$\mathcal{N}=4$ SYM THEORY.

$U(N)$ GAUGE THEORY.

$(A_\mu)_\alpha^{\ \beta}$; $(\phi^i)_\alpha^{\ \beta}$; $(\psi^a_\alpha \ , \ \bar{\psi}_{a\dot\beta})$

$$W \propto \mathrm{Tr}\left[X\,[Y, Z]\right]$$

$Q_{m\alpha}$; $\bar{Q}^m_{\dot\alpha}$

$SU(4)$: 4, $\bar{4}$ } Two 4 d reps

$SO(6)$: Spinor Chiral (4), Spinor Antichiral $(\bar{4})$

Fund (4) of $SU(4)$ $\longleftrightarrow$ Chiral spinor

Anti fund $(\bar{4})$ of $SU(4)$ $\longleftrightarrow$ Antichiral spinor.

ϕ^{ij} ; $\phi^{ij} = \epsilon^{ijkm} \bar{\phi}^*_{km}$.

$(\bar{\phi}^*_{km}) = (\phi^{km})^*$

$$Q_{m\alpha}\, \phi^{ij} = \delta_m^{\ i} \psi^j_\alpha - \delta_m^{\ j} \psi^i_\alpha$$

$$Q_{m\alpha}\, \psi^l_\beta = f_{\alpha\beta}\, \delta_m^{\ l} + [\phi_{mn}, \phi^{nl}]\, \epsilon_{\alpha\beta}.$$

$$Q_{m\alpha}\, \bar{\psi}_{\dot\beta j} = D_{\alpha\dot\beta}\, \phi_{mj}$$

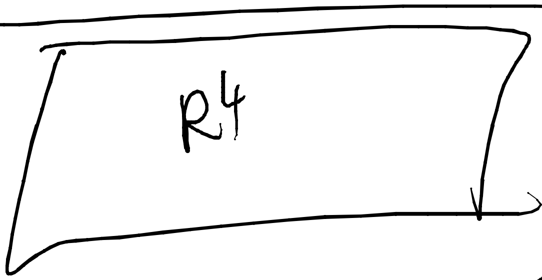
$$Q_{m\alpha}\, A_{\beta\dot\beta} = \epsilon_{\alpha\beta}\, \bar{\psi}_{m\dot\beta}$$

$$P_\mu \longleftrightarrow K_\mu .$$

$$\partial_\mu \qquad \frac{\partial}{\partial\left(\frac{1}{x^\mu}\right)} \qquad \left(\frac{x^\mu}{x^2}\right)$$

$$\{S, S\} \sim K \qquad \left(\{Q, Q\} \sim P\right).$$

$$\{Q_{m\alpha}, S^n_\beta\} \sim \delta^m_n \epsilon_{\alpha\beta} D + \delta^n_m \underline{d_{\alpha\beta}} + \epsilon_{\alpha\beta} I^n_m$$

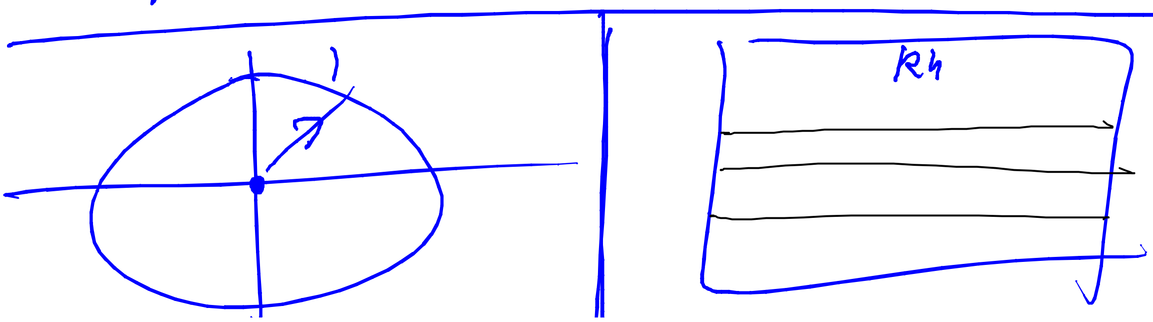


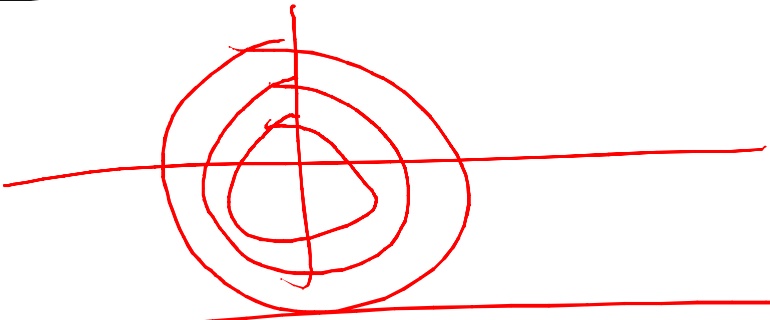
$$ds^2 = \left(dr^2 + r^2 d\Omega_3^2\right) = \cancel{r^2}\left[\frac{dr^2}{r^2} + d\Omega_3^2\right] = \left(d\tau^2 + d\Omega_3^2\right) \qquad r = e^\tau$$

$$r\frac{d}{dr} \longleftrightarrow \frac{d}{d\tau}$$

State Operator map

Operators $(R^4) \longleftrightarrow$ States on S^3





$e^{i\omega t} \longleftrightarrow e^{-i\omega \tau}$.

$t \longleftrightarrow -t$

$P_\mu^\dagger = K_\mu$

$\{Q, Q\} \sim P$

$\{S, S\} \sim K$

$Q^\dagger = S$

$\{Q, S\} \sim D + I + J$

$S|\psi\rangle = 0$ $\quad (K|\psi\rangle)$ $\quad SO(6) \times SO(4) \times \mathrm{Dil.}$

$\langle\psi|\{Q, S\}|\psi\rangle = \langle\psi|SQ|\psi\rangle$.

$\langle\psi|D + I + J|\psi\rangle$ ~ Positive definite.

$D \geq f(I, J)$

Operator ↪ (Traceless Sym Rep of SO(6) with k indices).

→ Scalar under SO(4)

↪ $\Delta = k$. SHORT OPERATOR

$$\mathrm{Tr}_R\left[\phi^{\mu_1} \cdots \phi^{\mu_n}\right] C_{\mu_1 \cdots \mu_n} \ldots$$

$$ds^2 = f^{-1/2}\left(\sum_{i=1}^{4} dx_i^2\right) + f^{1/2}\left[dr^2 + r^2 d\Omega_5^2\right].$$

$$f = 1 + \frac{R^4}{r^4} \longrightarrow \frac{R^4}{r^4}.$$

$$\boxed{v = \frac{r}{R^2}}$$

$\alpha' \to 0$ $(R^2 \to 0)$

$$\left(R^2 = \alpha'(gN4\pi)^{1/2}\right)$$

$\alpha' \to 0$; at fixed v ; $r \to 0$

$$ds^2 = R^2\left[v^2 dx_i^2 + \frac{dv^2}{v^2} + d\Omega_5^2\right]$$

$$ds^2 = \left(v^2 dx^2 + \frac{dv^2}{v^2}\right)$$

$[AdS_5]$ (Euclidean)

$$\boxed{R^2 = \alpha'(4\pi g N)^{1/2}}$$

IIB string Theory ON THIS SPACE

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$N=4$ YM

$\cancel{g^2_{YM} \sim g_s}$

$\cancel{R^4/\alpha'^2 \sim \lambda}$

$g^2_{YM} \neq g_s$

$\lambda \; \cancel{R^4/\alpha'^2} \neq \left(\frac{R^4}{\alpha'^2}\right)$

$\boxed{\frac{1}{\sqrt{\alpha'}} \sim \lambda^{1/4}}$

$\boxed{\frac{1}{R} \leftrightarrow 1}$