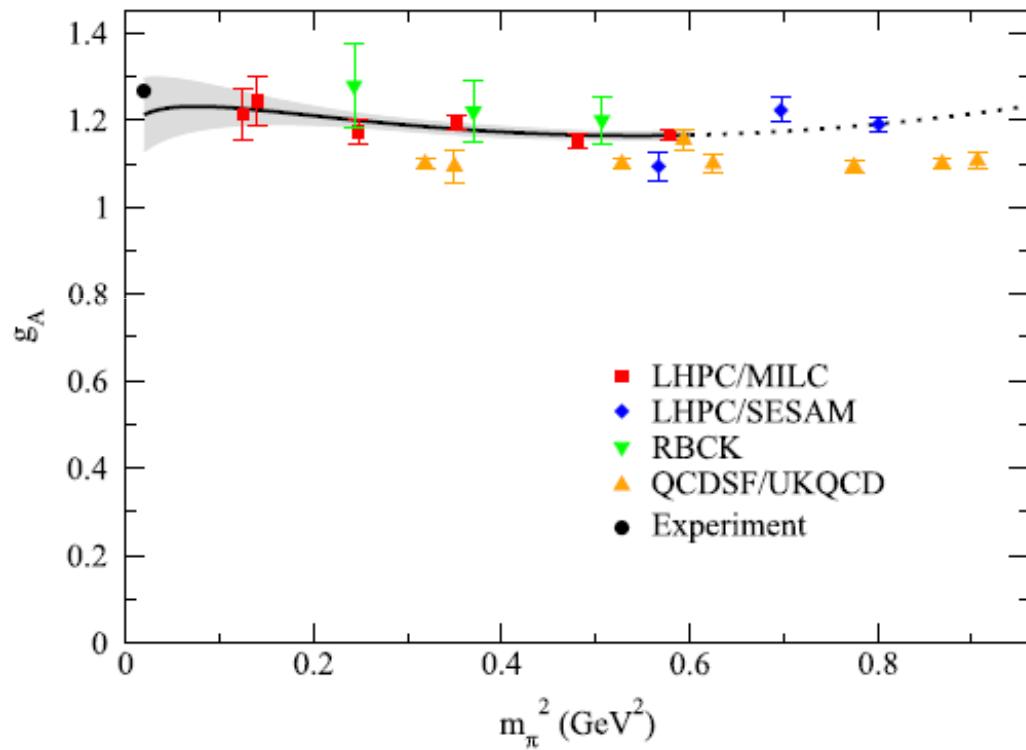
■ C. Alexandrou, *et. al.*, PRD76 (2007)

■ RBC-UKQCD, PoS Lattice2008



# 軸性結合定数

■ R. G. Edwards, *et. al.*, PRL96 (2006)

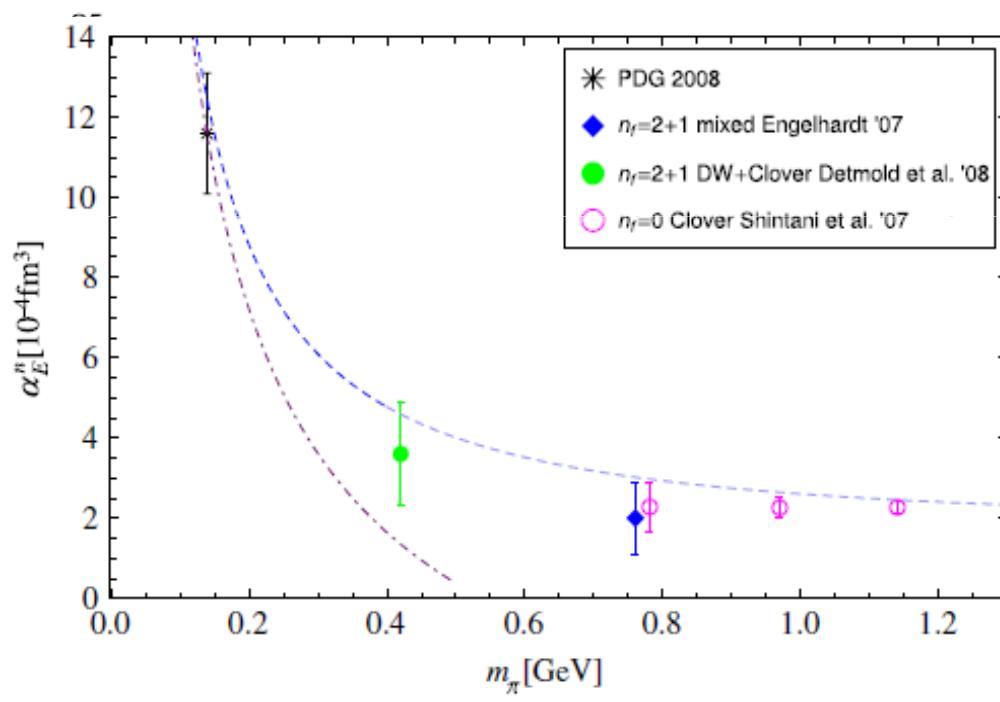


...

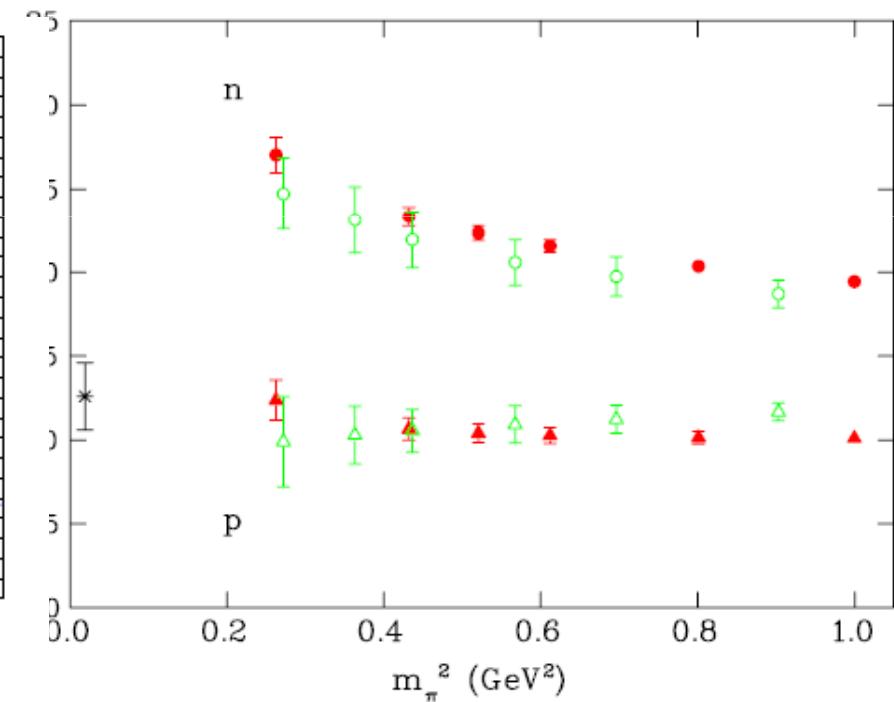
# 電氣分極率, 磁氣分極率

■ J. C. Christensen, *et. al.*, PRD**72** (2005)

■ F. X. Lee, *et. al.*, PLB**627** (2005)



■ Ph. Hägler, Phys. Rep. **490** (2009)

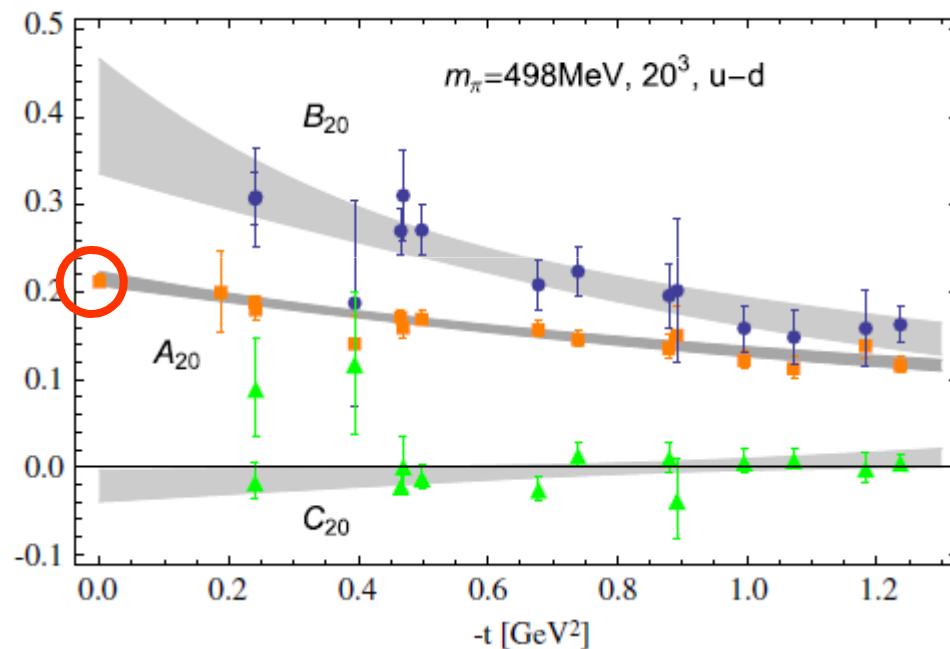


quench 近似

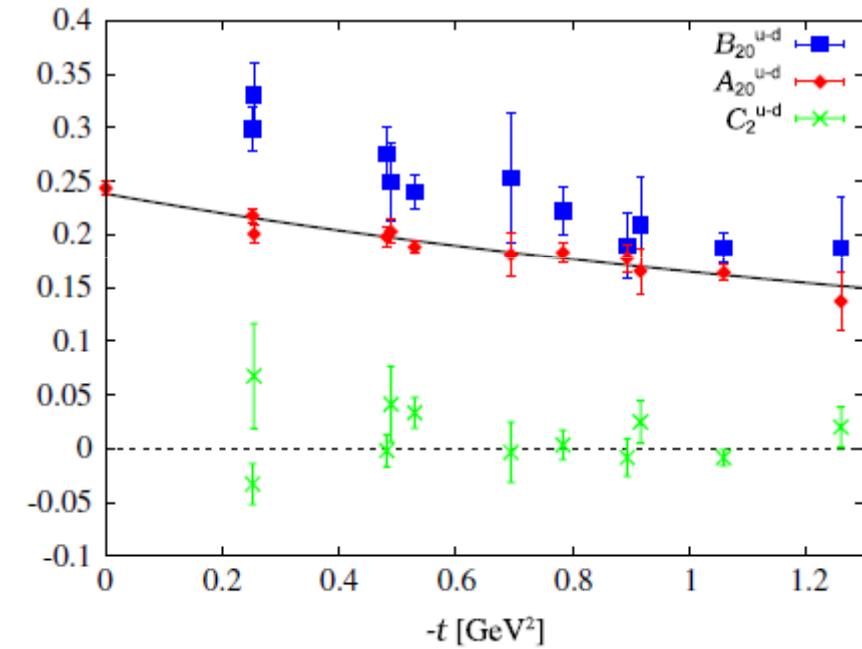
# GPDの2次モーメント

■ LHPC, PRD77 (2008)

■ QCDSF-UKQCD, PoS Lattice2007



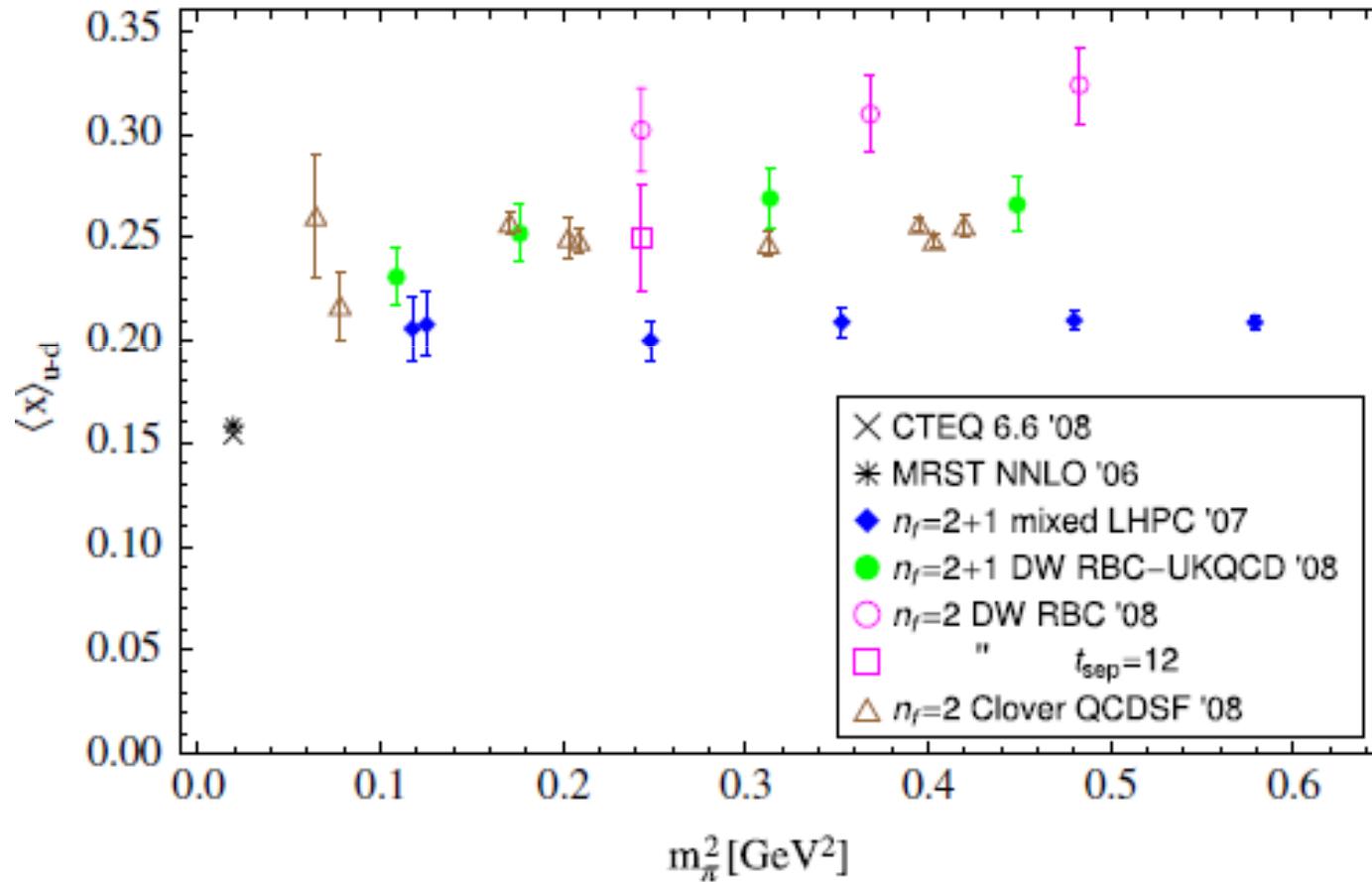
$N_F=2+1$ , DW valence on staggered sea



$N_F=2$ , Wilson-clover

# 運動量比の期待値

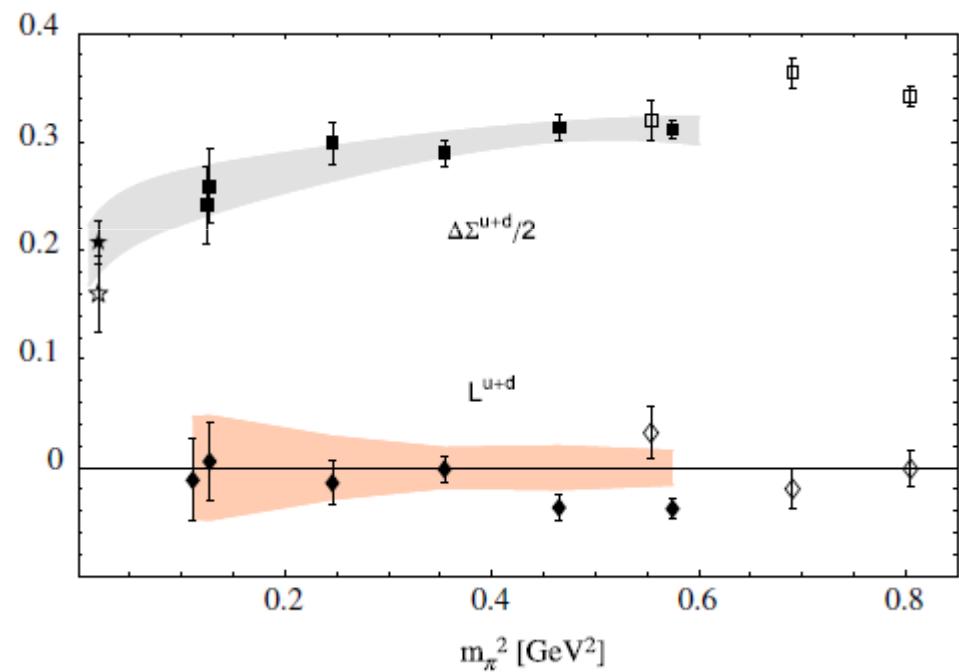
Ph. Hägler, Phys. Rep. 490 (2009)



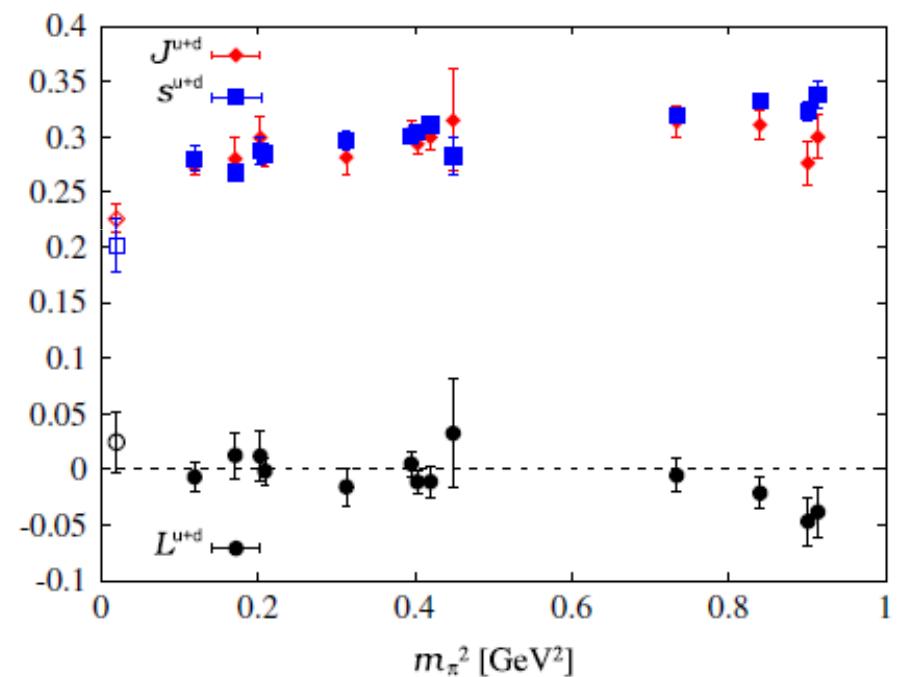
# 核子スピンのクォークからの寄与

LHPC, PRD77(2008)

QCDSF-UKQCD, PoS Lattice2007



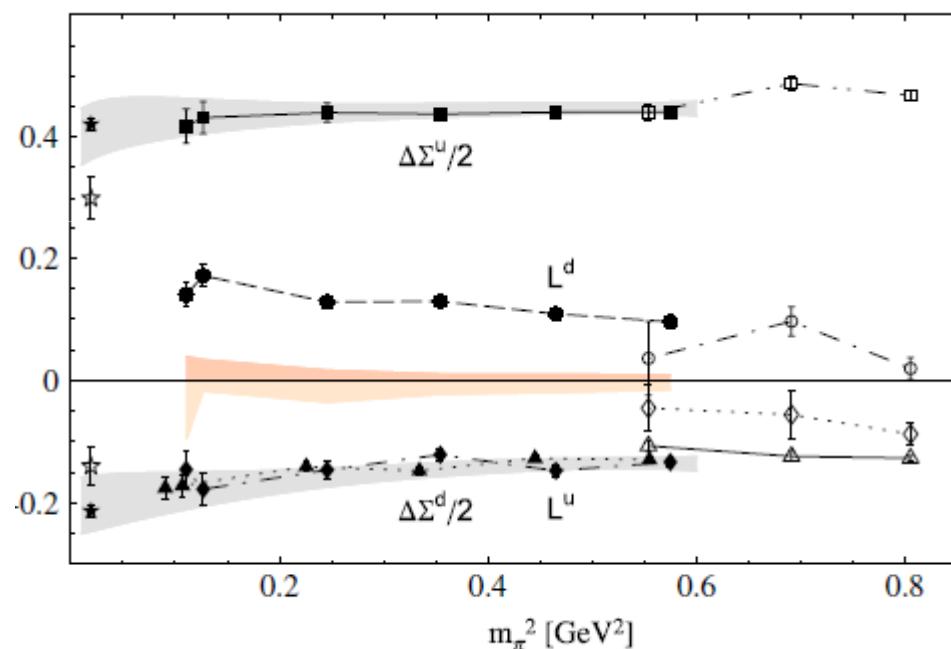
$N_F=2+1$ , DW valence on staggered sea



$N_F=2$ , Wilson-clover

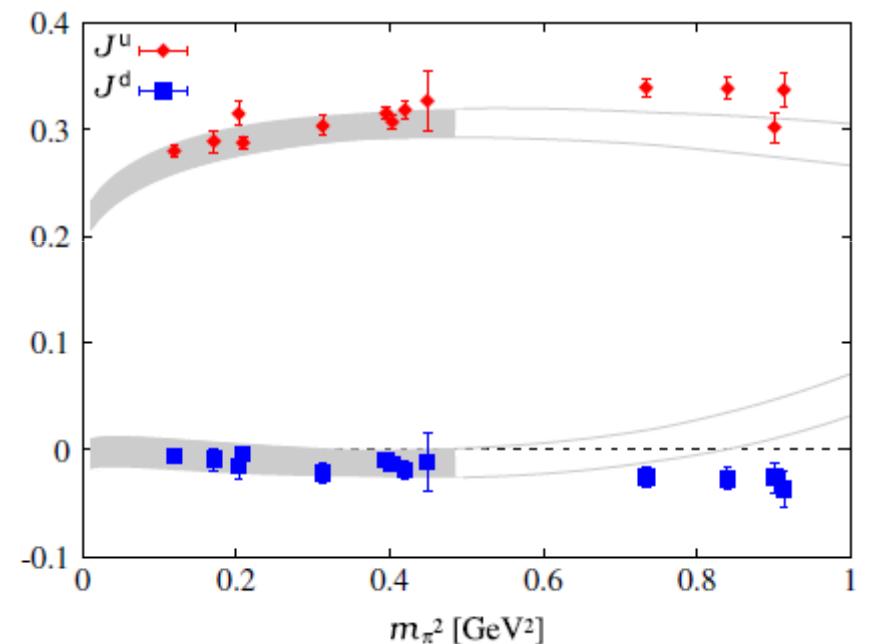
# クオーカスピンのフレーノバー依存性

LHPC, PRD77(2008)



$N_F=2+1$ , DW valence on staggered sea

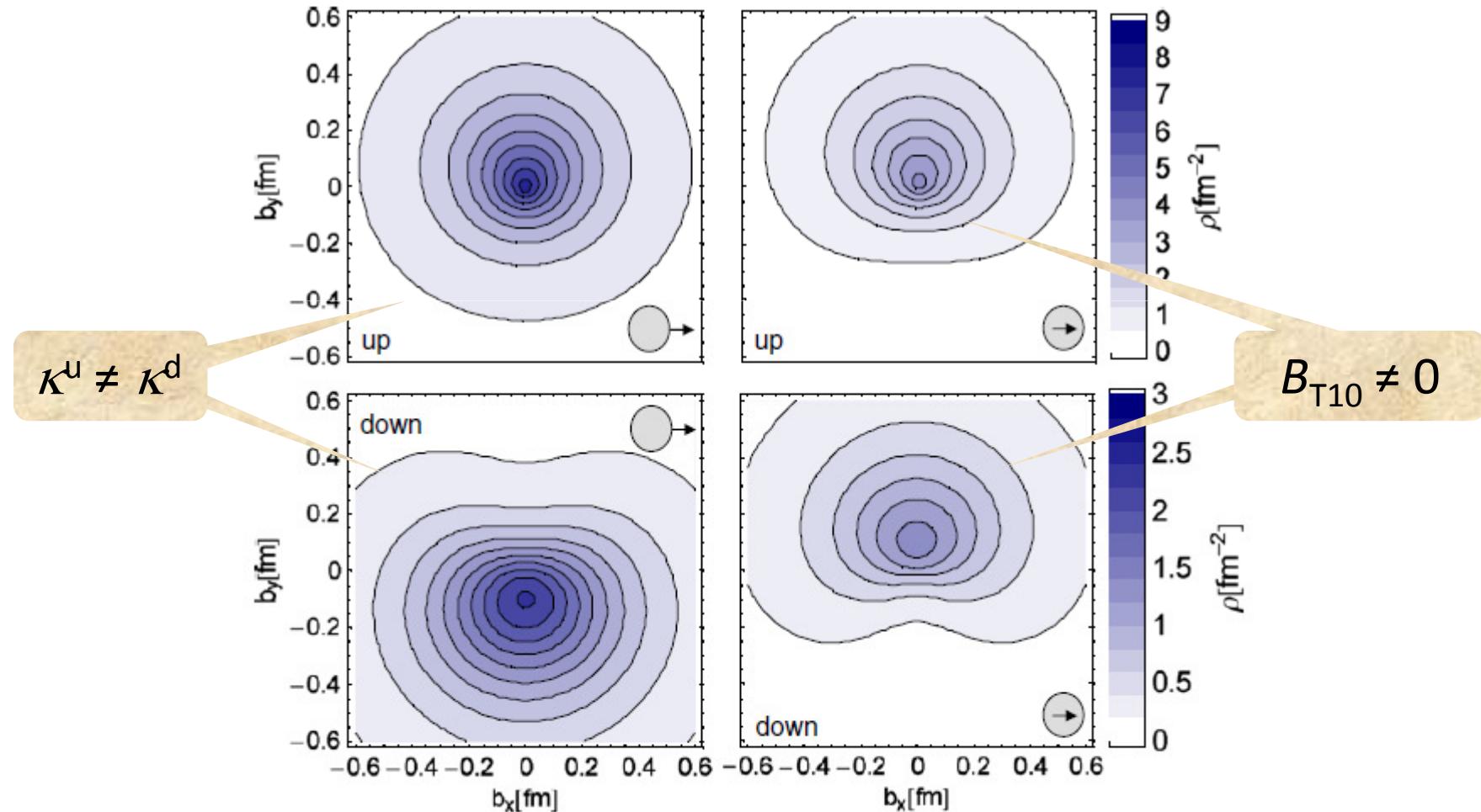
QCDSF-UKQCD, PoS Lattice2007



$N_F=2$ , Wilson-clover

# 横偏極スピン構造

QCDSF-UKQCD, PRL 98 (2007)



# グルーオンの寄与

- グルーオンのスピンへの寄与

J. E. Mandula, PRL 65 (1990)

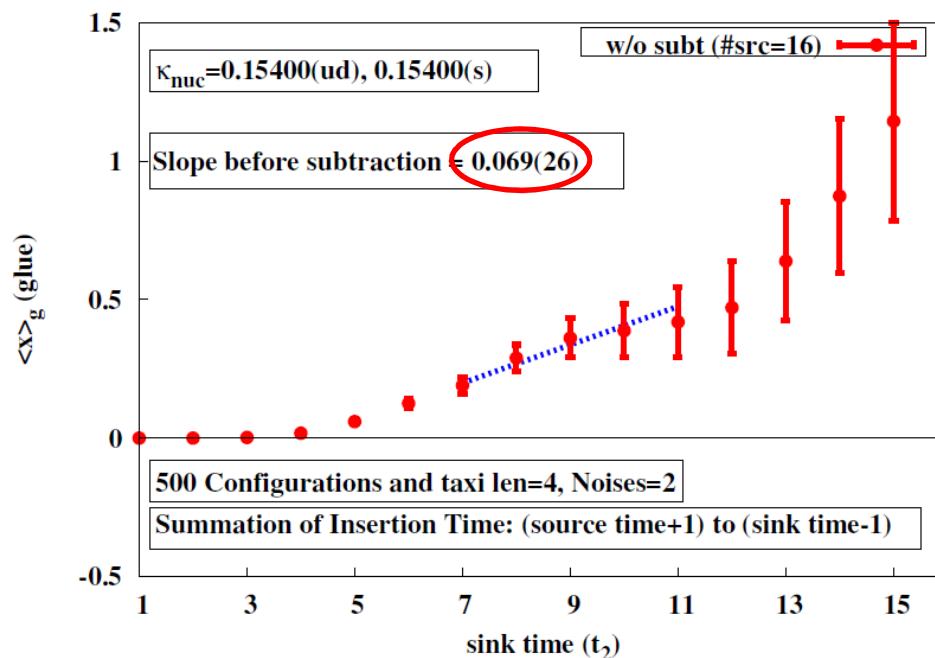
$$\Delta g = - \frac{\text{Tr} P + i \gamma_5 s' \langle 0 | \Psi(x_4) \mathbf{s} \cdot \mathbf{K}_\mu(y_4) \bar{\Psi}(z_4) | 0 \rangle}{\text{Tr} \langle 0 | \Psi(x_4) \bar{\Psi}(z_4) | 0 \rangle}$$

$K_i = 2\epsilon_{ijk} \text{Tr} A_j F_{k4}$

Separation		Forward	$\Delta g$
$x_4 - y_4$	$y_4 - z_4$		
4	3	$-0.158 \pm 0.671$	$+0.008 \pm 0.191$
3	4	$-0.292 \pm 0.367$	$+0.009 \pm 0.124$
3	3	$-0.097 \pm 0.397$	$+0.001 \pm 0.114$

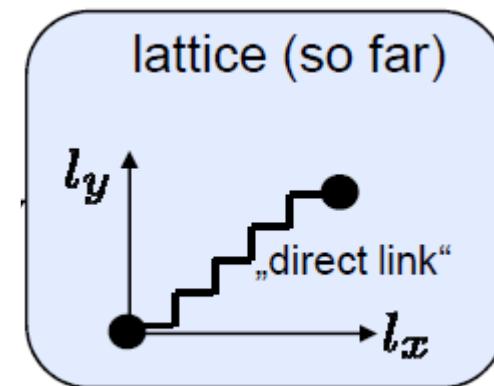
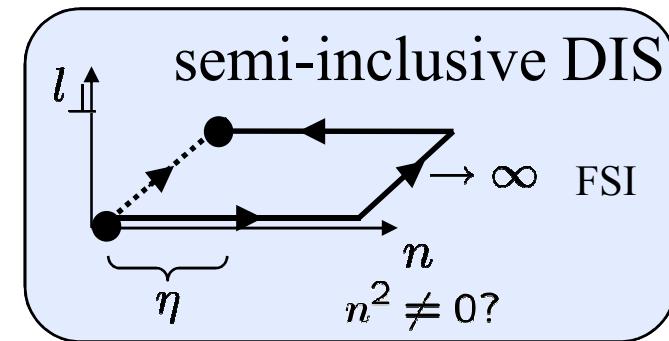
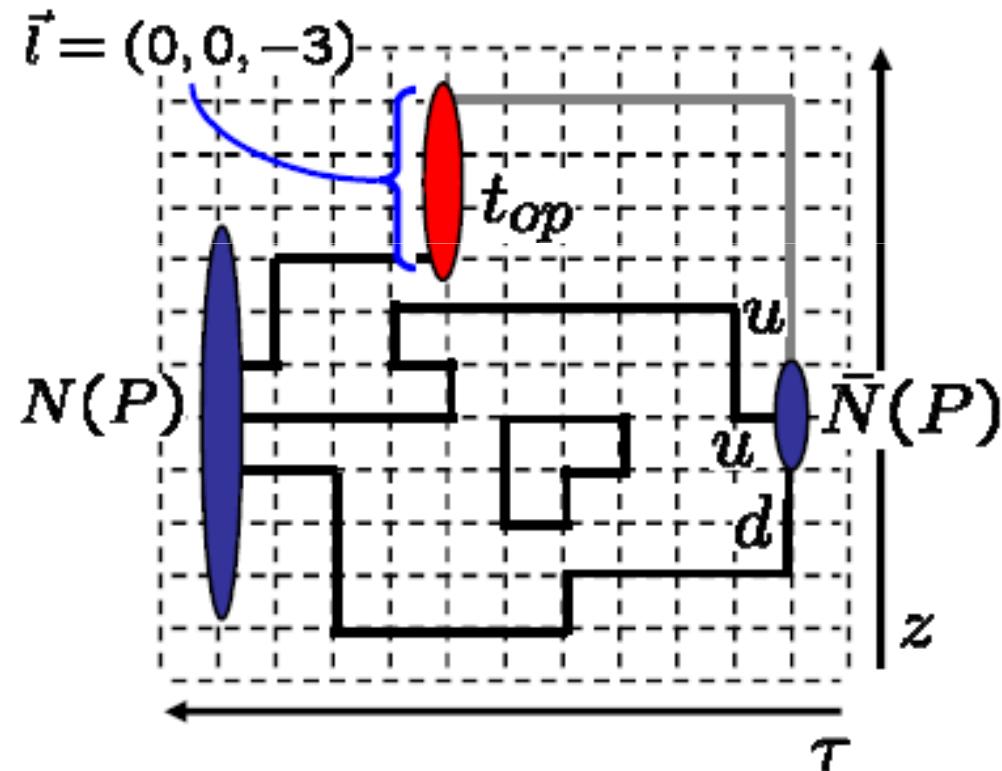
- グルーオンの運動量比

T. Doi, et al., PoS Lattice2008



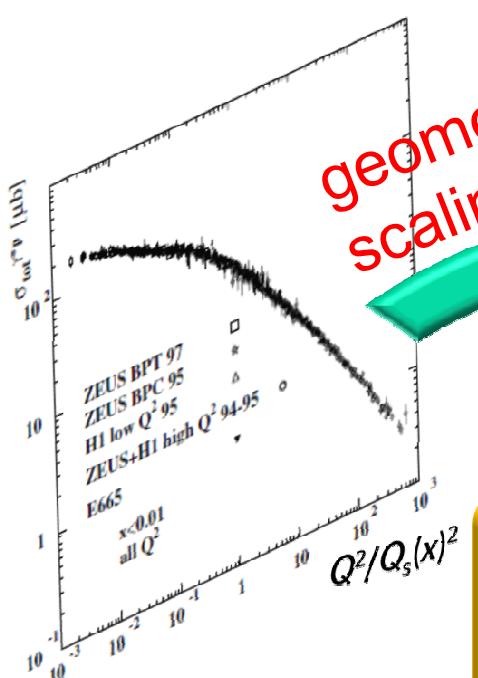
# TMD on the lattice

Ph. Hägler, Talk @ HESI2010, YITP

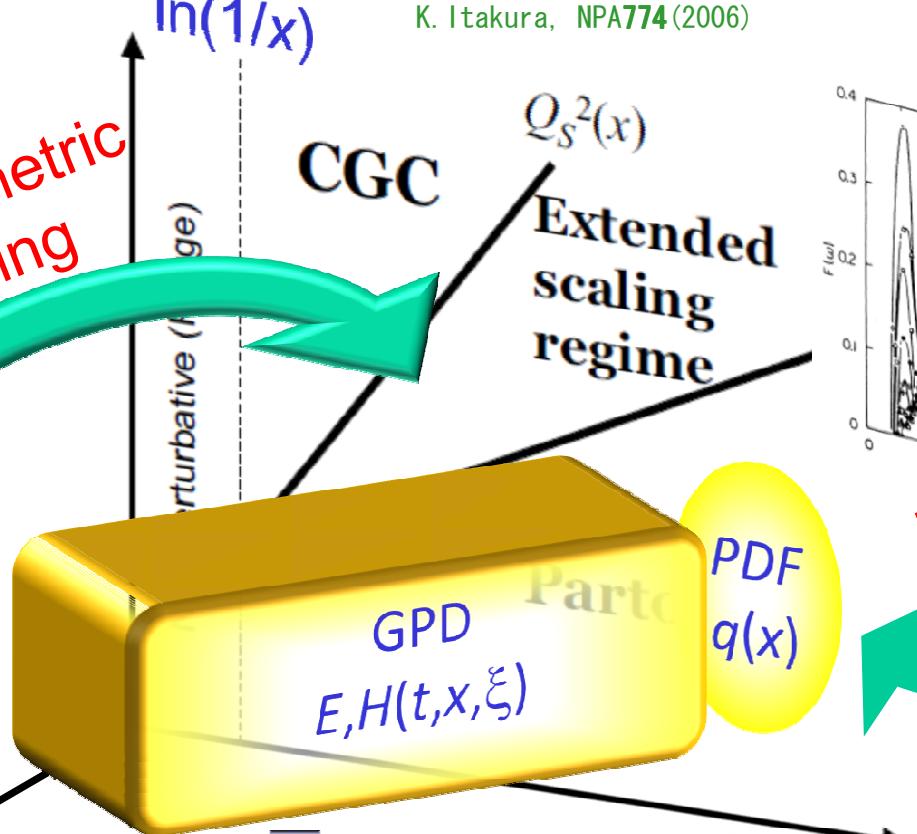
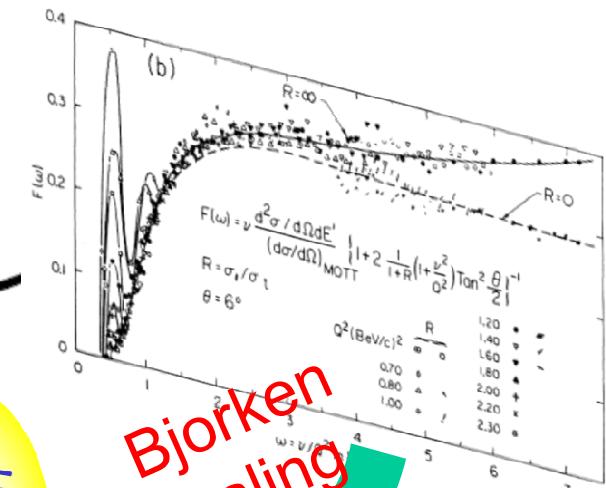


# 新機軸としての非前方度

■ A. M. Staśto, K. Golec-Biernat,  
J. Kwieciński, PRL86 (2001)



■ E. Iancu, K. Itakura, L. McLerran, NPA708 (2002) ;  
K. Itakura, NPA774 (2006)



$$|t| = |\Delta^2|$$

(一般化) 形状因子  
 $F_1(t), A_{11}(t), \dots$

$$\int dx x^{n-1}$$

Parton

PDF  
 $q(x)$

$Q_S^2(x)$   
CGC  
Extended  
scaling  
regime

GPD  
 $E, H(t, x, \xi)$