微小系の非平衡熱力学と量子コヒーレンス

布能 謙　（分子研）

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based on:
H. Tajima and K. Funo
Phys. Rev. Lett. **127**, 190604 (2021)
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2023年2月16日-17日

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熱力学：マクロからミクロへ

マクロの領域で動作する熱機関

- 古典熱力学 (19th and 20th century)

- どのような熱機関であってもカルノー効率を超えた動作はできないことを予測
- 現実には動作のスピード(仕事率)も重要だが、古典熱力学は平衡状態に関する理論であり、時間や動力学へ言及することができない

熱力学：マクロからミクロへ

大きいスケール

マクロの領域で動作する熱機関

- 古典熱力学 (19th and 20th century)

一分子領域で動作する熱機関

- ゆらぎの熱力学 (21st century ~)

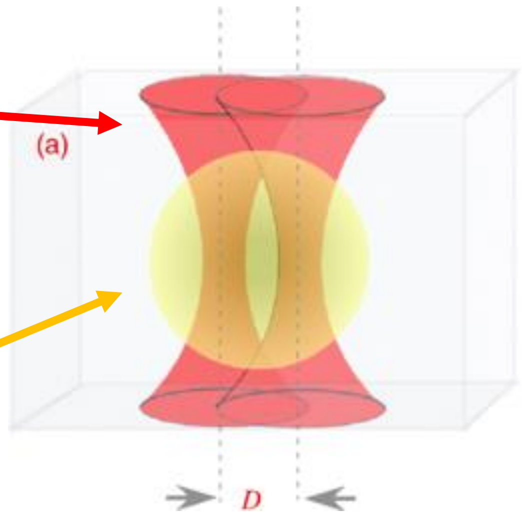
熱ゆらぎ

関本謙「ゆらぎのエネルギー論」
U. Seifert, Rep. Prog. Phys. (2012)

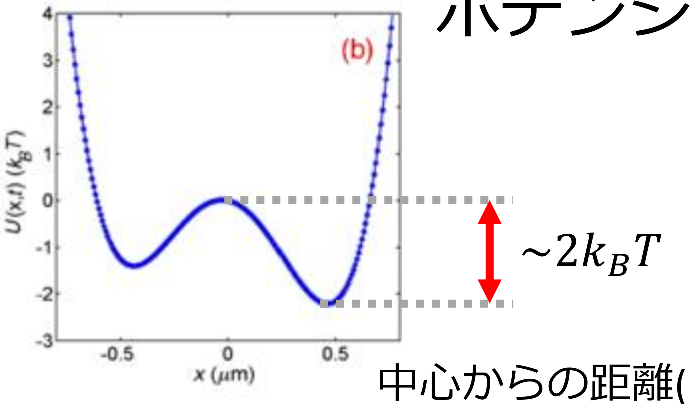
小さいスケール

２つの
レーザービーム

トラップされて
いるビーズ
(ポリスチレン粒子)



ポテンシャル



中心からの距離(μm)

S. Ciliberto, PRX (2017)

熱力学：マクロからミクロへ

マクロの領域で動作する熱機関

- 古典熱力学 (19th and 20th century)

一分子領域で動作する熱機関

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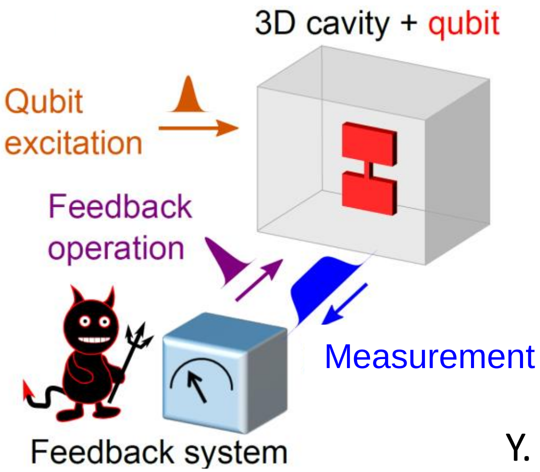
一原子/キュービット領域で動作する熱機関

- 量子熱力学

例：超電導量子ビット系



Y. Masuyama, et al., Nat. Comm. (2018)

大きいスケール

熱ゆらぎ

量子ゆらぎ

小さいスケール

熱力学：マクロからミクロへ

大きいスケール

マクロの領域で動作する熱機関

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熱ゆらぎ

一分子領域で動作する熱機関

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量子ゆらぎ

一原子/キュービット領域で動作する熱機関

- 量子熱力学

- 動力学とエネルギー学がコンシステントに融合した理論体系
- 非平衡・有限時間の熱力学的解析が可能

小さいスケール

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1. 古典熱力学から21世紀の熱力学へ
2. 量子開放系のダイナミクスと熱力学の第一・第二法則
3. 熱力学とコヒーレンス
4. 結果:
   散逸・熱流トレードオフ関係におけるコヒーレンスの効果
5. 熱機関への応用:
   有限パワー・カルノー効率の漸近的達成
6. 結論

量子開放系のダイナミクス

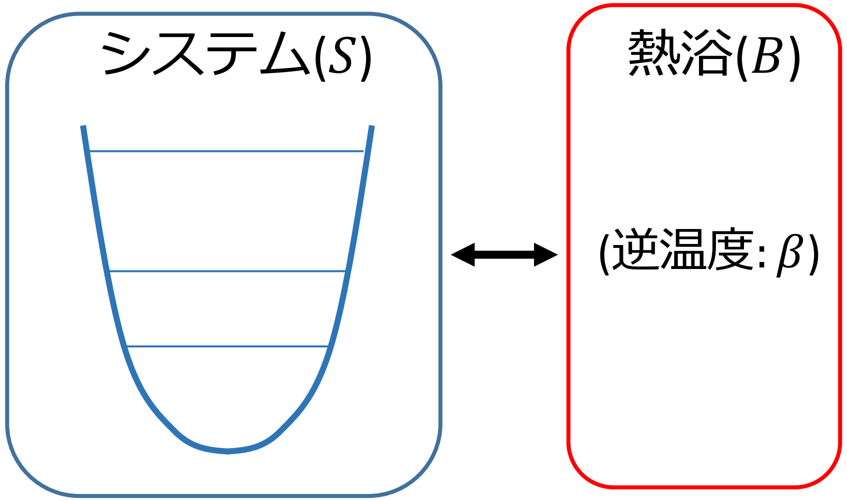
開放系の時間発展（システムS+熱浴B）

$$\rho_S(\tau) = \mathrm{Tr}_B\left[\underline{U_\tau^{SB}}\left(\underline{\rho_S(0)} \otimes \underline{\rho_B^\beta}\right)\underline{\left(U_\tau^{SB}\right)}^\dagger\right]$$

全体系($S+B$)の
ユニタリ時間発展

システムの
初期状態

熱浴の初期状態
(逆温度βのギブス分布)



- 厳密な記述だが、時間発展を解析的・数値的に解くのは困難　[cf. 非マルコフ効果]
- 多くの実験系では熱浴の影響はマルコフ的（メモリー効果が無い）

“マルコフ的”なダイナミクスの最も一般的な形

→ Gorini-Kossakowski-Sudarshan-Lindblad (GKSL)マスター方程式

$$\partial_t \rho_S(t) = \mathcal{L}(t)\rho_S(t)$$

物理的に妥当と思われる仮定（弱結合、ボルン・マルコフ・セキュラー近似）からも
GKSLの形になるマスター方程式が導かれる

量子開放系のダイナミクス

GKSLマスター方程式

$$\partial_t \rho = \mathcal{L}(t) = \underline{-\frac{i}{\hbar}[H,\rho]} + \underline{\mathcal{D}[\rho]}$$

ユニタリー時間発展の部分　熱浴によるデコヒーレンス・散逸の効果

ディシペーター

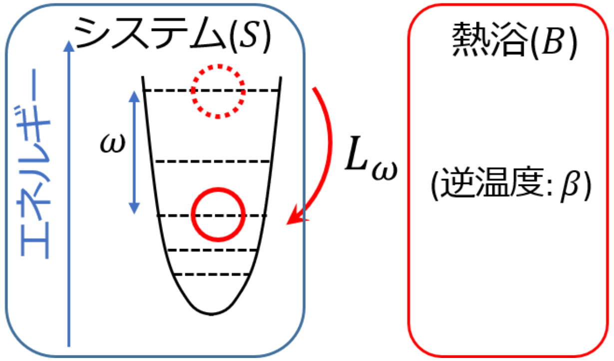
$$\mathcal{D}[\rho] = \sum_\omega \gamma(\omega)\left(L_\omega \rho L_\omega^\dagger - \frac{1}{2}\left\{L_\omega^\dagger L_\omega, \rho\right\}\right)$$

GKSLマスター方程式は“マルコフ的”なダイナミクスの最も一般的な形を与える
→ 量子熱力学ではさらに熱力学的にコンシステントな条件を課す

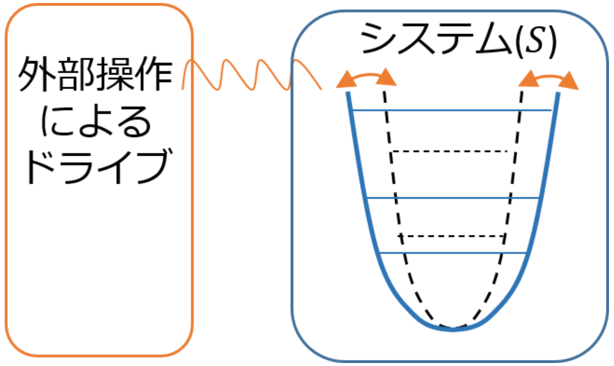
定常解が熱浴と同じ温度のギブス分布(平衡分布)

- L_ω : エネルギー固有状態$\Pi_{e+\omega}$からΠ_eへのジャンプ
- 詳細釣り合い: $\gamma(\omega)/\gamma(-\omega) = \exp[\beta\omega]$

(通常の弱結合、ボルン・マルコフ・セキュラー近似による導出は上記の条件を満たす)



外部からの古典的な操作によってハミルトニアン$H(t)$が時間的に変化する状況も取り扱う（この場合、各時刻ごとに上記の条件を課す）

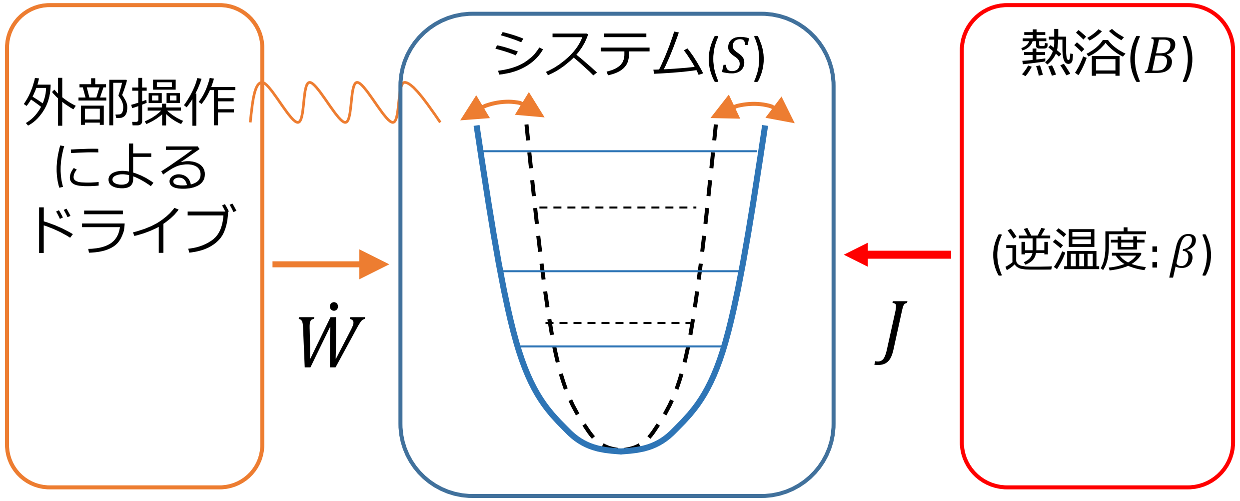


→ 熱力学的にコンシステントなダイナミクスの構造の上に
仕事・熱・エントロピー生成などのオブザーバブルを定義する

熱力学的なオブザーバブルと熱力学の法則

- 熱力学第一法則（エネルギー保存）

$$\underbrace{\partial_t(\mathrm{Tr}[H\rho])}_{\text{内部エネルギー変化 }\dot{E}} = \underbrace{\mathrm{Tr}[(\partial_t H)\rho]}_{\text{仕事流 }\dot{W}} + \underbrace{\mathrm{Tr}[H\partial_t\rho]}_{\text{熱流 }J}$$



外部操作によるドライブ

$\dot{W}$

システム(S)

J

熱浴(B)

(逆温度: β)

物理的解釈

- 仕事流 $\dot{W} = \mathrm{Tr}[(\partial_t H)\rho]$

  外部操作でハミルトニアンをドライブしたときのエネルギー変化

- 熱流 $J = \mathrm{Tr}[H\underbrace{\partial_t\rho}_{\text{マスター方程式}}]$

  $\partial_t\rho = -\frac{i}{\hbar}[H,\rho] + \mathcal{D}[\rho]$ を代入

  $J = \mathrm{Tr}[H\mathcal{D}[\rho]]$: 熱浴と相互作用したときの散逸によるエネルギー変化

熱力学的なオブザーバブルと熱力学の法則

・熱力学第二法則（エントロピー増大）

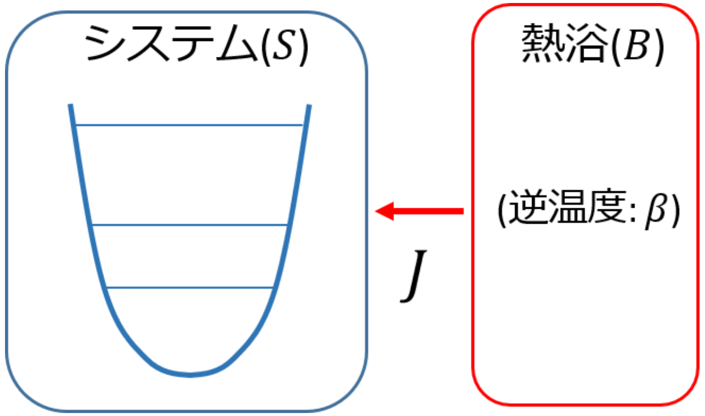
エントロピー生成率（$S+B$全体系のエントロピー増加）

$$\dot{\sigma}(\rho) = \underline{\dot{S}(\rho)}\ \underline{-\beta J(\rho)}$$

システムのvon Neumannエントロピー
$S(\rho) = -\mathrm{Tr}[\rho \ln \rho]$の増加率

熱浴のエントロピー増加率
（cf. $\delta S = \delta E/T$）

$\dot{S}_B = -\beta J$



- Spohnの表現（の一般化）

$$\dot{\sigma}(\rho) = \lim_{dt\to 0}\frac{1}{dt}\left(D[\rho(t)||\rho^{\beta}_{\mathrm{Gibbs}}(t)] - D[\rho(t+dt)||\rho^{\beta}_{\mathrm{Gibbs}}(t)]\right) \geq 0$$

- $\rho^{\beta}_{\mathrm{Gibbs}}(t)$: システムのハミルトニアン$H(t)$に対応するギブス分布（熱平衡状態）
- $D[\rho||\sigma] = \mathrm{Tr}[\rho(\ln\rho - \ln\sigma)]$: 相対エントロピー（２つの状態の“距離”）

熱平衡状態からのずれによる散逸、不可逆性の定量化

[$\dot{\sigma} \geq 0$の証明には、時間発展がマルコフ的、$\rho^{\beta}_{\mathrm{Gibbs}}(t)$が定常解であることと相対エントロピーの単調性を用いる]

目次

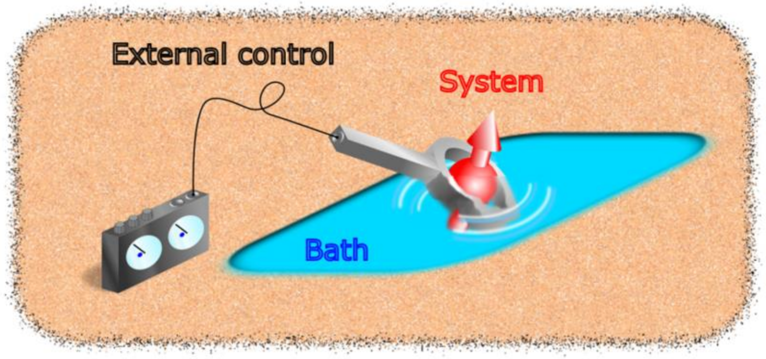
Understanding fundamental thermodynamic relations

Quantum thermodynamics:

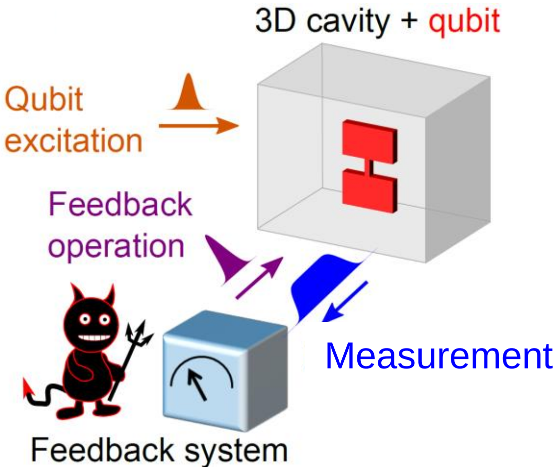
- Understanding fundamental limits (no-go theorems) on the controllability and manipulation of quantum devices

- Understanding a deep connection between work, heat, and information in quantum systems

- Understanding potential quantum advantages in thermodynamics



from [PRE 95, 012136 (2017)]



from [Nat. Comm. 9, 1291 (2018)]

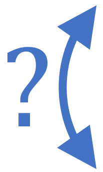
Quantum advantage in thermodynamics

- **Quantum coherence** improves the *performance of quantum devices* in many scenarios, e.g., superdense coding, Grover's algorithm, quantum sensing

- It is still unclear if there exists a nontrivial "quantum advantages" in thermodynamics

  - Coherence that is built up during a heat engine cycle acts as quantum friction **(disadvantage)**

    - Linear response: Brandner, Bauer, Seifert, PRL (2017)
    - experimentally relevant model: Karimi and Pekola, PRB (2016)



  - Some studies show that coherence improves the performance of heat engines **(advantage)**

    Uzdin, Levy, Kosloff, PRX (2015)

classification of coherence advantages/disadvantages in thermodynamics

Main topic of this study

trade-off relations between “current” and “dissipation”

e.g. Joule heating $W = RI^2$

- thermodynamic uncertainty relations

  Barato, Seifert, PRL 2015

  (precision of) general current ⇔ entropy production

- open system quantum speed limits

  probability current ⇔ entropy production

  Shiraishi, Funo, Saito, PRL, 2018
  Funo, Shiraishi Saito, NJP, 2019

- power-efficiency trade-off relation in heat engines

  Shiraishi, Saito, Tasaki, PRL 2016

  output power ⇔ thermodynamic efficiency

➢ trade-off relation between heat current and entropy production

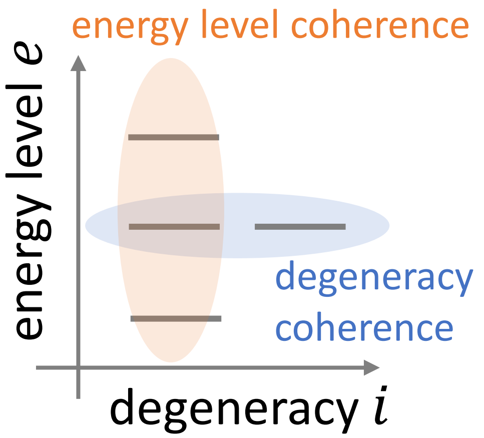
coherence effect? not fully explored!

2 types of coherence

main aim of this research:
understand the effect of coherence on current-dissipation trade-off relation

for later purpose, we introduce 2 types of coherence:

1. coherence between different energy levels
2. coherence between degenerate states



	general ρ	block-diagonal $\rho_{\mathrm{bd}} = \sum_e \Pi_e \rho \Pi_e$	strictly-diagonal $\rho_{\mathrm{sd}} = \sum_{e,i} \Pi_{e,i} \rho \Pi_{e,i}$
energy level coherence	✓	X	X
degeneracy coherence	✓	✓	X

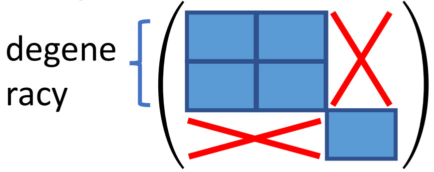
projection to energy level e

$$\Pi_e = \sum_i \Pi_{e,i}$$

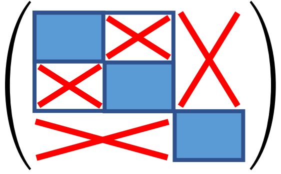
projection to $|e,i\rangle$ state

$$\Pi_{e,i} = |e,i\rangle\langle e,i|$$

ρ_{bd} :block-diagonal



ρ_{sd} :strictly-diagonal



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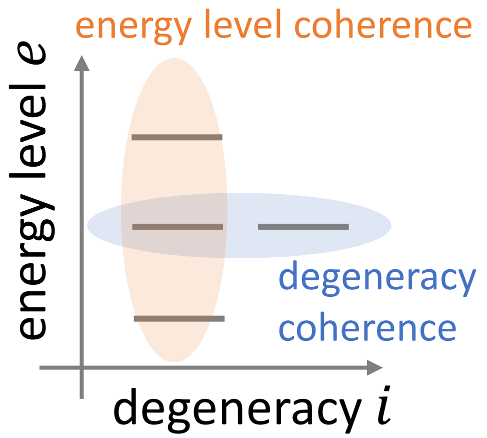

compare these two states

	general ρ	block-diagonal $\rho_{\mathrm{bd}} = \sum_e \Pi_e \rho \Pi_e$	strictly-diagonal $\rho_{\mathrm{sd}} = \sum_{e,i} \Pi_{e,i} \rho \Pi_{e,i}$
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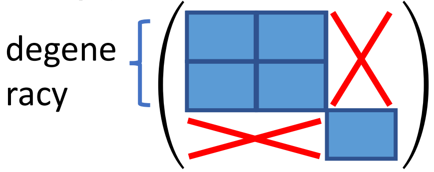
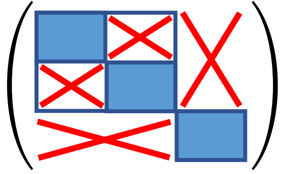


ρ_{sd} :strictly-diagonal



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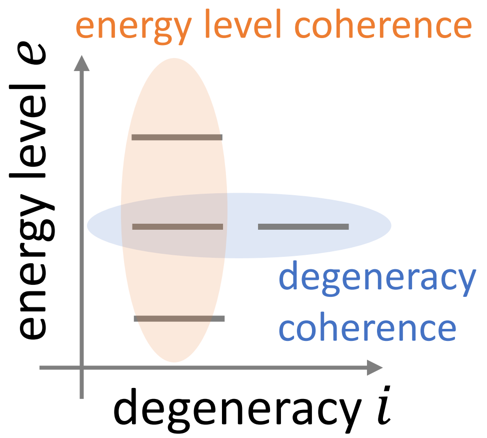

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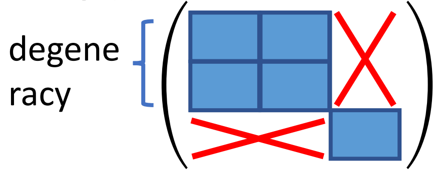
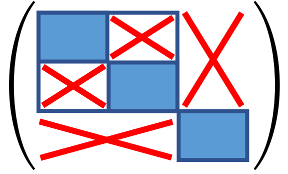


ρ_{sd} :strictly-diagonal



目次

Main results: coherence effect on current-dissipation ratio

heat current: $J(\rho) = \mathrm{Tr}[H\partial_t\rho]$

entropy production: $\dot{\sigma}(\rho) = \dot{S}(\rho) - \beta J(\rho)$

current-dissipation (CD) ratio

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)}$$

← electrical conductance: $\frac{1}{R} = \frac{I^2}{W}$ (cf. Joule heating)

- we want large heat current and small entropy production for having high-performance thermodynamic devices (e.g. heat engines, refrigerators)
- higher CD ratio is preferred, but there exists an upper limit (CD trade-off relation)

	general ρ	block-diagonal $\rho_{\mathrm{bd}} = \sum_e \Pi_e \rho \Pi_e$	strictly-diagonal $\rho_{\mathrm{sd}} = \sum_{e,i} \Pi_{e,i} \rho \Pi_{e,i}$
energy level coherence	✓	✗	✗
degeneracy coherence	✓	✓	✗

In the following, we derive the CD trade-off relation and discuss how $J^2/\dot{\sigma}$ depends on coherence ($\rho, \rho_{\mathrm{bd}}, \rho_{\mathrm{sd}}$)

Main results: coherence effect on current-dissipation (CD) ratio

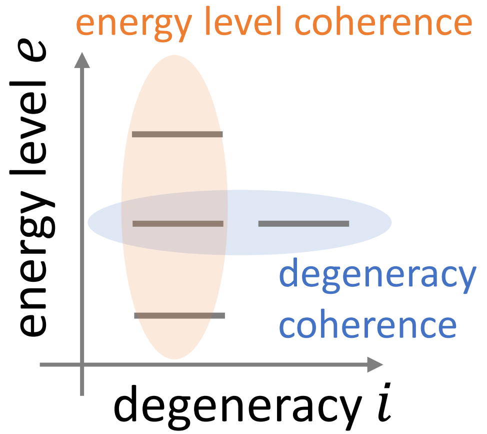
main result 1: energy level coherence *always* reduces the CD ratio → **disadvantage**

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \leq \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})}$$

(e.g., conductance)

- note: a stronger relation can be obtained, $J(\rho) = J(\rho_{\mathrm{bd}})$ and $\dot{\sigma}(\rho) \geq \dot{\sigma}(\rho_{\mathrm{bd}})$
- heat bath cannot utilize and only destroys energy level coherence
- if the Hamiltonian is non-degenerate, coherence would always lower the performance

	general ρ	block-diagonal $\rho_{\mathrm{bd}} = \sum_e \Pi_e \rho \Pi_e$	strictly-diagonal $\rho_{\mathrm{sd}} = \sum_{e,i} \Pi_{e,i} \rho \Pi_{e,i}$
energy level coherence	✓	X	X
degeneracy coherence	✓	✓	X



Main results: coherence effect on current-dissipation (CD) ratio

main result 1: energy level coherence *always* reduces the CD ratio → **disadvantage**

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \le \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})}$$

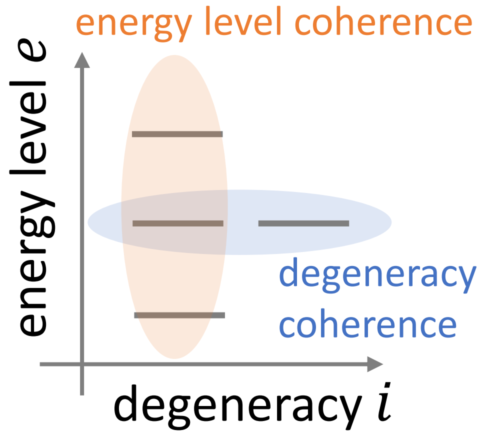
(e.g., conductance)

main result 2: degeneracy coherence allows increasing the CD ratio → **advantage**

$$\frac{J(\rho_{\mathrm{sd}})^2}{\dot{\sigma}(\rho_{\mathrm{sd}})} \le \frac{A_{\mathrm{cl}}}{2} \quad \text{and} \quad \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})} \le \underbrace{\frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2}}_{\text{also bounds CD ratio for } \rho \text{ (using result 1)}}$$

coherence effect!

	general ρ	block-diagonal $\rho_{\mathrm{bd}} = \sum_e \Pi_e \rho \Pi_e$	strictly-diagonal $\rho_{\mathrm{sd}} = \sum_{e,i} \Pi_{e,i} \rho \Pi_{e,i}$
energy level coherence	✓	X	X
degeneracy coherence	✓	✓	X



Main results: coherence effect on current-dissipation (CD) ratio

main result 1: energy level coherence *always* reduces the CD ratio → **disadvantage**

(e.g., conductance)

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \leq \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})}$$

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$$\frac{J(\rho_{\mathrm{sd}})^2}{\dot{\sigma}(\rho_{\mathrm{sd}})} \leq \frac{A_{\mathrm{cl}}}{2} \qquad \text{and} \qquad \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})} \leq \underbrace{\frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2}}_{\text{also bounds CD ratio for } \rho \text{ (using result 1)}}$$

coherence effect!

- physical interpretation of A: *instantaneous heat fluctuation*

  classical (ρ_{sd}-dependent) part: A_{cl}

  coherence effect part: $\boldsymbol{A_{qm} = (collective\ jumps) \times (degeneracy\ coherence)}$

- when bath *collectively* acts on *coherent* degenerate states, we can improve the CD ratio up to $A_{qm}/2$ → **advantage!**

Main results: scaling analysis of the current-dissipation ratio

$$\frac{J(\rho_{\mathrm{sd}})^2}{\dot{\sigma}(\rho_{\mathrm{sd}})} \le \frac{A_{\mathrm{cl}}}{2} \qquad \text{and} \qquad \frac{\overset{O(N)}{J(\rho_{\mathrm{bd}})^2}}{\underset{O(1)}{\dot{\sigma}(\rho_{\mathrm{bd}})}} \le \frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2} \longleftarrow O(N^2)$$

Next, we discuss the scaling behavior of the CD trade-off relation

- suppose that A_{qm} is an $O(N^2)$ quantity   N: number of degeneracy or particle number

  we find that $J = O(N)$ and $\dot{\sigma} = O(1)$ scaling is possible

when $A_{qm} = \mathrm{O}(N^2)$ via $(O(N)\ \textit{degeneracy coherence} \times O(N)\ \textit{collective jumps})$
⇒ “$\underbrace{\text{macroscopic heat flows}}_{O(N)}$ but the entropy production remains at $\underbrace{\text{microscopic order}}_{O(1)}$ ”

In the following, we show a toy model example that realizes the above observation

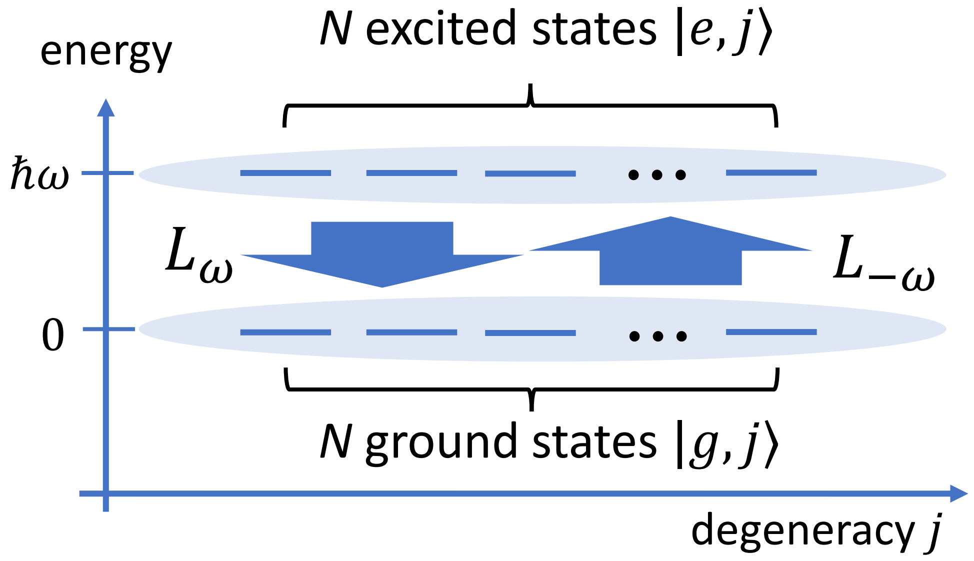
example: 2N-state model that realizes $A_{\mathrm{qm}} = O(N^2)$ scaling

system Hamiltonian:

$$H = \sum_{j=1}^{N} \hbar\omega \left|\mathrm{e}, j\right\rangle \left\langle \mathrm{e}, j\right| ,$$

dissipator:

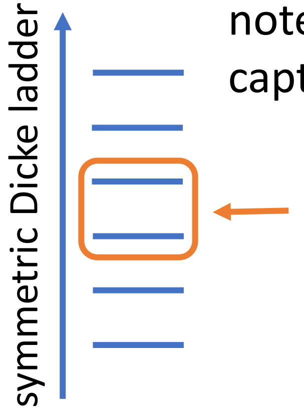
$$\mathcal{D}[\rho] = \sum_{\Omega=\omega,-\omega} \gamma(\Omega)(L_\Omega \rho L_\Omega^\dagger + \{L_\Omega^\dagger L_\Omega, \rho\})$$



jump operators: *correlated decays/excitation*

$$L_\omega = \sum_{j,j'} \sigma_-^{j,j'} \quad L_{-\omega} = \sum_{j,j'} \sigma_+^{j,j'}$$

$$\sigma_+^{j,j'} = \left|\mathrm{e}, j\right\rangle \left\langle \mathrm{g}, j'\right| , \ \ \sigma_-^{j,j'} = \left|\mathrm{g}, j\right\rangle \left\langle \mathrm{e}, j'\right|$$



note: essential feature of this model can be captured by a n-qubit super-radiance model

use these two Dicke states
(via a technique called superabsorption)

S. Kamimura, et al., PRL 2022

example: 2N-state model that realizes $A_{\mathrm{qm}} = O(N^2)$ scaling

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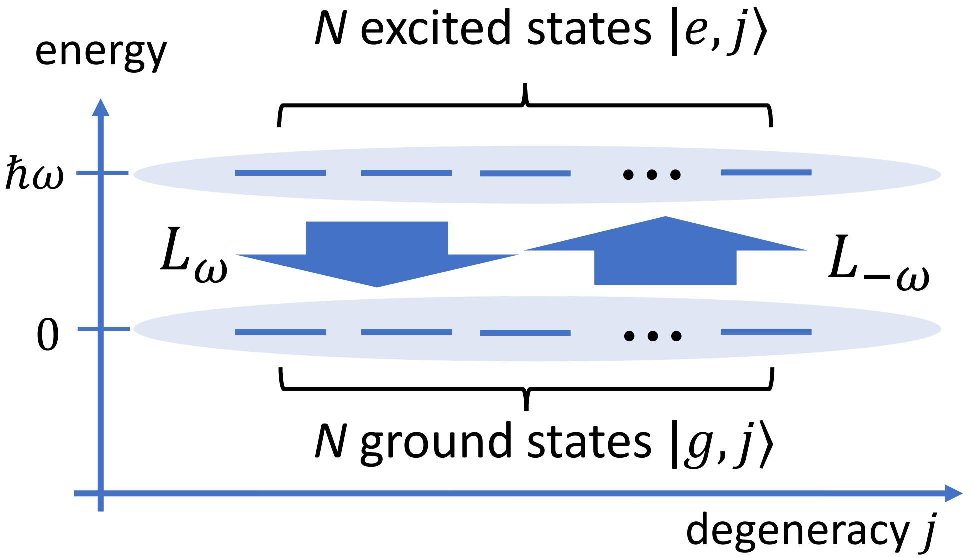

jump operators: *correlated decays/excitation*

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$$\sigma_+^{j,j'} = \left|\mathrm{e}, j\right\rangle \left\langle \mathrm{g}, j'\right| , \quad \sigma_-^{j,j'} = \left|\mathrm{g}, j\right\rangle \left\langle \mathrm{e}, j'\right|$$

$$|g, +\rangle \coloneqq \frac{\sum|g, j\rangle}{\sqrt{N}} \qquad |e, +\rangle \coloneqq \frac{\sum|e, j\rangle}{\sqrt{N}}$$

consider the following initial state (contains $O(N)$ coherence)

$$\rho_0 \coloneqq p_g|g, +\rangle\langle g, +| + p_e|e, +\rangle\langle e, +|$$

$$\frac{p_g}{p_e} = \left(1 + \frac{1}{N}\right) \mathrm{e}^{\beta\hbar\omega}$$

straightforward calculation shows that

$$A_{\mathrm{qm}} = O(N^2) \quad \& \quad J(\rho_0) = O(N) \quad \dot{\sigma}(\rho_0) = O(1)$$

➡ **realization of a "dissipation-less" heat current**

example: 2N-state model that realizes $A_{\rm qm}=O(N^2)$ scaling

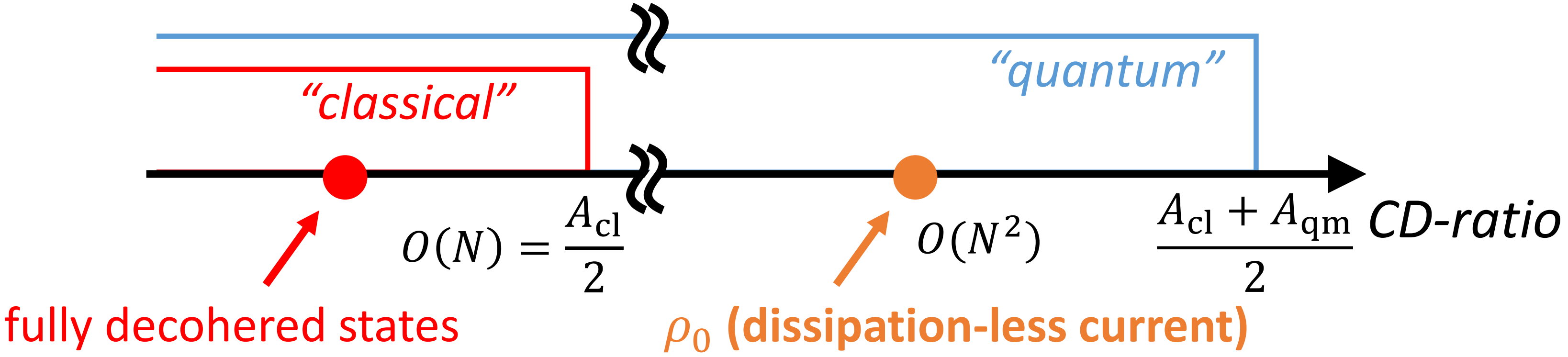
at least for this 2N-state model, we can rigorously show that

$$A_{\rm cl}=O(N)\ \textit{for any state}\ \rho$$

- by noting the relation $\frac{\overset{O(N^2)}{\boxed{J(\rho_{\rm sd})^2}}}{\dot{\sigma}(\rho_{\rm sd})}\le\overset{O(N)}{\boxed{\frac{A_{\rm cl}}{2}}}$, the above statement implies that fully decohered states (i.e., $\rho_{\rm sd}$) cannot produce a dissipation-less current:

$$J(\rho_{\rm sd})=O(N)\ \ ➡\ \ \dot{\sigma}(\rho_{\rm sd})=O(N)$$

- note that the CD ratio for the dissipation-less current is $J(\rho_0)^2/\dot{\sigma}(\rho_0)=O(N^2)$, and it goes beyond the “classical” bound:



Summary of the main results

H. Tajima and K. Funo
Phys. Rev. Lett. **127**, 190604 (2021)

main result 1: energy level coherence *always* reduces the CD ratio **(disadvantage)**

(Current-Dissipation)

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \leq \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})}$$

main result 2: degeneracy coherence allows increasing the CD ratio **(advantage)**

$$\frac{J(\rho_{\mathrm{sd}})^2}{\dot{\sigma}(\rho_{\mathrm{sd}})} \leq \frac{A_{\mathrm{cl}}}{2} \quad \text{and} \quad \frac{J(\rho_{\mathrm{bd}})^2}{\dot{\sigma}(\rho_{\mathrm{bd}})} \leq \frac{A_{\mathrm{cl}} + \boxed{A_{\mathrm{qm}}}}{2}$$

coherence effect!

main result 3: large degeneracy coherence + collective jump mechanism
→ effective cancellation of the CD trade-off relation

macroscopic heat flows but the entropy production remains vanishingly small

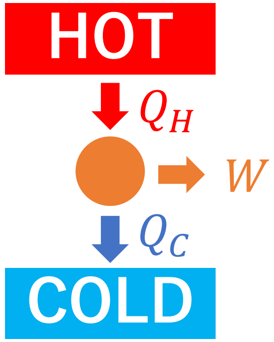
$$J(\rho_0) = O(N) \quad \text{and} \quad \dot{\sigma}(\rho_0) = O(1)$$

in the following, we discuss how these results are related to the performance of heat engines

目次

Application to heat engines

Heat engines, refrigerators



Output power and heat-to-work conversion efficiency

$$P = \frac{W}{\tau} \quad \text{and} \quad \eta = \frac{W}{Q_H}$$

universal upper bound: Carnot efficiency

$$\eta \leq \eta_{\text{Car}} = \left(1 - \frac{T_C}{T_H}\right)$$

・Carnot efficiency can be reached for infinitely slow cycles

⟶ power vanishes

is it possible to produce finite power and reach Carnot efficiency?

classical limit: power-efficiency trade-off relation

power-efficiency trade-off relation

Shiraishi, Saito, Tasaki, PRL 2016

$$O(1) \quad \boxed{P} \le \overset{O(N)}{\boxed{\overline{\Theta}}}\beta_C\eta\overset{O(1/N)}{\boxed{\left(\eta_{\mathrm{Car}}-\eta\right)}}$$

from the above theorem, finite-time Carnot engine is not possible:

$$\eta = \eta_{\mathrm{Car}} \quad \Longrightarrow \quad P = 0$$

how about asymptotic realization of the Carnot efficiency with finite power? **No**

→ For classical models with reasonable assumptions, $\overline{\Theta}$ scales at most $O(N)$

$$\eta = \eta_{\mathrm{Car}} - O(1/N) \quad \Longrightarrow \quad P = \underline{O(1)}$$

cannot be $O(N)$, and producing
finite power is not possible

➡ **however, by using quantum effects, we could overcome this problem**

asymptotic realization of finite-power Carnot engine via coherence

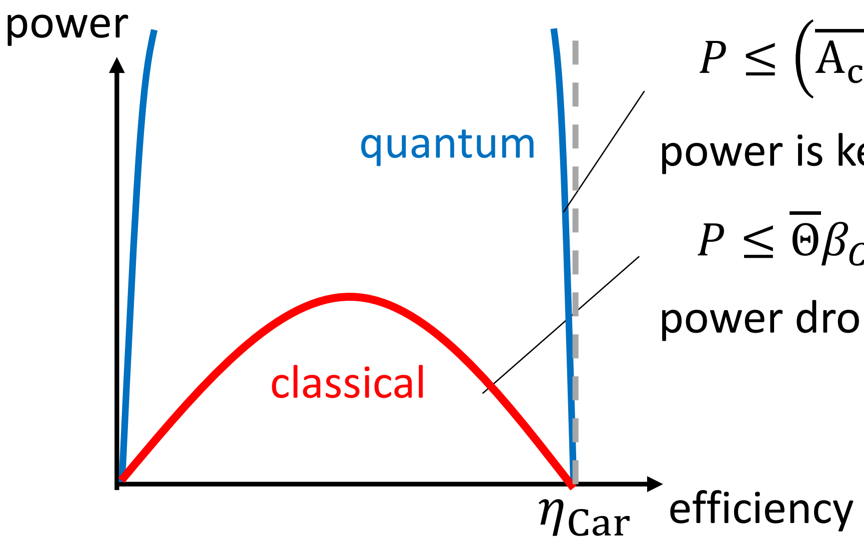
In quantum systems, we can derive a power-efficiency trade-off relation

$$\underbrace{\boxed{P}}_{O(N)} \leq \left(\overline{\mathrm{A}_{\mathrm{cl}}} + \underbrace{\boxed{\overline{\mathrm{A}_{\mathrm{qm}}}}}_{O(N^2)}\right) c\beta_L \eta \underbrace{\boxed{(\eta_{\mathrm{Car}} - \eta)}}_{O(1/N)}$$

Tajima, Funo, PRL 2021

The coefficient $\overline{\mathrm{A}_{\mathrm{qm}}}$ can grow $O(N^2)$ with coherence + collective jump effects
→ this would realize fundamental scaling difference between classical and quantum systems

$$\overline{\mathrm{A}_{\mathrm{qm}}} = O(N^2) \Longrightarrow \boldsymbol{\eta = \eta_{Car} - O(1/N)} \textbf{ and } \boldsymbol{P = O(N)} \textbf{ is possible!}$$



$$P \leq \left(\overline{\mathrm{A}_{\mathrm{cl}}} + \overline{\mathrm{A}_{\mathrm{qm}}}\right) c\beta_C\, \eta(\eta_{\mathrm{Car}} - \eta)$$

power is kept high even if η becomes close to η_{Car}

$$P \leq \overline{\Theta}\beta_C\, \eta(\eta_{\mathrm{Car}} - \eta)$$

power drops down quickly to 0 as η becomes close to η_{Car}

asymptotic realization of a finite-power Carnot engine is possible in quantum systems [cf. 2N-state model]

Coherence effect on the performance of heat engines

larger value of the current-dissipation (CD) ratio is preferred for having good heat engines

(e.g., conductance)

from the obtained CD trade-off relation, we find the following intuitions:

1. Energy level coherence lowers the engine performance

   ➡ **for nondegenerate systems, coherence is always bad**

2. Degeneracy coherence can be utilized to improve the engine performance

   ✓ 2N-state model effective finite-time Carnot engine

   ✓ 2qubit super-radiance model heat engine (cf. Tajima-Funo 2021)

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macroscopic heat flows but the entropy production remains vanishingly small

$$J(\rho_0) = O(N) \quad \text{and} \quad \dot{\sigma}(\rho_0) = O(1)$$

this scaling realizes **asymptotic finite-time Carnot engine:** $\eta = \eta_{\mathrm{Car}} - O(1/N)$ **and** $P = O(N)$