KEK 素核宇・物性 連携研究会　2021年3月29日(月)
高エネルギー加速器機構（オンライン）

超伝導及び電荷密度波系における ヒッグスモードと相転移ダイナミクス

THE UNIVERSITY OF TOKYO

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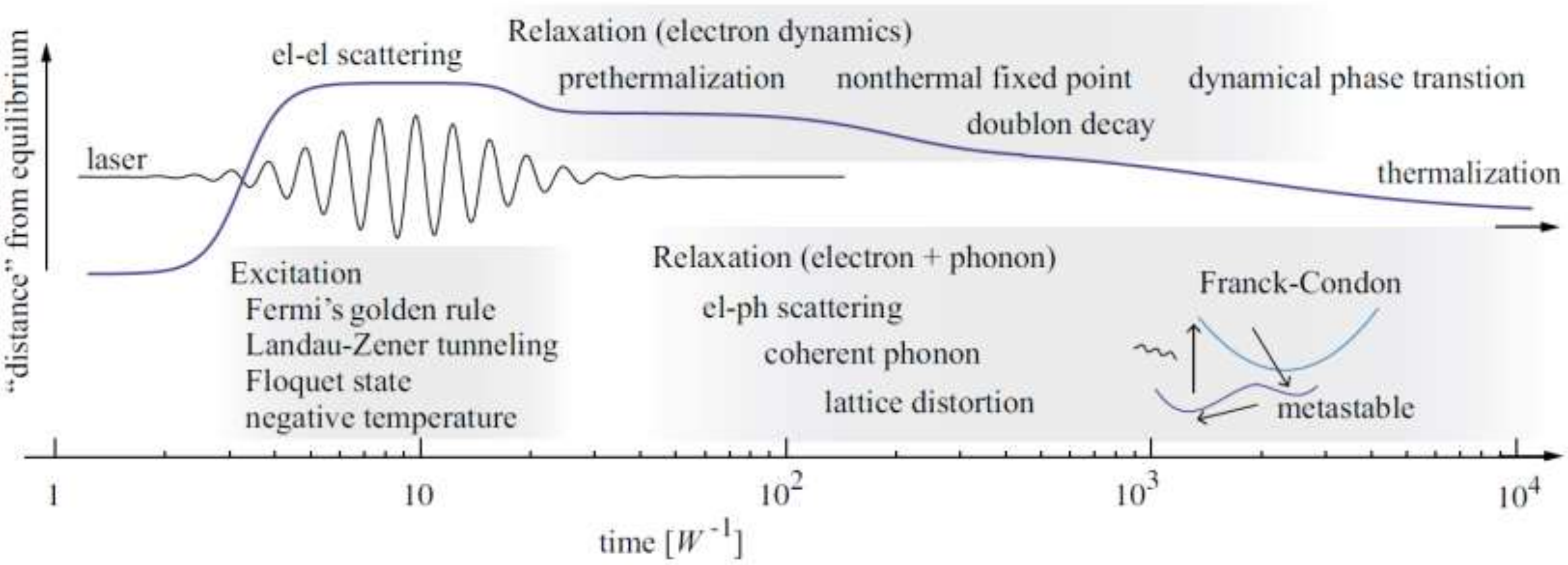
強相関電子系の光励起非平衡ダイナミクス

REVIEWS OF MODERN PHYSICS, VOLUME 86, APRIL–JUNE 2014

Nonequilibrium dynamical mean-field theory and its applications

H. Aoki, N. Tsuji, M. Eckstein, M. Kollar, T. Oka, P. Werner,
Rev. Mod. Phys. **86**, 779(2014)



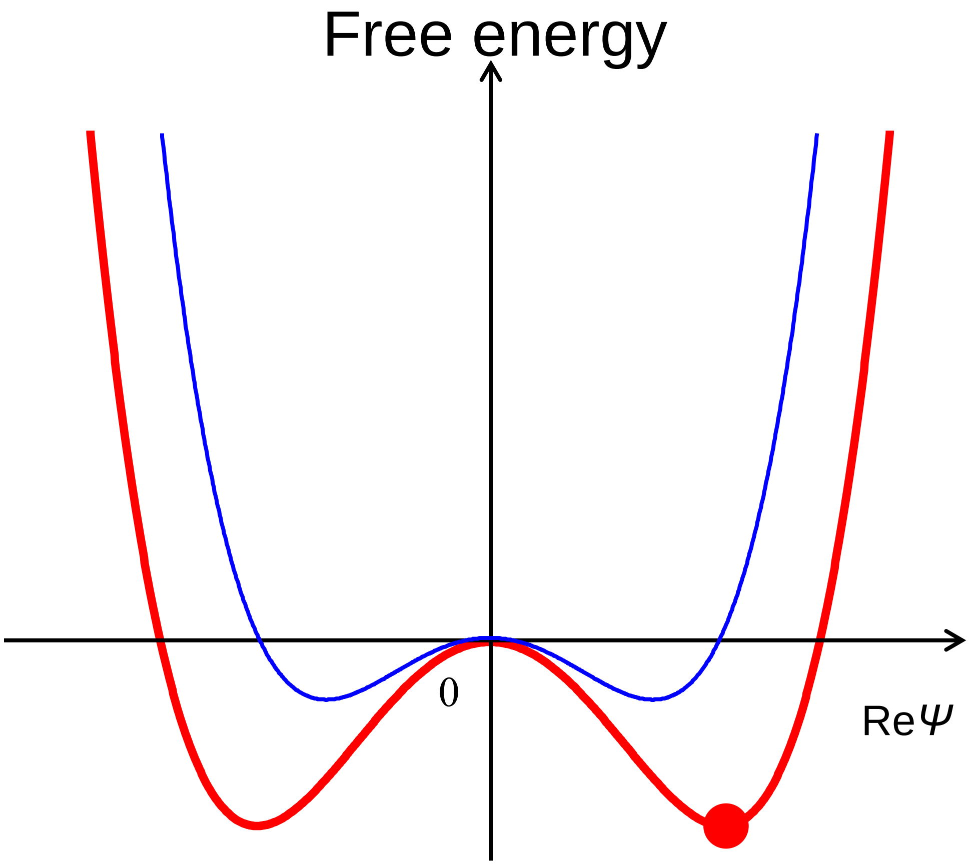
T. Oka and S. Kitamura, "**Floquet Engineering of Quantum Materials**",
Ann. Rev. Cond. Mat. Phys. 10, 387 (2019)

量子クエンチ問題

電子間相互作用を Uを高速にクエンチしたら….

$\tau_\Delta \sim \hbar/\Delta$ （Δ:秩序変数）

⇨ 秩序変数の振動が生じる(ヒッグスモード)



秩序変数の非平衡ダイナミクス

Volkov *et al*., Sov. Phys. JETP 38, 1018 (1974).
Barankov *et al*., PRL 94, 160401 (2004).
Yuzbashyan *et al*., PRL 96, 230404 (2006).
Gurarie *et al*., PRL 103, 075301 (2009).
Podolsky, PRB84, 174522 (2011).
A. P. Schnyder et al., PRB84, 214513 (2011)
N. Tsuji et al., PRB 88,165115 (2013).
N. Tsuji *et al*., PRL 110, 136404 (2013).

⋮

$$\frac{\Delta(t)}{\Delta_\infty} = 1 + a\frac{\cos(2\Delta_\infty t + \pi/4)}{\sqrt{\Delta_\infty t}}$$

最近のレビュー：R. Shimano and N. Tsuji, Ann. Rev. Cond. Mat. Phys.**11**, 103-124 (2020).

アンダーソンの擬スピン表示

$$\left|\Psi_{\rm BCS}\right\rangle = \prod_{\mathbf{k}}\left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{+}_{\mathbf{k}\uparrow} c^{+}_{-\mathbf{k}\downarrow}\right)\left|0\right\rangle$$

Anderson, Phys. Rev. 112, 1900 (1958)

Pseudospin up : (*k*, -*k*) both empty

Pseudospin down: (*k*, -*k*) both occupied

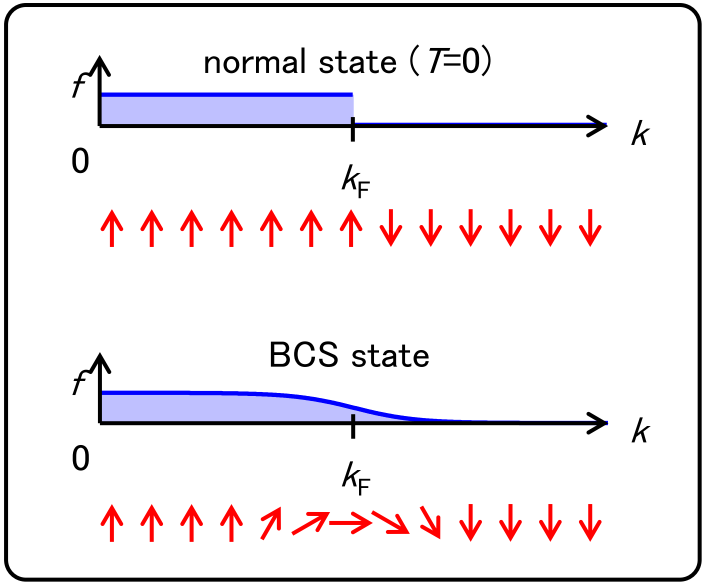
$$\mathcal{H}^{\rm BCS} = \sum_{k} \boldsymbol{b}_{k}^{\rm eff} \cdot \boldsymbol{\sigma}_{k}$$

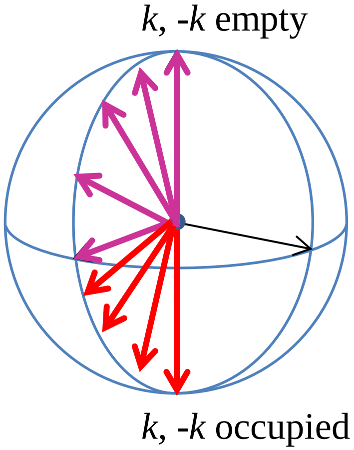
$$\boldsymbol{b}_{k}^{\rm eff} = \left(-\Delta',\ -\Delta'',\ \varepsilon_{k}\right)$$

: effective magnetic field for *k*

$$\Delta = \Delta' + i\,\Delta'' = U\sum_{k}\left(\sigma_{k}^{x} + i\sigma_{k}^{y}\right)$$

$$\boxed{\frac{d}{dt}\boldsymbol{\sigma}_{k} = i\left[\mathcal{H}^{\rm BCS}, \boldsymbol{\sigma}_{k}\right] = 2\boldsymbol{b}_{k}^{\rm eff} \times \boldsymbol{\sigma}_{k}}$$





Time evolution of BCS state= motion of pseudospins under effective magnetic field

アンダーソンの擬スピン表示

The BCS Hamiltonian and ground state

P.W. Anderson, PR 112, 1900 (1958)

$$H^{BCS} = 2\sum_{k\sigma} \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \Delta^* \sum_{k} c^\dagger_{-k\downarrow} c^\dagger_{k\uparrow} - \Delta \sum_{k} c_{-k\downarrow} c_{k\uparrow}$$

$$|\Psi_{\mathrm{BCS}}\rangle = \prod_{k} \left(u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}\right)|0\rangle$$

Here we introduce the pseudospin:

$$\sigma_k = \frac{1}{2}\Psi^\dagger_k \tau \Psi_k = \frac{1}{2}\begin{pmatrix} \Psi^\dagger_k \tau^x \Psi_k \\ \Psi^\dagger_k \tau^y \Psi_k \\ \Psi^\dagger_k \tau^z \Psi_k \end{pmatrix}$$

where $\tau = (\tau^x, \tau^y, \tau^z)$ are the Pauli matrices and

$$\Psi_k = (c_{k\uparrow}, c^\dagger_{-k\downarrow})$$

is the Nambu spinor.

Then the BCS Hamiltonian can be written in a simple form as

$$H^{BCS} = 2\sum_{k} \boldsymbol{b}_k \cdot \boldsymbol{\sigma}_k$$

where b_k is the pseudo magnetic fi

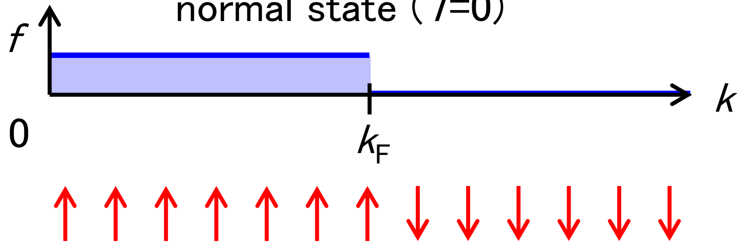
$$\boldsymbol{b}_k = (-\Delta', -\Delta'', \varepsilon_k)$$

$$\Delta = \Delta' + i\Delta'' = V\sum_{k}(\sigma^x_k + i\sigma$$

Pseudospin up : $(k, -k)$ both occupied

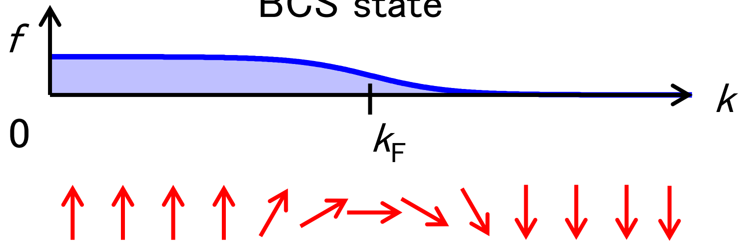
Pseudospin down: $(k, -k)$ both empty

normal state (T=0)



all ↑ (k<k_{F}) all ↓ (k>k_{F})

BCS state



superposition of ↑ & ↓ near k_{F}

擬スピンの時間発展：ブロッホ方程式

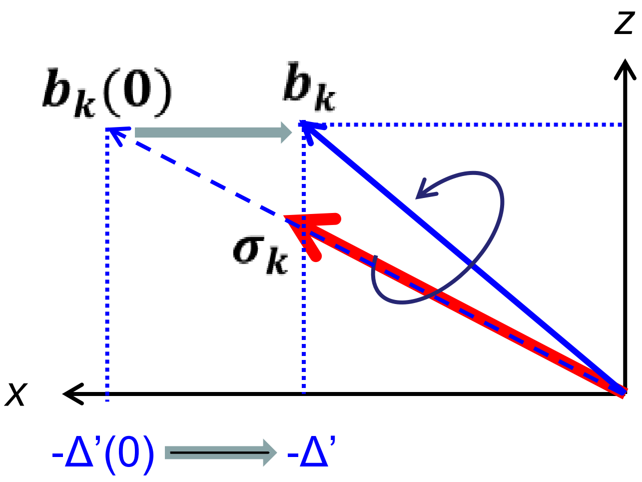
$$\frac{d}{dt}\boldsymbol{\sigma}_{\boldsymbol{k}} = -i[H^{BCS}, \boldsymbol{\sigma}_{\boldsymbol{k}}] = 2\boldsymbol{b}_{\boldsymbol{k}} \times \boldsymbol{\sigma}_{\boldsymbol{k}}$$

$$\Delta(t) = \Delta'(t) + i\Delta''(t) = V\sum_{k}\left(\sigma_{\boldsymbol{k}}^{x}(t) + i\sigma_{\boldsymbol{k}}^{y}(t)\right)$$

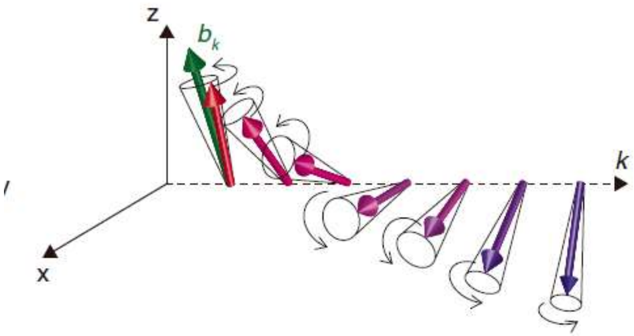
$$\boldsymbol{b}_{\boldsymbol{k}}(\boldsymbol{t}) = (-\Delta'(\boldsymbol{t}), -\Delta''(\boldsymbol{t}), \varepsilon_{\boldsymbol{k}})$$

Time evolution of BCS state is described by the motion of pseudospins under effective magnetic field

Let's consider that Δ' is suddenly quenched at t=0.



Each pseudospin $\boldsymbol{\sigma}_{\boldsymbol{k}}$ starts the precession around the



秩序変数のクエンチダイナミクス

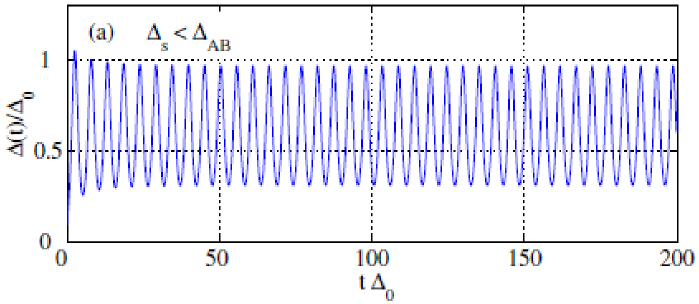
Quench Problem:

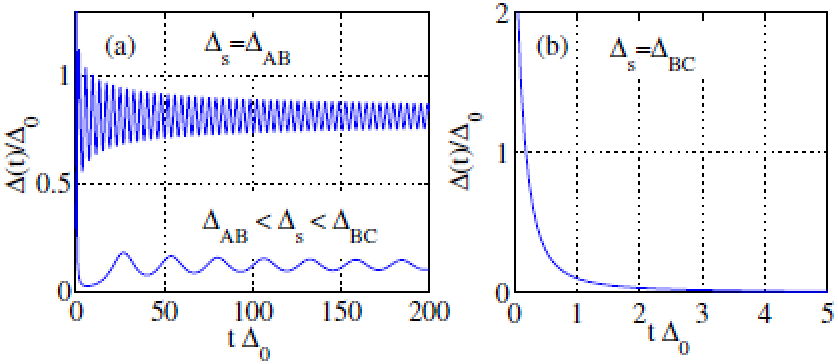
rapid switching of the orientation of b_k^{eff}

$$\frac{d}{dt}\boldsymbol{\sigma}_{\mathbf{k}} = 2\mathbf{b}_{\mathbf{k}}^{\mathrm{eff}} \times \boldsymbol{\sigma}_{\mathbf{k}}$$

$$\Delta'(t) + i\Delta''(t) = -V\sum_{\mathbf{k}}(\sigma_{\mathbf{k}}^{x}(t) + i\sigma_{\mathbf{k}}^{y}(t))$$

$$\mathbf{b}_{\mathbf{k}}^{\mathrm{eff}} = \left(-\Delta'(t),\ -\Delta''(t),\ \varepsilon_{\mathbf{k}}\right)$$





Order parameter change induced by external perturbation

= change in the orientation of b_k^{eff}

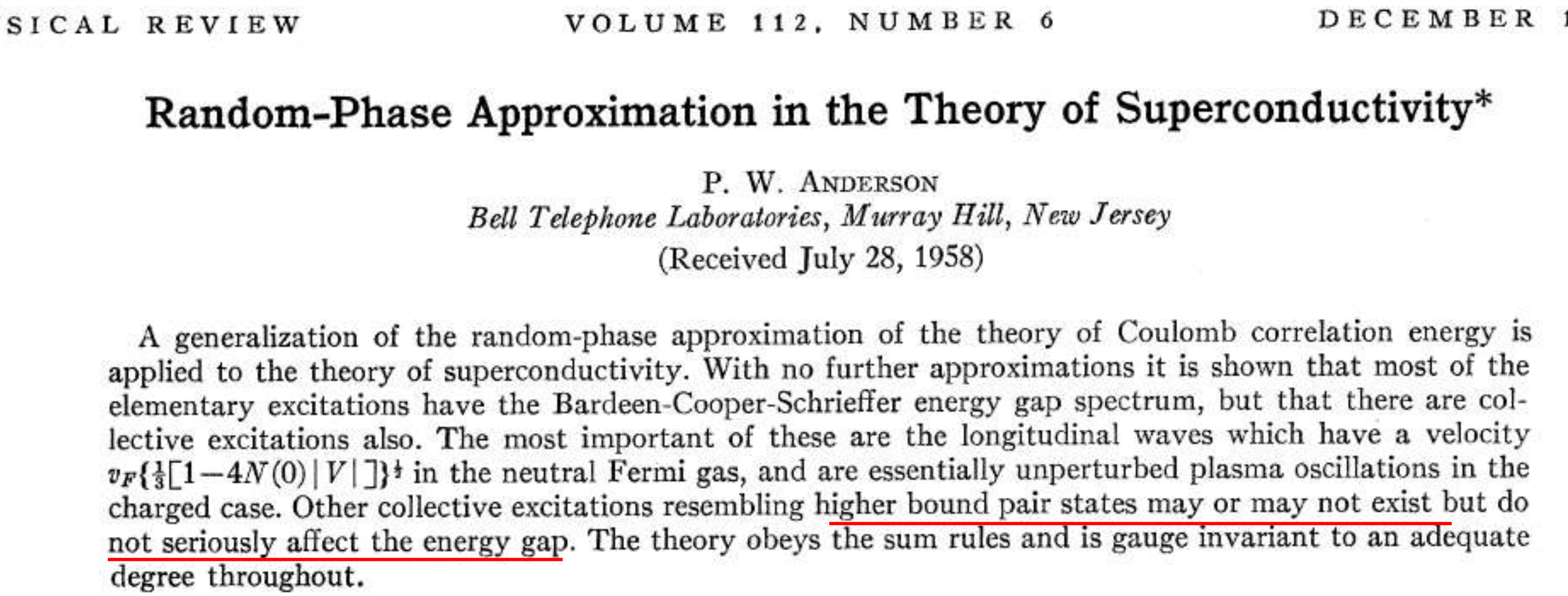
⇨ Collective precession of the pseudospin

= order parameter oscillation (Higgs mode)

Barankov and Levitov,

PRL **96**, 230403 (2006)

超伝導体の”ヒッグス”モード



SICAL REVIEW VOLUME 112, NUMBER 6 DECEMBER

Random-Phase Approximation in the Theory of Superconductivity*

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

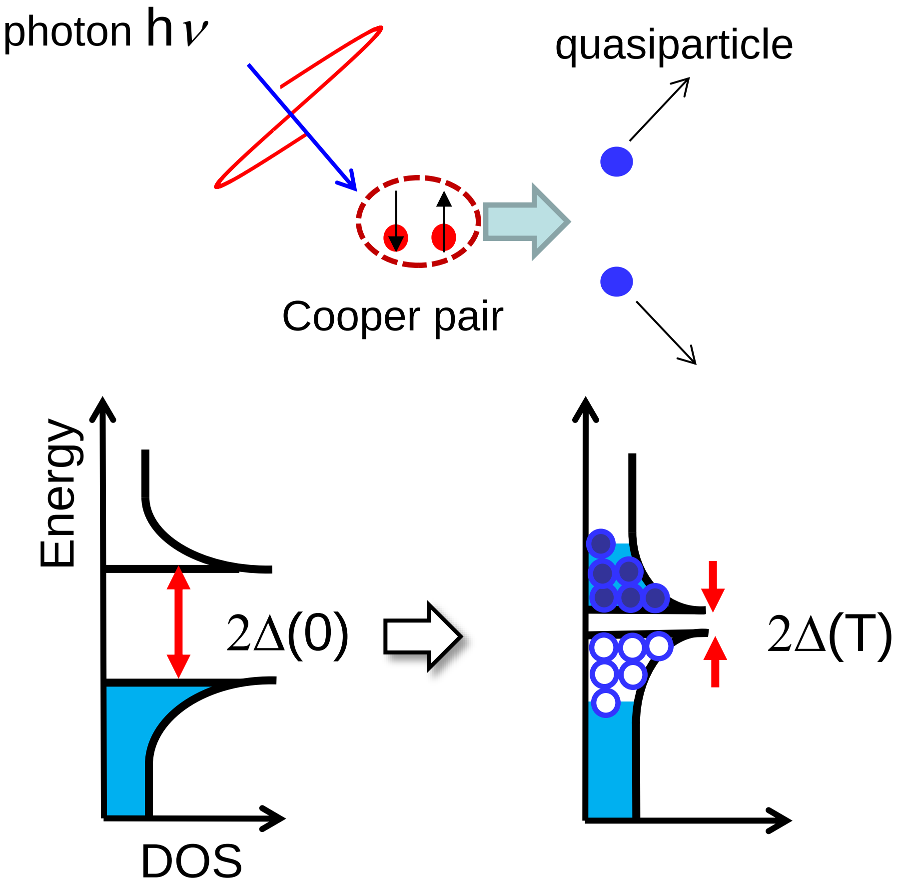
(Received July 28, 1958)

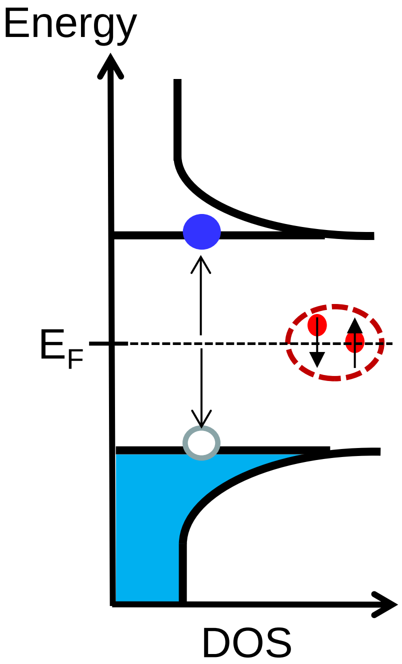
A generalization of the random-phase approximation of the theory of Coulomb correlation energy is applied to the theory of superconductivity. With no further approximations it is shown that most of the elementary excitations have the Bardeen-Cooper-Schrieffer energy gap spectrum, but that there are collective excitations also. The most important of these are the longitudinal waves which have a velocity $v_F\{\frac{1}{3}[1-4N(0)|V|]\}^{\frac{1}{2}}$ in the neutral Fermi gas, and are essentially unperturbed plasma oscillations in the charged case. Other collective excitations resembling higher bound pair states may or may not exist but do not seriously affect the energy gap. The theory obeys the sum rules and is gauge invariant to an adequate degree throughout.

Physical Review 1958

相互作用クエンチの代わりに…

超短THz電磁波パルスで瞬間的に準粒子を注入する



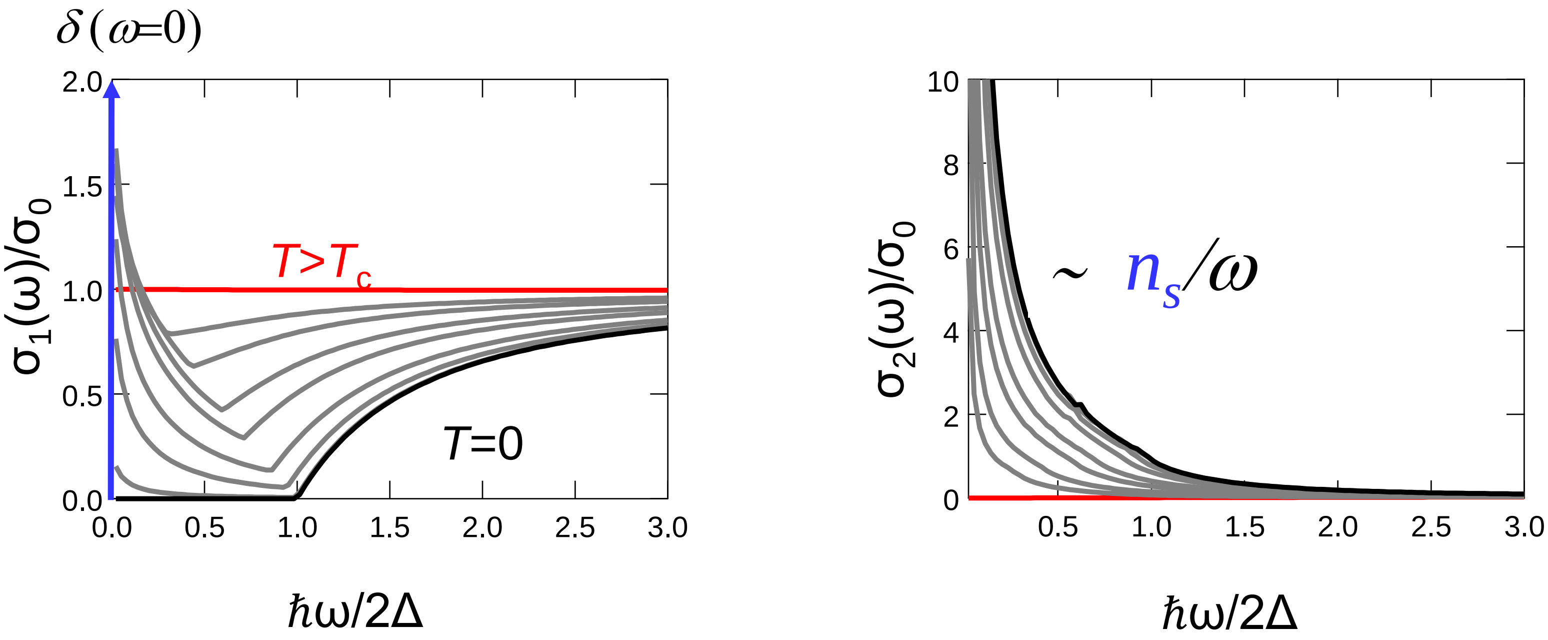


超伝導ギャップΔは準粒子分布と自己無撞着に決まっている。

$$\Delta = V \int_{\Delta}^{\hbar\omega_D} d\varepsilon \frac{\Delta}{\sqrt{\varepsilon^2 - \Delta^2}}[1 - 2f(\varepsilon)]$$

どうやって秩序変数のダイナミクスを見る？

超伝導体の光学スペクトル (BCS理論)



2Δ：超伝導ギャップエネルギー(~meV): 秩序変数

光吸収スペクトルでギャップ構造の時間変化を見る。

THz ポンプTHzプローブ分光

Sample

$\mathrm{Nb_{0.8}Ti_{0.2}N}$ film (12nm)/Quartz

$T_{\mathrm{C}} = 8.5$ K,

$2\Delta(T=4\ \mathrm{K}) = 3.0$ meV = 0.72 THz

response time : $\tau_{\Delta} = \Delta^{-1} \sim 2.8$ ps

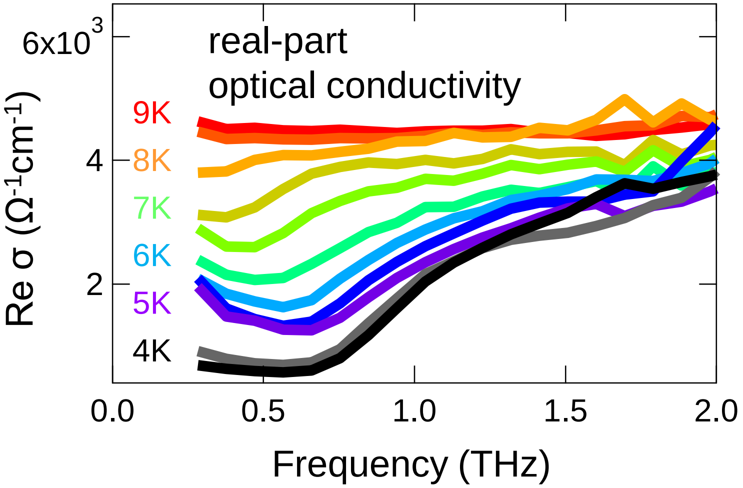
THz pump pulse

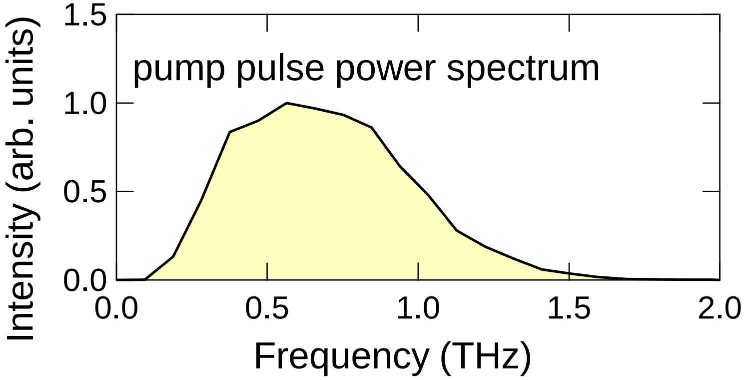
Center frequency 0.7THz～2Δ

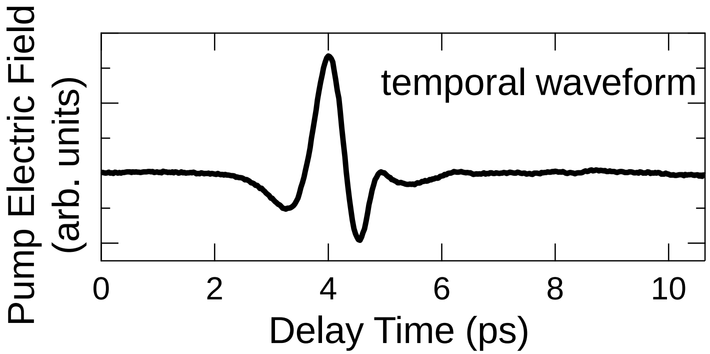
pulse width: $\tau_{\mathrm{pump}} \sim 1.5$ ps

$\tau_{\mathrm{pump}}/\tau_{\Delta}$ ～0.57 < 1

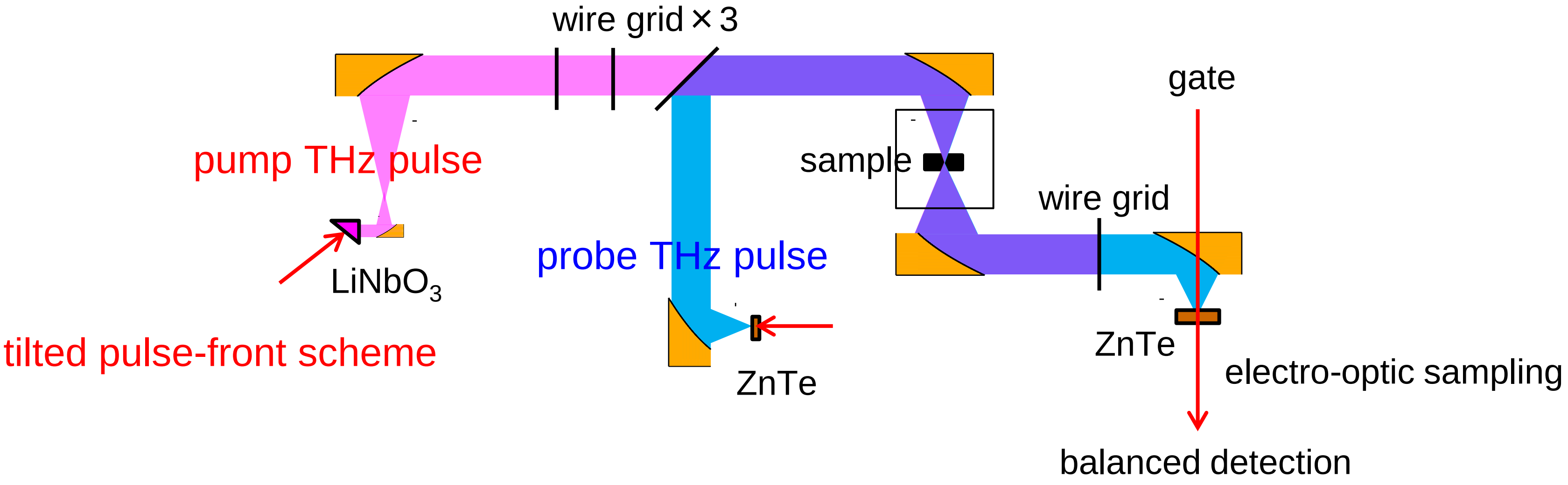
nonadiabatic excitation condition

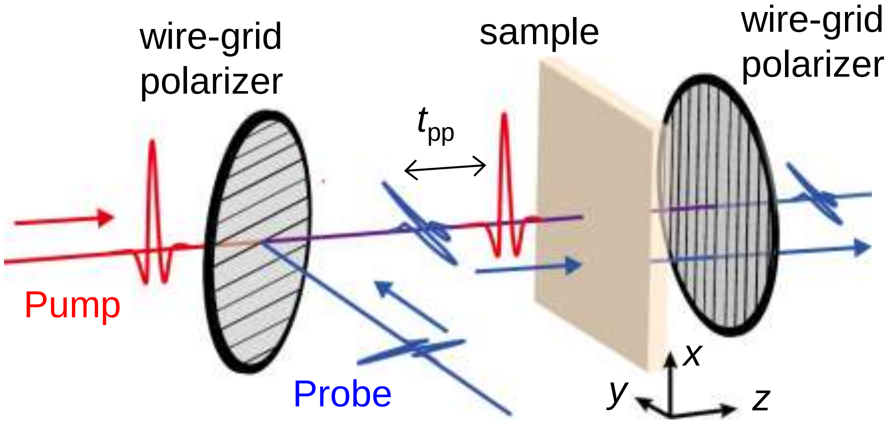






THz ポンプTHzプローブ分光





Pump　：$E_{\mathrm{pump}}//x$

Probe：$E_{\mathrm{probe}}//y$

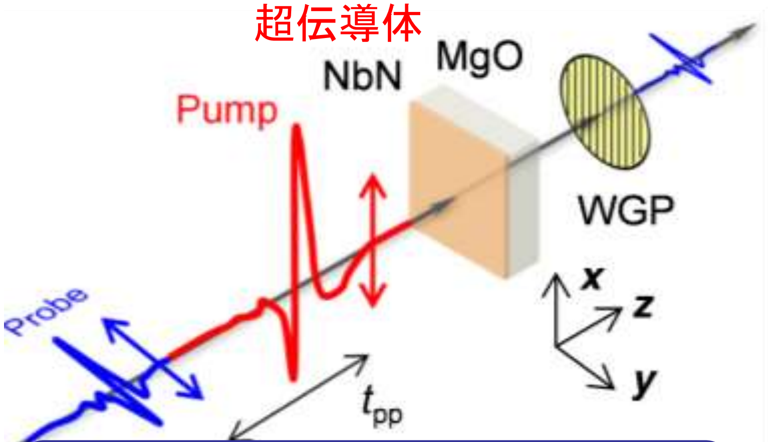
t_{pp}: pump-probe delay

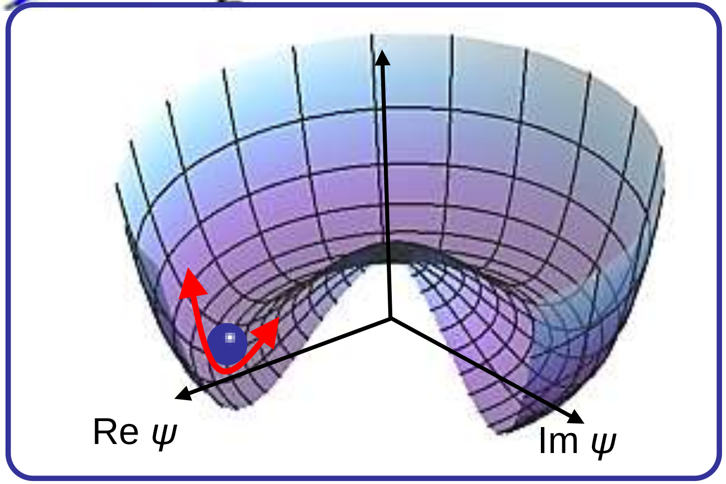
Transmitted probe THz electric field:

Free space EO sampling

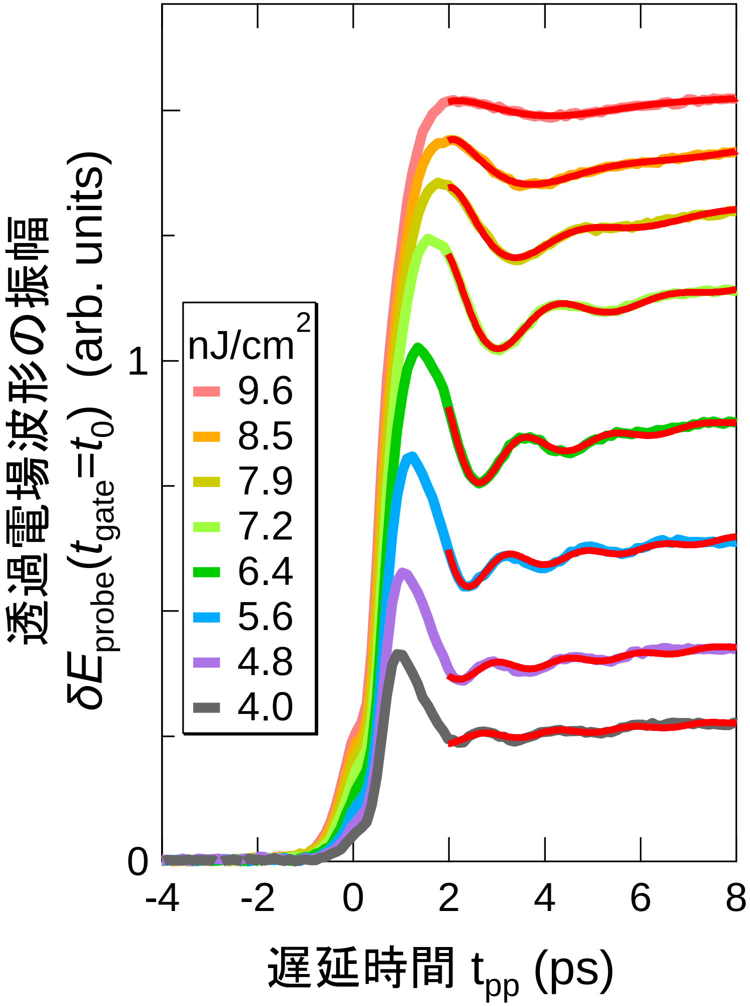
t_{gate}: gate pulse delay

超伝導体の”ヒッグス“モード観測に成功



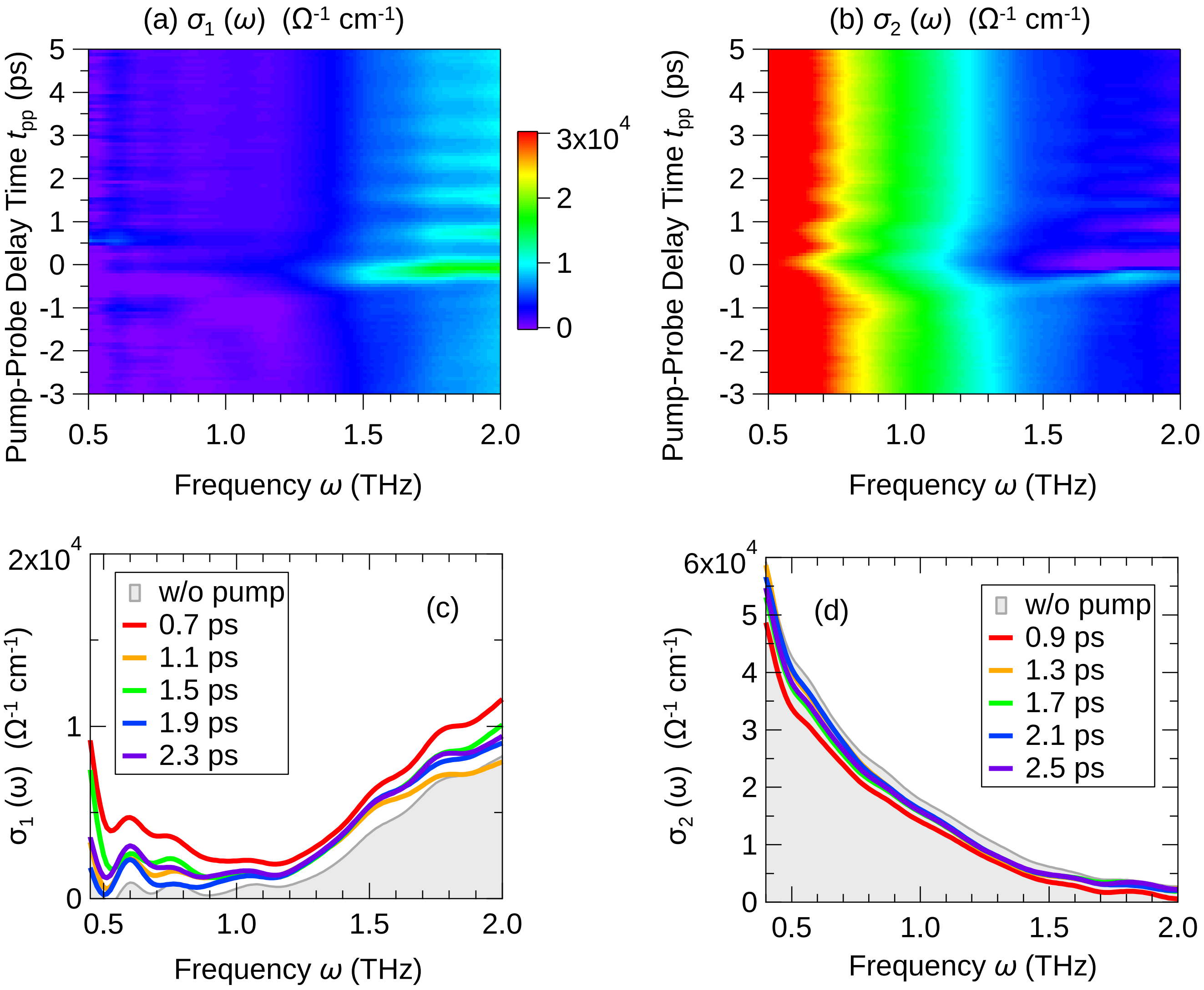


$$\frac{\Delta(t)}{\Delta_\infty} = 1 + a\frac{\cos(2\Delta_\infty t + \pi/4)}{\sqrt{\Delta_\infty t}}$$

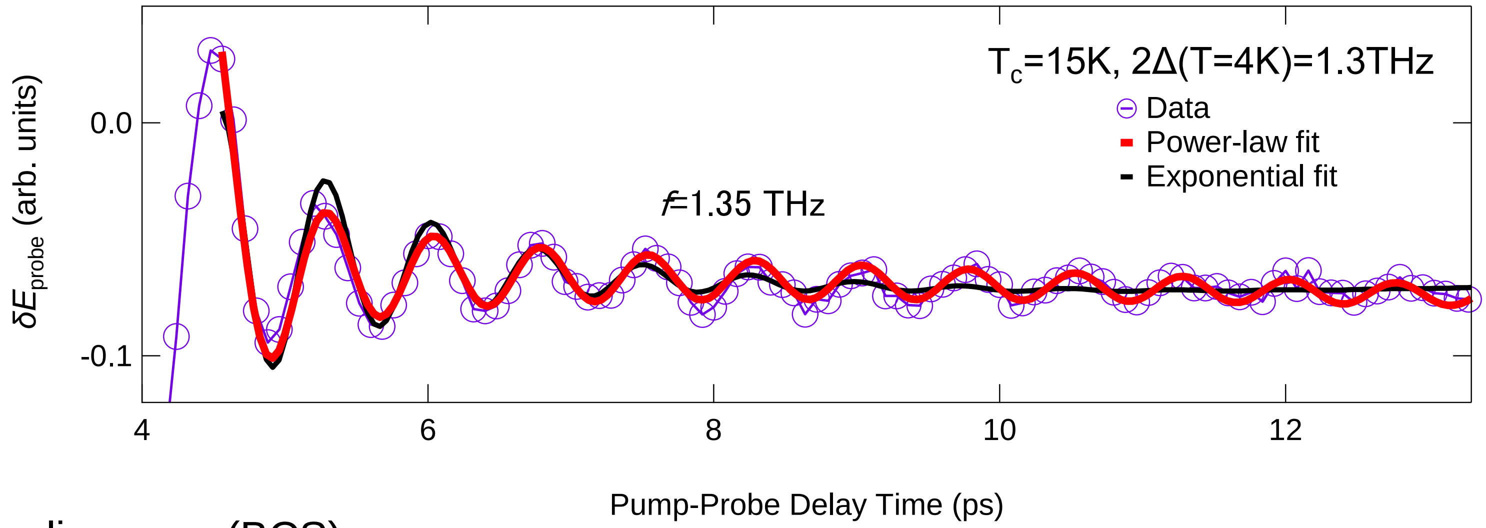


R. Matsunaga et al., Phys. Rev. Lett. **111**, 057002 (2013).

光学伝導度スペクトルのダイナミクス



ヒッグスモードの減衰



Weak coupling case (BCS)

$$\frac{\Delta(t)}{\Delta_\infty} = 1 + a\frac{\cos(2\Delta_\infty t + \pi/4)}{\sqrt{\Delta_\infty t}}$$

Volkov *et al*., Sov. Phys. JETP 38, 1018 (1974).

Yuzbashyan *et al*., PRL 96, 097005 (2006).

exponential decay

$$\delta E_{\mathrm{probe}}(t_{\mathrm{pp}}) = C + A\exp\left(-\frac{t}{\tau}\right)\cos(2\pi f t_{\mathrm{pp}} + \phi)$$

τ= 1.3 ps χ^2= 3.6 × 10^{-4}

power-law decay

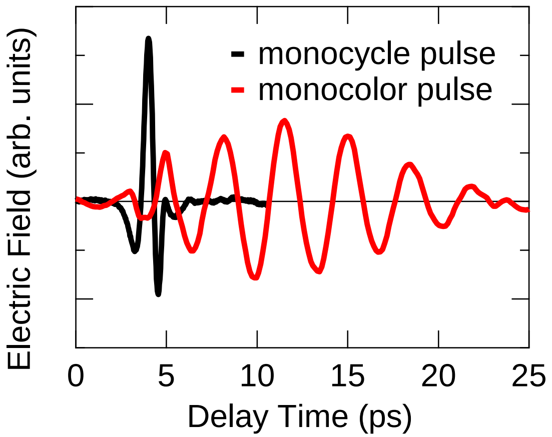
$$\delta E_{\mathrm{probe}}(t_{\mathrm{pp}}) = C + \frac{A}{(t_{\mathrm{pp}} - t_0)^b}\cos(2\pi f t_{\mathrm{pp}} + \phi)$$

b = 0.71 χ^2= 2.8 × 10^{-4}

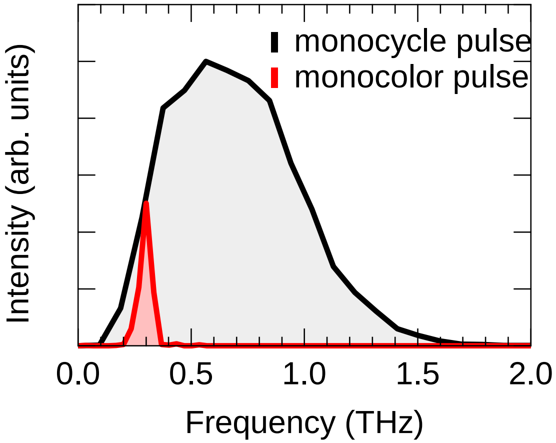
QuenchからDriveへ

Quasi-monochromatic THz pulse（0.3THz, pulsewidth～13ps）

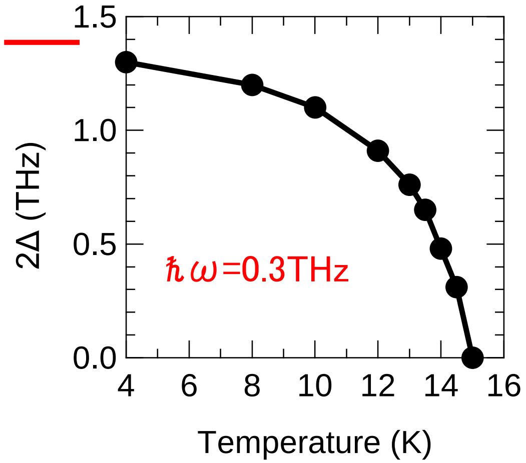
E-field waveform



Power Spectrum



Photon energy vs BCS gap



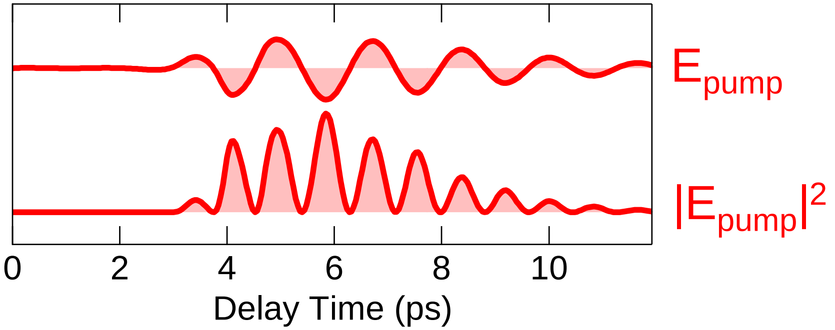
How does the BCS ground state respond to the strong electromagnetic field with $\hbar\omega < 2\Delta$?

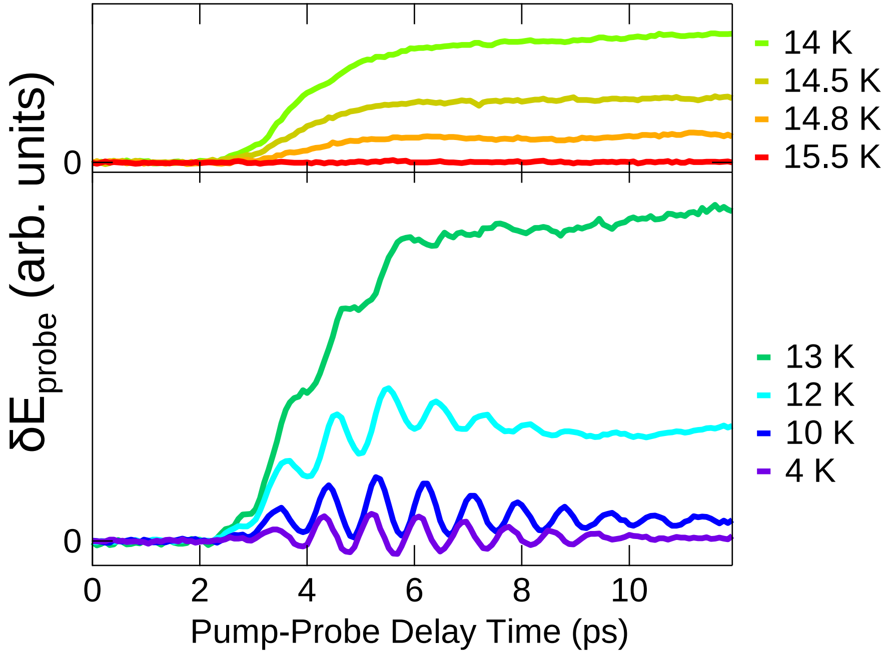
マルチサイクルTHz波照射下の秩序変数の振舞い

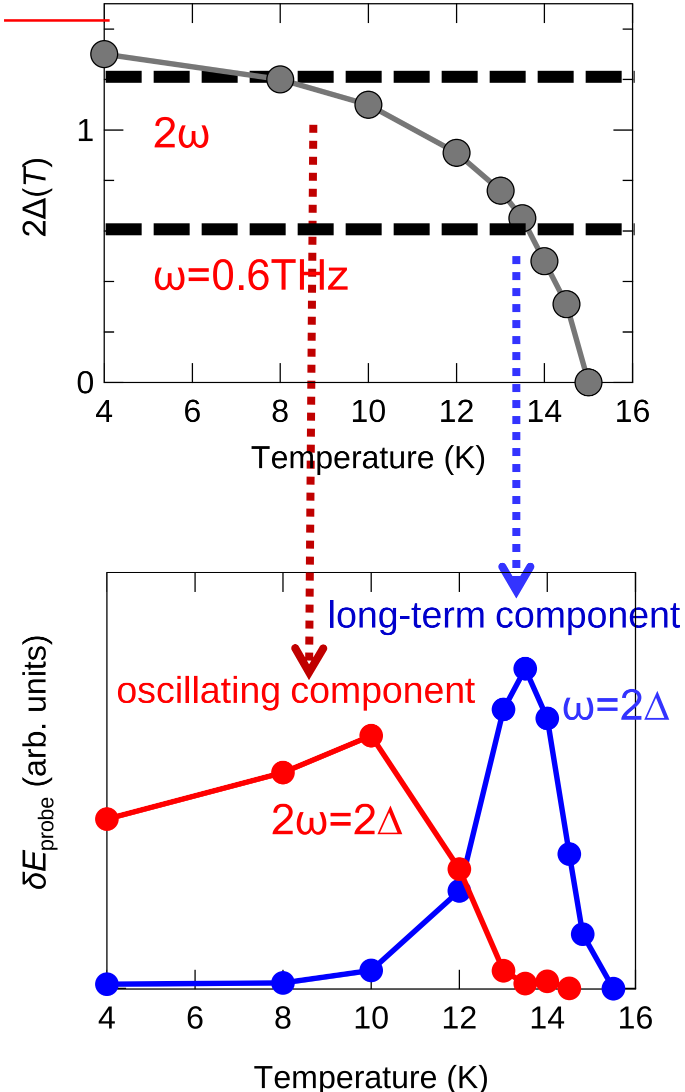
ω=0.6THz

E=3.5 kV/cm @ peak

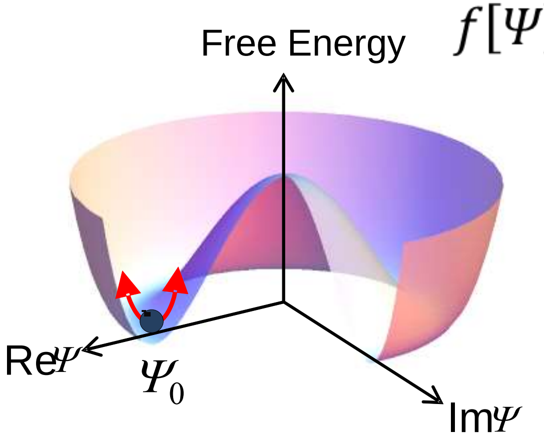
R. Matsunaga et al., Science 345, 1145 (2014)







Ginzburg-Landau picture



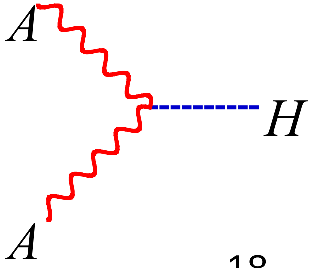
$$f[\Psi] = f_0 + a|\Psi(\boldsymbol{r})|^2 + \frac{b}{2}|\Psi(\boldsymbol{r})|^4 + \frac{1}{2m^*}|(-i\nabla - e^*\boldsymbol{A})\Psi(\boldsymbol{r})|^2$$

$a < 0$ $\quad \Psi(\mathrm{r}) = [\Psi_0 + \mathrm{H}(\mathrm{r})]\mathrm{e}^{\mathrm{i}\theta(\mathrm{r})}$

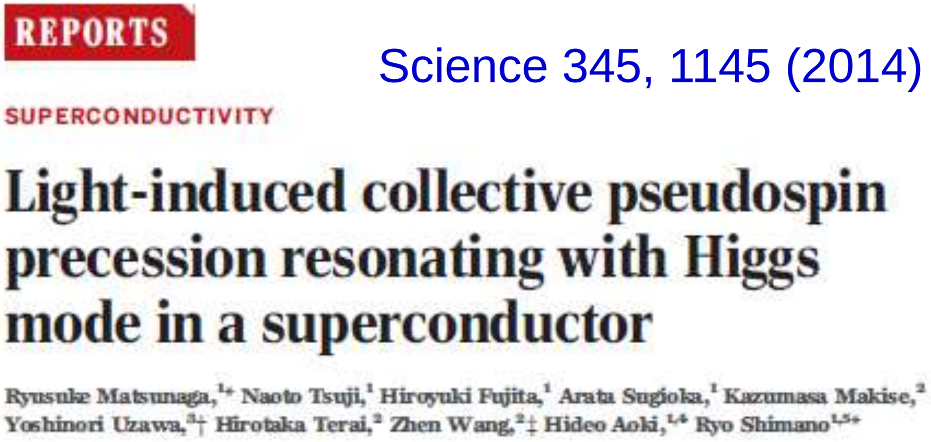
$$f = -2aH^2 + \frac{1}{2m^*}(\nabla H)^2 + \frac{e^{*2}}{2m^*}\left(\boldsymbol{A} - \frac{1}{e^*}\nabla\theta\right)^{\mathbf{2}}(\Psi_0 + H)^2 + \cdots$$

Local gauge transformation $\quad \boldsymbol{A}' = \boldsymbol{A} - \nabla\theta/e^* \quad \boldsymbol{A}' \to \boldsymbol{A}$

$$f = -2aH^2 + \frac{1}{2m^*}(\nabla H)^2 + \frac{e^{*2}\Psi_0^2}{2m^*}\boldsymbol{A}^2 + \boxed{\frac{e^{*2}\Psi_0}{m^*}\boldsymbol{A}^2 H} + \cdots$$



ヒッグスモードの光駆動



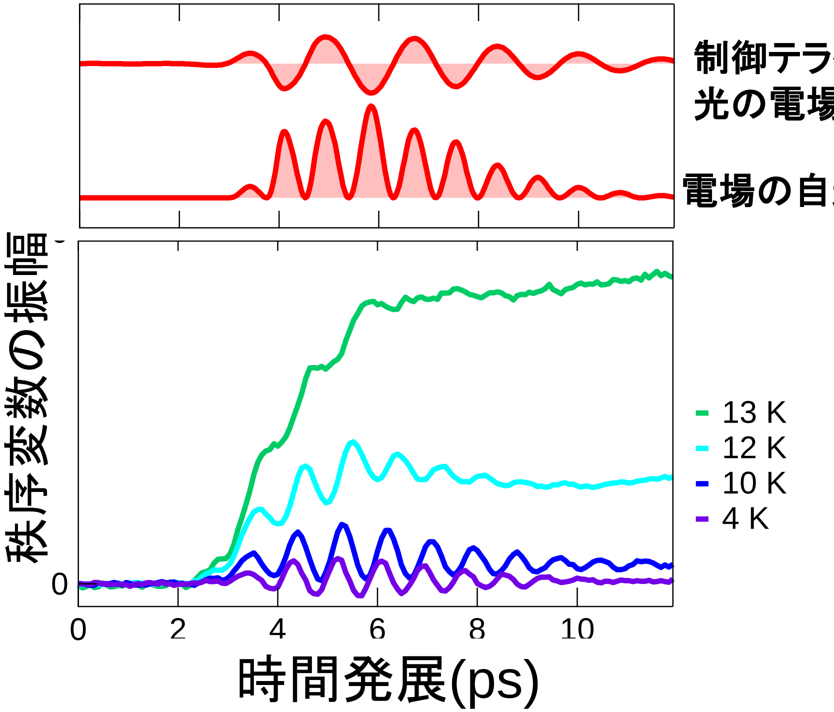
REPORTS

SUPERCONDUCTIVITY

Light-induced collective pseudospin precession resonating with Higgs mode in a superconductor

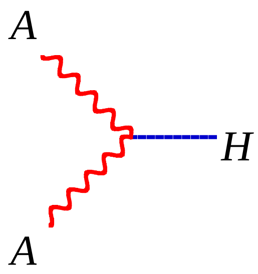
Ryusuke Matsunaga,[1*] Naoto Tsuji,[1] Hiroyuki Fujita,[1] Arata Sugioka,[1] Kazumasa Makise,[2] Yoshinori Uzawa,[3†] Hirotaka Terai,[2] Zhen Wang,[2‡] Hideo Aoki,[1,4] Ryo Shimano[1,5*]

Science 345, 1145 (2014)

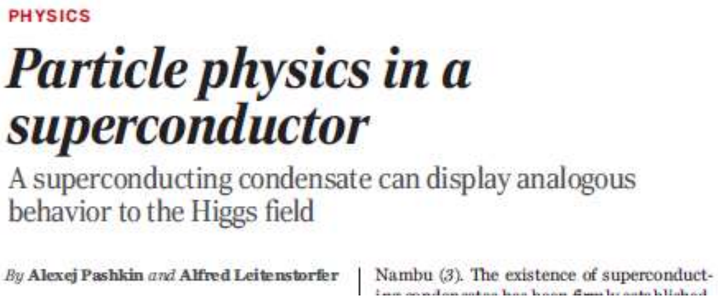


制御テラヘルツ光の電場成分

電場の自乗波形



Science,
Perspective in Physics

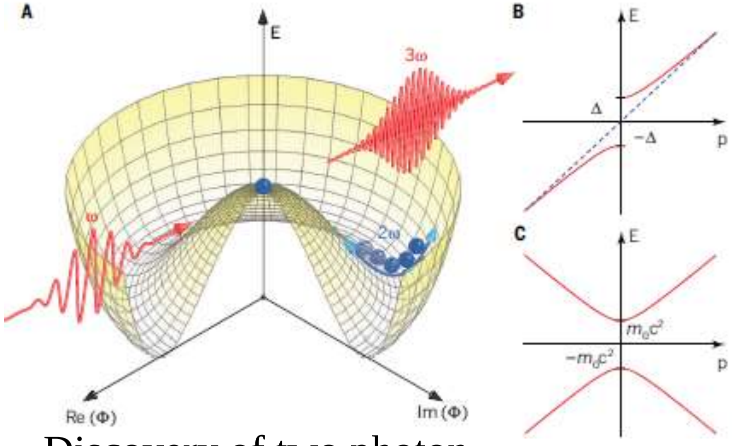


PHYSICS

Particle physics in a superconductor

A superconducting condensate can display analogous behavior to the Higgs field

By Alexej Pashkin *and* Alfred Leitenstorfer | Nambu (3). The existence of superconduct-



Discovery of two photon coupling with Higgs mode

R. Matsunaga et al., Science **345**, 1145 (2014).

第三高調波発生

Current density

$$\boldsymbol{j}(t) = e\sum_{\boldsymbol{k}} \boldsymbol{v}_{\boldsymbol{k}-\boldsymbol{A}} n_{\boldsymbol{k}} = e\sum_{\boldsymbol{k}} \frac{\partial \varepsilon_{\boldsymbol{k}-e\boldsymbol{A}(t)}}{\partial \boldsymbol{k}}\left(\sigma^{z}_{\boldsymbol{k}}(t)+\frac{1}{2}\right)$$

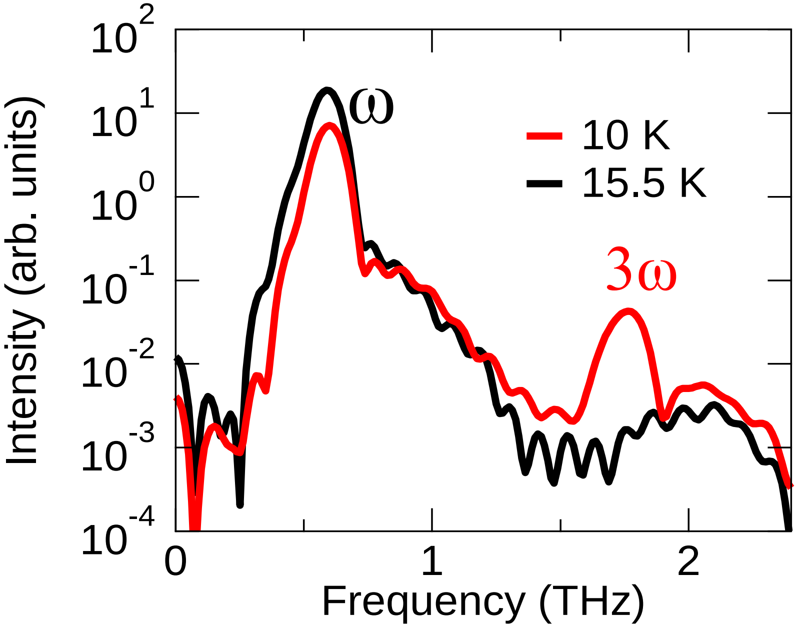
$$\sim \boldsymbol{j}_{\mathrm{linear}}(t) - \underline{\frac{e^{2}\Delta}{U}\boldsymbol{A}(t)\delta\Delta(t)}$$

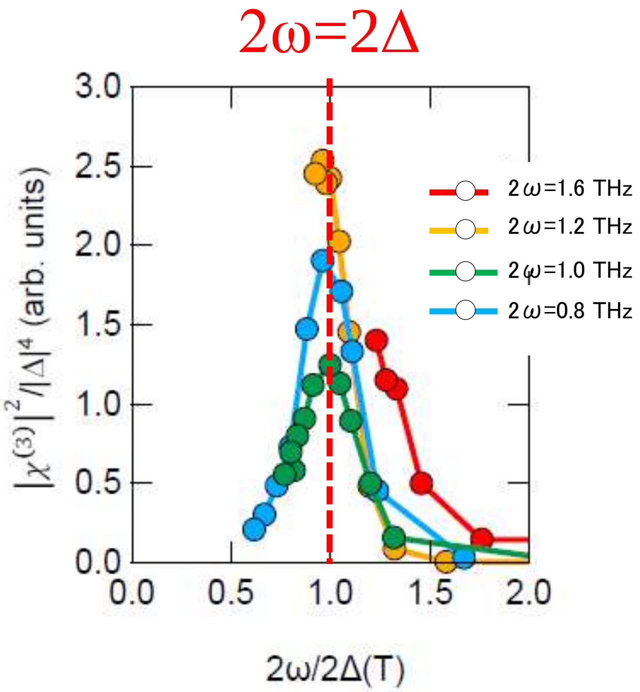
London equation for nonlinear current j_{nl}

$$\begin{array}{l} \delta\Delta(t) \sim e^{i2\omega t}, \\ A(t) \sim e^{i\omega t} \end{array} \quad \Longrightarrow \quad j(t) \sim e^{i3\omega t}$$

Does superconductor emit THz third harmonics?

第三高調波発生

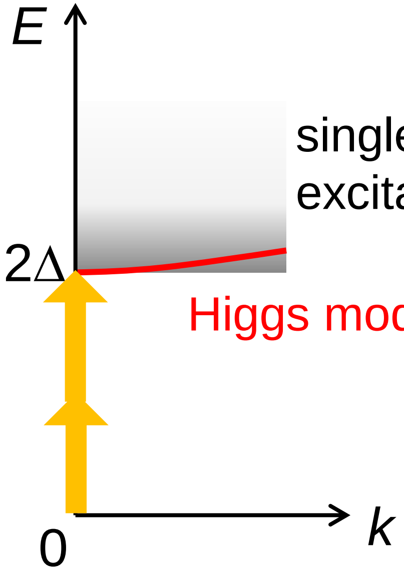

Intensity (arb. units)
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
ω
10 K
15.5 K
3ω
0
1
2
Frequency (THz)


2ω=2Δ
$|\chi^{(3)}|^2/|\Delta|^4$ (arb. units)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2ω=1.6 THz
2ω=1.2 THz
2ω=1.0 THz
2ω=0.8 THz
2ω/2Δ(T)

それって本当にHiggsなの？

T. Cea, C. Castellani, and L. Benfatto,
Phys. Rev. B93, 180507 (2016)

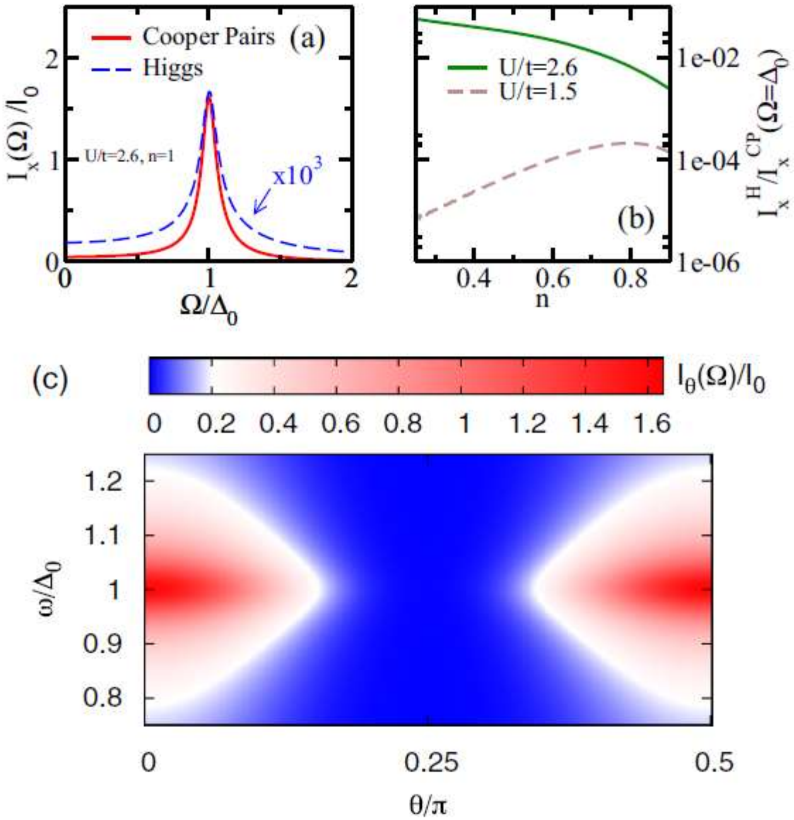
BCS with 2D square lattice model



BCS mean field:
Higgs << Charge density fluctuation

$<\Delta\Delta>$ $<\rho\rho>$

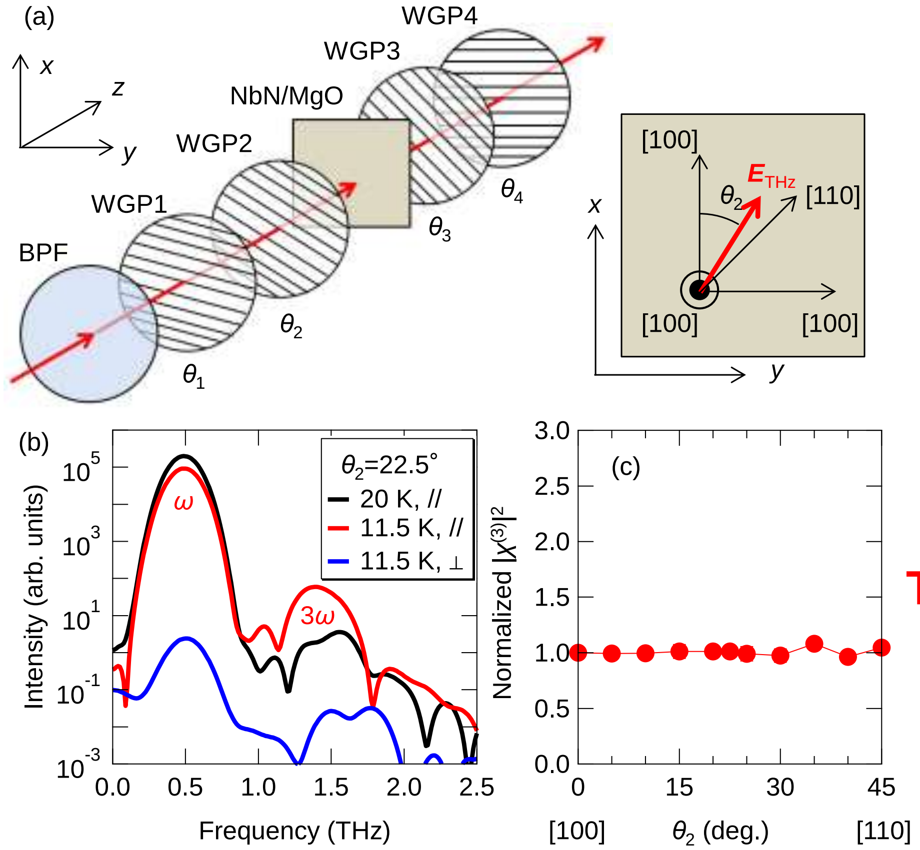
$$\frac{I_{Higgs}}{I_{CDF}} \sim \left(\frac{\Delta}{V}\right)^4$$



Pump polarization dependence

第三高調波の偏光依存性

R. Matsunaga, et al. Phys. Rev. B 96, 020505(R) (2017).

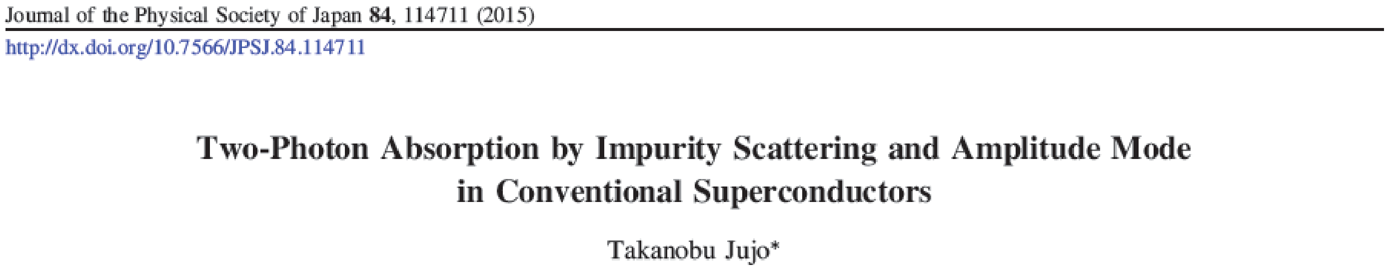


Totally isotropic!

Polarization of THG is always in parallel with the incident light polarization and its intensity is irrespective to the crystal axis.

The origin of THG is dominated by Higgs.

不純物散乱の効果



Journal of the Physical Society of Japan **84**, 114711 (2015)

http://dx.doi.org/10.7566/JPSJ.84.114711

Two-Photon Absorption by Impurity Scattering and Amplitude Mode in Conventional Superconductors

Takanobu Jujo*

Mattis-Bardeen model analysis

Y. Murotani and RS, PRB**99**, 224510(2019)

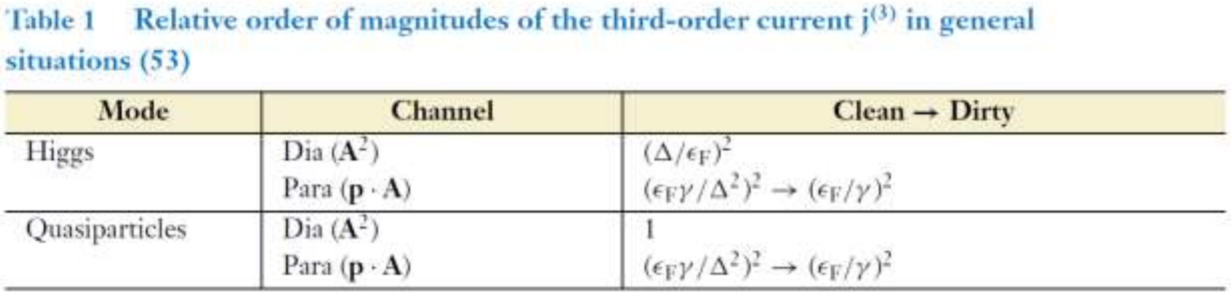
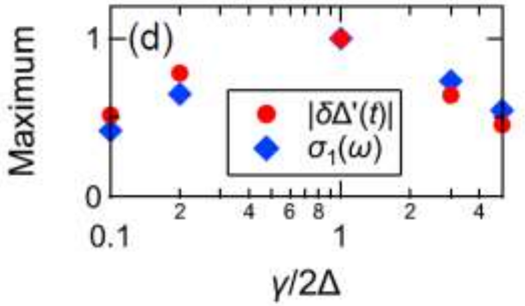
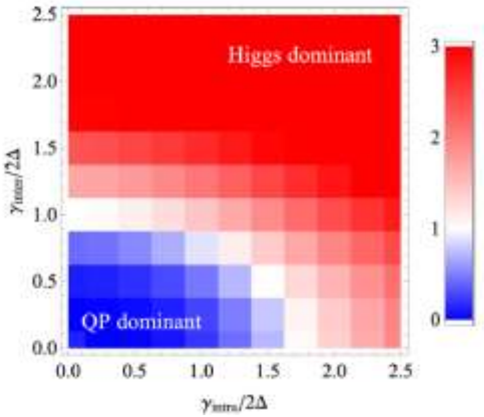


Table 1 Relative order of magnitudes of the third-order current $j^{(3)}$ in general situations (53)

Mode	Channel	Clean → Dirty
Higgs	Dia ($\mathbf{A}^2$)	$(\Delta/\epsilon_F)^2$
	Para ($\mathbf{p}\cdot\mathbf{A}$)	$(\epsilon_F\gamma/\Delta^2)^2 \to (\epsilon_F/\gamma)^2$
Quasiparticles	Dia ($\mathbf{A}^2$)	1
	Para ($\mathbf{p}\cdot\mathbf{A}$)	$(\epsilon_F\gamma/\Delta^2)^2 \to (\epsilon_F/\gamma)^2$



N. Tsuji and Y. Nomura,
Phys. Rev. Res. **2**, 043029(2020)



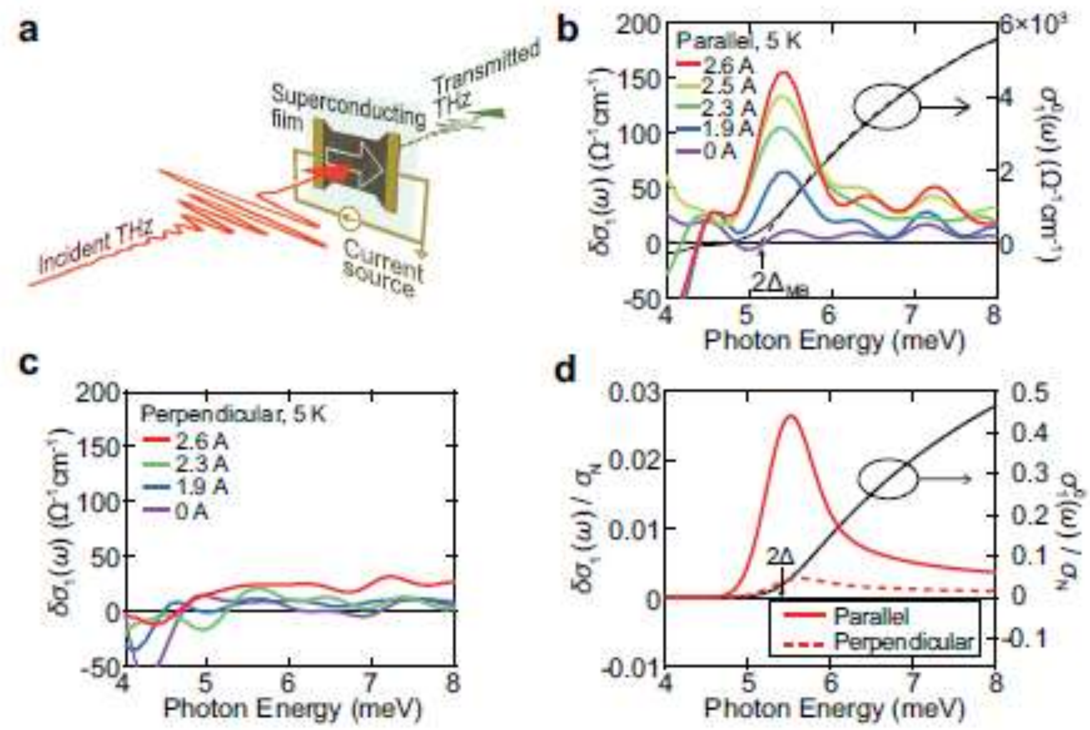
電流注入すると線形吸収でHiggsが見える！

A. Moor et al., Phys. Rev. Lett. 118, 047001 (2017).

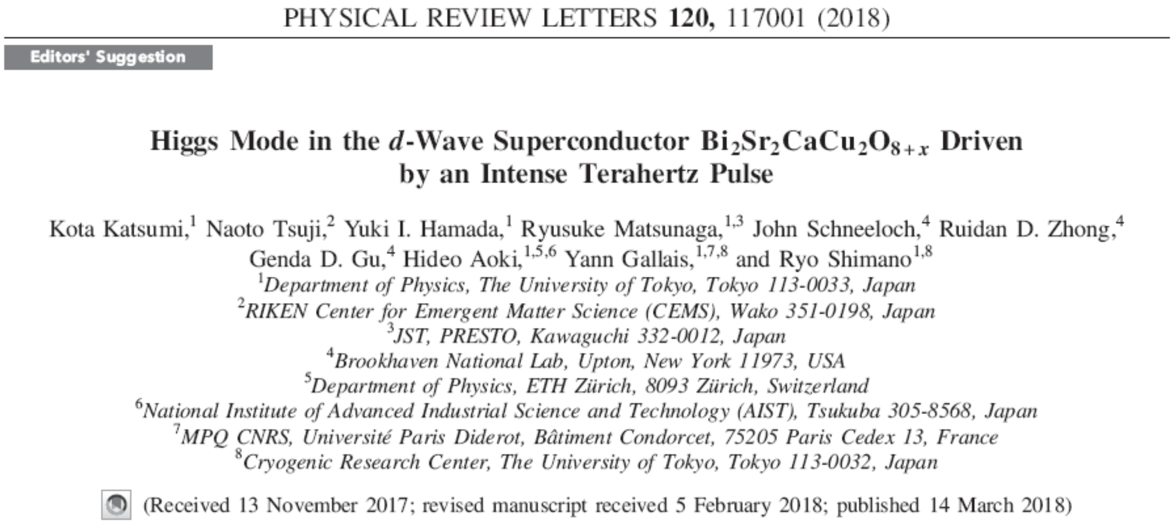
$$A(t)^2=(\underline{A_0}+A_\omega e^{i\omega t})^2=A_0^2+\boxed{2A_0A_\omega e^{i\omega t}}+A_\omega^2e^{2i\omega t}$$

Injected Supercurrent THz field Linear coupling between A and H field

S. Nakamura, et al., Phys. Rev. Lett. 122, 257001 (2019).



銅酸化物高温超伝導体への展開



PHYSICAL REVIEW LETTERS **120,** 117001 (2018)

Editors' Suggestion

Higgs Mode in the *d*-Wave Superconductor $\mathrm{Bi_2Sr_2CaCu_2O_{8+x}}$ Driven by an Intense Terahertz Pulse

Kota Katsumi,[1] Naoto Tsuji,[2] Yuki I. Hamada,[1] Ryusuke Matsunaga,[1,3] John Schneeloch,[4] Ruidan D. Zhong,[4]
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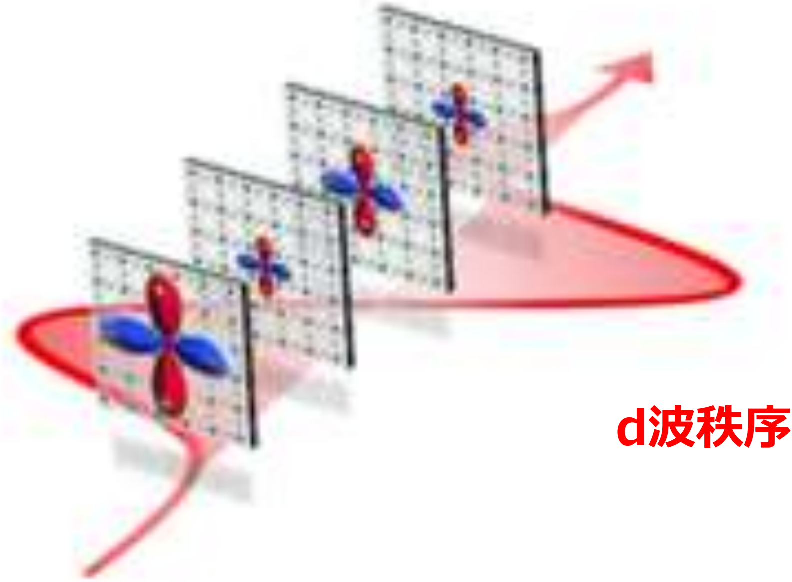
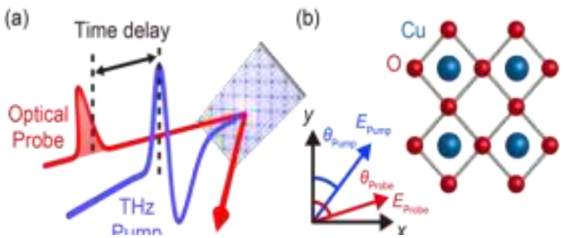


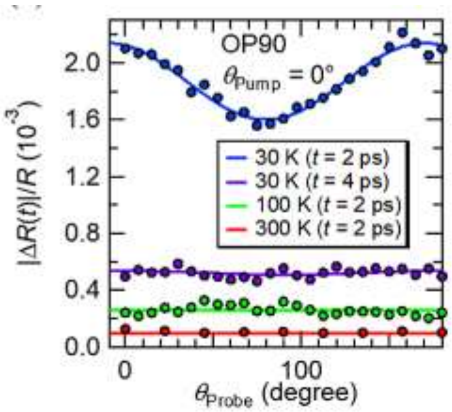
d波秩序変数が揺れる。

銅酸化物高温超伝導体のヒッグスモード

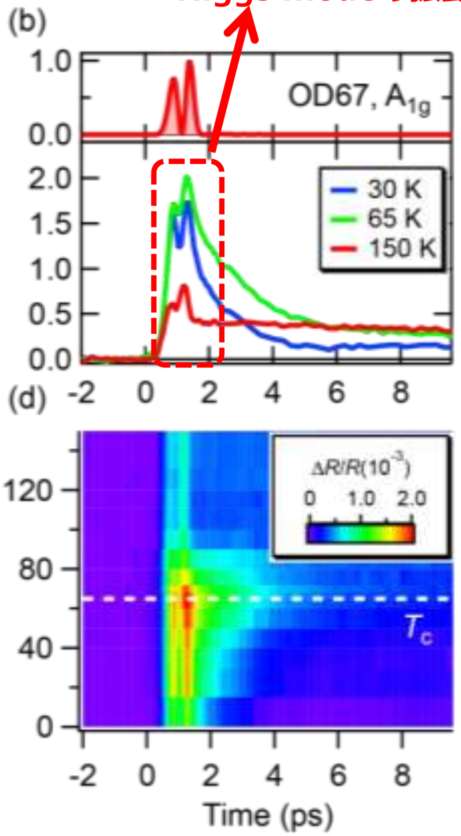
K. Katsumi et al., Phys. Rev. Lett. **120**, 117001 (2018).

$Bi_2Sr_2CaCu_2O_{8+x}$



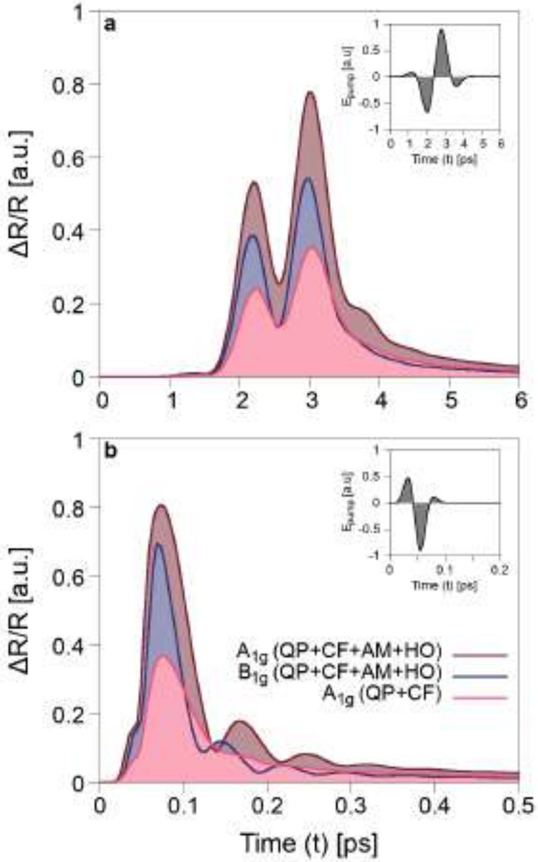


Higgs modeの振動



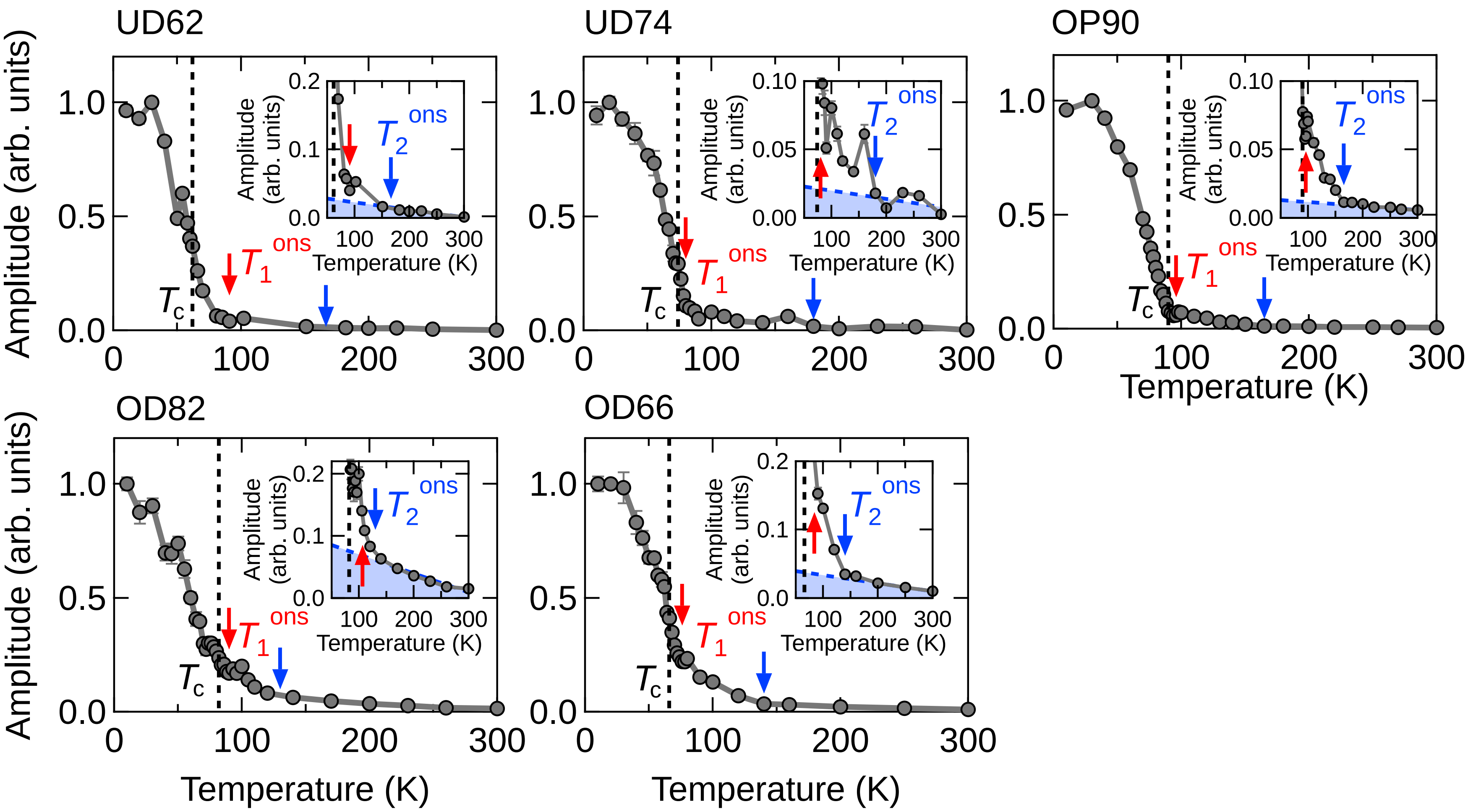
ヒッグスモードの振動

理論

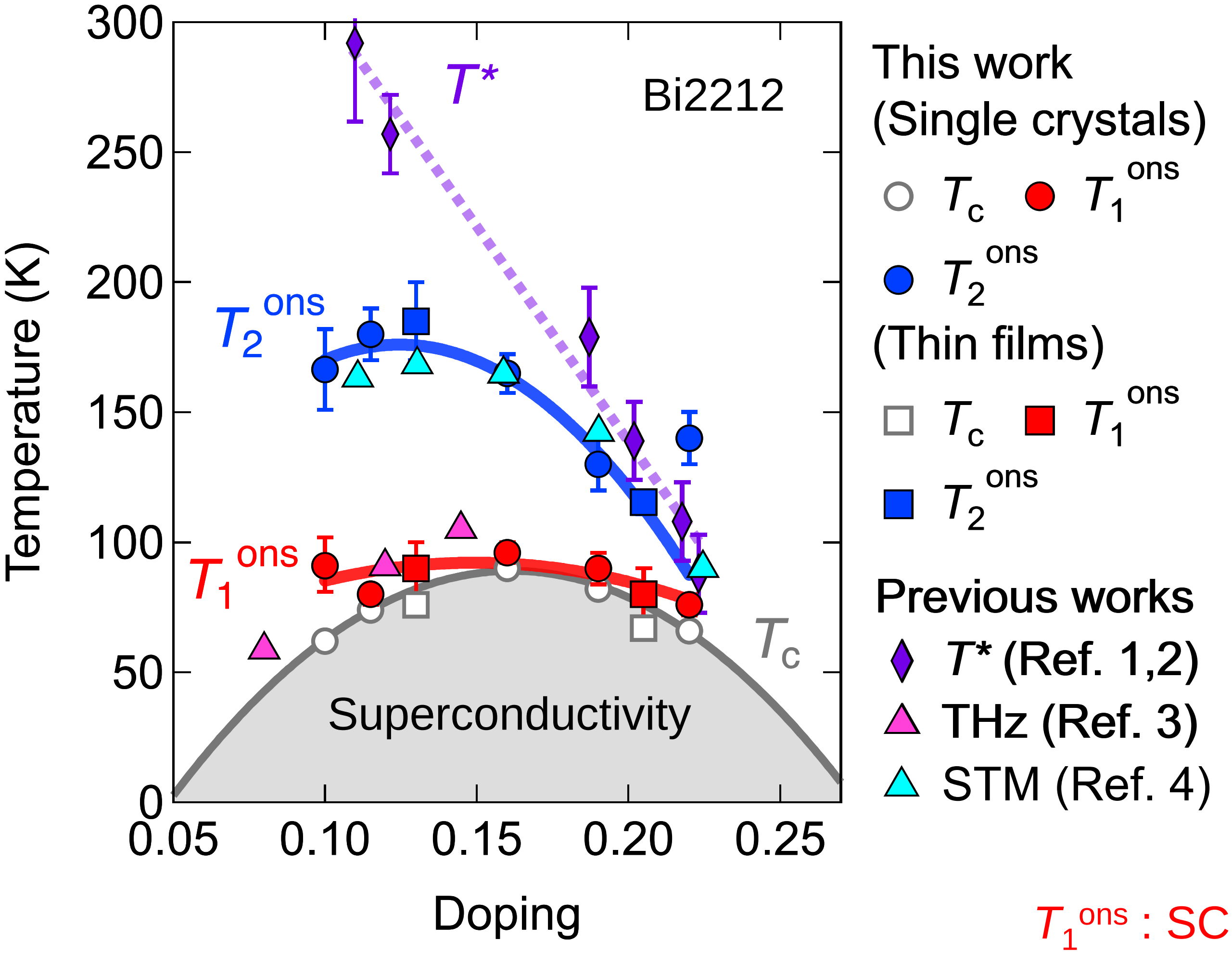


M. Puviani et al.,
arXiv:2012.01922v1

振動成分の温度依存性

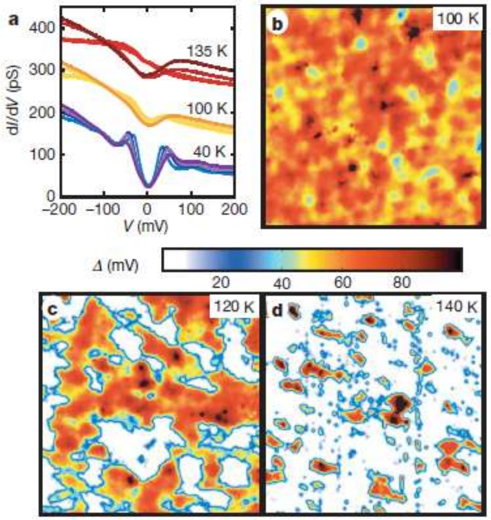

UD62
UD74
OP90
OD82
OD66
Amplitude (arb. units)
Temperature (K)
Amplitude (arb. units)
T_c
T_1^{ons}
T_2^{ons}
0.0
0.5
1.0
0
100
200
300
0.0
0.1
0.2
0.00
0.05
0.10

ヒッグスから見る超伝導クーパー対形成温度



K. Katsumi *et al.*, PRB **102** 054510 (2020)

STM



K. K. Gomes et al.
Nature 2007

T_1^{ons} : SC phase fluctuation

T_2^{ons} : Preformed Cooper pairs

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