量子計算と場の量子論

Masazumi Honda

(本多正純)

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お題:

量子コンピュータでの場の量子論のダイナミクス解明へ向けた話

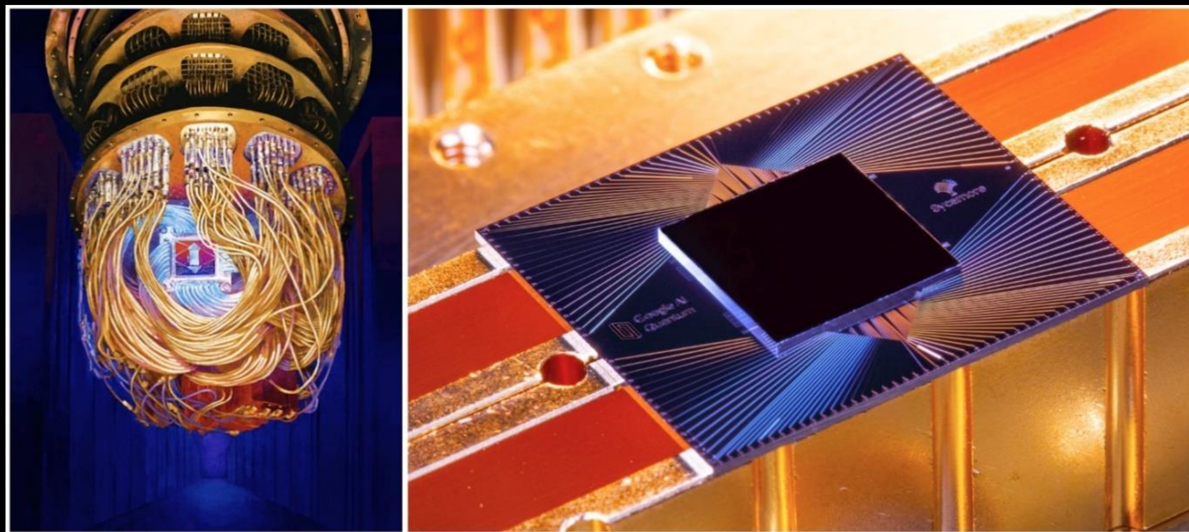
特に、これまでの例だけでなく

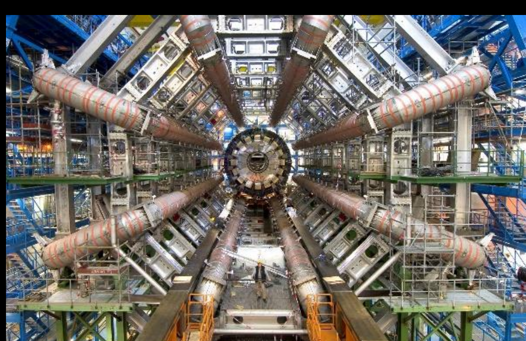
- ・より将来へ向けた展望
- ・そのための乗り越えるべき壁
- ・新たに参入しようとする人へ向けた挑戦状

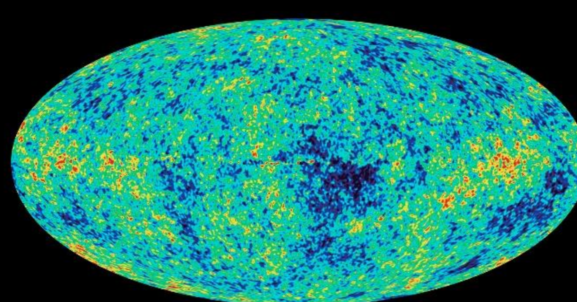
本講演:

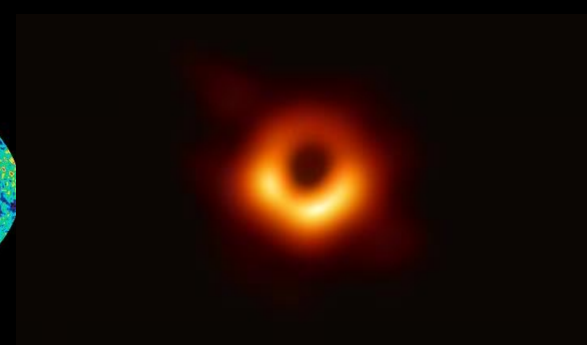
- ・(場の量子論に関して) 何ができるようになりそうか
- ・そのためにどうするべきか

できるようになりそうなこと (期待)









etc...

量子計算機でできるようになりそうなこと: (詳細は後述)

従来の数値的アプローチ(=モンテカルロ法)では問題(=符号問題)がある状況のシミュレーション

符号問題が起きやすい物理的状況:

- ・トポロジカル項をもつ系 (ex. Chern-Simons項, theta項)
- ・フェルミオン系に化学ポテンシャルを入れた場合
  —— これが原因でQCD(量子色力学)の相図が未完成
- ・実時間系 —— 言うまでもなく重要

課題

- 一定以上の性能をもつ量子計算機が必要
  (qubit数, fidelity, etc...)
- 量子計算機で効率的に解ける問題の開拓

課題

・一定以上の性能をもつ量子計算機が必要

(qubit数, fidelity, etc...)

・量子計算機で効率的に解ける問題の開拓

やるべきこと: (ハードの発展を待つ以外に)

・ベンチマーク・必要な性能の見積り

・手法の開発・改善 → 必要な性能を下げる

—— 演算子形式での格子上の場の理論の整備

—— 特にゲージ理論の適切な正則化方法の模索

・場の量子論 → 量子計算？

—— 新たな誤り訂正方法の提案など

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Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi\ \mathcal{O}(\phi)e^{-S[\phi]} \rightarrow \int d\phi\ \mathcal{O}(\phi)\underbrace{e^{-S(\phi)}}_{\text{probability}}$$

②

Sign problem in Monte Carlo simulation

Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \Rightarrow \int d\phi \ \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\sharp(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

Sign problem in Monte Carlo simulation

Markov Chain Monte Carlo:

$$\int d\phi \; \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

can't directly apply when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$

Sign problem in Monte Carlo simulation

Markov Chain Monte Carlo:

$$\int d\phi\ \mathcal{O}(\phi)\underbrace{e^{-S(\phi)}}_{\text{probability}}$$

can't directly apply when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$

Naïve way to avoid = reweighting:

$$\langle \mathcal{O}(\phi) \rangle = \frac{\int D\phi\ \mathcal{O}(\phi) e^{-S[\phi]}}{\int D\phi\ e^{-S[\phi]}} = \frac{\int D\phi\ \mathcal{O}(\phi) e^{-S[\phi]}}{\int D\phi\ \left|e^{-S[\phi]}\right|} \frac{\int D\phi\ \left|e^{-S[\phi]}\right|}{\int D\phi\ e^{-S[\phi]}}$$

$$= \frac{\langle \mathcal{O}(\phi) \cdot \mathrm{phase}(e^{-S}) \rangle_{\mathrm{no-phase}}}{\langle \mathrm{phase}(e^{-S}) \rangle_{\mathrm{no-phase}}}$$

Sign problem in Monte Carlo simulation

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$$= \frac{\langle \mathcal{O}(\phi) \cdot \mathrm{phase}(e^{-S}) \rangle_{\mathrm{no-phase}}}{\langle \mathrm{phase}(e^{-S}) \rangle_{\mathrm{no-phase}}}$$

For highly oscillating integral, $\sim \frac{0}{0}$ ⇨ needs huge statistics

"sign problem"

Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \; \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " *much worse*

In operator formalism,

sign problem is absent from the beginning

($^{\exists}$various approaches within framework of path integral formalism but I'll skip it)

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Quantum computers do this job?

"Regularization" of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

⟶ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  — Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  — Hilbert sp. at each site is ∞ dimensional
  (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  — ∞ dimensional Hilbert sp. in higher dimensions

Citation history of "Hamiltonian Formulation of Wilson's Lattice Gauge Theories" by Kogut-Susskind

(totally 2285 at this moment)

(1+1)d free Dirac fermion

Continuum:

$$\psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} \qquad \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2$$

$$H = \int dx \left[-i\bar{\psi}\gamma^1\partial_1\psi + m\bar{\psi}\psi \right]$$

$$= \int dx \left[-i(\psi_u^\dagger \partial_1 \psi_d + \psi_d^\dagger \partial_1 \psi_u) + m(\psi_u^\dagger\psi_u - \psi_d^\dagger\psi_d) \right]$$

$\Downarrow$

Lattice (w/ N sites and spacing a)**:** *"Staggered fermion"* [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

ψ_u → odd site

ψ_d → even site

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n\right) + m\sum_{n=1}^{N}(-1)^n\,\chi_n^\dagger\chi_n$$

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

Jordan-Wigner transformation

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}},\ \{\chi_m, \chi_n\} = 0$$

This is satisfied by the operator:

[Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Then the system is mapped to the spin system:

$$\hat{H} = \frac{w}{2}\sum_{n=1}^{N-1}\left(X_n X_{n+1} + Y_n Y_{n+1}\right) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n$$

Now we can apply quantum algorithms to QFT!

Scalar field theory

Continuum Hamiltonian:

$$H = \int d^d\boldsymbol{x} \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi)\right]$$

$$\int d^dx \to a^d \sum_n ,$$

$$\partial_\mu\phi(x) \to \Delta_\mu\phi(x_n) \equiv \frac{\phi(x_n+ae_\mu)-\phi(x_n)}{a}$$

Lattice Hamiltonian (simplest):

$$H = a^d \sum_n \left[\frac{1}{2}\Pi_n^2 + \frac{1}{2}\sum_i(\Delta_i\phi_n)^2 + V(\phi_n)\right]$$

$$[\phi(\boldsymbol{x_m}), \Pi(\boldsymbol{x_n})] = i\delta_{\boldsymbol{m},\boldsymbol{n}}$$

technically the same as multi-particle QM

Regularization for single particle QM

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\text{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;=\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\Longrightarrow \quad \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

regularize!

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\Big|_{\text{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a} + \hat{a}^\dagger)\Big|_{\text{regularized}}$$

$$\hat{p}\Big|_{\text{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a} - \hat{a}^\dagger)\Big|_{\text{regularized}}$$

Regularization for single particle QM (Cont'd)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\, |n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle \cdots |b_0\rangle \qquad K \equiv \log_2 \Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \otimes_{\ell=0}^{K-1} \left(|b'_\ell\rangle\langle b_\ell|\right)$$

either one of

$$|0\rangle\langle 0| = \frac{1_2 - \sigma_z}{2}, \qquad |1\rangle\langle 1| = \frac{1_2 + \sigma_z}{2},$$

$$|0\rangle\langle 1| = \frac{\sigma_x + i\sigma_y}{2}, \qquad |1\rangle\langle 0| = \frac{\sigma_x - i\sigma_y}{2}$$

Pure Maxwell theory

Continuum:

$$\mathcal{H} = \frac{1}{2} E_i^2 + \frac{1}{2} B_i^2 \qquad \partial_i E^i = 0$$

Lattice:

$$\mathcal{H} = \frac{a^d}{2} \sum_{\boldsymbol{n},i} L_{\boldsymbol{n},i}^2 + \mathrm{Re} \sum_{\mathrm{plaquette}} \sum_{i<j} \prod_{P \in \mathrm{plaquette}} U_P$$

$$[U_{\boldsymbol{m},i}, L_{\boldsymbol{n},j}] = i \delta_{ij} \delta_{\boldsymbol{m},\boldsymbol{n}}$$

Gauss law:

$$\sum_i (L_{\boldsymbol{n}+\boldsymbol{e}_i,i} - L_{\boldsymbol{n},i}) = 0$$

Ex. (1+1)d pure Maxwell theory w/ θ

Continuum:

$$\mathcal{L} = \frac{1}{2g^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} \quad \xrightarrow{\Pi = \frac{1}{g^2}\dot{A} + \frac{\theta}{2\pi}} \quad \mathcal{H} = \frac{1}{2}\left(\Pi - \frac{\theta}{2\pi}\right)^2$$

Lattice:

$$H = \frac{g^2 a}{2} \sum_n \left(L_n + \frac{\theta}{2\pi}\right)^2 \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

Gauss law:

$$L_{n+1} - L_n = 0$$

- open b.c.

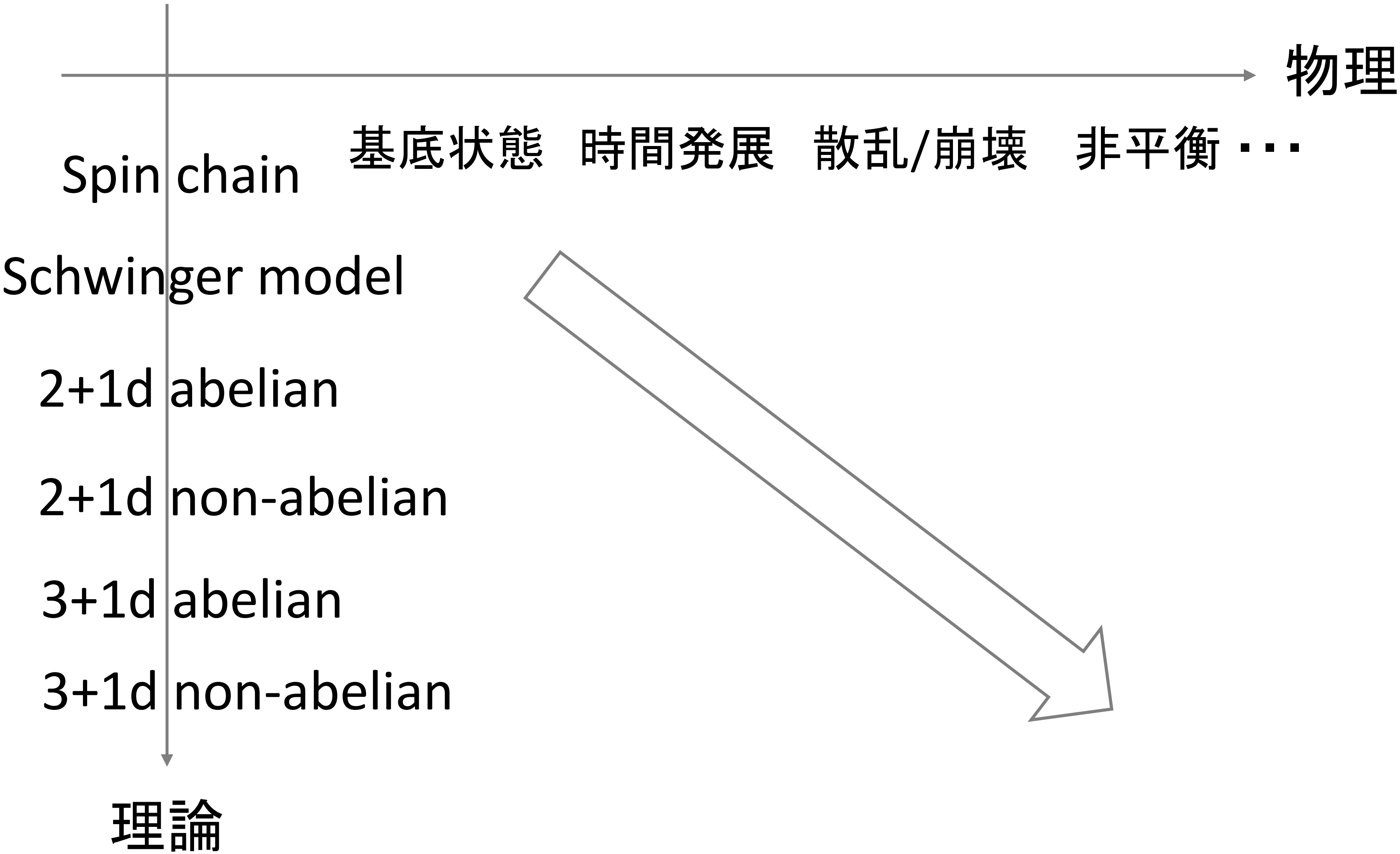
$$L_n = L_{n-1} = L_{n-2} = \cdots = L_1 = (b.c.)$$

- p.b.c.

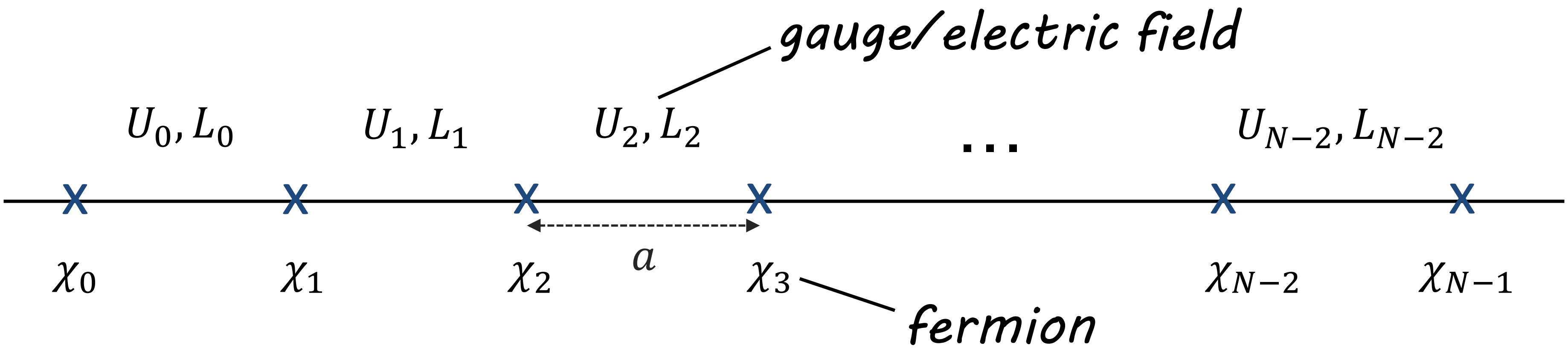
$$L_n = L_{n-1} = \cdots = L_1 = \cdots = L_{n+1} = L_n$$

one d.o.f. remains

分野の大体の研究の流れ (?)



Charge-q Schwinger model



$$H = J\sum_{n=0}^{N-2}\left(L_n + \frac{\theta_0}{2\pi}\right)^2 - \mathrm{i}w\sum_{n=0}^{N-2}\left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.}\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n$$

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Physical states are subject to Gauss law:

$$(L_n - L_{n-1})|\mathrm{phys}\rangle = q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right]|\mathrm{phys}\rangle$$

"$\nabla \cdot \vec{E}\,(x)$" "$\rho(x)$ "

Schwinger model as qubits

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q \sum_{j=1}^{n} \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

3. Map to spin system: $\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$ (X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

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"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$H = J \sum_{n=0}^{N-2} \left[q \sum_{i=0}^{n} \frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi} \right]^2 + \frac{w}{2} \sum_{n=0}^{N-2} \left[X_n X_{n+1} + Y_n Y_{n+1} \right] + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

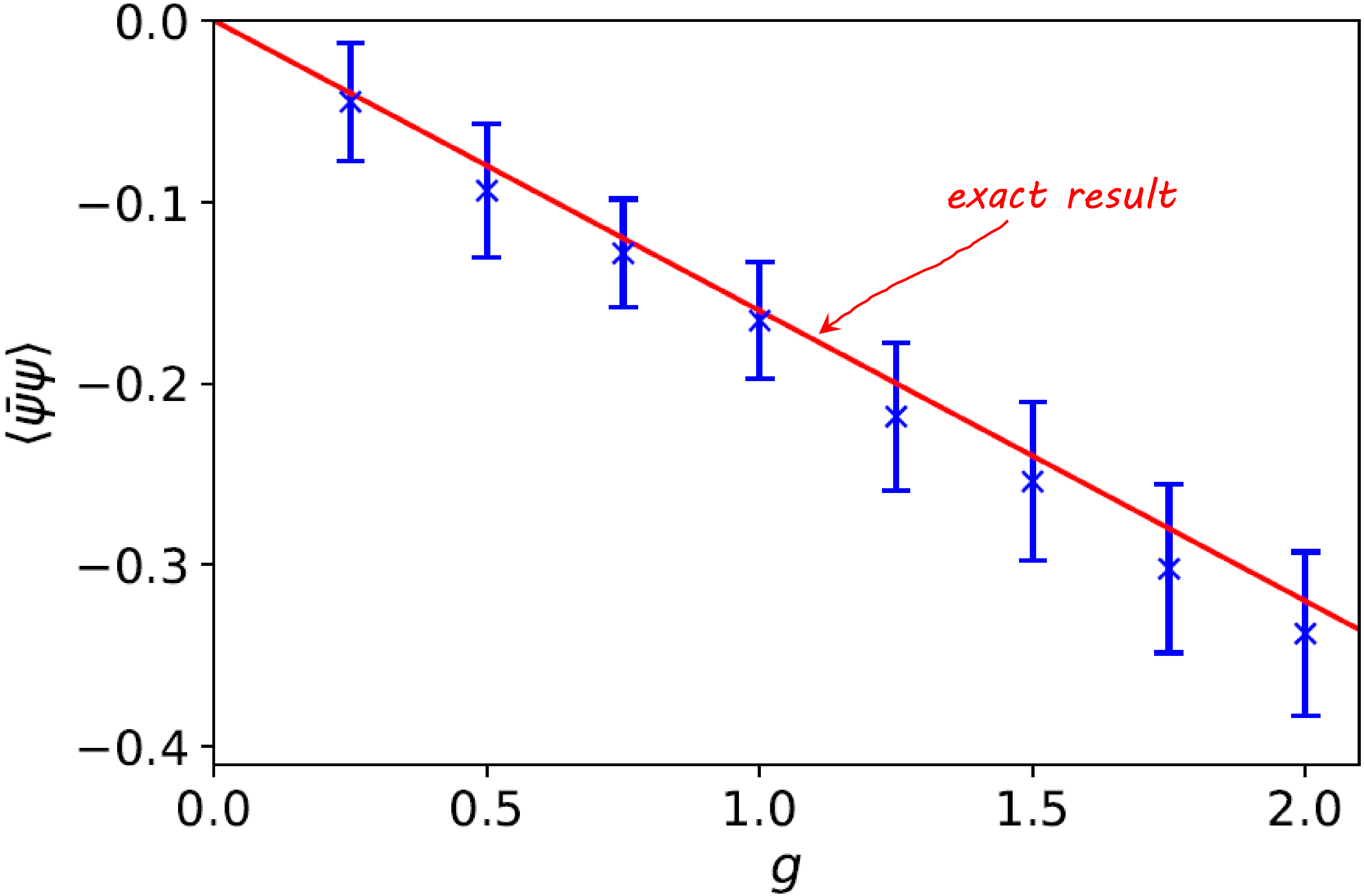
Qubit description of the Schwinger model !!

Ground state expectation value in massless case

$T = 100, \delta t = 0.1, N_{\max} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

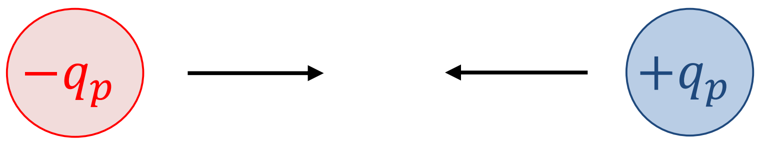
(after continuum limit)



Screening versus Confinement

Let’s consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

Coulomb law in 1+1d
||
confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$

screening

- massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right)$$ *screening*

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right)x \qquad (m \ll g,\ |x| \gg 1/g\ )$$

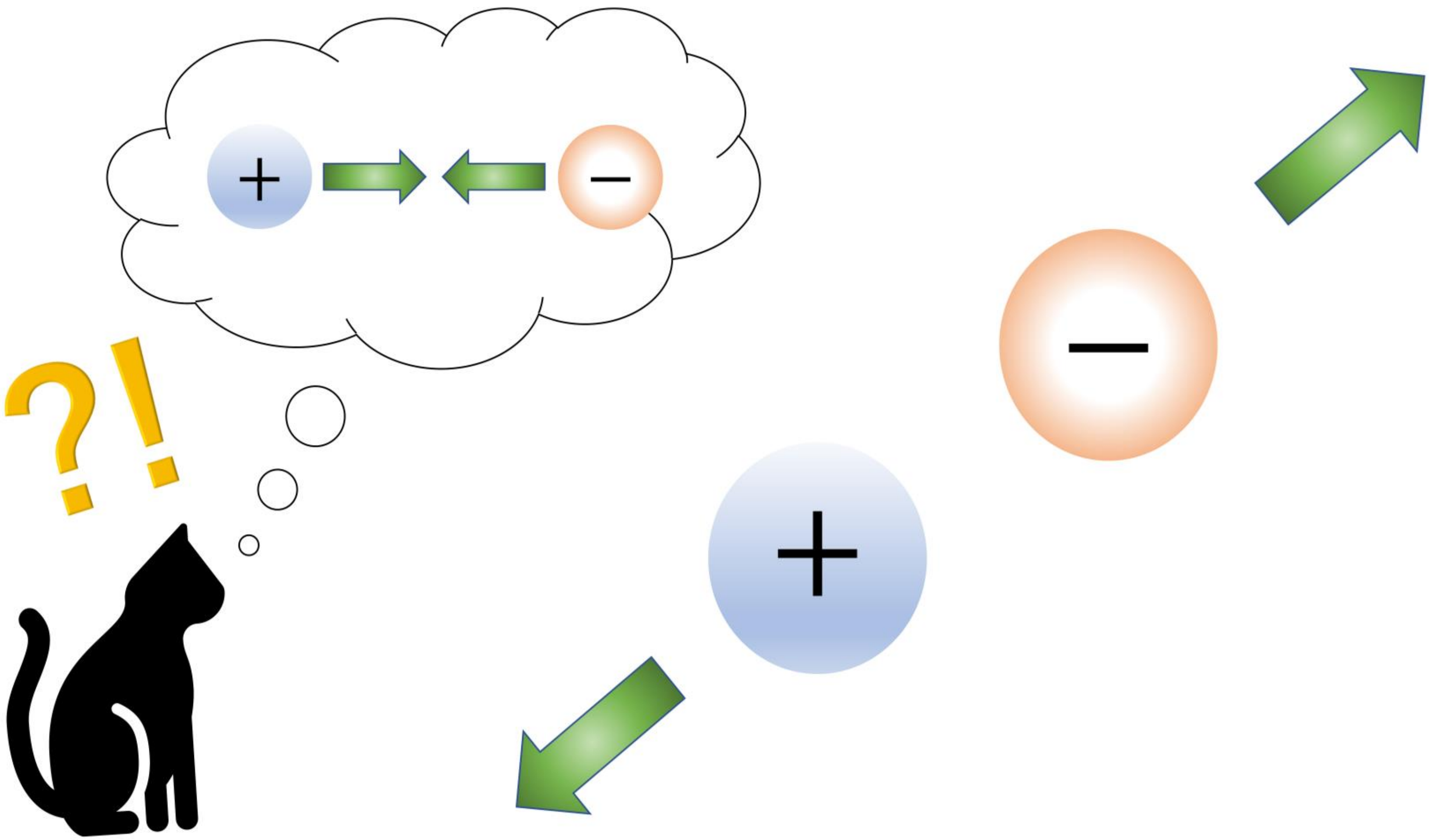
$$\begin{cases} = \text{Const.} & \text{for } q_p/q = \mathbf{Z} \\ \propto x & \text{for } q_p/q \neq \mathbf{Z} \end{cases}$$

= Const. for $q_p/q = \mathbf{Z}$ *screening*

$\propto x$ for $q_p/q \neq \mathbf{Z}$ *confinement?*

*but sometimes **negative** slope!*

That is, as changing the parameters…



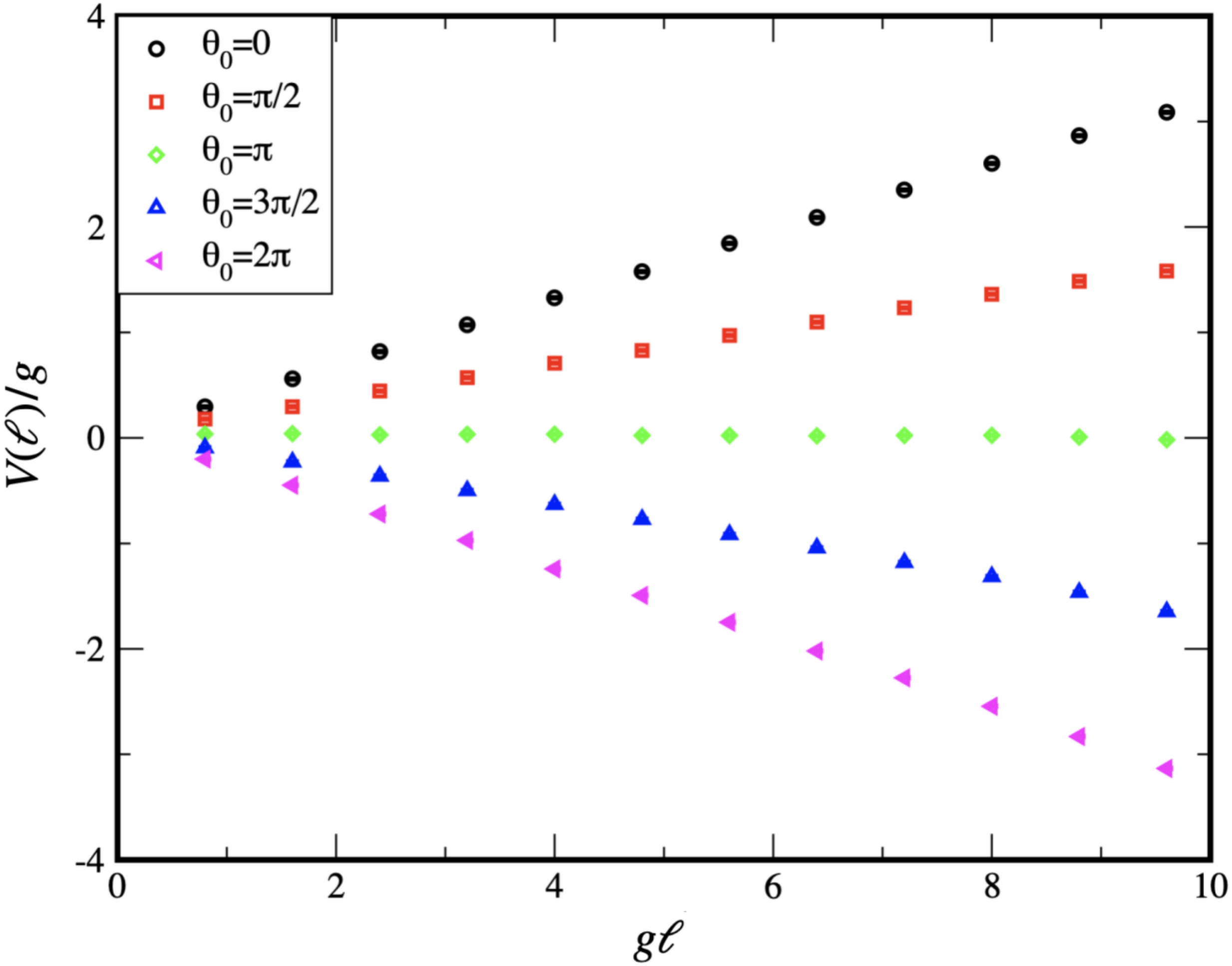
Let’s explore this aspect by quantum simulation!

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki '21]

[cf. MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

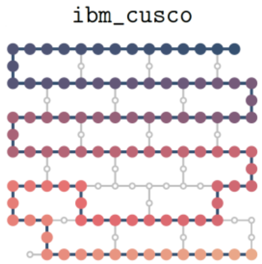


Sign(tension) changes as changing θ-angle!!

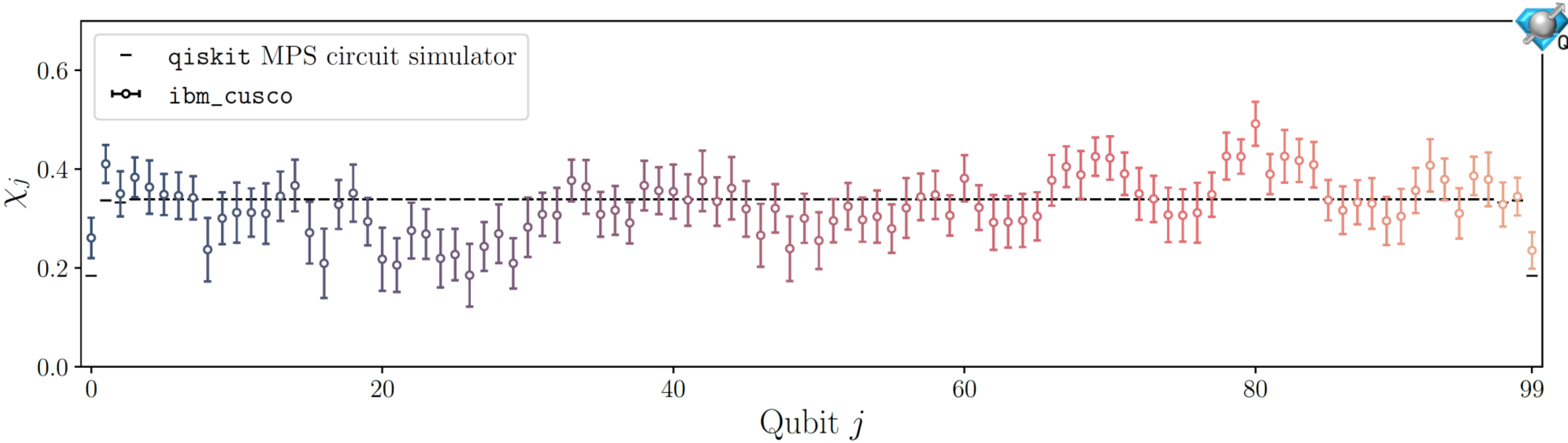
100 qubit simulation of Schwinger model

(127-qubit device: **ibm_cusco** w/ error mitigation)

[Farrel-Illa-Ciavarella-Savage '23]



Ground state exp. of local chiral condensate :



Other simulations of Schwinger model

- decay of massive vacuum under time evolution

  [cf. Martinez etal. **Nature** 534 (2016) 516-519]

- quenched dynamics of θ [Nagano-Bapat-Bauer '23]

- Schwinger model in open quantum system

  [De Jong-Metcalf-Mulligan-Ploskon-Ringer-Yao '20, de Jong-Lee-Mulligan-Ploskon-Ringer-Yao '21, Lee-Mulligan-Ringer-Yao '23]

- 112 qubit simulation of meson propagation

  [Farrell-Illa-Ciavarella-Savage '24]

- finding energy spectrum [MH-Ghim, work in progress]

- finite temperature [Itou-Sun-Pedersen-Yunoki '23]

etc...

Scattering in Thirring model

Thirring model on lattice:

[Chai-Crippa-Jansen-Kuhn-Pascuzzi-Tacchino-Tavernelli '23]

$$H = \sum_{n=0}^{N-1} \left(\frac{i}{2a} \left(\xi_{n+1}^\dagger \xi_n - \xi_n^\dagger \xi_{n+1} \right) + (-1)^n m \, \xi_n^\dagger \xi_n \right) + \sum_{n=0}^{N-1} \frac{g(\lambda)}{a} \xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1},$$

Scattering in Thirring model

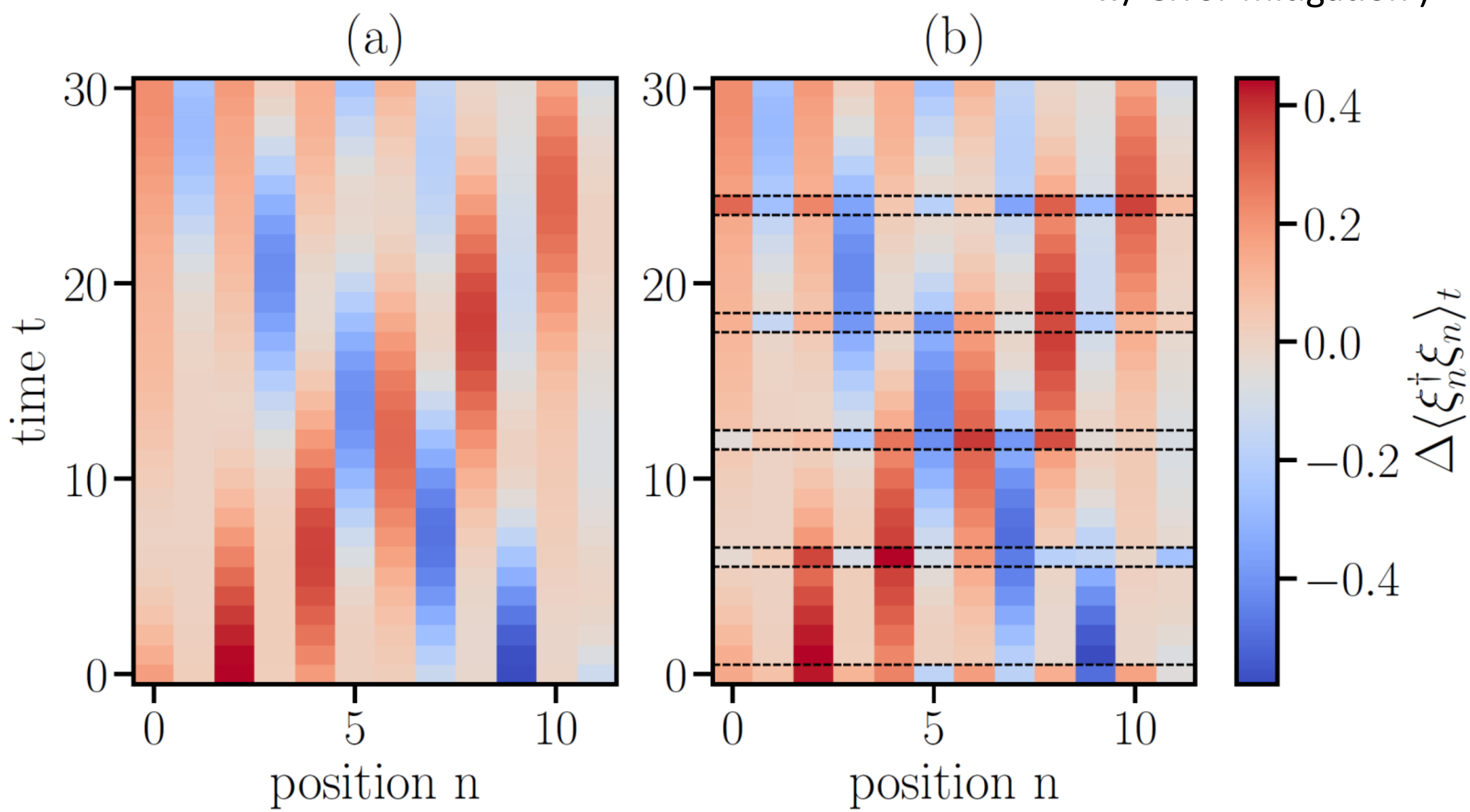
Thirring model on lattice:

[Chai-Crippa-Jansen-Kuhn-Pascuzzi-Tacchino-Tavernelli '23]

$$H = \sum_{n=0}^{N-1} \left(\frac{i}{2a} \left(\xi_{n+1}^\dagger \xi_n - \xi_n^\dagger \xi_{n+1} \right) + (-1)^n m \, \xi_n^\dagger \xi_n \right) + \sum_{n=0}^{N-1} \frac{g(\lambda)}{a} \xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1},$$

Particle density of two wave packets:

(12-qubit device: **ibm_peekskill** w/ error mitigation)



On higher dimensional fermion

Go to higher dimensions!

[MH, work in progress]

⇩

1st step: find a nice way to map 2d fermion to spins

Problem in naïve approach:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

- 1d

$$\chi_{n+1}^{\dagger}\chi_n \xrightarrow{\text{Jordan-Wigner}} {}^{\exists}X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

local

- 2d

On higher dimensional fermion

Go to higher dimensions! [MH, work in progress]

⇩

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- 1d

$$\chi_{n+1}^\dagger \chi_n \xrightarrow{\text{Jordan-Wigner}} \exists X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

local

- 2d ($N \times N$ square lattice)

Relabeling site (i,j) like 1d label (say $n = i + Nj$),

$$\chi_{(i,j+1)}^\dagger \chi_{(i,j)} = \chi_{I+N}^\dagger \chi_I \xrightarrow{\text{JW}} \exists X_{I+N}X_I \prod_{i=I+1}^{I+N-1} Z_i \text{ , etc...}$$

(cf. $\mathcal{O}(\log N)$ for Bravyi-Kitaev trans.) *non-local*

Application of a new map to field theory

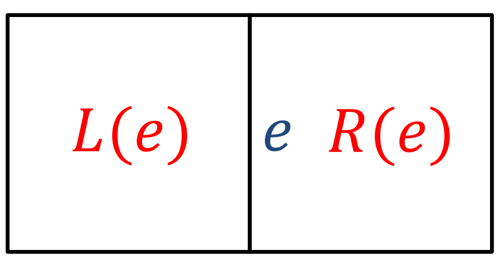
[Chen-Kapustin-Radicevic '17]

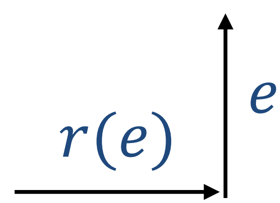
2 Majorana fermions on face $\Longleftrightarrow$ Spin op. on edge

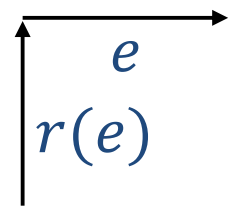
$$(-1)^{F_f} = -i\gamma_f \gamma'_f \longleftrightarrow W_f. \qquad S_e = i\gamma_{L(e)}\gamma'_{R(e)} \longleftrightarrow U_e$$

where $$W_f = \prod_{e \subset f} Z_e. \qquad U_e = X_e Z_{r(e)}.$$

"Gauss law" constraint at site v: $$W_{\mathrm{NE}(v)} \prod_{e \supset v} X_e = 1.$$







ex.) $$H = t\sum_e (c^\dagger_{L(e)} c_{R(e)} + c^\dagger_{R(e)} c_{L(e)}) + \mu \sum_f c^\dagger_f c_f.$$

$\Longrightarrow$ $$H = \frac{t}{2}\sum_e X_e Z_{r(e)}(1 - W_{L(e)}W_{R(e)}) + \frac{\mu}{2}\sum_f (1 - W_f)$$ *local*

Some other applications

- Efficient simulation of (2+1)d U(1) gauge th. [Kane-Grabowska-Nachman-Bauer '22]
- Inflation (scalar in curved spacetime) [Liu-Li '20]
- Chiral fermion [Hayata-Nakayama-Yamamoto '23]
- Quantum group approach to Non-abelian gauge th. [Zache-Gonzalez-Cuadra-Zoller '23, Hayata-Hidaka '23]
- Conformal bootstrap [Bao-Liu '18]
- Dark sector showers [Chigusa-Yamazaki '22, Bauer-Chigusa-Yamazaki '23]
- Measurement-based quantum computation [Okuda-Sukeno '22]
- quantum machine learning [Nagano-Miessen-Onodera-Tavernelli-Tacchino-Terashi '23, etc…]
- String/M-theory [Gharibyan-Hanada-MH-Liu '20]

etc…

Contents

1. Introduction

2. QC for QFT

3. QFT for QC (?)

4. Summary

relations between QEC & gauge theory

Motivations

[Spirit may be similar to Rajput-Roggaro-Wiebe '21, Gustafson-Lamm '23, etc...]

1. $^{\exists}$ explicit examples

   ex.) Toric code = $\boldsymbol{Z_2}$ lattice gauge theory [Kitaev '97]

2.

3.

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relations between QEC & gauge theory

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2. Conceptual similarities:

   QEC = redundant description of logical qubits

   Gauge theory = redundant description of physical states

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relations between QEC & gauge theory

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3. Nature = Gauge theory & Nature = Quantum computer

   ⟹ Gauge theory may know something on QEC?

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   Gauge theory = redundant description of physical states

3. Nature = Gauge theory & Nature = Quantum computer

   ⟹ Gauge theory may know something on QEC?

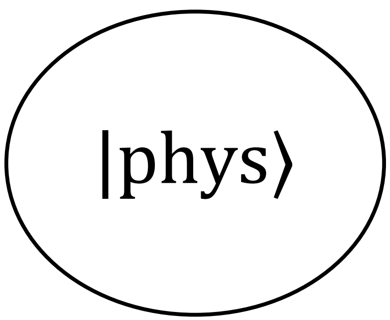
4. $^{\exists}$proposals on relations among QEC & concepts in HEP

   ex.) Holography, Black hole, CFT, Renormalization group

   [Almheiri-Dong-Harlow '14, Hayden-Preskill '07, Dymarsky-Shapere '20, Kawabata-Nishioka-Okuda '22, Furuya-Lashkari-Moosa '21, etc...]

Gauge theory on QC w/ error correction

When we don't solve Gauss law before simulation…



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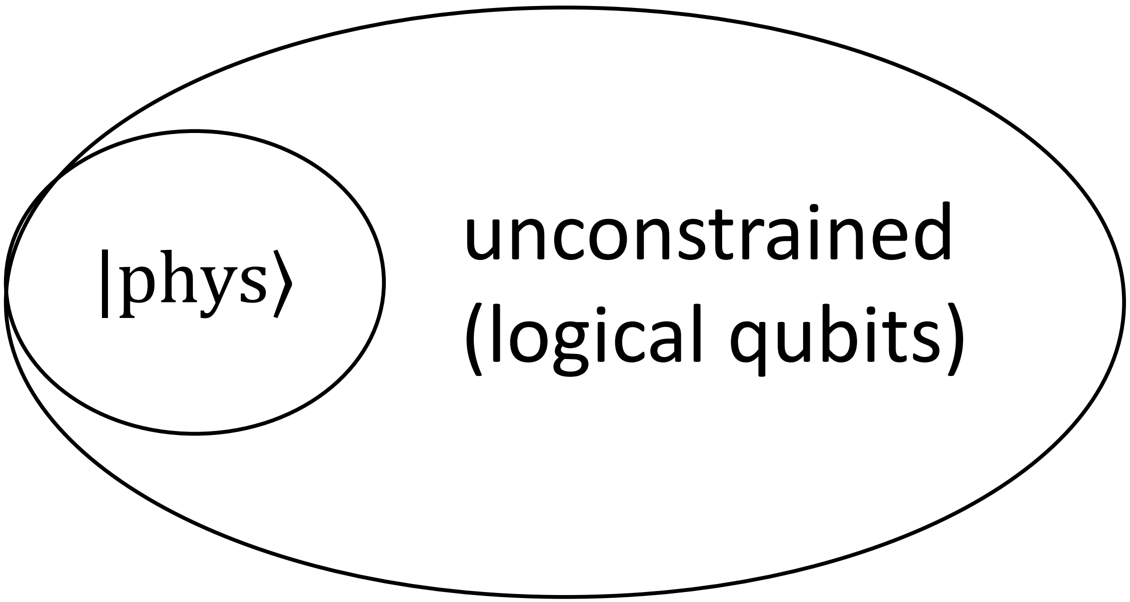

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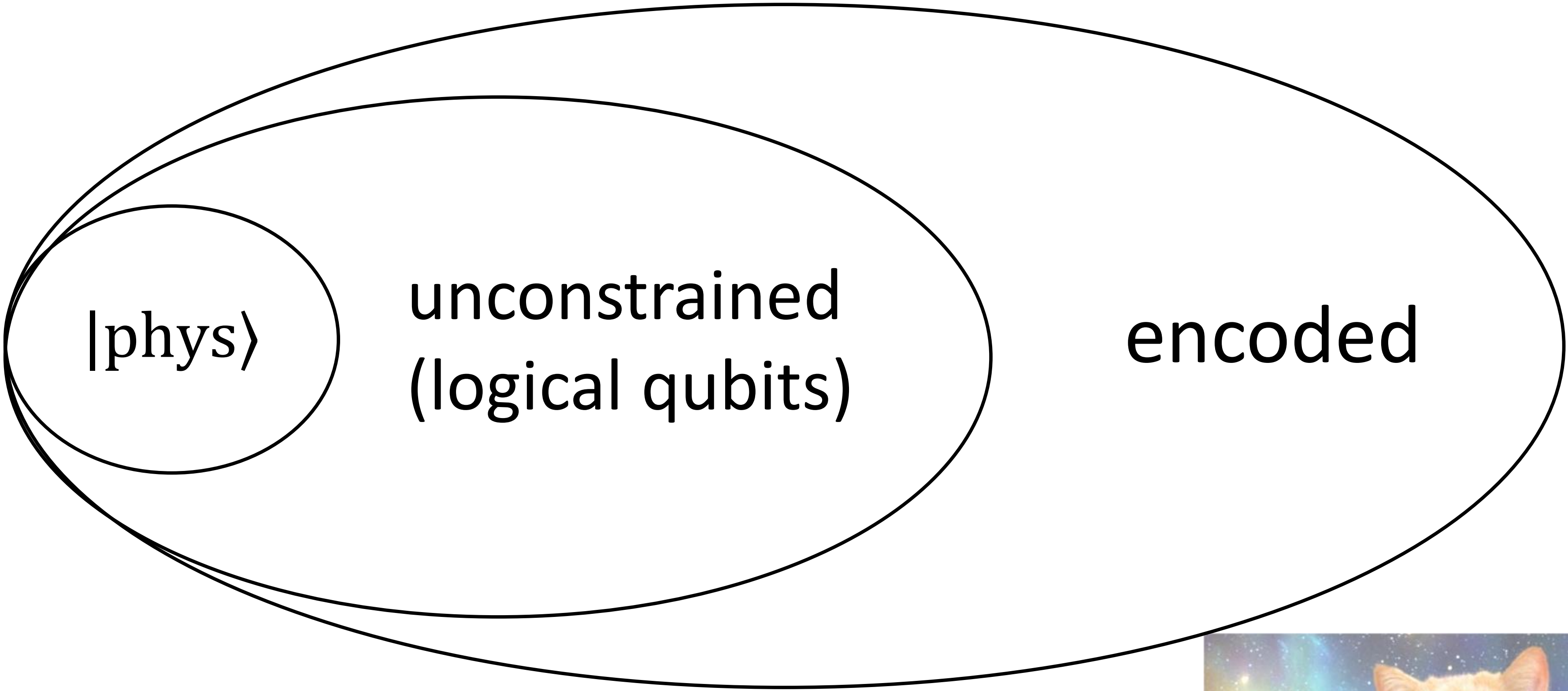


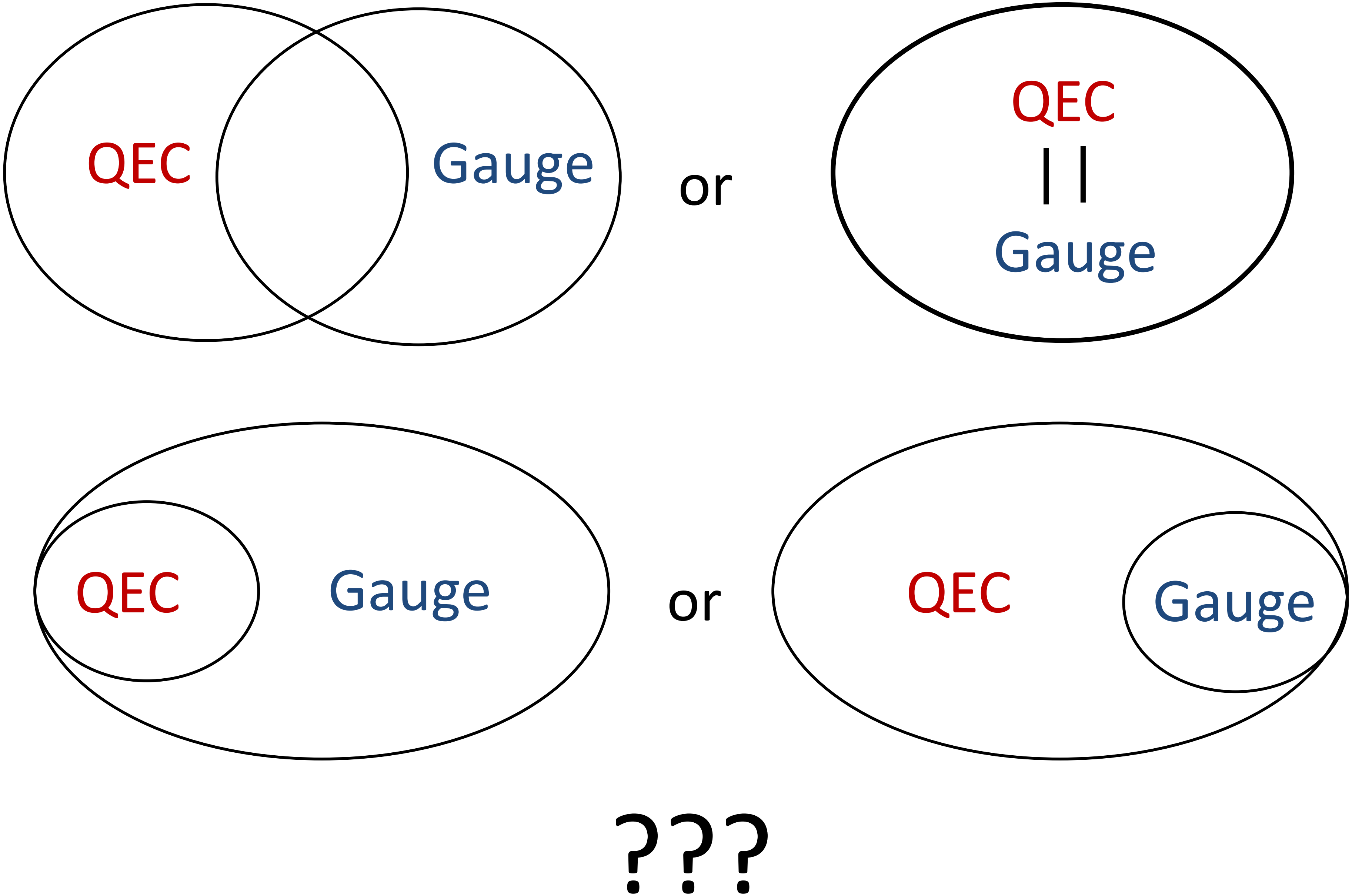
redundancy^2 !!

What I'm doing...

[MH, work in progress]

to make dictionary for classes of codes/gauge theories:

QEC	Gauge theory
errors	unphysical op. (& excitation)
logical qubits	physical states (w/ low energy)
"no error conditions" (stabilizer)	Gauss law (& min[energy])
logical op.	gauge invariant op.
ancilla for recovery	additional matter
⋮	⋮


QEC
Gauge
or
QEC
||
Gauge
QEC
Gauge
or
QEC
Gauge
???

QFT as a generator of error correcting code?

Toric code

- Lattice model interpreted as QEC
- Low energy effective theory = QFT (BF theory)

QFT ↔ Lattice model ↔ QEC

Idea : if we get something new in one of them,
then try to fill the other parts

ex.) "Dipolar" generalization of Toric code [Pace-Wen '22]

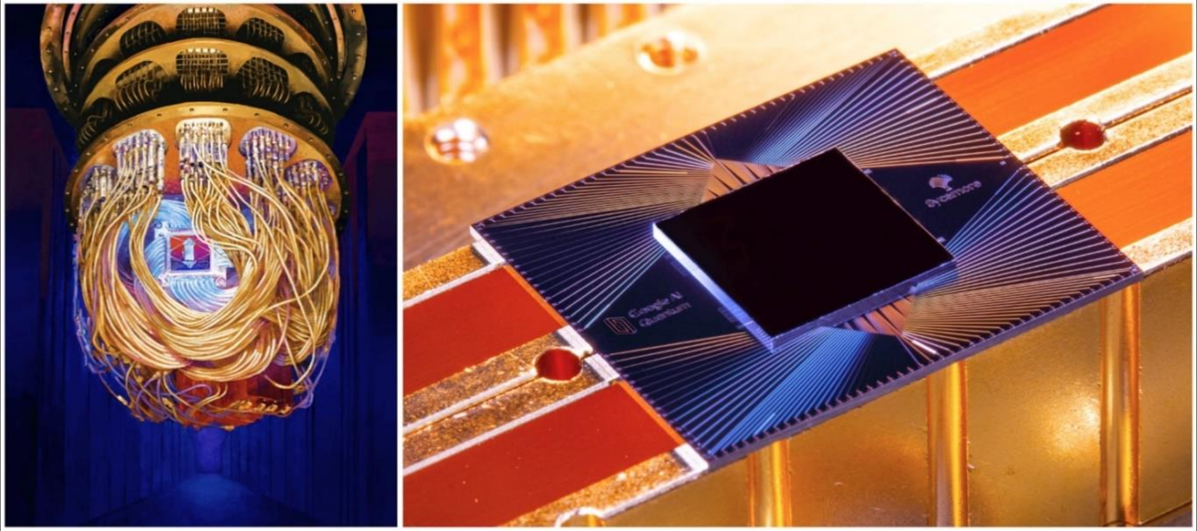
⇩

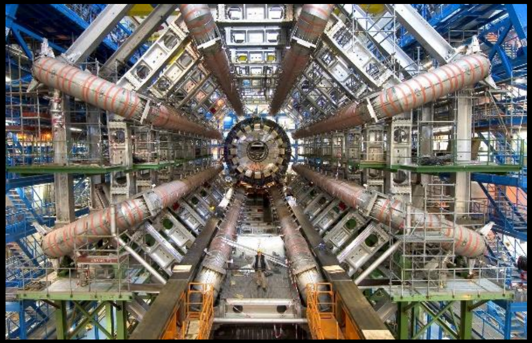
corresponds to a "layer" of BF theory w/ some rule

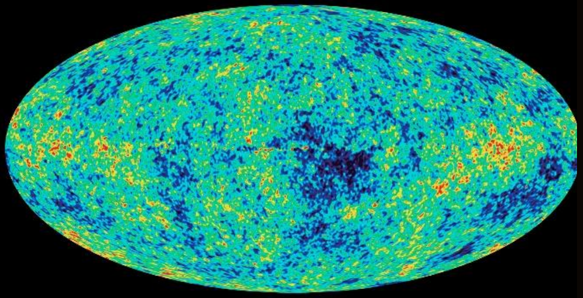
[Ebisu-MH-Nakanishi '23]

Summary

できるようになりそうなこと (期待)







etc...

課題

・一定以上の性能をもつ量子計算機が必要

(qubit数, fidelity, etc...)

・量子計算機で効率的に解ける問題の開拓

やるべきこと:　(ハードの発展を待つ以外に)

・ベンチマーク・必要な性能の見積り

・手法の開発・改善 → 必要な性能を下げる

—— 演算子形式での格子上の場の理論の整備

—— 特にゲージ理論の適切な正則化方法の模索

・場の量子論 → 量子計算？

—— 新たな誤り訂正方法の提案など

Thanks!