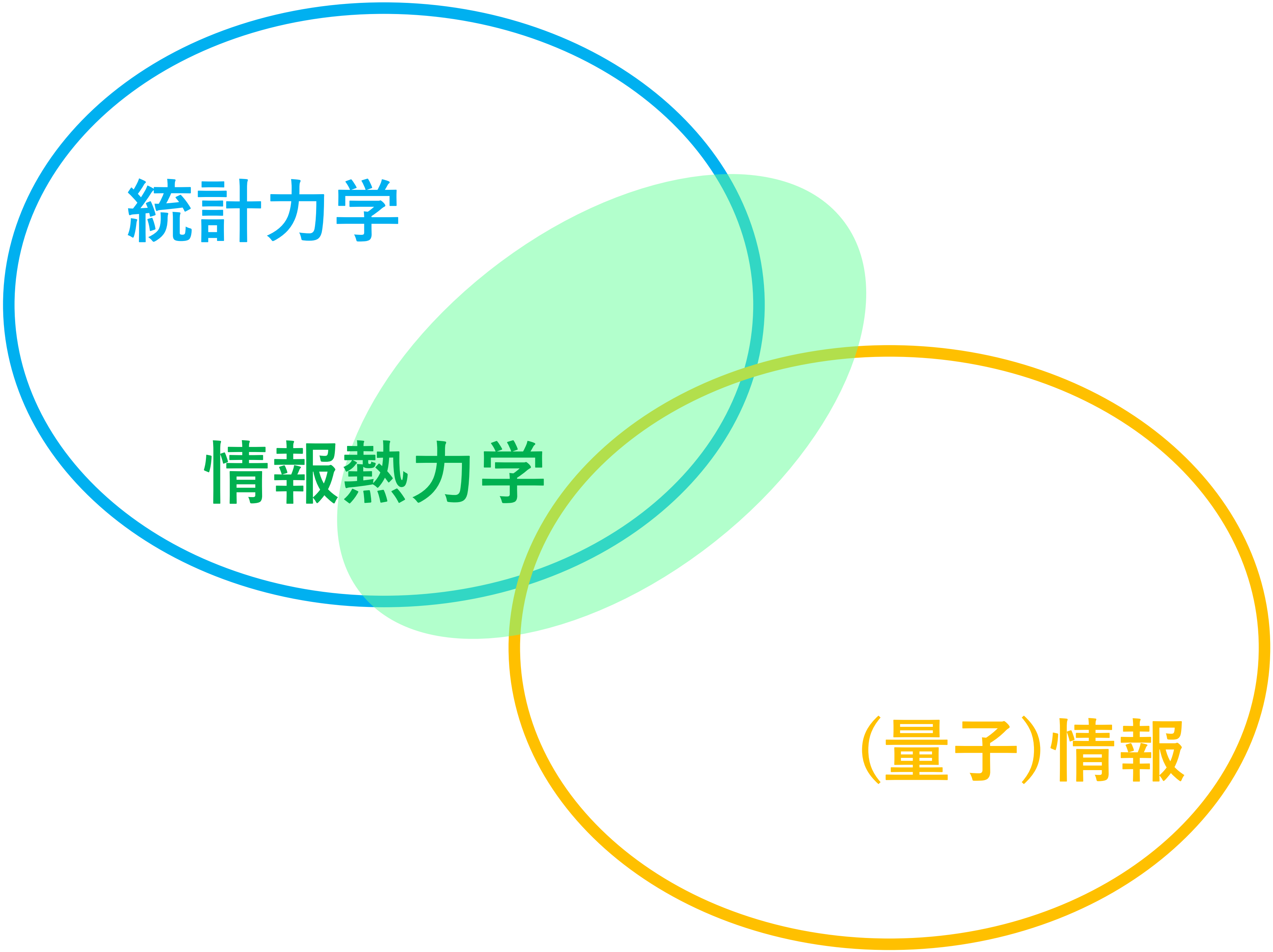
量子連続制御下の情報熱力学

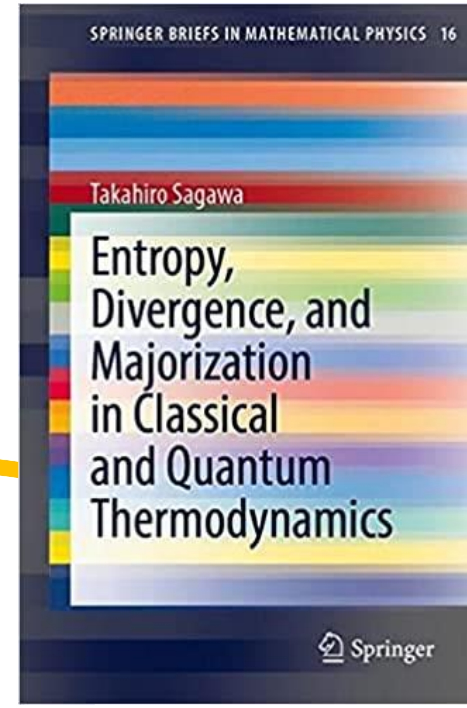
沙川 貴大
東京大学大学院 工学系研究科

2023年2月16日
KEK 素核研・物構研 連携研究会


統計力学
情報熱力学
(量子)情報

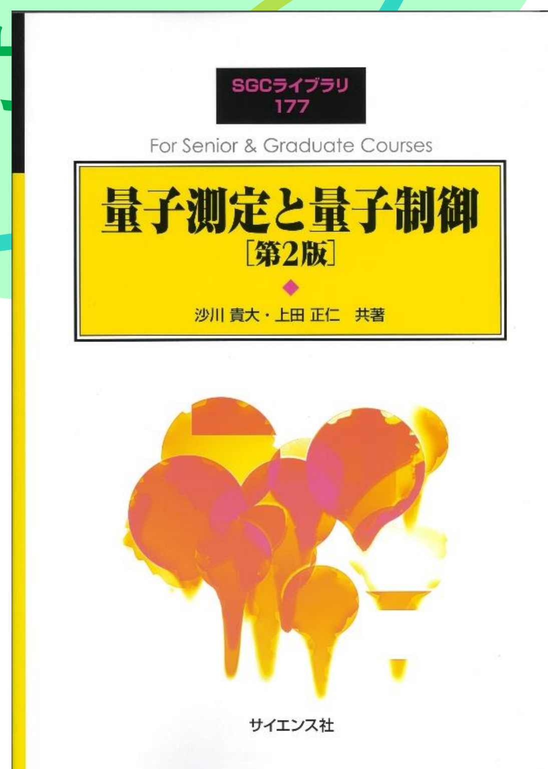
arXiv:2007.09974

2022年6月発売



2022年3月発売

熱力



2022年6月発売

(量子)情報

Outline

- Introduction
- Generalized second laws: Classical
- Generalized second laws: Quantum
- Main result: Continuous quantum feedback
- Summary and references

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- **Introduction**
- Generalized second laws: Classical
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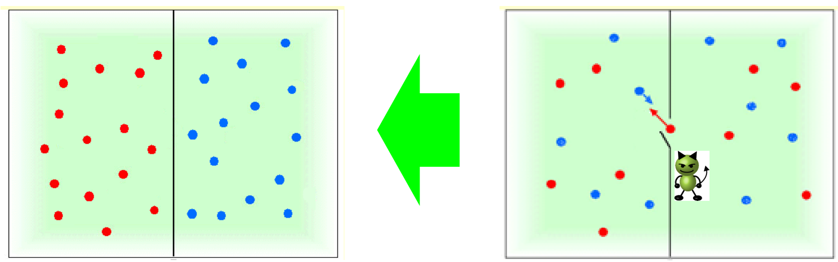
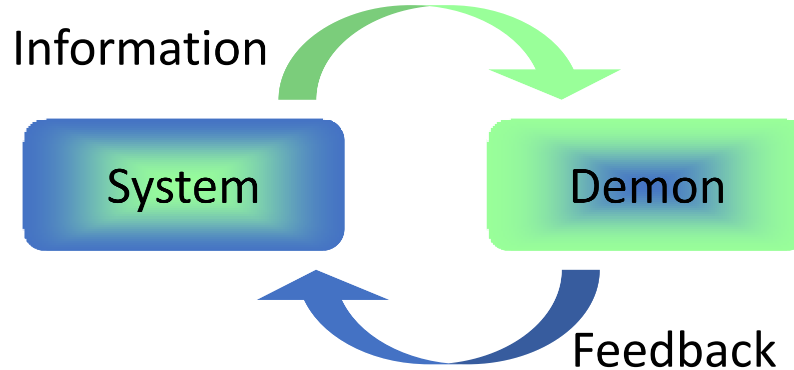
Information thermodynamics





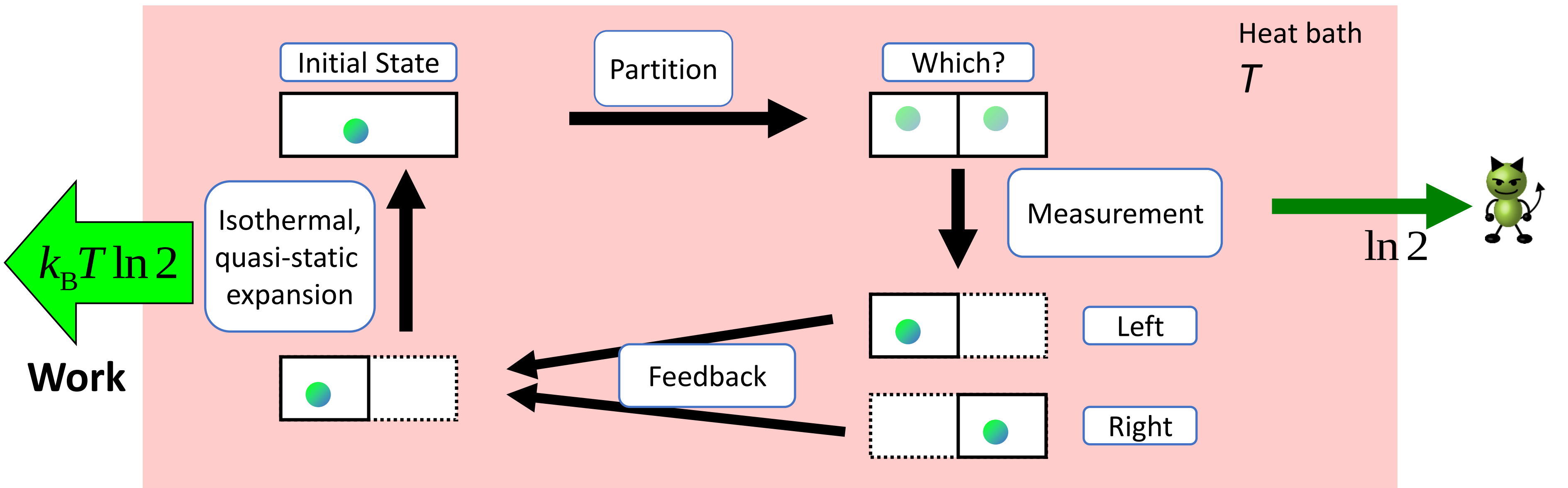
Information processing at the level of thermal fluctuations

- ✓ Foundation of the second law of thermodynamics
- ✓ Application to nanomachines and nanodevices

Review: J. M. R. Parrondo, J. M. Horowitz, & T. Sagawa, *Nature Physics* **11**, 131-139 (2015).

Szilard engine (1929)

L. Szilard, Z. Phys. **53**, 840 (1929)



Free energy: $F = E - TS$

Increase

Decrease by feedback

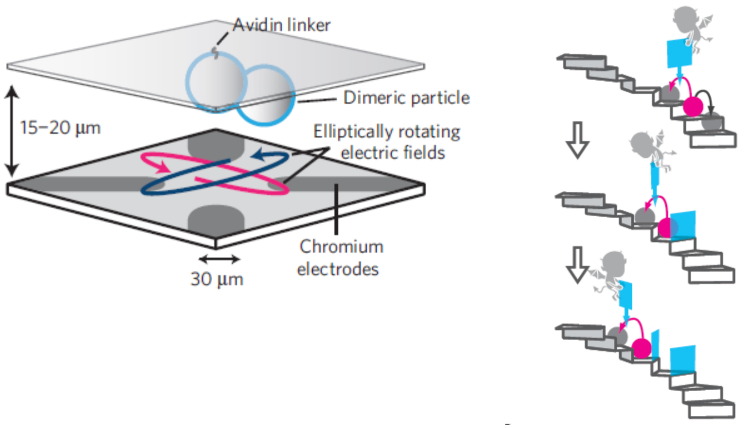
Can control physical entropy by using information

Experimental realizations

- With a colloidal particle
  Toyabe, TS, Ueda, Muneyuki, & Sano, Nature Physics (2010)

  Efficiency: 30%

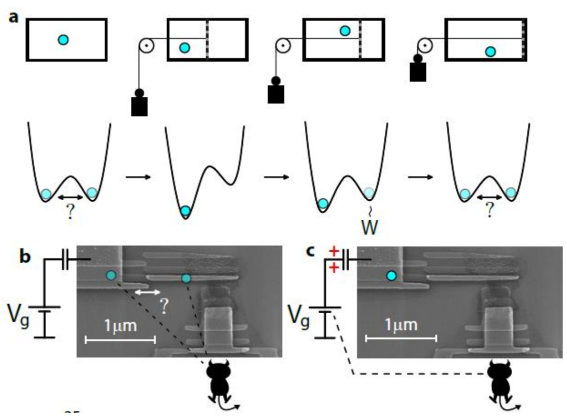
  Validation of $\left\langle e^{-\beta(W-\Delta F)} \right\rangle = \gamma$



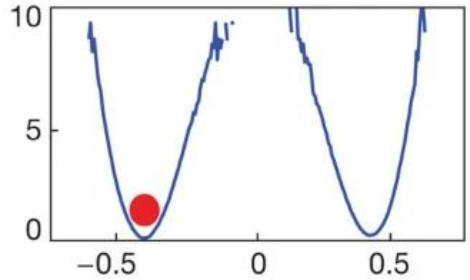
- With a single electron
  Koski, Maisi, TS, & Pekola, PRL (2014)

  Efficiency: 75%

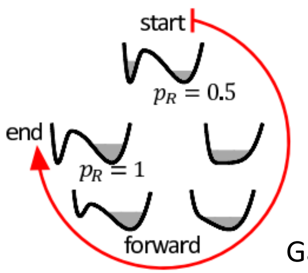
  Validation of $\left\langle e^{-\beta(W-\Delta F)-I} \right\rangle = 1$



Colloidal particle

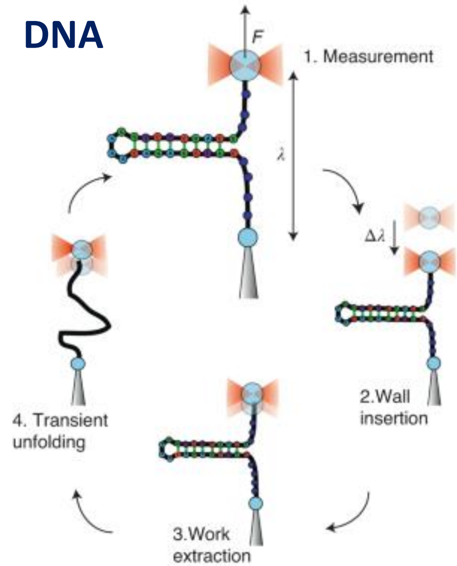


Bérut et al., Nature (2012)



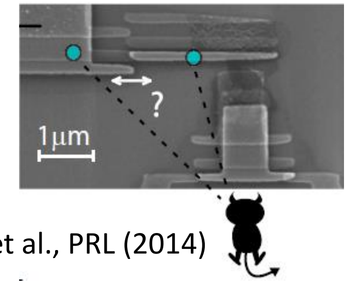
Gavrilov & Bechhoefer, PRL (2016)

DNA

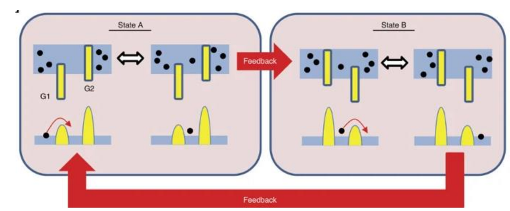


Ribezzi-Crivellari & Ritort, Nature Phys. (2019)

Single electron

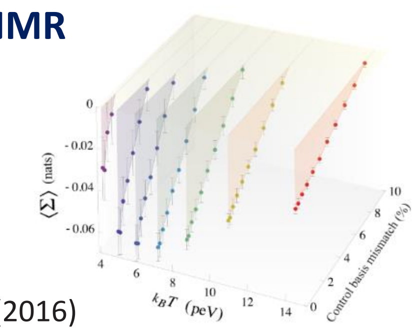


Koski et al., PRL (2014)



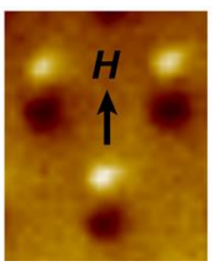
Chida et al., Nature Commu. (2017)

NMR



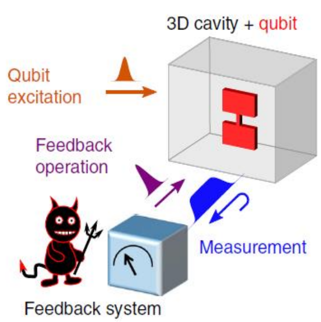
Camati et al., PRL (2016)

Nanomagnet



Hong et al., Science Adv. (2016)

Superconducting qubit

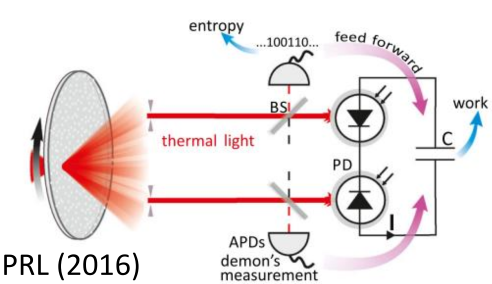


Masuyama et al., Nat. Commu. (2018)

Cottet et al., PNAS (2017)

Naghiloo et al., PRL (2018)

Photon



Vidrighin et al., PRL (2016)

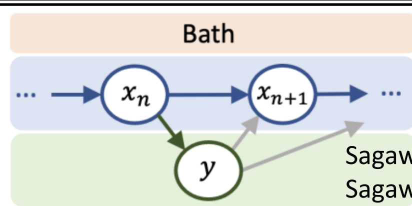
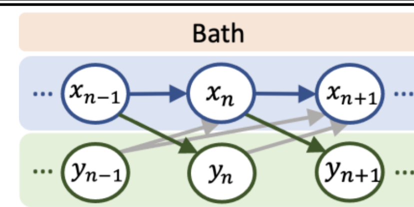
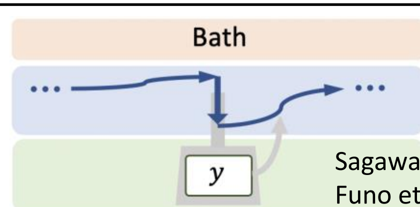
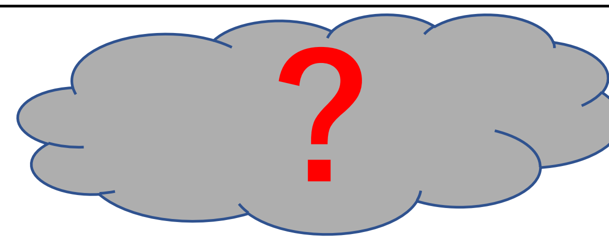
and more…

Second law under various feedback setups

The generalized SL: $\langle\sigma\rangle \geq -\langle\, i\,\rangle$

The generalized FT: $\left\langle e^{-\sigma - i}\right\rangle = 1$

i : Information obtained by measurement

	Single measurement and feedback	Continuous measurement and feedback
Classical system	 Sagawa&Ueda PRL 2010 Sagawa&Ueda PRL 2012 **i: mutual information**	 Sagawa&Ueda PRE 2012 Ito&Sagawa PRL 2013 **i: transfer entropy**
Quantum system	 Sagawa&Ueda PRL 2008 Funo et. al. PRE 2013 **i: QC-mutual information**	 **The present work**

T. Yada, N. Yoshioka, and T. Sagawa, Phys. Rev. Lett. **128**, 170601 (2022)

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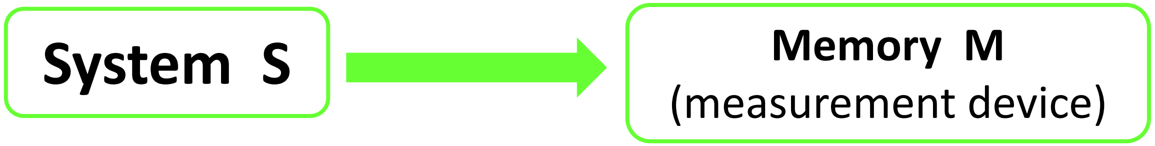
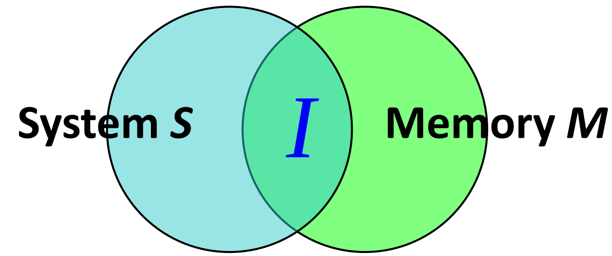
Mutual information



Measurement with stochastic errors



$$I(S:M) \equiv H(S) + H(M) - H(SM)$$

Shannon information $H = -\sum_k p_k \ln p_k$

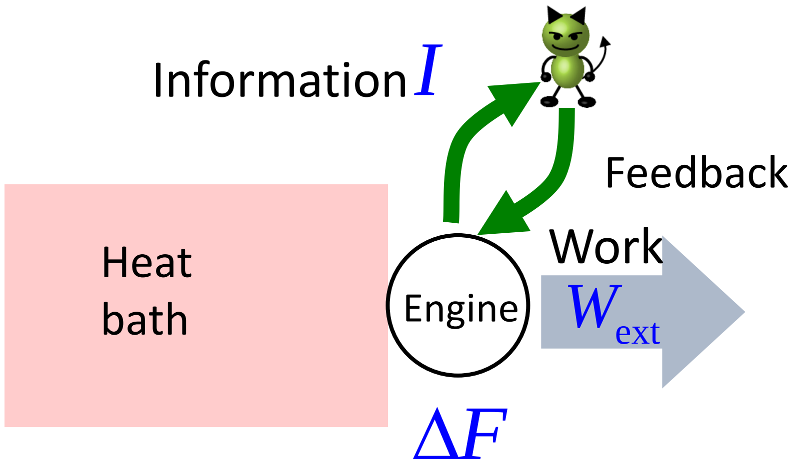
$$0 \leq I \leq H(M)$$

No information

No error

Correlation between S and M

Generalized second law with feedback



TS and M. Ueda, PRL **100**, 080403 (2008).
TS and M. Ueda, PRL **104**, 090602 (2010).

$$\Rightarrow \quad W_{\text{ext}} \leq -\Delta F + k_{\text{B}} T I$$

The upper bound of the work extracted by the demon is bounded by the mutual information.

The equality is achieved in the thermodynamically reversible limit

Generalized Jarzynski equality

With feedback control

$$\left\langle e^{-\beta(W-\Delta F)-I} \right\rangle = 1$$

TS and M. Ueda, PRL **104**, 090602 (2010)

Stochastic mutual information:

$$I(x:y) = \ln \frac{P(x,y)}{P(x)P(y)}$$

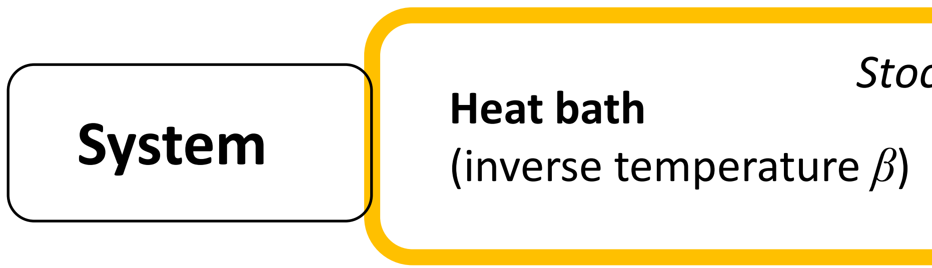
Mutual information:

$$\langle I \rangle = \sum_{xy} P(x,y) \ln \frac{P(x,y)}{P(x)P(y)}$$

Reproduce the generalized second law:

$$-W_{\mathrm{ext}} = \langle W \rangle \geq \Delta F - k_{\mathrm{B}} T \langle I \rangle$$

Entropy production



Stochastic heat: q

Stochastic Shannon entropy:

$$s(x) \equiv -\ln P(x)$$

x: system's state

$$\langle \dot{s} \rangle = \dot{S}$$

$$\langle \dot{q} \rangle = \dot{Q}$$

$$\langle \dot{\sigma} \rangle = \dot{\Sigma}$$

***Stochastic* entropy production rate:** $\dot{\sigma} \equiv \dot{s} - \beta \dot{q}$

Time integral: $\sigma \equiv \int \dot{\sigma} dt = \Delta s - \beta q$

Fluctuation theorem

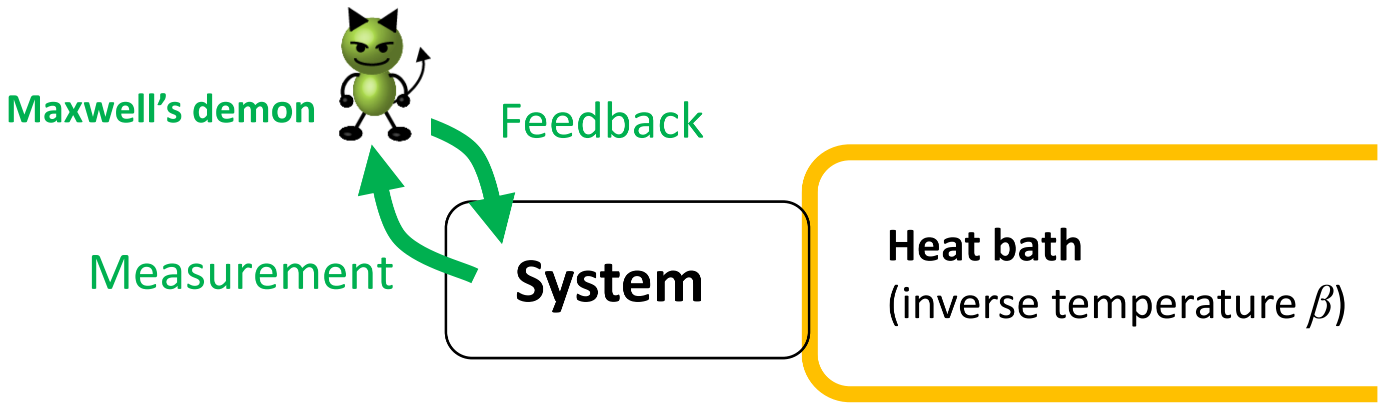
$$\langle \exp(-\sigma) \rangle = 1$$

Second law:

$$\langle \sigma \rangle \geq 0$$

Seifert, PRL (2005)

Generalized fluctuation theorem



Stochastic **mutual information:** $i(x:y) \equiv \ln \dfrac{P(x,y)}{P(x)P(y)}$

x: system's state
y: demon's outcome

Generalized fluctuation theorem: $\langle \exp(-\sigma - i) \rangle = 1$

TS and M. Ueda, PRL **104**, 090602 (2010)

Reproduce the generalized second law:

$$\Rightarrow \quad \langle \sigma \rangle \geq -\langle i \rangle \qquad \langle i \rangle = I$$

Two approaches to *continuous* information flow

- **"Transfer entropy"** approach
  - ✓ Applicable to non-Markovian dynamics
  - ✓ Second law is weaker in Markovian dynamics

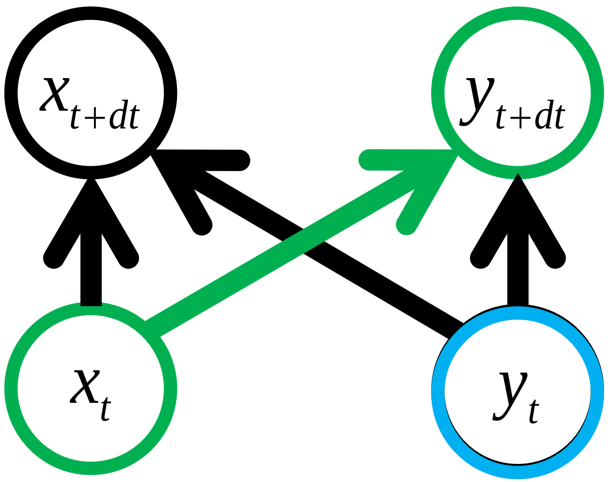
Sagawa & Ueda, Phys. Rev. E (2012)
Ito & Sagawa, Phys. Rev. Lett. (2013)

- **"Information flow"** approach
  - ✓ Not applicable to non-Markovian dynamics
  - ✓ Second law is stronger in Markovian dynamics

Horowitz & Esposito, Phys. Rev. X (2014)
Shiraishi & Sagawa, Phys. Rev. E (2015)

Transfer entropy

Directional information transfer between two systems



Transfer entropy:
Directional information transfer
from X to Y
during time t and $t + dt$

Conditional mutual information

$$\dot{T}_{X\to Y} \equiv \frac{1}{dt} I\left(X_t : Y_{t+dt} \mid \underline{Y_t}\right)$$

$$\equiv \frac{1}{dt} \sum_{x_t y_t y_{t+dt}} P(x_t, y_t, y_{t+dt}) \ln \frac{P(x_t, y_{t+dt} \mid y_t)}{P(x_t \mid y_t) P(y_{t+dt} \mid y_t)}$$

$$\geq 0$$

T. Schreiber, PRL **85**, 461 (2000)

Fluctuation theorem with transfer entropy

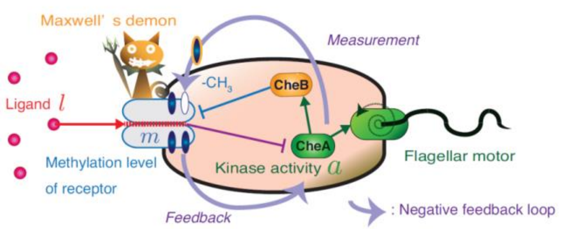
Stochastic **transfer entropy** $\dot{\tau}_{X \to Y} \equiv \frac{1}{dt} \ln \frac{p(x_t, y_{t+dt} \mid y_t)}{p(x_t \mid y_t) p(y_{t+dt} \mid y_t)}$ $\dot{T}_{X \to Y} = \langle \dot{\tau}_{X \to Y} \rangle$

Fluctuation theorem: $\langle \exp(-\sigma_X - \tau_{X \to Y}) \rangle = 1$

Second law: $\langle \sigma_X \rangle \geq -\langle \tau_{X \to Y} \rangle$

S. Ito & T. Sagawa, PRL **111**, 180603 (2013).

Application to biochemical signal transduction:
S. Ito & T. Sagawa, Nat. Commu. **6**, 7498 (2015)



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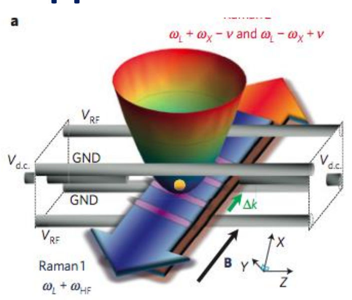
Quantum thermodynamics

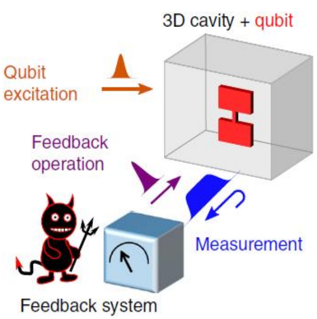
Various experimental platforms for quantum thermodynamics

Trapped ions



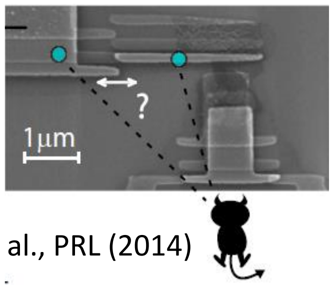
An et al., Nat. Phys. (2015)

Superconducting qubit



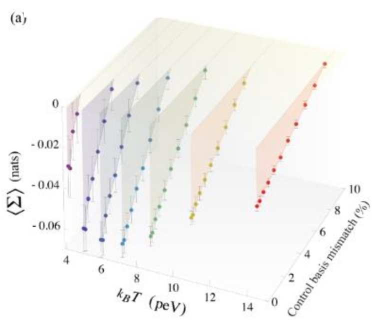
Masuyama et al.,
Nat. Commu. (2018)

Single electron



Koski et al., PRL (2014)

NMR



Camati et al., PRL (2016)

Thermodynamics with quantum measurement and feedback?

QC-mutual information

Information flow from **Q**uantum system to **C**lassical outcome by quantum measurement

$$I_{\mathrm{QC}} \equiv S(\rho) - \sum_k p_k S(\rho_k)$$

ρ : measured density operator

$p_k = \mathrm{tr}[\rho M_k^\dagger M_k]$: probability of obtaining outcome k

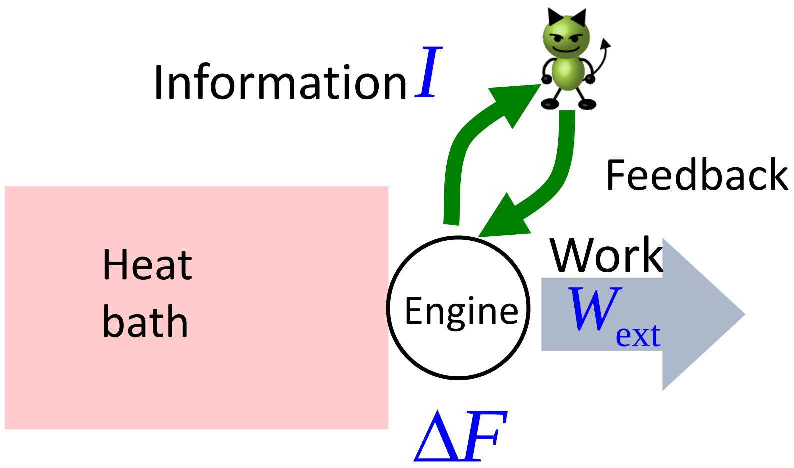
$\rho_k = \frac{1}{p_k} M_k \rho M_k^\dagger$: post-measurement state with outcome k

$$0 \leq I_{\mathrm{QC}} \leq H \qquad H = -\sum_k p_k \ln p_k$$

No information (left equality); Error-free & classical (right equality)

H. J. Groenewold, Int. J. Theor. Phys. **4**, 327 (1971).
M. Ozawa, J. Math. Phys. **27**, 759 (1986).
TS and M. Ueda, PRL **100**, 080403 (2008).

Generalized second law with *quantum* feedback



TS and M. Ueda, PRL **100**, 080403 (2008)

$$W_{\mathrm{ext}} \leq -\Delta F + k_{\mathrm{B}} T I_{\mathrm{QC}}$$

The upper bound of the work extracted by the demon is bounded by the **QC-mutual information**.

Generalized Quantum Jarzynski Equality

Initial state: $\rho_{\rm ini} = e^{\beta(F_{\rm ini} - H_{\rm ini})} = \sum_i e^{\beta(F_{\rm ini} - E^i_{\rm ini})} |\varphi_i\rangle\langle\varphi_i|$

Let $p(i) := e^{\beta(F_{\rm ini} - E^i_{\rm ini})}$

For simplicity, suppose that the measurement is performed on $\rho_{\rm ini}$ with Kraus operators $\{M_k\}$

Post-measurement state with outcome k: $\rho_k = \frac{M_k \rho_{\rm ini} M_k^\dagger}{p_k} = \sum_{i'} p(i'|k) |\varphi^k_{i'}\rangle\langle\varphi^k_{i'}|$

Apply k-dependent unitary (feedback) U_k

Stochastic QC-mutual information: $i_{\rm QC} \coloneqq \ln p(i'|k) - \ln p(i)$

➡ The ensemble average equals the QC-mutual information: $\langle i_{\rm QC} \rangle = I_{\rm QC}$

Generalized quantum Jarzynski equality: $\left\langle e^{-\beta(W - \Delta F) - i_{\rm QC}} \right\rangle = 1$

K. Funo, Y. Watanabe, & M. Ueda, PRE **88**, 052121 (2013)

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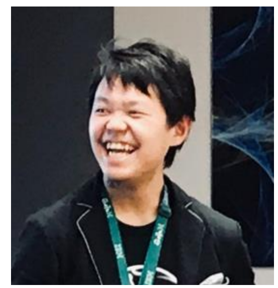
Collaborators

Toshihiro
Yada



Nobuyuki
Yoshioka

T. Yada, N. Yoshioka, and T. Sagawa,
Phys. Rev. Lett. **128**, 170601 (2022)

EDITORS' SUGGESTION

Quantum Fluctuation Theorem under Quantum Jumps with Continuous Measurement and Feedback

A generalized fluctuation theorem is derived for quantum systems undergoing continuous measurement and feedback.

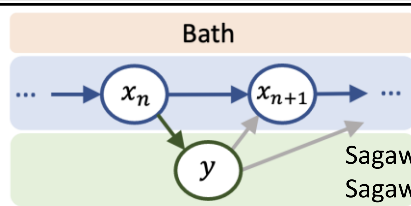
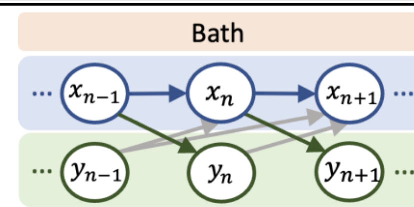
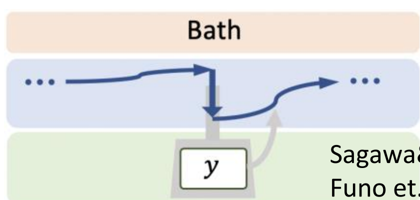
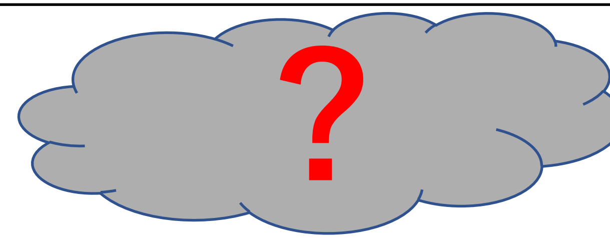
Toshihiro Yada, Nobuyuki Yoshioka, and Takahiro Sagawa
Phys. Rev. Lett. **128**, 170601 (2022)

What was missing?

The generalized SL: $\langle \sigma \rangle \geq -\langle\, i \,\rangle$

The generalized FT: $\left\langle e^{-\sigma - i} \right\rangle = 1$

***i* : Information obtained by measurement**

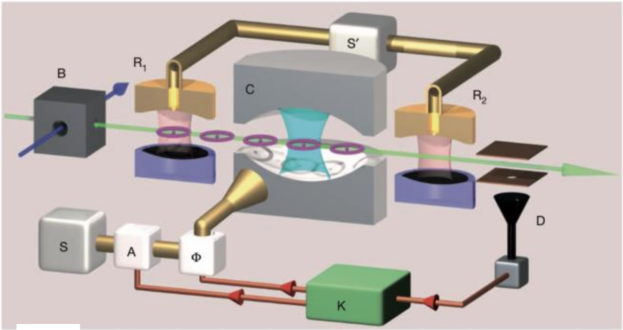
	Single measurement and feedback	Continuous measurement and feedback
Classical system	 Sagawa&Ueda PRL 2010 Sagawa&Ueda PRL 2012 **i: mutual information**	 Sagawa&Ueda PRE 2012 Ito&Sagawa PRL 2013 **i: transfer entropy**
Quantum system	 Sagawa&Ueda PRL 2008 Funo et. al. PRE 2013 **i: QC-mutual information**	 **The present work**

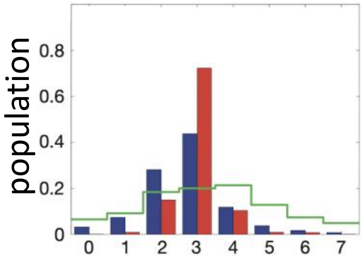
T. Yada, N. Yoshioka, and T. Sagawa, Phys. Rev. Lett. **128**, 170601 (2022)

Quantum continuous feedback control

Cavity QED

C. Sayrin et. al.,
Nature **477**, 73-77, (2011)





Prepare the photon number state $|n = 3\rangle$

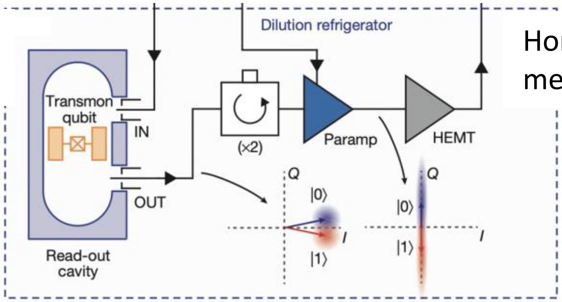
- initial state
- State at fixed time
- Successfully feedback-controlled state

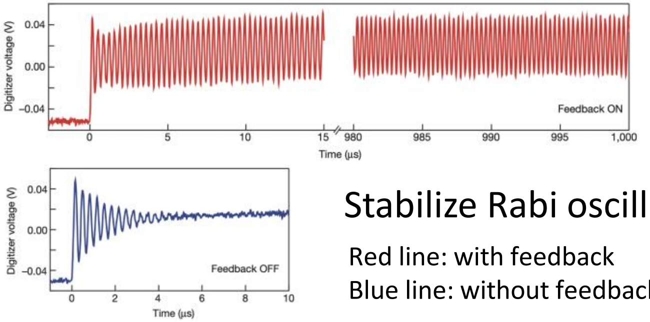
Circuit QED

R. Vijay et. al.,
Nature **490**, 77-80 (2012)

Feedback signal

Homodyne measurement





Stabilize Rabi oscillation

Red line: with feedback
Blue line: without feedback

Prepare and stabilize desired quantum states.

Summary of our main results

T. Yada, N. Yoshioka, and T. Sagawa, Phys. Rev. Lett. **128**, 170601 (2022)

The generalized SL

$$\langle \sigma \rangle \geq -\langle i_{\mathrm{QC}} \rangle$$

Measurement & feedback / System	Single	Continuous
Classical	mutual information	transfer entropy
Quantum	QC-mutual information	**QC-transfer entropy**

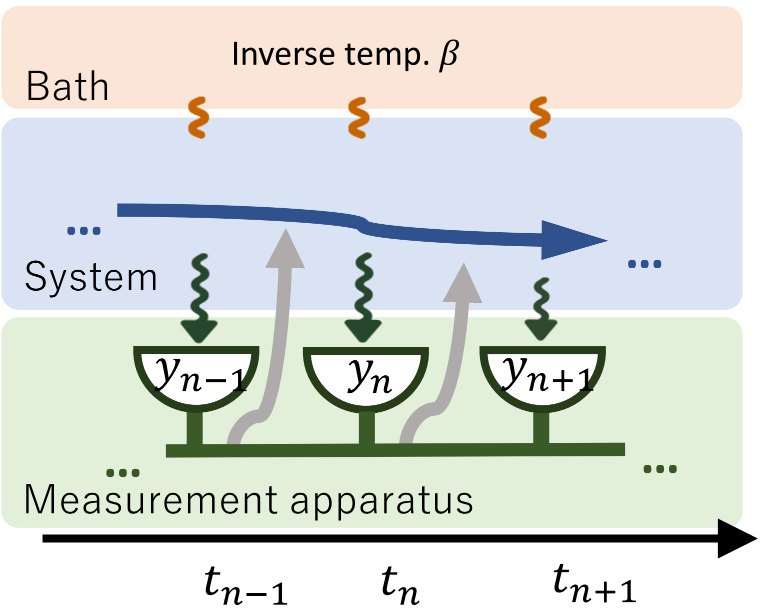
The generalized FT

$$\left\langle e^{-\sigma - i_{\mathrm{QC}}} \right\rangle = 1$$

$\langle i_{\mathrm{QC}} \rangle$: QC-transfer entropy

- **Newly introduced**
- **Total information transfer by continuous measurement**

Setup



Time discretization $\quad t_n \equiv n \cdot \Delta t$

Take continuous time limit $\Delta t \to 0, \tau$ const.

Stochastic master equation

$$\rho_{t_{n+1}}^{Y_{n+1}} = \rho_{t_n}^{Y_n} + \Delta t \Big\{ -i[H_{t_n} + h_{t_n}, \rho_{t_n}^{Y_n}] + \sum_d \mathcal{D}[L_d]\rho_{t_n}^{Y_n}$$
$$+ \sum_y -\frac{1}{2}\{M_y^\dagger M_y, \rho_{t_n}^{Y_n}\} + \mathrm{Tr}[M_y \rho_{t_n}^{Y_n} M_y^\dagger]\rho_{t_n}^{Y_n} \Big\} + \sum_y \Delta N_y \mathcal{G}[M_y]\rho_{t_n}^{Y_n}$$

$$\left(\mathcal{D}[c]\rho \equiv c\rho c^\dagger - \frac{1}{2}\{c^\dagger c, \rho\},\ \mathcal{G}[c]\rho \equiv \frac{c\rho c^\dagger}{\mathrm{Tr}[c\rho c^\dagger]} - \rho \right)$$

Measurement outcome

y_n: Measurement result at t_n

$\mathrm{Y_n}$: Result until t_n (i.e., $(y_1, y_2, \dots, y_n)$)

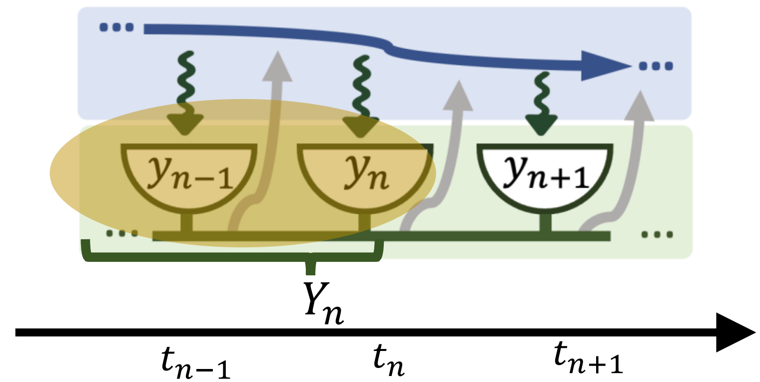
Continuous feedback

Change Hamiltonian $H_{t_n} + h_{t_n}$ according to Y_n

H_{t_n}: System Hamiltonian $\quad h_{t_n}$: External driving

QC-transfer entropy

$$\langle i_{\mathrm{QC}} \rangle \equiv \sum_{n=0}^{N-1} \sum_{Y_n} P[Y_n] \mathcal{J}_{\mathrm{QC}}(\rho_{t_n}^{Y_n} : y_{n+1})$$



- QC-mutual information quantifies the information obtained by the measurement in $[t_n, t_{n+1})$

  $\mathcal{J}_{\mathrm{QC}}(\rho_{t_n} : y) \equiv S(\rho_{t_n}) - \sum_{y=1}^{m} p_y S(\rho_{t_{n+1}}^{y})$

- Conditioned on the past measurement outcomes Y_n

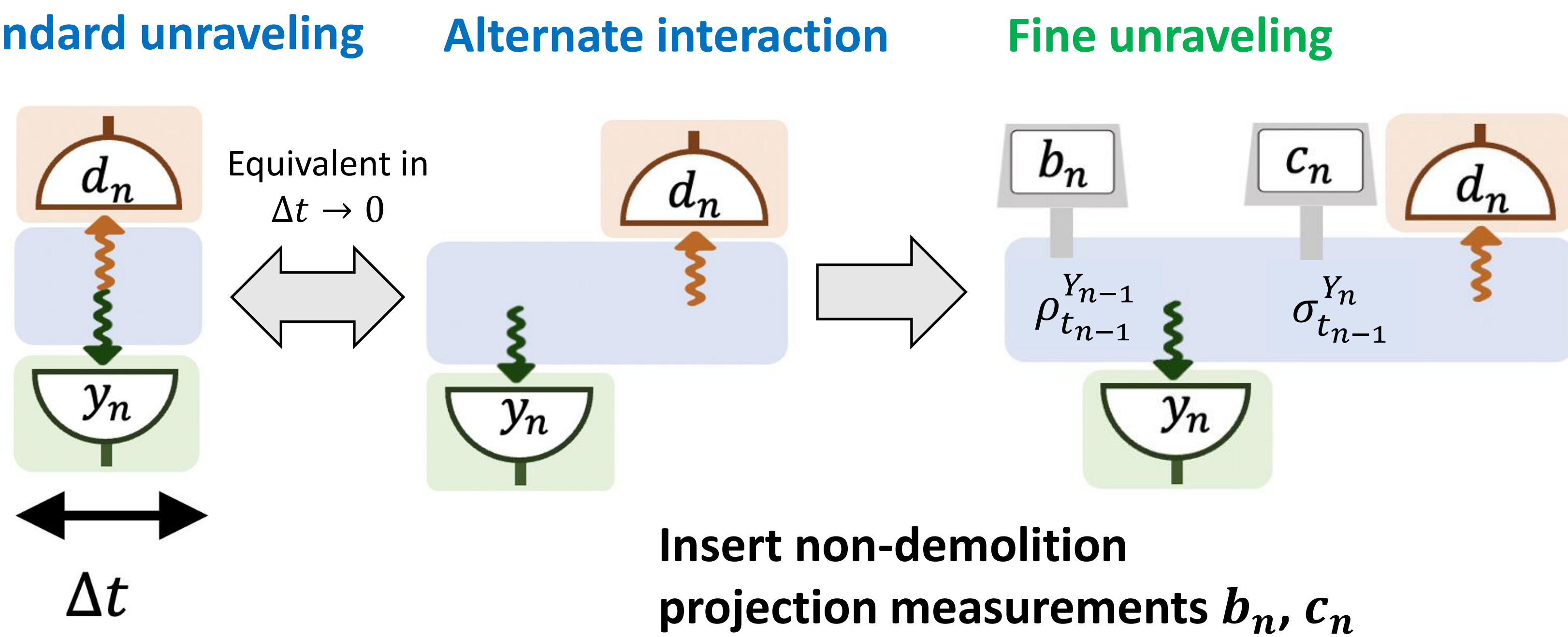
cf. **transfer entropy**

$\langle i_{\mathrm{TE}} \rangle \equiv \sum_{n=1}^{N} I(x_n, y_{n+1} | Y_n)$

System \ Measurement & feedback	Single	Continuous
Classical	mutual information	transfer entropy
Quantum	QC-mutual information	**QC-transfer entropy**

Conditional accumulation

Fine unraveling



$$\rho_{t_{n-1}}^{Y_{n-1}} \equiv \sum_{b_n} p^{Y_{n-1}}(b_n)\, |b_n\rangle\langle b_n|$$
$$\sigma_{t_{n-1}}^{Y_n} \equiv \sum_{c_n} p^{Y_n}(c_n)\, |c_n\rangle\langle c_n|$$

do not destroy the state at the ensemble level

Stochastic QC-transfer entropy:

$$i_{\mathrm{QC}} \equiv \sum_{n=1}^{N} - \ln p^{Y_{n-1}}(b_n) + \ln p^{Y_n}(c_n)$$

Experiment-numerics hybrid method

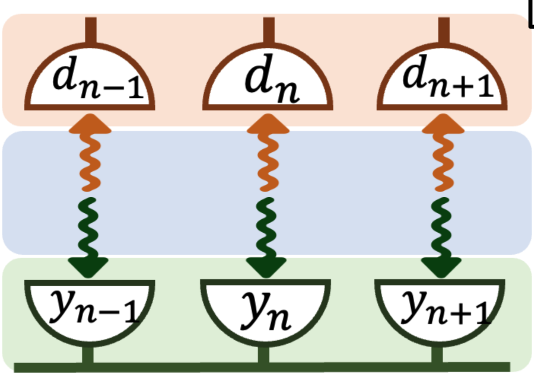
Verification of the generalized FT $\langle e^{-\sigma - i_{\mathrm{QC}}} \rangle = 1$ in real experiments

✕ Direct sampling of the fine trajectory

○ Sample the standard trajectories & Perform auxiliary numerical calculation

Standard trajectory ψ_τ

Measurement outcomes $\{y_n\}_{n=1}^N$,
Heat-bath dissipation $\{d_n\}_{n=1}^N$

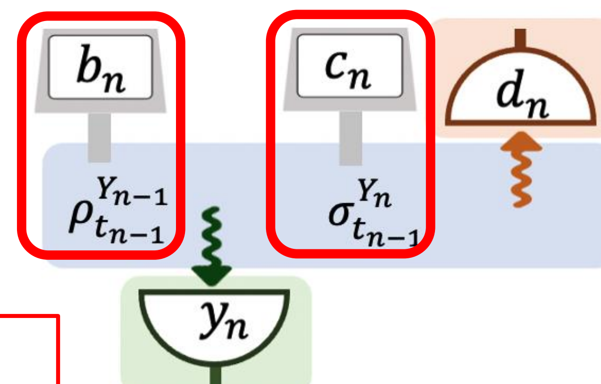


Experimental sampling ψ_τ

Numerically insert PMs π_τ

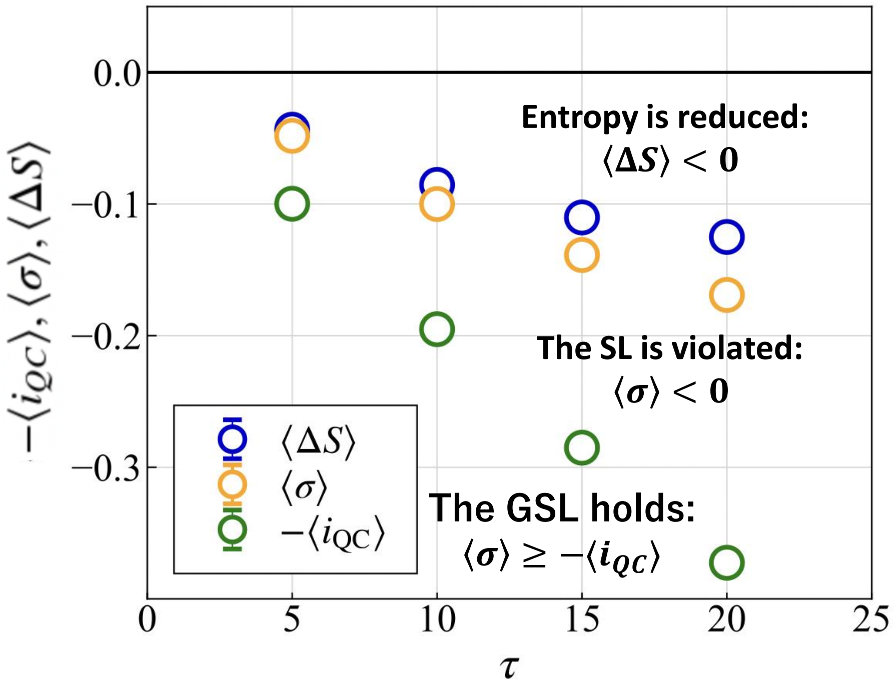
Fine trajectory (ψ_τ, π_τ)

Inserted projection measurements
$\pi_\tau \equiv \{b_n, c_n\}_{n=1}^N$

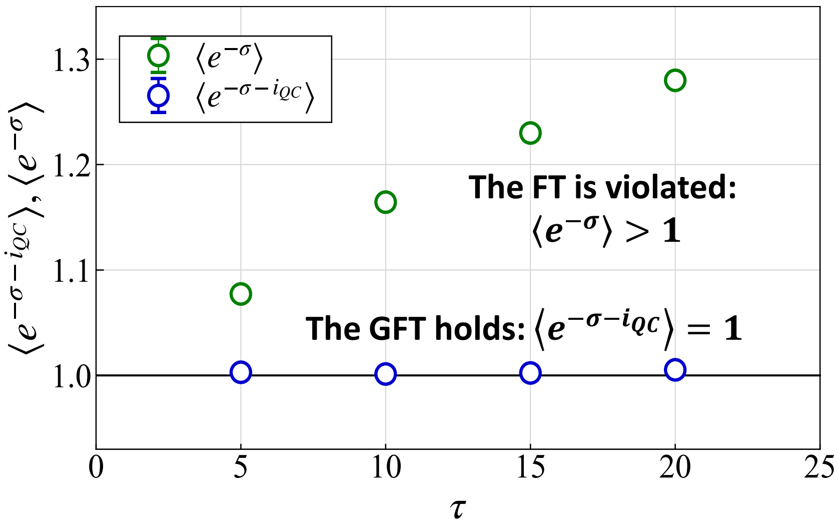


Numerical demonstration

Generalized second law (GSL)



Generalized fluctuation theorem (GFT)



(Error bars are hidden by the plots.)

Outline

- Introduction
- Generalized second laws: Classical
- Generalized second laws: Quantum
- Main result: Continuous quantum feedback
- **Summary and references**

Summary

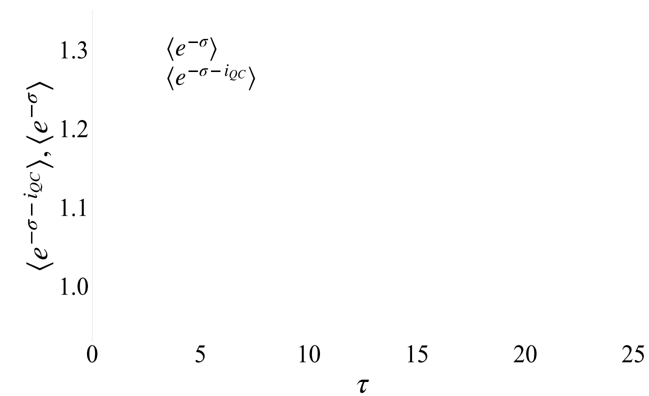
In quantum systems under continuous measurement and feedback, we have

- generalized the SL and FT
- introduced the QC-transfer entropy
- proposed the experiment-numerics hybrid method

$$\langle \sigma \rangle \geq -\langle i_{\mathrm{QC}} \rangle \qquad \left\langle e^{-\sigma - i_{\mathrm{QC}}} \right\rangle = 1$$

System \ Measurement & feedback	Single		Continuous
Classical	**mutual information**	→	**transfer entropy**
Quantum	**QC-mutual information**	→	**QC-transfer entropy**

Conditional accumulation



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Main result of this talk:

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