# Hydrodynamic fluctuations of chiral matter

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We study the effects of hydrodynamic fluctuations on the chiral transport properties using newly-development non-equilibrium effective field theory. We show that the hydrodynamic fluctuation leads to a finite correction to the static vector conductivity. This correction is a generalization of the long-time tail in the presence of an emergent IR scale due to the finite axial relaxation rate or external magnetic field.

## Introduction

- Chiral Matter
  - QGP, Electroweak plasma, Weyl semimetals, Core-collapse supernovae
  - Novel transport phenomena, e.g., Chiral Magnetic Effect (CME) [1]
- Hydrodynamic fluctuation effects
  - Long-time tail phenomena [2]
  - Renormalization of transport coefficients

[1] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, PRD ('08)
[2] Y. Pomeau and P. Resibois, Physics Reports (1975)

## Short summary

- Hydrodynamic fluctuation effects of chiral matter
- Universal one-loop formula of conductivity

$$\frac{\text{(Hydrodynamic fluctuation)}}{\text{(Bare conductivity)}} = \mathscr{N}(gTu)^2 q_*^3$$

$g^{-2} = \chi T = \langle \Delta n^2 \rangle$ : Equilibrium fluctuation

$u$ : Nonlinear couplings

$q_* \ll l_{\rm mfp}^{-1}$ : IR emergent scale
- Axial damping rate: $q_{*r}$
- Magnetic field: $q_{*B}$

C.f., Long time tail: $q_{*\omega}$

## Setup and formulation

- Long-range and long-time behavior of chiral matter under $\boldsymbol{B}$
- Slow variables: $n_{\rm V}, n_{\rm A}$
- Slow relaxation rate of the axial charge: $r \ll \tau_{\rm mic}^{-1}$
- Non-equilibrium effective field theory [3]
- Symmetries (full and approximate):
  - U(1)V and approx. U(1)A
  - Dynamical KMS symmetry
  - CPT symmetry

[3] M. Crossley, P. Glorioso, and H. Liu ('17)

## Effective Field Theory Approach

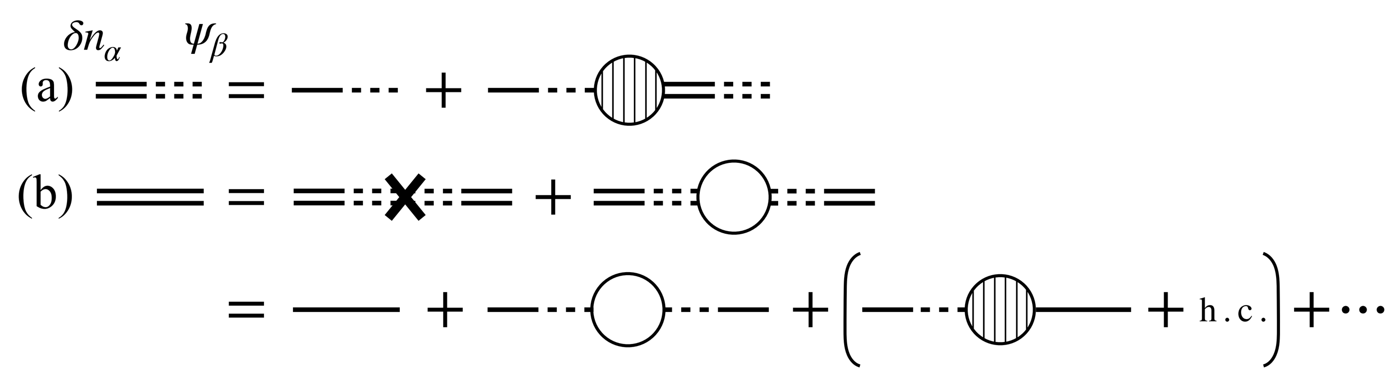
- Lagrangian

$$\mathscr{L} = \sum_\alpha (\partial_t \psi_\alpha)(\delta n_\alpha) - \sum_\alpha D_\alpha \left[(\nabla\psi_\alpha)\cdot\nabla\delta n_\alpha - {\rm i}(\nabla\psi_\alpha)^2\right]$$
$$+\boldsymbol{v}_{\rm A}\cdot(\nabla\psi_{\rm V})\delta n_{\rm A} + \boldsymbol{v}_{\rm V}\cdot(\nabla\psi_{\rm A})\delta n_{\rm V} - r\left[\psi_{\rm A}\delta n_{\rm A} - {\rm i}(\psi_{\rm A})^2\right]$$

- $D_\alpha, r, \boldsymbol{v}_\alpha$ : Expanded with respect to $\langle n_\alpha\rangle_{\rm eq}, \delta n_\alpha$
- $D = \dfrac{\sigma}{\chi}, \quad \boldsymbol{v} = \dfrac{C|\boldsymbol{B}|}{\chi},$ for both $\alpha = {\rm V, A}$ at equilibrium
- Anomaly matching in Schwinger Keldysh formalism [4]

- Green functions: $G^{\rm rr}_{\alpha\beta} = \left\langle \delta n_\alpha(x)\delta n_\beta(0)\right\rangle, \quad G^{\rm ra}_{\alpha\beta} = \left\langle \delta n_\alpha(x)\psi_\beta(0)\right\rangle$

- Dyson equation:




- Kubo formula: $\lim_{\boldsymbol{k}\to\boldsymbol{0}} \dfrac{\beta}{2}\dfrac{\omega^2}{\boldsymbol{k}^2} G^{\rm rr}_{\rm VV} = \sigma_{{\rm V}\perp}(\omega) + [\sigma_{{\rm V}\parallel}(\omega) - \sigma_{{\rm V}\perp}(\omega)](\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{B}})^2$

$$\sigma_{{\rm V}\perp}(\omega) = \sigma_{\rm V} + \Delta\sigma_{{\rm V}\perp}(\omega)$$

$$\sigma_{{\rm V}\parallel}(\omega) = \Delta\sigma_{{\rm V}\parallel}(\omega) + \frac{\boldsymbol{v}^2_{\rm ren}(\omega)\chi}{r_{\rm ren}(\omega)}$$

[4] P. Glorioso, H. Liu, and S. Rajagopal ('19)

## Results

- Parameter sets: (I) $\langle n_{\rm V}\rangle_{\rm eq} \neq 0, \langle n_{\rm A}\rangle_{\rm eq} = 0, r \neq 0$
  (II) $\langle n_{\rm V}\rangle_{\rm eq} = 0, \langle n_{\rm A}\rangle_{\rm eq} \neq 0, r = 0$

(I) Vector charge background with axial relaxation

$$\sigma_{{\rm V}\parallel}(0) = \frac{\boldsymbol{v}^2\chi}{r}\left[1 - \frac{g^2T^2\left(2u_n^2 + u_r^2\right)q_{*r}^3}{8\sqrt{2}\pi}\right]$$

$$q_{*r} = \sqrt{\frac{r}{D}}, \quad \underbrace{u_n = \frac{1}{\chi}\frac{\partial n}{\partial\mu\partial\mu}, \quad u_\gamma = \frac{1}{r}\frac{\partial r}{\partial\mu}}_{\text{Finite at } \langle n_{\rm V}\rangle \neq 0}$$

(II) Axial charge background without relaxation effects

$$\frac{\Delta\sigma_{{\rm V}\parallel}(0)}{\sigma} = -\frac{g^2T^2u_\sigma^2 q_{*r}^3}{48\pi}, \quad \Delta\sigma_{{\rm V}\perp}(0) = \frac{\Delta\sigma_{{\rm V}\parallel}(0)}{4}$$

$$q_{*B} = \frac{|\boldsymbol{v}|}{D}, \quad \underbrace{u_\sigma = \frac{1}{\sigma}\frac{\partial\sigma}{\partial\mu}}_{\text{Finite at } \langle n_{\rm A}\rangle \neq 0}$$

- C.f., Long-time tail phenomenon (two conserved charges without anomaly)

$$\sigma_{\rm LTT}(\omega) = \sigma\left(1 + \frac{g^2T^2u_\sigma^2 q_{*\omega}^3}{32\pi}\right), \quad q_{*\omega} = \sqrt{\frac{\omega}{D}}$$

Correspondence to [5] up to overall sign (See "Discussion")

- Origin of $q_*$s : Poles of the hydrodynamic-loop integrals:

$$\int\frac{{\rm d}\boldsymbol{q}}{(2\pi)^3}\frac{1}{\omega + {\rm i}r + 2{\rm i}D\boldsymbol{q}^2}, \quad \int\frac{{\rm d}\boldsymbol{q}}{(2\pi)^3}\frac{D\boldsymbol{q}^2 q_i q_j}{2[(D\boldsymbol{q}^2)^2 + (\boldsymbol{v}\cdot\boldsymbol{q})^2]} \longrightarrow \text{Finite cutoff-indep. corrections}$$

Chiral magnetic wave [6,7]

(Valid at $q_* \ll l_{\rm mfp}^{-1}$ )

<u>Discussion: Ward-Takahashi identity and multiplicative noises</u>

$$\lim_{\boldsymbol{k}\to\boldsymbol{0}}\frac{\beta}{2}\frac{\omega^2}{\boldsymbol{k}^2}G^{\rm rr}_{\rm VV} = \lim_{\boldsymbol{k}\to\boldsymbol{0}}\frac{\beta}{2}\left\langle(\hat{\boldsymbol{k}}\cdot\boldsymbol{j}^{\rm r})(\hat{\boldsymbol{k}}\cdot\boldsymbol{j}^{\rm r})\right\rangle = \lim_{\boldsymbol{k}\to\boldsymbol{0}}{\rm Im}\,\frac{\beta}{2\omega}{\rm i}\left\langle(\hat{\boldsymbol{k}}\cdot\boldsymbol{j}^{\rm r})(\hat{\boldsymbol{k}}\cdot\boldsymbol{j}^{\rm a})\right\rangle \quad \cdots(\#)$$

Fluctuation dissipation relation
(Multiplicative noise effects [8])

$$\boldsymbol{j}^{\rm r}_{\rm V} = -\sigma_{\rm V}\nabla\mu_{\rm V} + 2{\rm i}T\sigma_{\rm V}\nabla\psi_{\rm V} + C\mu_{\rm A}\boldsymbol{B},$$

$$\boldsymbol{j}^{\rm a}_{\rm V} = \partial_t\left(\sigma_{\rm V}\nabla\psi_{\rm V} + C\psi_{\rm A}\boldsymbol{B}\right) + C\nabla\times\left(\mu_{\rm A}\nabla\psi_{\rm V} + \mu_{\rm V}\nabla\psi_{\rm A} + \boldsymbol{E}\psi_{\rm A}\right)$$

(#) Breaks down if $\sigma_{\rm V}(\delta n)$ is not taken into account at 3 places consistently

[5] P. Kovtun (2015), [6] G. M. Newman ('06), [7] D. E. Kharzeev and H. U. Yee ('11) [8] J. Chao and T. Schäfer ('21)