KEK連携研究会
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東京大学
THE UNIVERSITY OF TOKYO

冷却原子気体における開放量子多体系の物理

中川 大也
(東京大学)

MN, N. Tsuji, N. Kawakami, and M. Ueda, PRL 124, 147203 (2020)

MN, N. Kawakami, and M. Ueda, PRL 126, 110404 (2021)

中川, 辻, 川上, 上田, 日本物理学会誌 第77巻2号, p88 (2022)

K. Honda, S. Taie, Y. Takasu, N. Nishizawa, MN, and Y. Takahashi, PRL 130, 063001 (2023)

T. Haga, MN, R. Hamazaki, and M. Ueda, arXiv:2211.14991

Outline

1. Introduction

    - Dissipation in cold atoms: open quantum many-body systems

    - Experimental advance & theoretical description

2. Dissipative Hubbard model: magnetism & exact solution

    [MN, N. Tsuji, N. Kawakami, and M. Ueda, PRL 124, 147203 (2020)]

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    [K. Honda *et al*., PRL 130, 063001 (2023)]

3. Incoherenton: quasiparticles of decoherence processes

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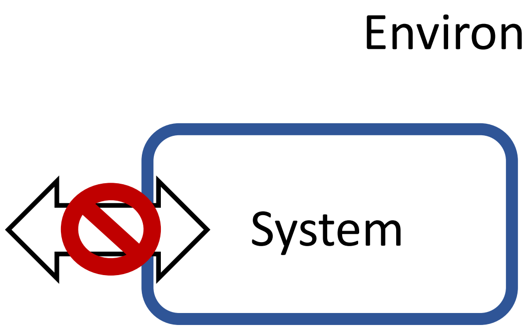
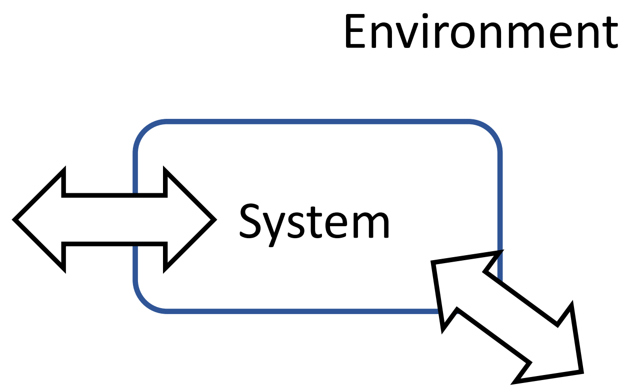
4. Summary

Open quantum system

Isolated quantum system **vs.** **Open quantum system**





- ◇ Unitary dynamics
- ◇ Pure state remains pure
- ◇ Schrödinger equation

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$$

- ◇ Non-unitary dynamics (**dissipation**)
- ◇ Mixed state: density matrix
- ◇ e.g., Quantum master equation

$$\frac{d\rho(t)}{dt} = \mathcal{L}\rho(t)$$

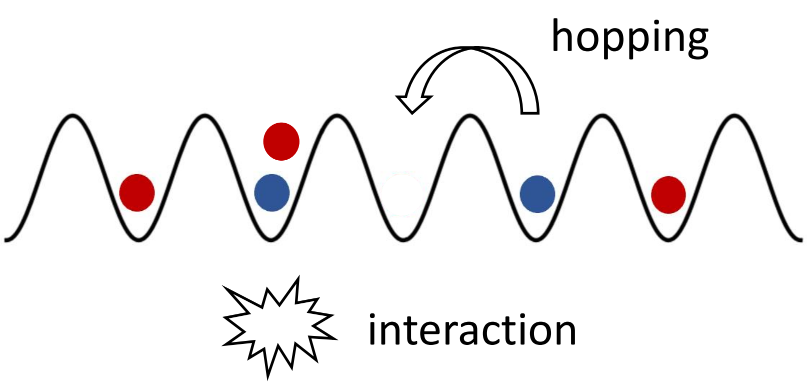
Ultracold atoms

Ultracold atoms

- ✓ Neutral atoms trapped in vacuum
- ✓ Dilute gas (density ∼10^{13} cm^{-3})
- ✓ Temperature ∼10 – 100 nK
- ✓ Short-range interaction between atoms
  (Long-range one can also be engineered)

[From webpage of I. Bloch's group]
https://www.quantum-munich.de/

Optical lattice potential



"Quantum simulation"
Atoms can simulate electrons in solids!

Ultracold atoms: advantages

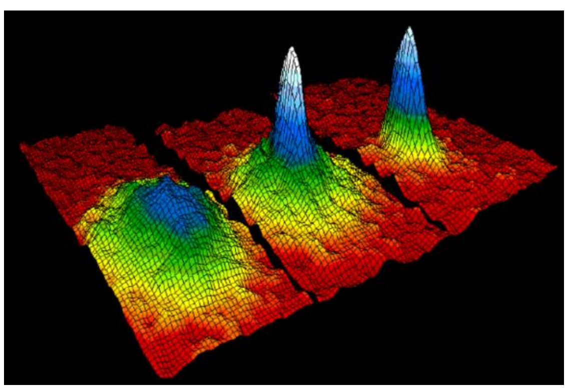
Advantage 1: Controllability

◇ Quantum statistics (bosons, fermions) ◇ Dimensionality (by trap potential)

◇ Interactions (repulsive or attractive: controlled by a Feshbach resonance)

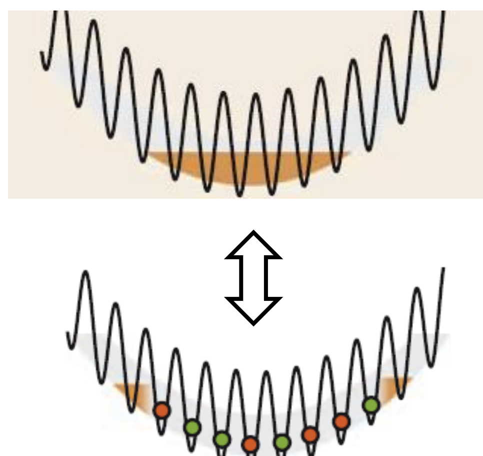
Atoms can behave as...

Superfluid



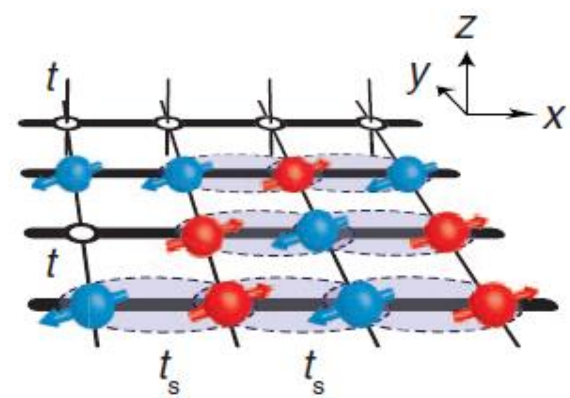
[BEC: Nobel prize (2001)]

Metal/Insulator



[Schneider *et al*., Science 322, 1520 (2008)]

Magnet



[Greif *et al*., Science 340, 1307 (2013)]

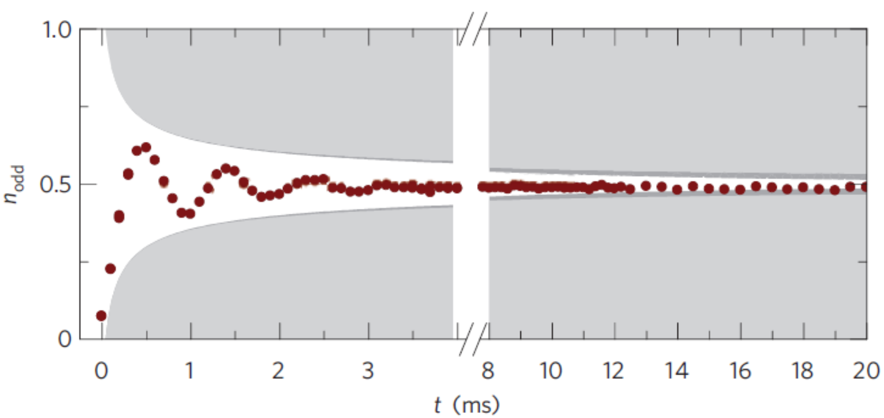
Ultracold atoms: advantages

Advantage 2: Resolution

Energy scale ∼ 100 nK ∼ 1 kHz

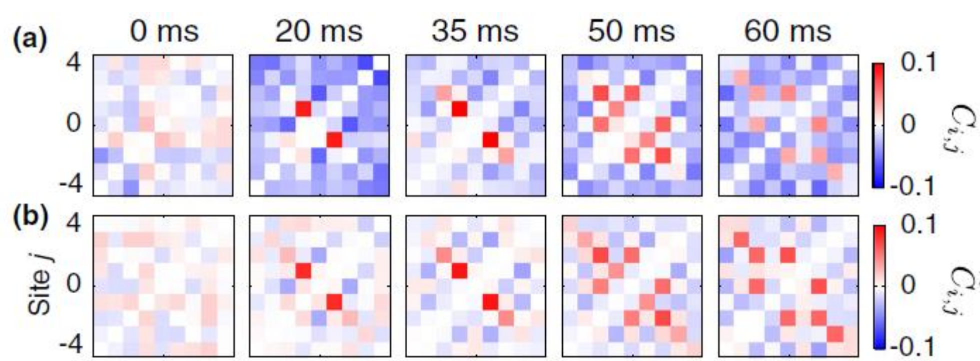
→ Time scale ∼ 1 msec

Real-time observation of quantum many-body dynamics



Thermalization dynamics

[Trotzky *et al.*, Nat. Phys. 8, 325 (2012)]

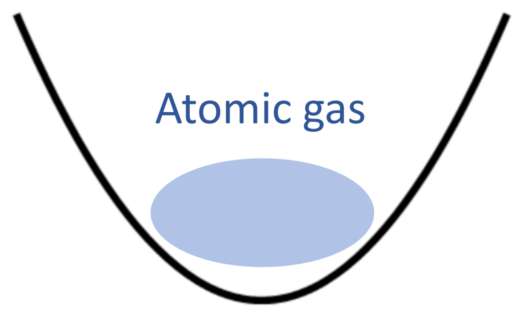


Propagation of
correlation & entanglement

[Fukuhara *et al.*, PRL 115, 035302 (2015)]

Dissipation in ultracold atoms?

Ultracold atoms → trapped in vacuum



Well isolated from environment!

Dissipation?

Dissipation in ultracold atoms

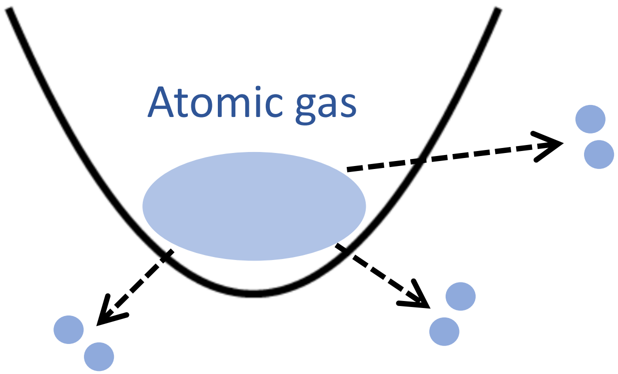
■ Examples of dissipation

✓ **Inelastic collision & atom loss**

e.g., collision between atoms in an excited (*e*) state (*g*: ground state)

$e + e \to g + g +$ large kinetic energy

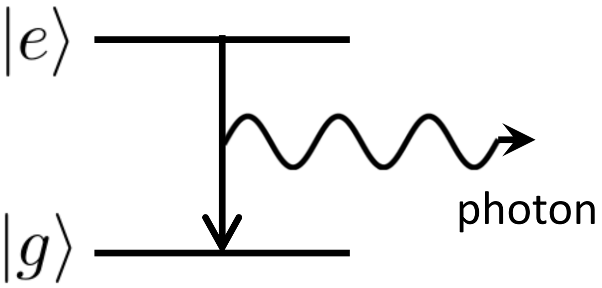
⇨ loss of atoms from trap



✓ **Spontaneous emission**

Decay from an excited state of an atom

by emitting a photon



Control of dissipation

Experimental development 1: <u>control</u> of dissipation

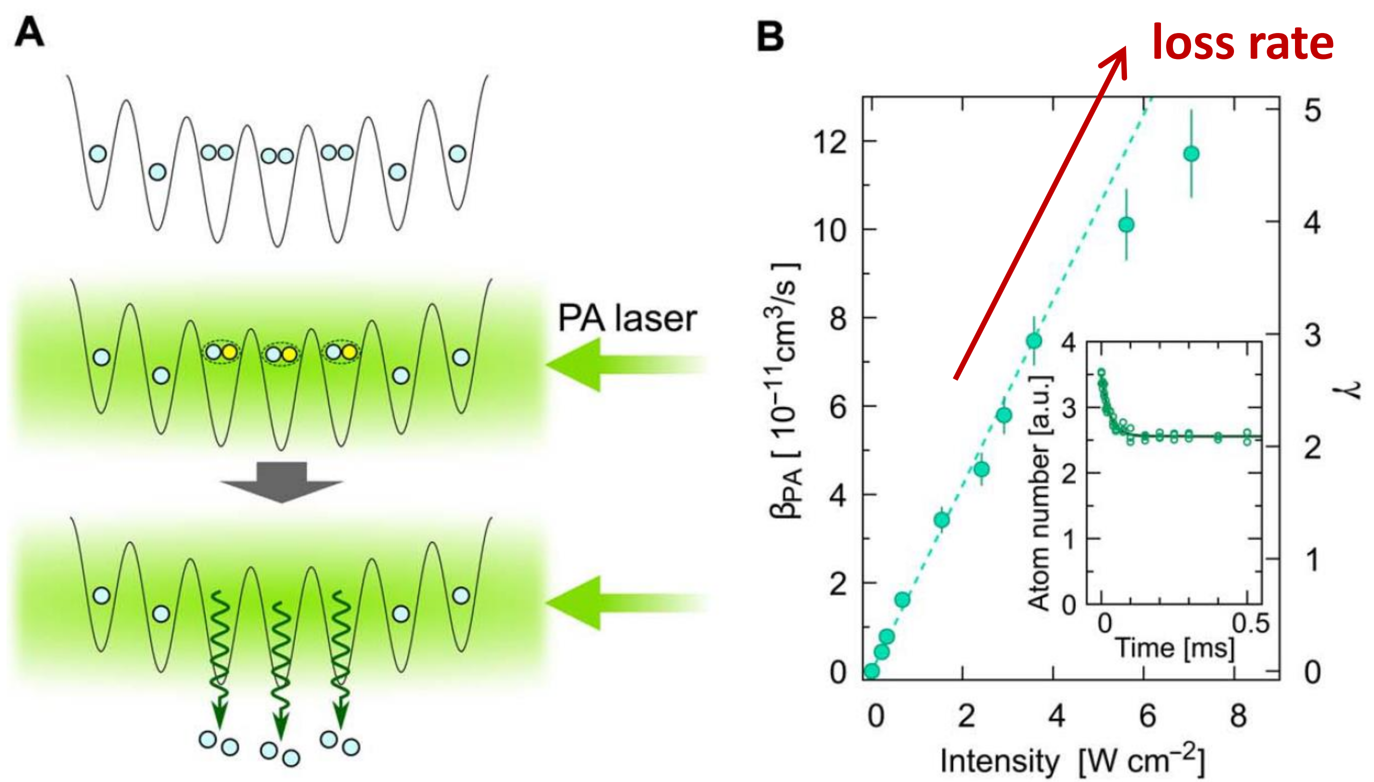


Photo-induced inelastic collisions

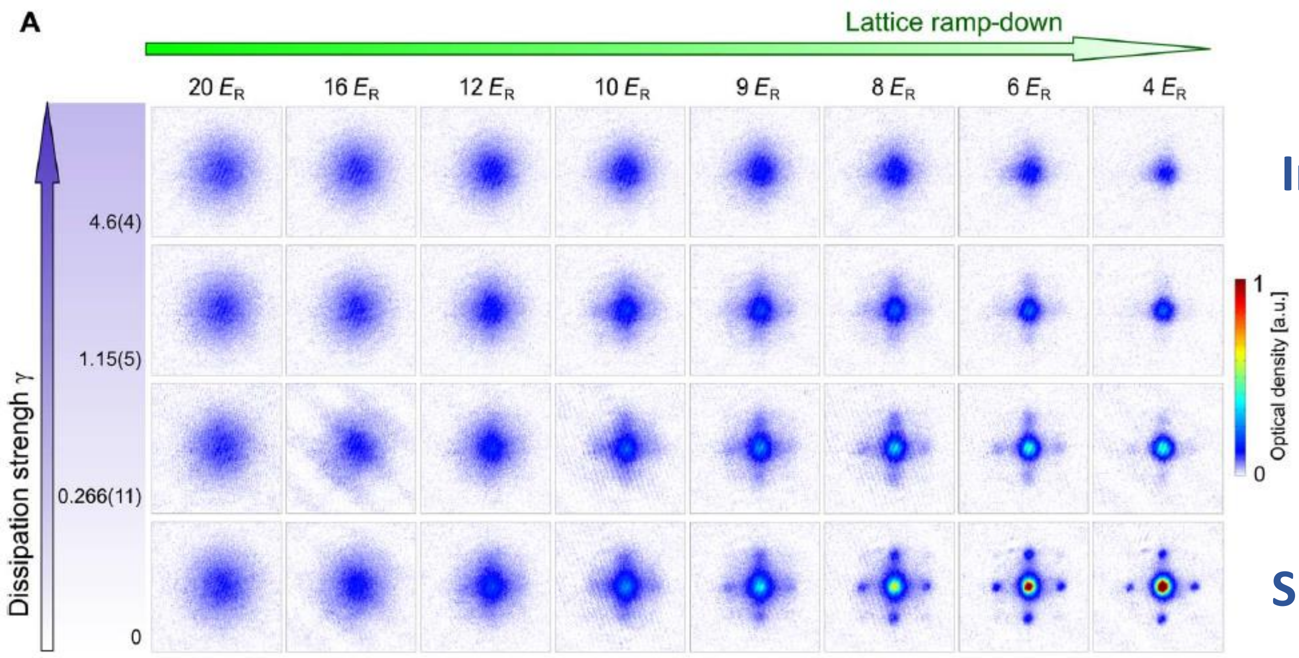
→ Controllable dissipation

[Tomita *et al.*, Sci. Adv. 3, e1701513 (2017)]

Quantum many-body physics meets dissipation

Experimental development 2: <u>many-body</u> systems

Increasing dissipation



Insulator

Superfluid

Insulator

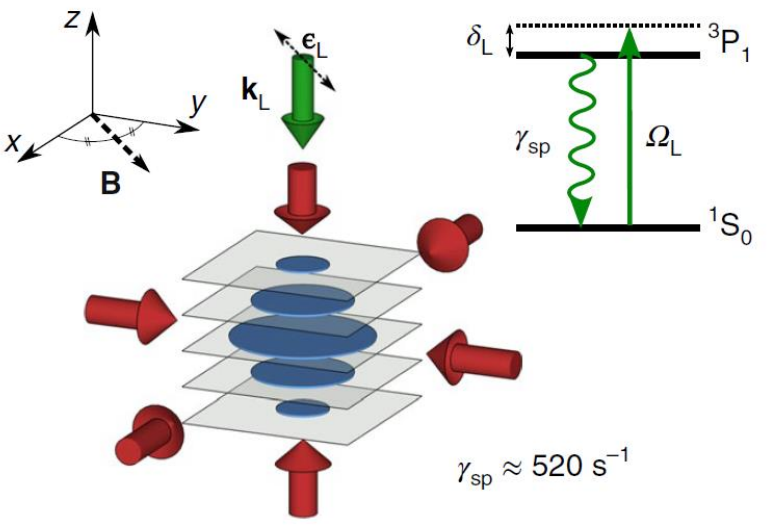
[Tomita *et al*., Sci. Adv. 3, e1701513 (2017)]

Effect of dissipation on

superfluid-insulator quantum phase transition

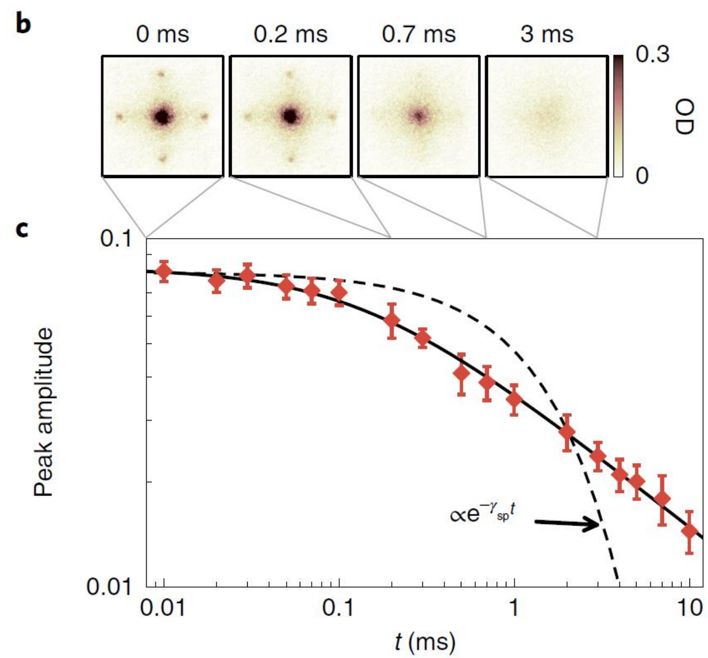
Controllable open quantum many-body systems

Control of dissipation & application to many-body systems



Bose-Hubbard system + laser excitation & spontaneous emission

[Bouganne *et al*., Nat. Phys. 16, 21 (2020)]



Power-law decoherence due to interaction

Large-scale, controllable, and strongly interacting open quantum many-body systems!

Motivation

Ultracold atoms

→ Controllable non-unitary dynamics of many-body systems!

Fundamental questions?

✓ **How to understand dissipative processes in many-body systems?**

e.g., How decoherence proceeds in many-body systems?

How a large-scale quantum computer operates under noise?

✓ **Non-equilibrium quantum phases triggered by dissipation?**

e.g., laser, time crystals, analogy with classical open systems,

magnetism + dissipation, superfluidity + dissipation, …

Toward quantum many-body physics of open systems

Theoretical description

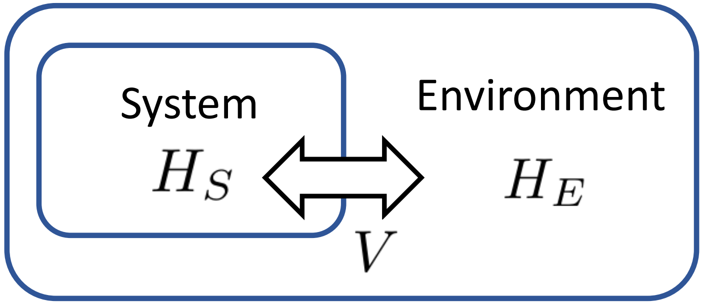
Markovianity of dissipative cold atoms

■ System + Environment Hamiltonian

$$H_{\text{tot}} = H_S + H_E + V$$

System: slow dynamics

Environment: fast relaxation



■ Trace out the environment → effective dynamics of the system

✓ Environment = atomic/photonic modes in the surrounding vacuum

✓ Separation of time scales

Atomic gas → ∼ 10^{-3} sec (energy scale ∼ 100 nK ∼ 1 kHz)

Environment → ∼ 10^{-14} sec

[= inverse of internal energy of atoms (∼ 10^{14} Hz: visible light)]

Environment immediately relaxes! No memory effect

Ultracold atoms: Markovian open system

Quantum master equation

■ General form of time-evolution eq. for Markovian quantum dynamics

[Gorini, Kossakowski, and Sudarshan, J. Math. Phys. 17, 821 (1976)]

[Lindblad, Commun. Math. Phys. 48, 119 (1976)]

Assumption: $$\frac{d\rho(t)}{dt} = \mathcal{L}\rho(t)$$

- ✓ $\rho(t)$: should be density matrix at any t (i.e., time evolution gives a CPTP map)
- ✓ RHS only depends on ρ at time t (No memory effect: **Markovian**)

⇩

Quantum master equation in GKSL (or Lindblad) form

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H,\rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right)$$

Hamiltonian part

Dissipation (L_α: Lindblad operator)

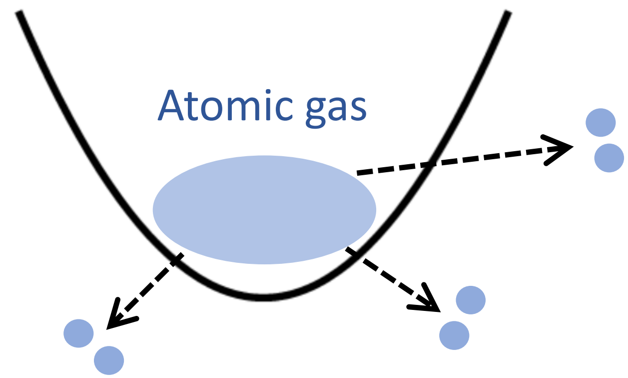
Example of Lindblad operators

■ Dissipation in ultracold atoms

✓ *k*-body loss (inelastic collision: *k* = 2)

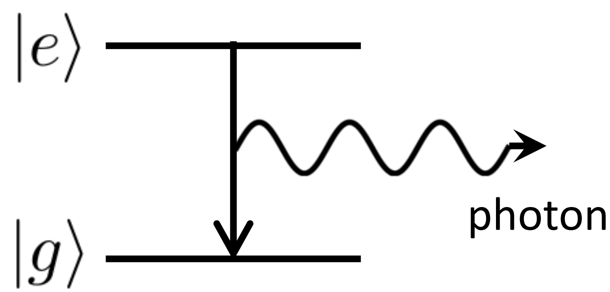
$$L_j = \sqrt{\gamma}(\psi_j)^k$$

Sec. 2



✓ Spontaneous emission

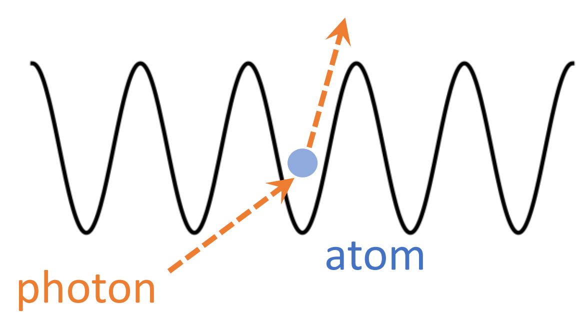
$$L_j = \sqrt{\gamma}\psi_j^{(g)\dagger}\psi_j^{(e)}$$



✓ Photon scattering → dephasing

$$L_j = \sqrt{\gamma}\psi_j^\dagger\psi_j$$

Sec. 3



Liouvillian spectrum

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_\alpha \left(L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}\right) \equiv \mathcal{L}\rho$$

Liouvillian superoperator (non-Hermitian)

■ Diagonalization of a Liouvillian

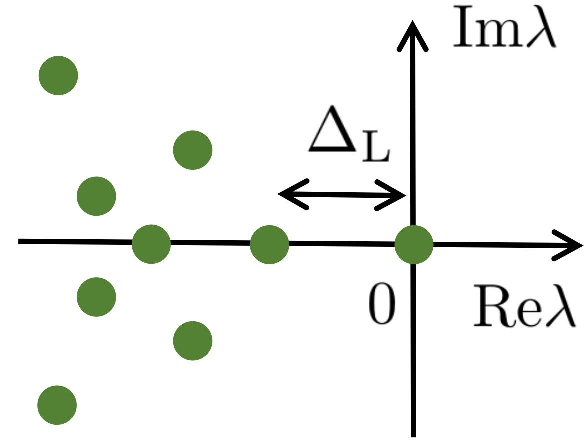
$$\mathcal{L}\rho_n = \lambda_n \rho_n \Rightarrow \rho(t) = \sum_n c_n e^{\lambda_n t} \rho_n$$

$\{\lambda_n\}_n$ **: Liouvillian spectrum**

■ General properties

- $\lambda_n \in \mathbb{C}$ **(non-Hermiticity)**, $\mathrm{Re}\lambda_n \leq 0$
- $\lambda_0 = 0$ **: steady state** $\rho(t \to \infty) = c_0 \rho_0$
- $\Delta_\mathrm{L} \equiv \min_{\lambda_n \neq 0}[-\mathrm{Re}\lambda_n]$ **: Liouvillian gap**

  → inverse of the relaxation time



[cf. Exceptional cases: Haga, MN *et al.*, PRL 127, 070402; Mori-Shirai, PRL 125, 230604]

Liouvillian spectrum

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_\alpha (L_\alpha \rho L_\alpha^\dagger - \frac{1}{2}\{L_\alpha^\dagger L_\alpha, \rho\}) \equiv \mathcal{L}\rho$$

Liouvillian superoperator (non-Hermitian)

■ Diagonalization of a Liouvillian

$$\mathcal{L}\rho_n = \lambda_n \rho_n \quad \Rightarrow \quad \rho(t) = \sum_n c_n e^{\lambda_n t} \rho_n$$

$\{\lambda_n\}_n$ **: Liouvillian spectrum**

Question:

How to diagonalize a Liouvillian of a many-body system?

Next: solvable examples

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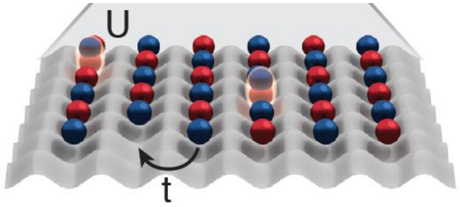
   [T. Haga, MN, R. Hamazaki, and M. Ueda, arXiv:2211.14991]

4. Summary

Hubbard model

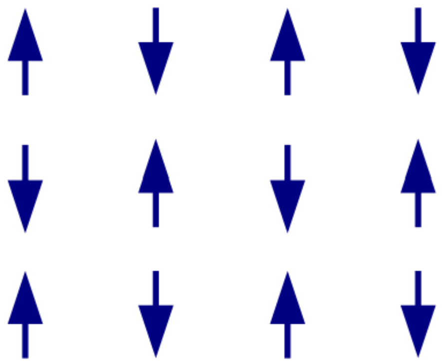
■ Fermi-Hubbard model: a prototypical example of many-body systems

$$H = -t \sum_{\langle i,j \rangle} \sum_{\sigma=\uparrow,\downarrow} (c^\dagger_{i,\sigma} c_{j,\sigma} + \mathrm{H.c.}) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

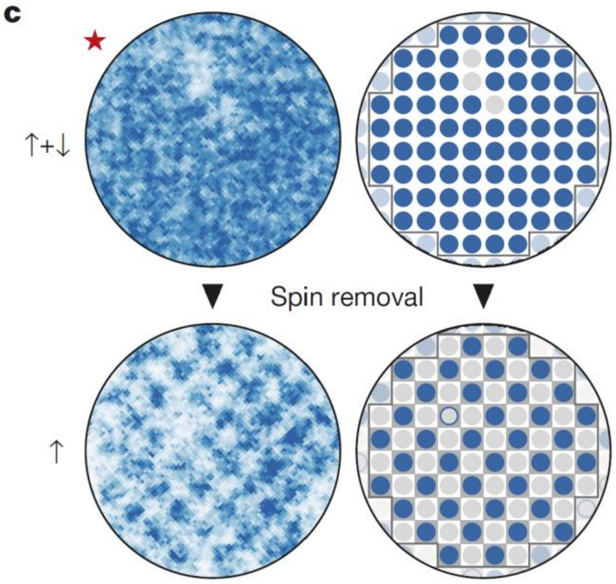


Hopping **Interaction** $(n_{j,\sigma} \equiv c^\dagger_{j,\sigma} c_{j,\sigma})$

■ Ultracold atoms → e.g., observation of magnetism of Mott insulators



Antiferromagnetic order



Direct observation by quantum-gas microscopy

[Mazurenko *et al.*, Nature 545, 462 (2017)]

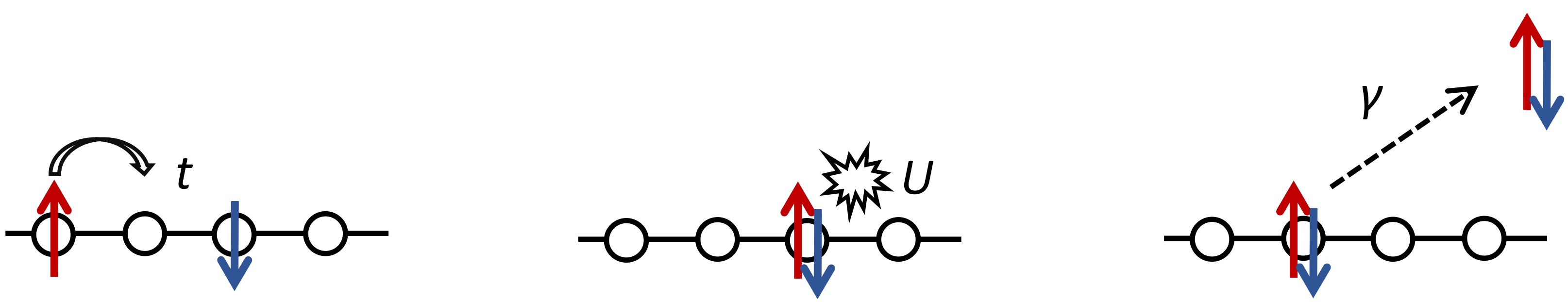
Setup

■ Model: 1D dissipative Fermi-Hubbard model

$$\frac{d\rho}{d\tau} = -i[H,\rho] + \sum_{j=1}^{L} (L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\})$$ quantum master eq.

$$H = -t \sum_{j=1}^{L} \sum_{\sigma=\uparrow,\downarrow} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \mathrm{H.c.}) + U \sum_{j=1}^{L} n_{j,\uparrow} n_{j,\downarrow},$$ Hubbard Hamiltonian

$$L_j = \sqrt{2\gamma} c_{j,\downarrow} c_{j,\uparrow}$$ 2-body loss at each site (inelastic collision)



hopping **interaction** **dissipation**

Setup

■ Model: 1D dissipative Fermi-Hubbard model

$$\frac{d\rho}{d\tau} = -i[H,\rho] + \sum_{j=1}^{L}(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\})$$ quantum master eq.

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$$L_j = \sqrt{2\gamma}c_{j,\downarrow}c_{j,\uparrow}$$ 2-body loss at each site (inelastic collision)

Liouvillian of the 1D dissipative Hubbard model

→ exactly solvable!

[MN, N. Kawakami, and M. Ueda, PRL 126, 110404 (2021)]

Solution of the Liouvillian

■ Step 1: Decomposition of the Liouvillian

$$\mathcal{L}\rho = -i[H,\rho] + \sum_j (L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\})$$

$$= -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger) + \sum_j L_j \rho L_j^\dagger$$

"Quantum jump" term
→ decreases particle #

Non-Hermitian Hubbard model

$$H_{\mathrm{eff}} = -t \sum_{j,\sigma} (c_{j,\sigma}^\dagger c_{j+1,\sigma} + \mathrm{H.c.}) + (U - i\gamma) \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

■ Step 2: Diagonalize the non-Hermitian Hamiltonian

$H_{\mathrm{eff}} |N,a\rangle = E_{N,a} |N,a\rangle$, (N: particle number → conserved quantity of H_{eff})

⇨

$$-i(H_{\mathrm{eff}} \varrho_{ab}^{(N,n)} - \varrho_{ab}^{(N,n)} H_{\mathrm{eff}}^\dagger) = \lambda_{ab}^{(N,n)} \varrho_{ab}^{(N,n)},$$

$$\lambda_{ab}^{(N,n)} = -i(E_{N,a} - E_{N+n,b}^*), \quad \varrho_{ab}^{(N,n)} = |N,a\rangle \langle N+n, b|$$

Solution of the Liouvillian

■ Step 3: Consider matrix elements in the particle-number basis

$$\sum_j L_j \varrho_{ab}^{(N,n)} L_j^\dagger = \sum_{c,d} A_{ab,cd}^{(N,n)} \varrho_{cd}^{(N-2,n)}$$

2-body loss
***N* → *N* - 2**

⇨ $$\mathcal{L} = \begin{pmatrix} \lambda^{(N,n)} & & \\ A^{(N,n)} & \lambda^{(N-2,n)} & \\ & A^{(N-2,n)} & \ddots \end{pmatrix}$$

Triangular structure!

[Torres, PRA 89, 052133 (2014)]

⇨ Liouvillian eigenvalue $\lambda_{ab}^{(N,n)} = -i(E_{N,a} - E_{N+n,b}^*),$

Liouvillian eigenoperator $\rho_{ab}^{(N,n)} = C_{ab}^{(N,n)} \varrho_{ab}^{(N,n)} + \sum_{N'=0}^{N-2} \sum_{a',b'} C_{a',b'}^{(N',n)} \varrho_{a'b'}^{(N',n)}$

Non-Hermitian Bethe ansatz

- Step 4: Solve the non-Hermitian Hubbard model

$$H_{\mathrm{eff}} = -t \sum_{j,\sigma} (c^{\dagger}_{j,\sigma} c_{j+1,\sigma} + \mathrm{H.c.}) + (U - i\gamma) \sum_{j} n_{j,\uparrow} n_{j,\downarrow}$$

complex-valued Hubbard interaction!

- 1D Hubbard model → solvable by Bethe ansatz [Lieb and Wu, PRL 20, 1445 (1968)]

→ can be extended to the non-Hermitian case!

Bethe eqs.

$$e^{ik_j L} = \prod_{\beta=1}^{M} \frac{\sin k_j - \lambda_\beta + iu}{\sin k_j - \lambda_\beta - iu}, \quad (j = 1, \cdots, N)$$

$$\prod_{j=1}^{N} \frac{\lambda_\alpha - \sin k_j + iu}{\lambda_\alpha - \sin k_j - iu} = -\prod_{\beta=1}^{M} \frac{\lambda_\alpha - \lambda_\beta + 2iu}{\lambda_\alpha - \lambda_\beta - 2iu}, \quad (\alpha = 1, \cdots, M)$$

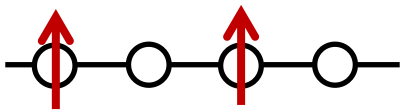
(N : # of particles, M : # of down spins, $u = (U - i\gamma)/(4t)$)

k_j : quasi-momentum, λ_α : spin rapidity

Steady state

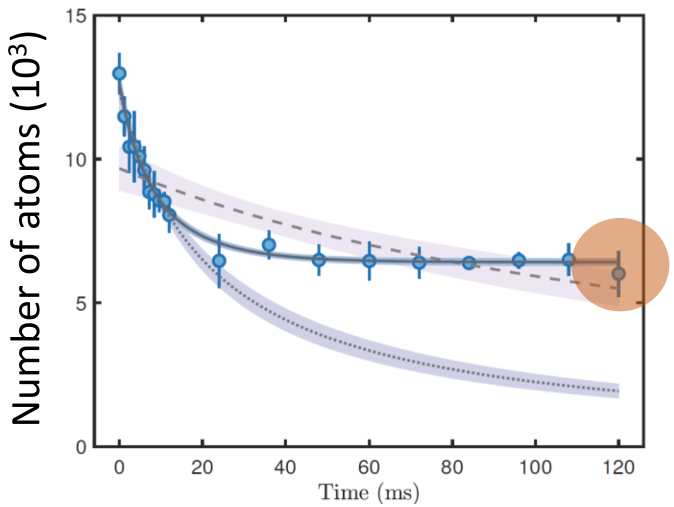
- Steady state: $\lambda = -i(E - E^*) = 0 \Leftrightarrow E \in \mathbb{R}$
- Vacuum $|0\rangle\langle 0|$ is a trivial steady state: $H_{\text{eff}}|0\rangle = 0$
- Other steady states?

Spin-polarized eigenstate $|\Psi\rangle\langle\Psi|, \; H_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$



→ No loss (Pauli exclusion): **ferromagnetic steady state!**

Spin SU(2) symmetry → Ferromagnetic steady states are degenerate



- Magnetization is conserved

  → Ferromagnetic, but S_z is fixed to initial value

- This result is not limited to 1D

Experiment: **non-vacuum steady state!**

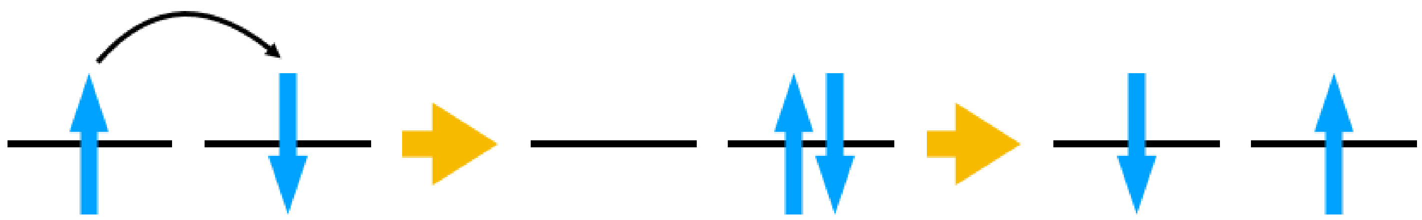
[Sponselee *et al.*, Quant. Sci. Tech. 4, 014002 (2018)]

Spin-exchange interaction

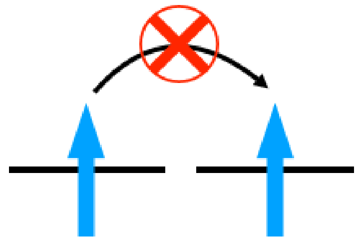
- Why is the steady state ferromagnetic?
- Standard origin of magnetism: **spin-exchange interaction**

  (2nd-order perturbation theory in the Hubbard model, $U >> t$)

◇ **Antiparallel spin configuration → lowers the energy**



◇ **Parallel spin configuration → cannot lower the energy (Pauli exclusion)**



Basic physics behind **antiferromagnetic** Heisenberg model

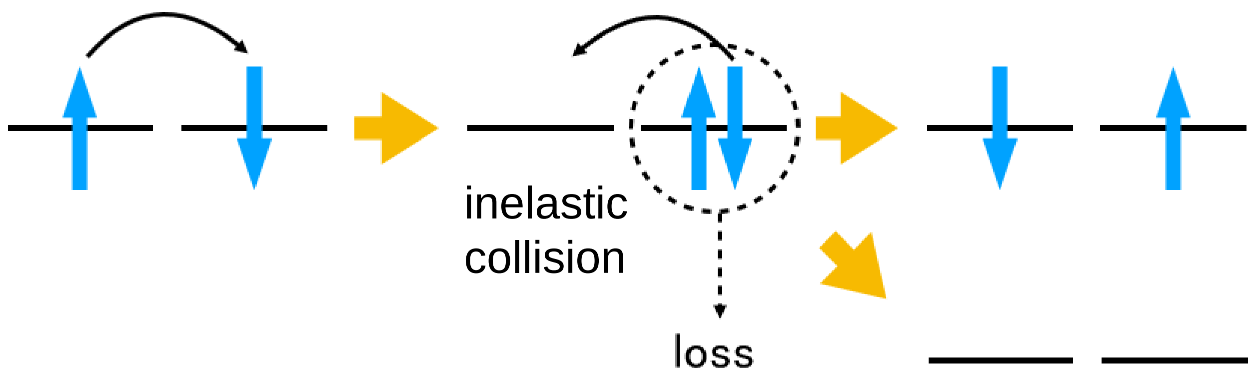
$$H = J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j \quad (J > 0)$$

Dissipative spin-exchange interaction

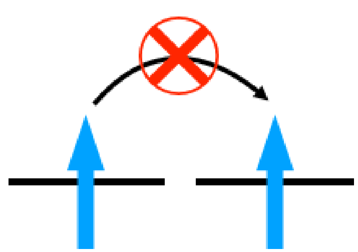
■ Spin-exchange interaction in the presence of **inelastic collisions**

[MN, N. Tsuji, N. Kawakami, and M. Ueda, PRL 124, 147203 (2020)]

◇ **Antiparallel spin configuration → lowers the energy,**
but has a finite lifetime



◇ **Parallel spin configuration → has higher energy, but is free from loss!**



Higher-energy states are stabilized by dissipation
→ Dissipation-induced ferromagnetism!

Liouvillian gap

■ Liouvillian eigenmodes near the steady state

→ **ferromagnetic spin-wave-like excitations**

Dispersion relation from Bethe ansatz ($\Delta P \simeq 0$)

$$\Delta E \simeq -\frac{t}{\pi u}\left(Q_0 - \frac{1}{2}\sin 2Q_0\right)\left(1-\cos\frac{\pi\Delta P}{Q_0}\right) \quad (Q_0 = \pi N/L)$$

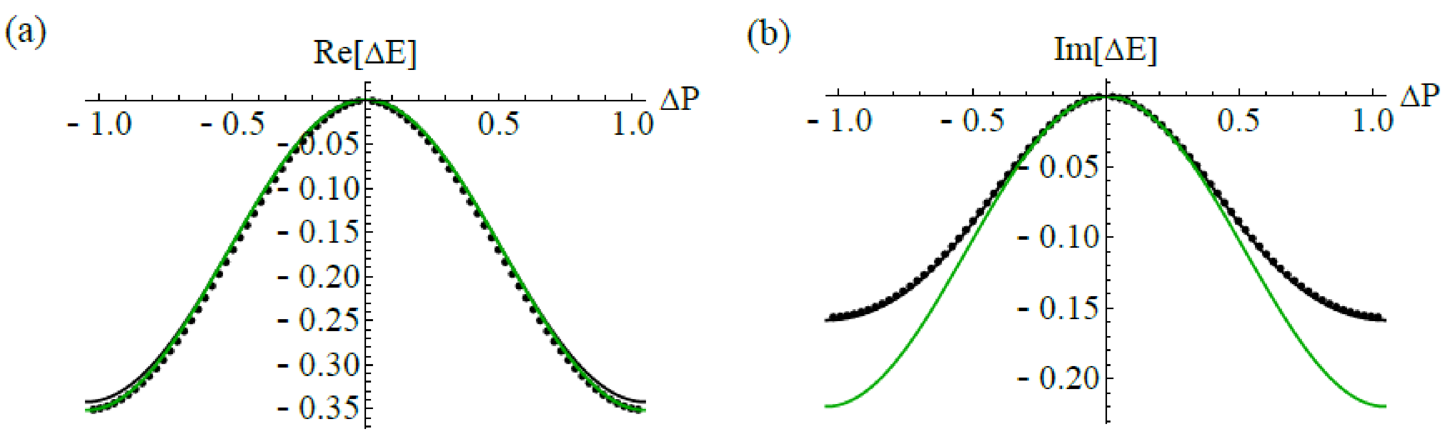
$$\propto (\Delta P)^2$$

gapless excitation!

Liouvillian gap

$$\Delta_{\rm L} \propto (\Delta P)^2 \sim \frac{1}{L^2}$$

Relaxation time τ_R diverges as $\tau_R \sim L^2$ in $L \to \infty$ limit

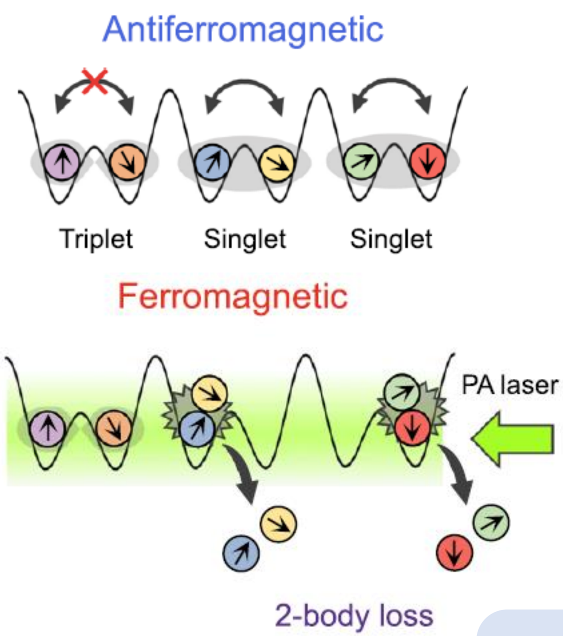


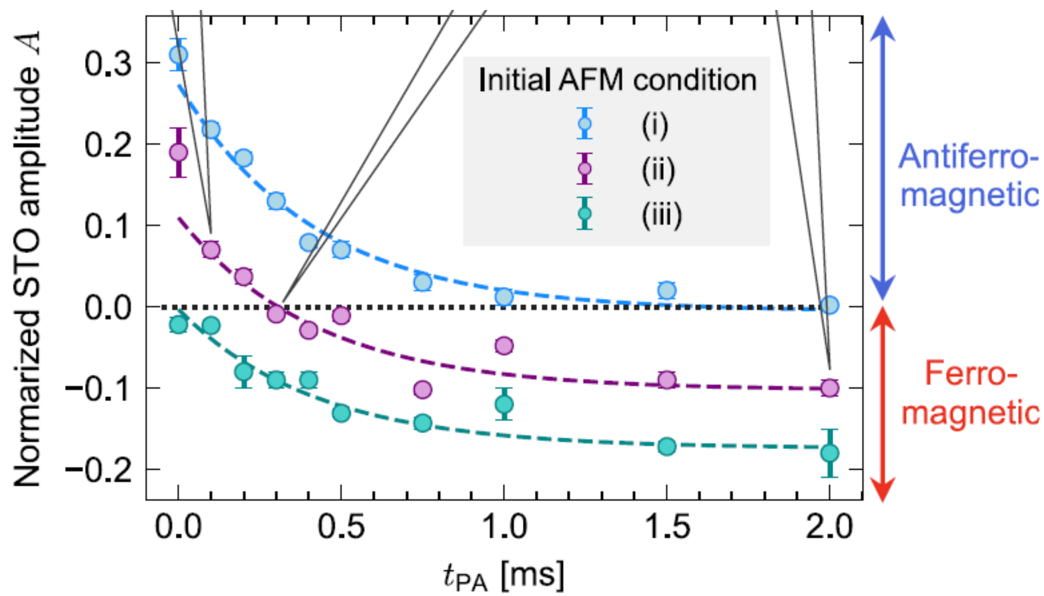
(Dot: numerics, L = 240, N = 80. Black line: numerics in $L \to \infty$. Green line: analytic, applicable to $\Delta P \simeq 0$)

Experiment

- Experimental realization (Takahashi group @ Kyoto university)
  [K. Honda, S. Taie, Y. Takasu, N. Nishizawa, MN, and Y. Takahashi, PRL 130, 063001 (2023)]
- ^{173}Yb atoms in a double-well optical lattice (SU(6) Hubbard model)

  photoassociation → controllable two-body loss





Sign reversal of spin correlations

→ Observation of dissipation-induced ferromagnetism!

Outline

1. Introduction

   - Dissipation in cold atoms: open quantum many-body systems

   - Experimental advance & theoretical description

2. Dissipative Hubbard model: magnetism & exact solution

   [MN, N. Tsuji, N. Kawakami, and M. Ueda, PRL 124, 147203 (2020)]

   [MN, N. Kawakami, and M. Ueda, PRL 126, 110404 (2021)]

   [K. Honda *et al*., PRL 130, 063001 (2023)]

3. Incoherenton: quasiparticles of decoherence processes

   [T. Haga, MN, R. Hamazaki, and M. Ueda, arXiv:2211.14991]

4. Summary

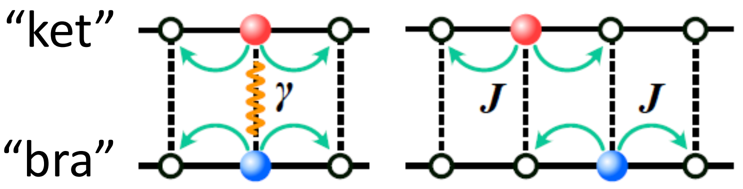
Ladder representation of a Liouvillian

■ Quantum master equation

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i(H_{\text{eff}}\rho - \rho H_{\text{eff}}^{\dagger}) + \sum_{\alpha} L_{\alpha}\rho L_{\alpha}^{\dagger},$$

Liouvillian

$$H_{\text{eff}} = H - \frac{i}{2}\sum_{\alpha} L_{\alpha}^{\dagger}L_{\alpha}$$



■ Ladder representation

$$\rho = \sum_{i,j} \rho_{ij} \left|i\right\rangle \left\langle j\right| \quad \Rightarrow \quad \left|\rho\right) = \sum_{i,j} \rho_{ij} \left|i\right\rangle \otimes \left|j\right\rangle$$

$$\mathcal{L} \quad \Rightarrow \quad \bar{\mathcal{L}} = -iH_{\text{eff}} \otimes I + iI \otimes H_{\text{eff}}^{*} + \sum_{\alpha} L_{\alpha} \otimes L_{\alpha}^{*}$$

“intrachain Hamiltonian”

“interchain interaction”

Dephasing model

- Example: spinless fermions + dephasing

$$\frac{d\rho}{dt} = \mathcal{L}\rho = -i[H,\rho] + \sum_{j=1}^{L}(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\}),$$

$$H = -J\sum_{j=1}^{L}(c_j^\dagger c_{j+1} + \mathrm{H.c.}),$$

1D tight-binding model

$$L_j = \sqrt{\gamma} c_j^\dagger c_j$$

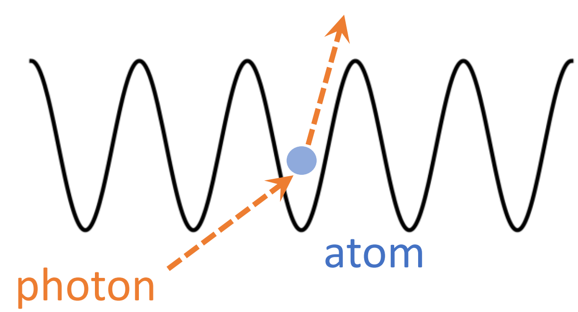
dephasing

- Experimental realization:

  Photon scattering from

  cold atoms in optical lattice

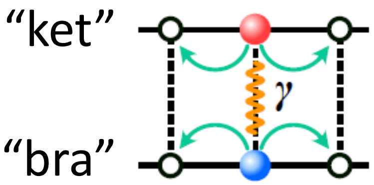
  [Pichler *et al*., PRA 82, 063605 (2010)]



Mapping to a non-Hermitian Hubbard model

- Ladder representation: “ket” = “↑ spin”, “bra” = “↓ spin”

$$\bar{\mathcal{L}} = -iH_{\text{eff}} \otimes I + iI \otimes H_{\text{eff}}^{*} + \sum_j L_j \otimes L_j^{*}$$



$$\Downarrow \quad U^{\dagger} c_{j,\downarrow} U = (-1)^{j} c_{j,\downarrow}$$

$$iU^{\dagger}\bar{\mathcal{L}}U = -J\sum_j \sum_{\sigma=\uparrow,\downarrow} (c_{j,\sigma}^{\dagger} c_{j+1,\sigma} + \text{H.c.}) - \frac{i\gamma}{2}\sum_{j,\sigma} n_{j,\sigma}$$

$$+ i\gamma \sum_j n_{j,\uparrow} n_{j,\downarrow}$$

intrachain Hamiltonian

interchain interaction

Spinless fermions + dephasing

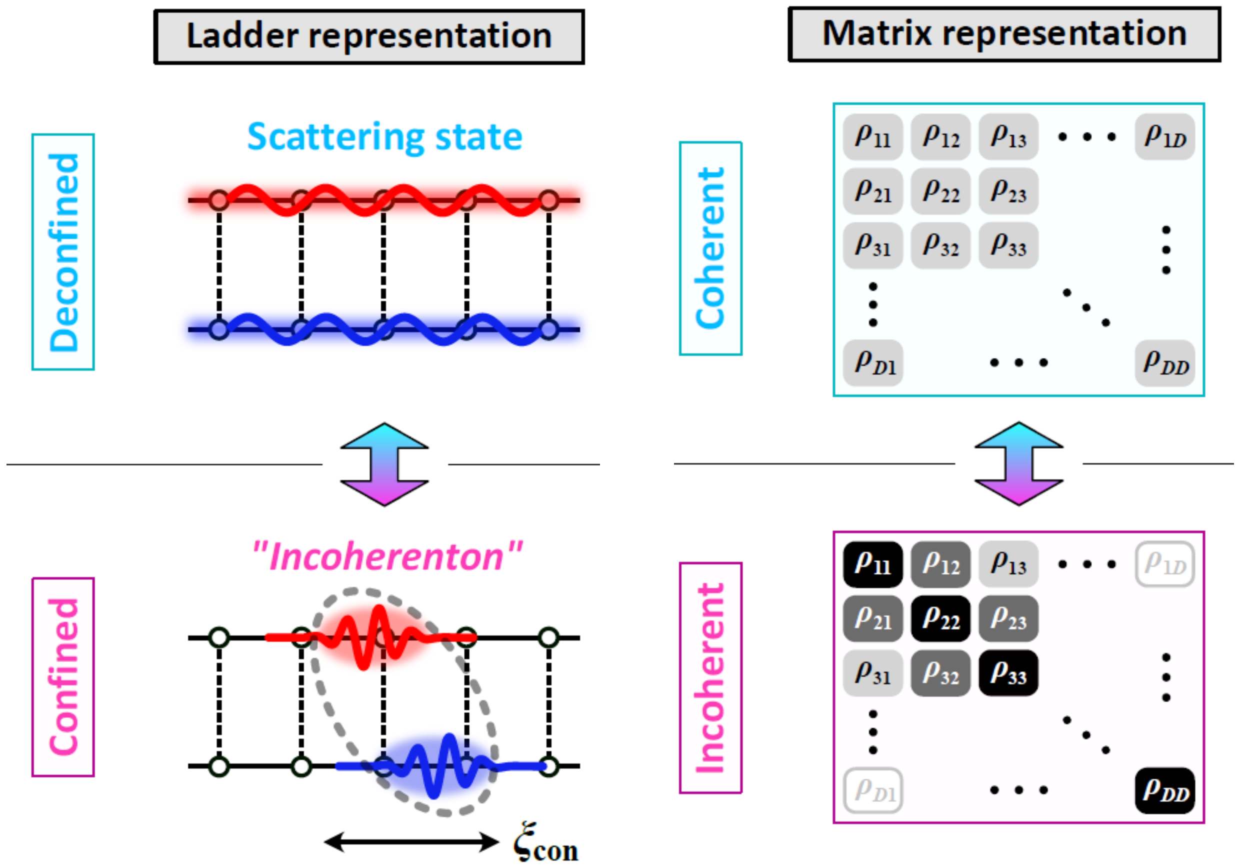
= Hubbard model with pure-imaginary interaction strength!

→ <u>Exactly solvable Liouvillian</u>

[Medvedyeva, Essler, and Prosen, PRL 117, 137202 (2016)]

Incoherenton

■ Interchain interaction → interchain bound state



bound state in ladder rep.
= exp. decay of
off-diagonal components
in the density matrix

Formation of an interchain bound state

= incoherent eigenmode ("incoherenton")

Bound state & its deconfinement

■ Bound state solution ("string solution" in Bethe ansatz)

Bethe eqs. $e^{ik_1 L} = \dfrac{\sin k_1 - \sin k_2 + 2iu}{\sin k_1 - \sin k_2 - 2iu}, \quad e^{ik_2 L} = \dfrac{\sin k_2 - \sin k_1 + 2iu}{\sin k_2 - \sin k_1 - 2iu},$

$L \to \infty$

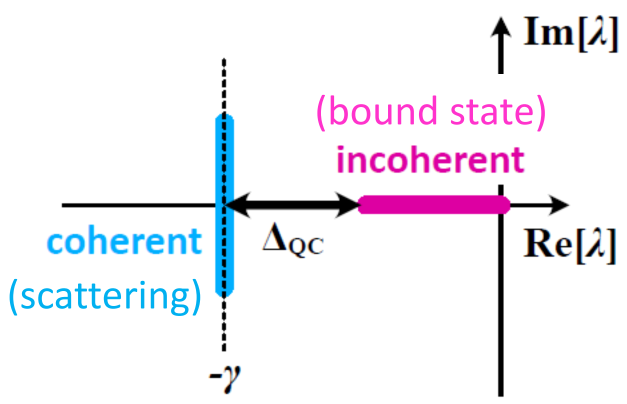
⇨ $\sin k_1 - \sin k_2 = 2iu \quad (u = i\gamma/(4J))$

■ The bound state exists only when $-\dfrac{\gamma}{4J} < \sin\dfrac{K}{2} < 0 \ \ (K = k_1 + k_2 - \pi)$

Single-particle Liouvillian eigenvalue
(can be generalized to many-body case)

$$\lambda = -\gamma + \sqrt{\gamma^2 - 16J^2 \sin^2(K/2)}$$

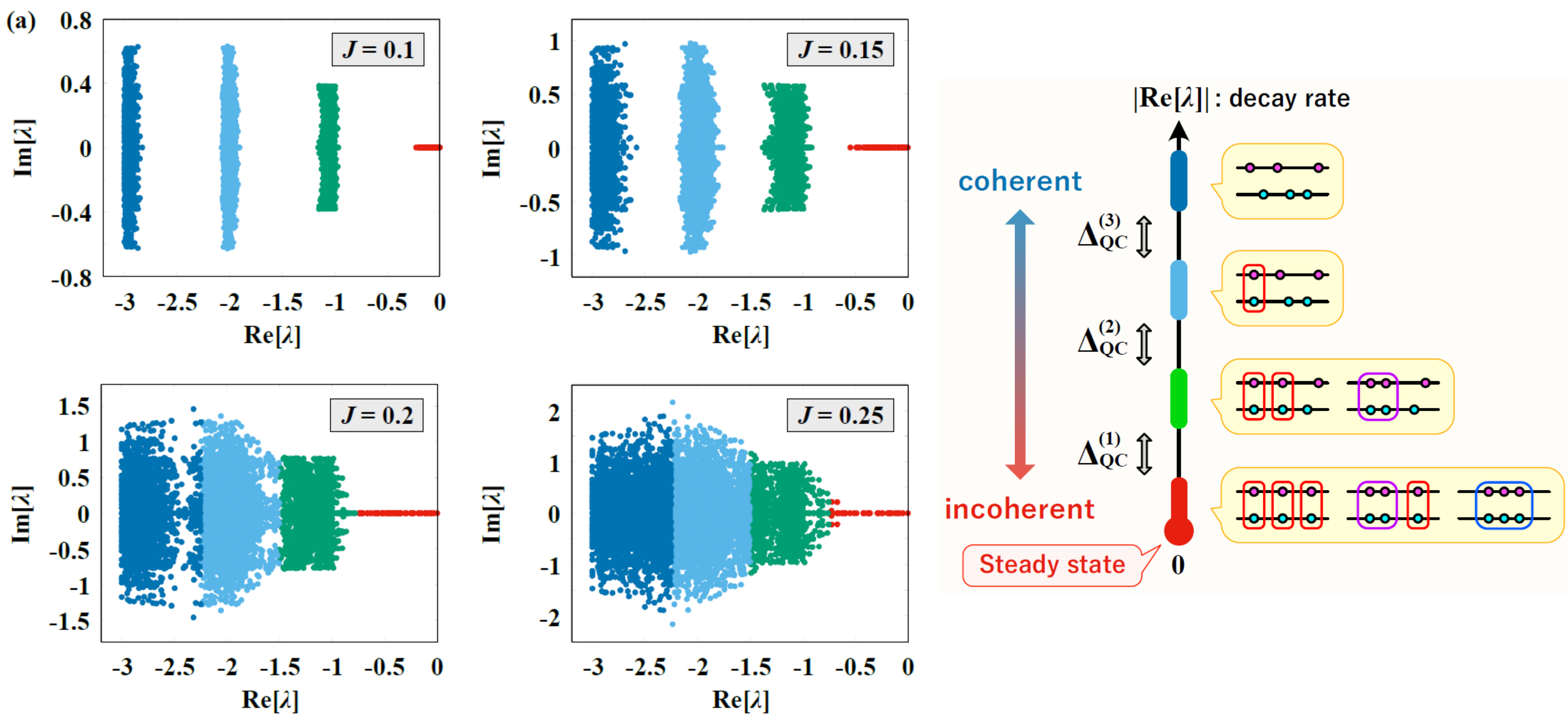
⇨ Gap $\Delta_{\mathrm{QC}} = \sqrt{\gamma^2 - 16J^2}$



(Minimal) binding energy of incoherenton = gap in Liouvillian spectrum

Hierarchy of eigenmodes

■ Liouvillian spectrum (L = 10, N = 3, γ = 1)



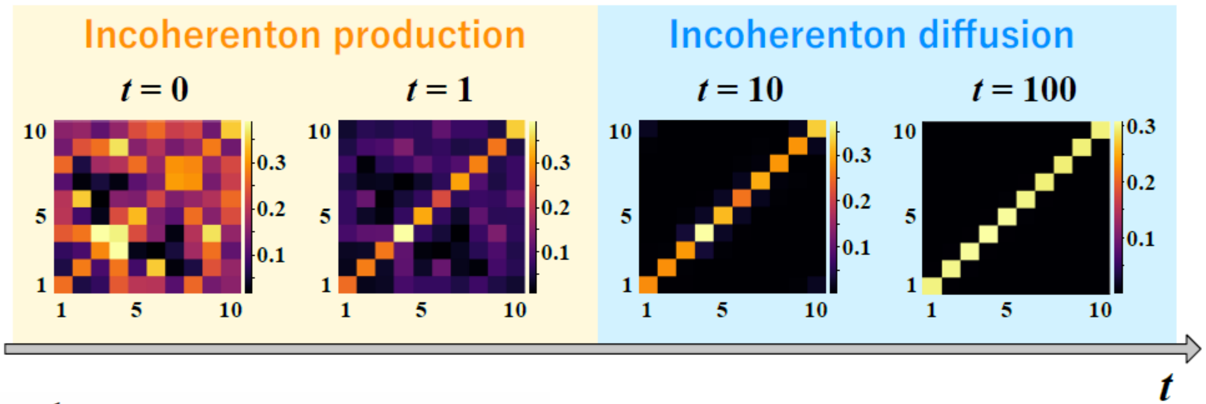
Eigenmodes are classified by the number of incoherentons

→ hierarchical structure of Liouvillian spectrum!

Hierarchical relaxation

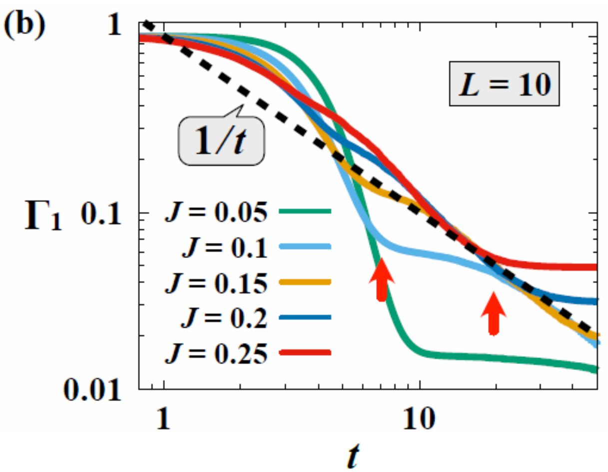
■ Dynamics of coherence (L = 10, N = 3, γ = 1, J = 0.1)

(a)



Two-stage relaxation of density matrix

(b)



$$\chi_1(t) = \sum_{j_1, j_2} |\rho_{j_1, j_2}(t)|(1 - \delta_{j_1, j_2}),$$

coherence

$$\Gamma_1(t) = -\frac{d}{dt} \ln \chi_1(t)$$

decay rate of coherence

Hierarchy of decay rates → different numbers of incoherentons

Many-body decoherence is interpreted in terms of incoherenton dynamics!

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4. Summary

Summary

■ Ultracold atoms → controllable open quantum many-body systems

■ Dissipative Hubbard model with two-body loss

- Exact solution based on the triangular structure of the Liouvillian

- Dissipation-induced ferromagnetism & experiment

■ Spinless fermions with dephasing

- Ladder representation of the Liouvillian

- Bound state between ket and bra d.o.f. = “incoherenton”

MN, N. Tsuji, N. Kawakami, and M. Ueda, PRL 124, 147203 (2020)

MN, N. Kawakami, and M. Ueda, PRL 126, 110404 (2021)

中川, 辻, 川上, 上田, 日本物理学会誌 第77巻2号, p88 (2022)

K. Honda, S. Taie, Y. Takasu, N. Nishizawa, MN, and Y. Takahashi, PRL 130, 063001 (2023)

T. Haga, MN, R. Hamazaki, and M. Ueda, arXiv:2211.14991