AdS/CFT CORRESPONDENCE

1. LARGE N LIMIT OF "MATRIX QUANTUM SYSTEMS"
2. MALDACENA'S ARGUMENT ($\mathcal{N}=4$ YANG MILLS / IIB String THEORY ON AdS_5/S^5
3. EUCLIDEAN SPACE [CORRELATION FUNCTIONS – IN GRAVITY)
4. ~~HILBERT SPA~~ LORENTZIAN ASPECTS
   - HILBERT SPACE
   - EXPECTATION VALUES IN STATES
   - [FLUID DYNAMICS] FINITE TEMPERATURE
5. OTHER EXAMPLES
   - BREAK CONFORMAL INV
   - (THEORY ON WV OF OTHER BRANES
   - AdS_3/CFT_2

LECTURE 1 (TOPIC 1)

Sym $M_i \to U M_i U^{-1}$

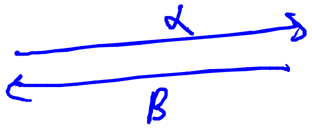
$$Z = \int DM_i \exp\left[-\frac{N}{\lambda} \mathrm{Tr}\, f(M_i)\right]$$

M_i are $\underline{N} \times N$ HERMITIAN MATRICES.

$M_i = A_\mu(x) \quad [\, i \equiv (\mu, x) \,] \quad (\text{Eg})$

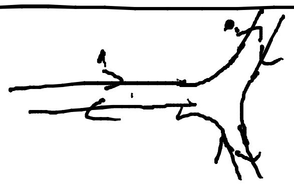
$Tr f = [\boxed{M_i} M_j \, g^{ij} + \cdots]$

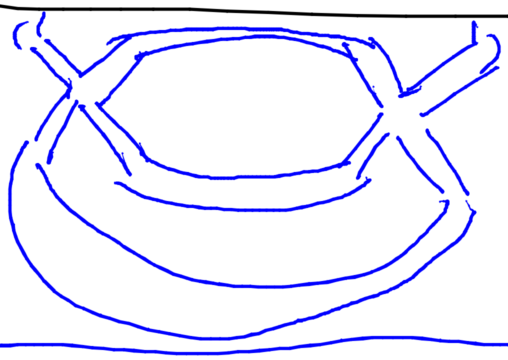
g^{ij}

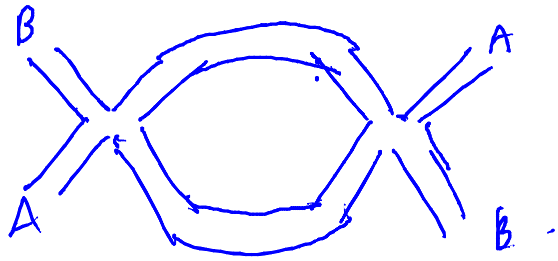


$(M^\alpha{}_\beta)$

$$Tr(M^3) = \overset{i}{M_\alpha{}^\beta} \, \overset{j}{M_\beta{}^\theta} \, \overset{k}{M_\theta{}^\alpha}$$



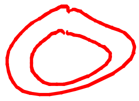




- Powers of λ. $\quad S = \frac{N}{\lambda} Tr f(m)$

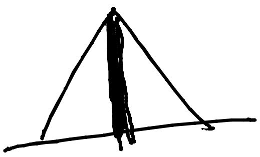
P Propagators ; V Vertices

$$\lambda^{P-V} = \lambda^{L} \quad (L = \#\text{ of Loops})$$



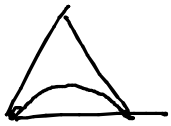
N^{V-P+F}

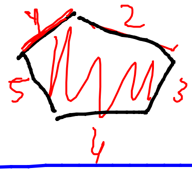
F = # INDEX LOOPS.

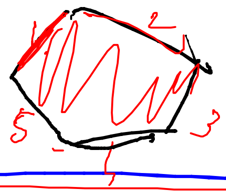


$V-P+F$

$V' = V+1$
$P' = P+2$
$F' = F+1$



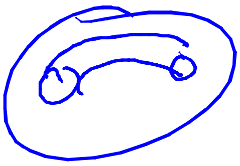




q sided Polygon

$V' = V$
$P' = P+q$
$F' = F+q-2$

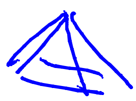
$V'-P'+F'$
$= (V-P+F) - 2$



$g = 0$
genus

Torus
$g = 1$

$V-P+F$ for the sphere



$V = 4$, $P = \frac{4\times 3}{2} = 6$; $F = 4$

$V-P+F = 2$

N^{2-2g} (g = genus of surface)

$$\left(\chi^L N^{2-2g}\right)$$

$1/N^2$ = GENUS COUNTING PARAMETER

$$\underbrace{c_1 \left(\mathrm{Tr}\, M^{k_1}\right)}_{O_1} + \underbrace{\alpha_2 \left(\mathrm{Tr}\, M^{k_2}\right)}_{O_2} \cdots \underbrace{\alpha_n M^{k_\alpha}_{k}}_{O_L}.$$

$$\alpha_1 = \tilde{\alpha}_1 N \quad \left(\tilde{\alpha}_1 = \frac{\alpha_1}{N}\right)$$

$$N\, \mathrm{Tr}\left(f(M, \tilde{\alpha})\right) \quad \Bigg| \quad \frac{\partial}{\partial \tilde{\alpha}_1} \frac{\partial}{\partial \tilde{\alpha}_2} \cdots \frac{\partial}{\partial \tilde{\alpha}_n} \ln Z = N^{2-2g}$$

EVERY INSERTION OF $\mathrm{Tr}(M^k)$ ADDITIONAL FACTOR OF $\#\underbrace{\left(\frac{1}{N}\right)}_{\text{cancels}} \times N^{2-2g}$

$$N^2 + \left(\frac{1}{N}\right)^2 = 1$$