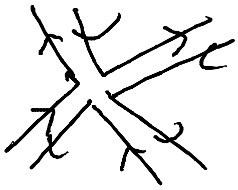
$$Z = \int dM_i \exp\left[-\frac{N}{\lambda} \mathrm{Tr} f(M_i)\right]$$

M_i were $N \times N$ Hermitian Matrices.

$\rightleftarrows$ 

$\lambda^{L-1} N^{2-2g}$ | $\underset{\sim}{\mathrm{Tr}\, M^k}$ | Additional factor of $1/N$.

Leading Large N (Planar limit).

$\ln Z = O(N^2)$ | k operator insertions $\sim N^{2-k}$.

$k=2$, N^0 ; 3pt fn $\sim \frac{1}{N}$

4pt fn $\sim \frac{1}{N^2}$

INTEGRAL is a 0+0 FIELD THEORY

a) $\int dC \equiv \int \prod_{i,j} d^2 C_{ij}$ (Integral over complex Matrices)

$\delta(C) = \prod_{i,j} \delta^2(C_{ij})$.

b) $\int (dA)_H \equiv \boxed{\int dA\, \delta(A - A^+)}$

$\equiv \int \prod_{i<j} d^2 A_{ij} \prod_i dA_{ii}$

$\delta_H(A) = \prod_{i<j} \delta^2(A_{ij}) \prod_i \delta(A_{ii})$.

$$A \to U A U^\dagger .$$

c) $\int (dC)_w = \int dC \, \delta_H (C C^\dagger - I)$.

$C \to C U \quad C \to V C$.

$$Z = \int (dA)_H \exp\left[-\frac{N}{\lambda} \operatorname{Tr} f(A) \right] .$$

$$\int dD \, (dU)_u \, \delta_H (U D U^\dagger - A) \, \Delta_{FP}(A) = 1 .$$

$\Delta_{FP}(A) = \Delta_{FP}(V A V^\dagger)$.
(FOR AN ARBITRARY UNITARY MATRIX V)

$A = V D_0 V^\dagger$. ; $\Delta_{FP}(A) = \Delta_{FP}(D_0)$

$$\frac{1}{\Delta_{FP}(A)} = \frac{1}{\Delta_{FP}(D_0)} = \int dD \, (dU)_u \, \delta (U D U^\dagger - D_0)$$

$U = I + iH$; $D = D_0 + \delta D$

$$\int d(\delta D) \, dH \, \delta \Big[\underbrace{i[H, D_0] + \delta D_0}_{N} \Big] .$$

$N_{ij} = i(d_i^0 - d_j^0) H_{ij}$ $(i \neq j)$ | $N_{ii} = (\delta D)_{ii}$

$$\frac{1}{\Delta_{FP}(A)} = \frac{1}{\Delta_{FP}(D_0)} = \frac{1}{\prod_{i<j} |d_i^0 - d_j^0|^2}$$

$$\overline{\Delta_{FP}(A)} - \overline{\Delta_{FP}(D_0)} - \overline{\prod_{i<j} |d_i^0 - d_j^0|^2}$$

$$\Delta_{FP}(A) = \prod_{i<j} |d_i^0 - d_j^0|^2$$

$$\int dD\, dU\, \delta(UDU^+ - A)\, \Delta_{FP}(A) = 1$$

$$\int dA\, \exp\left[-\frac{N}{\lambda} \mathrm{Tr} f(A)\right].$$

$$\int dD\, \exp\left[-\frac{N}{\lambda} \mathrm{Tr} f(D)\right] \prod_{i<j} |d_i - d_j|^2$$

$$\int \prod d(d_i)\, \exp\left[-\frac{N}{\lambda} \sum_i f(d_i) + 2\sum_{i\neq j} \ln |d_i - d_j|\right] -$$

$$-\frac{N}{\lambda} f'(d_i) + \sum_j \frac{2}{d_i - d_j} = 0$$

$$\rho(x) = \frac{1}{N} \sum_j \frac{1}{x - d_j}$$

$$\rho^2(x) = \frac{1}{N^2} \sum_j \frac{1}{(x-d_j)^2} + \frac{1}{N^2} \sum_{i\neq j} \frac{1}{(x-d_i)(x-d_j)}$$

$$\frac{1}{(x-a)(x-b)} = \frac{1}{(a-b)}\left[\frac{1}{(x-a)} - \frac{1}{(x-b)}\right]$$

$$= \left(\frac{1}{N^2} \sum_j \frac{1}{(x-d_j)^2} + \frac{2}{N^2} \sum_k \frac{1}{(d_j - d_k)} \frac{1}{(x-d_j)}\right)$$

$$\rho^2(x) = \frac{1}{N}\sum_j \frac{f'(d_j)}{(x-d_j)}$$

$$\lambda \rho^2(x) = \frac{1}{N}\sum_j \frac{f'(x)}{x-d_j} + \frac{1}{N}\sum_j \frac{f'(d_j)-f'(x)}{x-d_j}.$$

$$\lambda \rho^2(x) = \rho(x) f'(x) + $$

Let f be a polynomial of degree q

$g(x)$ of degree $q-2$

$$\lambda \rho^2(x) = \rho(x) f'(x) + g^{q-2}(x)$$

$$\rho(x) = \frac{1}{2}\left(\frac{f'(x)}{\lambda} \pm \sqrt{\frac{(f'(x))^2}{\lambda^2} + 4 g^{q-2}(x)}\right)$$

$$\rho(x) = \frac{1}{2}\left(\frac{f'(x)}{\lambda} - \sqrt{\frac{(f'(x))^2}{\lambda^2} + 4 g^{q-2}(x)}\right)$$

$f(x) = x^2/2$ | $q = 2$, $q-2 = 0$

$f'(x) = x$

$$\rho(x) = \frac{1}{2}\left(\frac{x}{\lambda} - \sqrt{\left(\frac{x}{\lambda}\right)^2 \cancel{+} a^2}\right)$$

$$= \frac{1}{2}\left(\frac{x}{\lambda} - \frac{x}{\lambda}\sqrt{1 \;\; \left(\frac{a\lambda}{x}\right)^2}\right)$$

$$= \frac{x}{\lambda} - \frac{x}{\lambda}\left[1 - \frac{1}{2}\frac{a^2\lambda^2}{x^2}\right]$$

$$= \frac{a^2\lambda}{4} \cdot \frac{1}{x}$$ | $a^2 = 4/\lambda$

$$\rho(x) = \frac{1}{2}\left[\frac{x}{\lambda} - \sqrt{\left(\frac{x}{\lambda}\right)^2 - \frac{4}{\lambda}}\right]$$

$$\rho(x) = \frac{1}{N}\sum_i \delta(x - d_i).$$

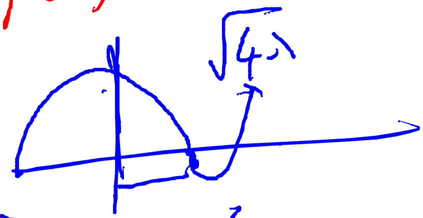
$$2i \quad \cancel{2\pi i}\sum_i 2\pi i\, h(d_i)$$

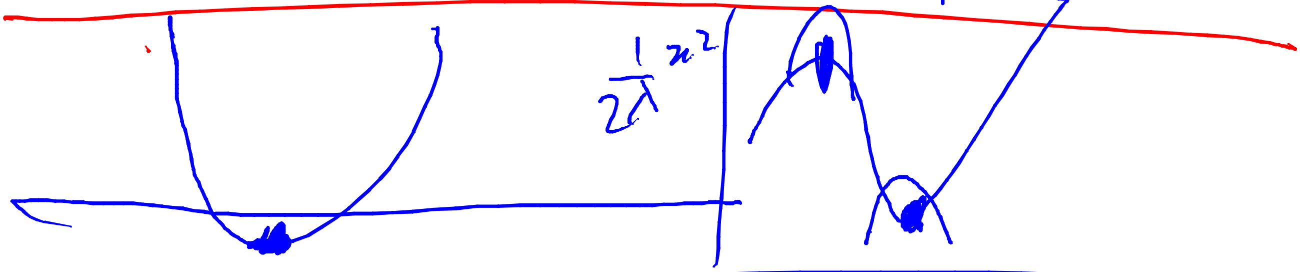
$$= 2\pi i \int \rho(x)\, h(x)$$

Square root −ve when ($x^2 < 4\lambda$)

$$2i\sqrt{\frac{4}{\lambda} - \left(\frac{x}{\lambda}\right)^2} = 2\pi i\, \rho(x)$$

$$\rho(x) = \frac{1}{\pi}\sqrt{\frac{4}{\lambda} - \left(\frac{x}{\lambda}\right)^2}$$





$$\frac{1}{N}\mathrm{Tr}(A^k) = \frac{1}{N}\sum_j d_j^k = \int x^k \rho(x)\, dx$$