Algebraic Bethe Ansatz

$$[Q_r, Q_s] = 0 \qquad r = 1 \ldots L$$

$Q_1 = \text{translation} = e^{iP}$ $\qquad Q_1|\psi\rangle = 1\cdot|\psi\rangle$

$Q_2 = \text{Hamiltonian}$

$$\mathrm{Tr}\, Z^{L-M} X^M \qquad J_3 = L-M$$

$$J_1 = M$$

$$\Delta = \underbrace{\Delta_0}_{L = J_1 + J_3} + 2g^2 E + \mathcal{O}(g^4)$$

$$H \cdot |\psi\rangle = E|\psi\rangle$$

$u_1\, u_2 \ldots u_M$ Bethe roots M eq's

$$\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right)^L = \prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$

$$\prod_{k=1}^{M}\left(\frac{u_k + \frac{i}{2}}{u_k - \frac{i}{2}}\right) = 1 = e^{iP} \qquad E = \sum_{k=1}^{M} \frac{1}{u_k^2 + \frac{1}{4}}$$

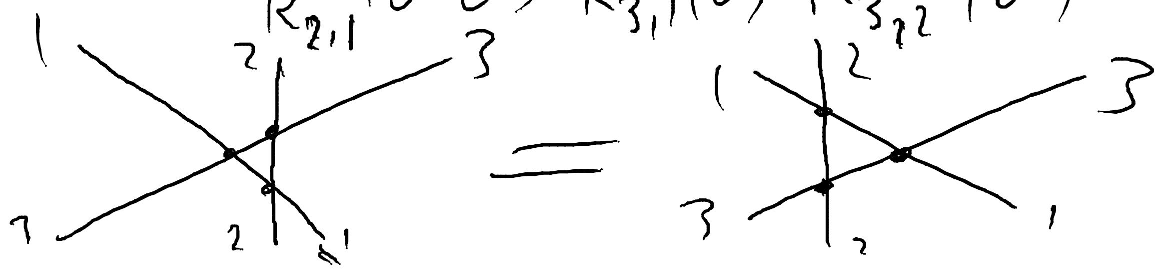
global $SU(2) \subset SU_R(4) \subset PSU(2,2|4)$

R-MATRIX

$$R(u) = u\,\mathbb{1}\otimes\mathbb{1} + i\,\mathbb{P} = \begin{pmatrix} u+i & & & \\ & u & i & \\ & i & u & \\ & & & u+i \end{pmatrix}$$

satisfies Yang-Baxter-equation

$$R_{3,2}(u')R_{3,1}(u)R_{2,1}(u-u') = R_{2,1}(u-u')\,R_{3,1}(u)\,R_{3,2}(u')$$

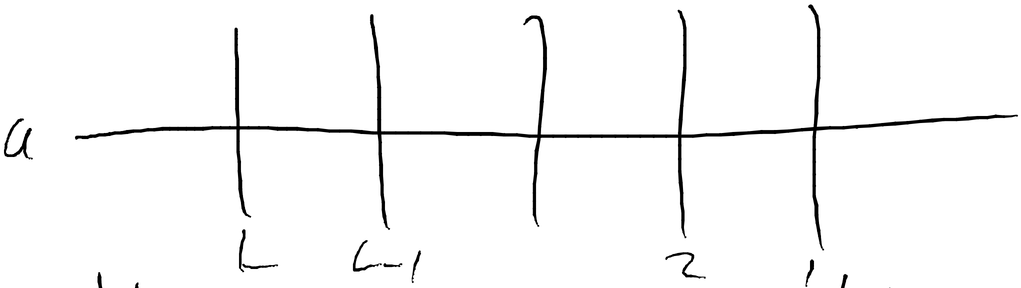


Lax-Operator:

$$\mathcal{L}_{a,n}(u) = R_{a,n}\left(u-\frac{i}{2}\right)$$

Monodromy Matrix:

$$\Omega(u) = \mathcal{L}_{a,1}(u)\,\mathcal{L}_{a,2}(u)\cdots\mathcal{L}_{a,L}(u)$$

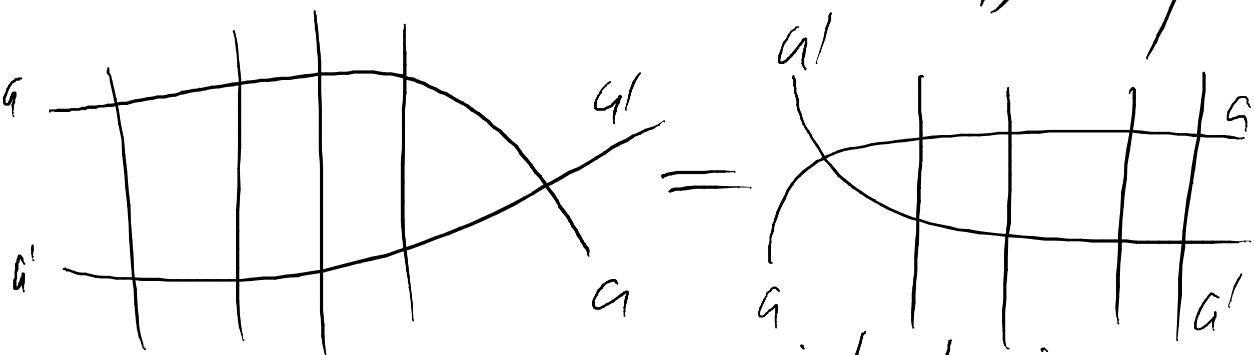


"monorail"

$$\mathbb{C}^2 \otimes \underbrace{\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2_1}_{L}$$

a = auxiliary !

$$\Omega(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$



intertwiner

$$\Omega_{a'}(u')\,\Omega_a(u)\,R_{aa'}(u-u') =$$

$$= R_{aa'}(u-u')\,\Omega_a(u)\,\Omega_{a'}(u')$$

take $\mathrm{Tr}_{a'} \cdot \mathrm{Tr}_a$

Transfer matrix:

$$T(u) = \mathrm{Tr}_a \Omega_a(u)$$

$$\Longrightarrow [T(u), T(u')] = 0$$

$$\log T(u) = \sum_{k=1}^{\infty} Q_k u^k$$

$$\Longrightarrow [Q_k, Q_m] = 0$$

$$Q_1 = i^{-L} T(\tfrac{i}{2}) \quad Q_2 = H$$

Solve $T(u)|\psi\rangle = t(u)|\psi\rangle$

Bethe Ansatz.

$$|\psi\rangle = B(u_1) \ldots B(u_M) \cdot |0\rangle$$

$$\Longrightarrow \text{- - - -} \Longrightarrow$$

$$t(u) = (u + \tfrac{i}{2})^L \prod_{j}^{M} \frac{u - u_j - i}{\quad}$$

$$T(u) = \left(u+\frac{i}{2}\right)^L \prod_{j=1}^{M} \frac{u-u_j-i}{u-u_j} +$$

$$+ \left(u-\frac{i}{2}\right)^L \prod_{j=1}^{M} \frac{u-u_j+i}{u-u_j}$$

Residue at $u=u_j$

$= 0$!

$$\Rightarrow \left(\frac{u_k+\frac{i}{2}}{u_k-\frac{i}{2}}\right)^L = \prod_{j=1}^{M} \frac{u_k-u_j+i}{u_k-u_j-i}$$

$$\mathrm{Tr}\, \cancel{Z}^{L-M} X^M = \mathcal{T}_1$$

$$\mathrm{Tr}\, Z D^S Z \qquad S := M$$

$\Delta_0 = S+2$

$L = 2$

$$\mathrm{Tr}\left[(D^{S_1} Z)(D^{S_2} Z) \dots (D^{S_L} Z)\right]$$

$$J_3 = L -$$

$$\sum S_i = M = S$$

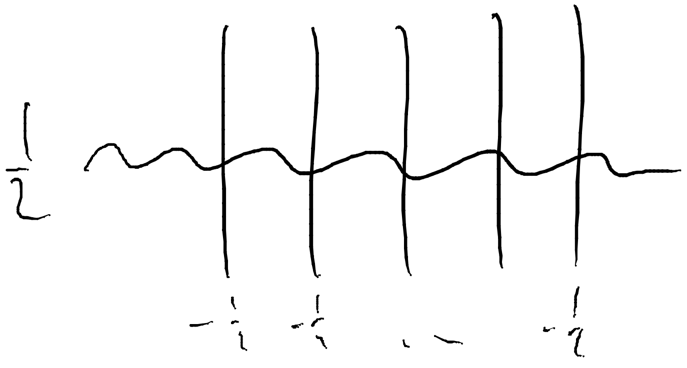
$$\uparrow = z \qquad \downarrow = x$$

$$\uparrow = z \quad \downarrow = Dz \quad \downarrow\downarrow = D^2 z$$

$$sl(2) \sim su(1,1) \subset su(2,2)$$

$$h = -\frac{1}{2} \qquad \subset psu(2,2|4)$$

$$sl(2) \qquad XXX_S$$



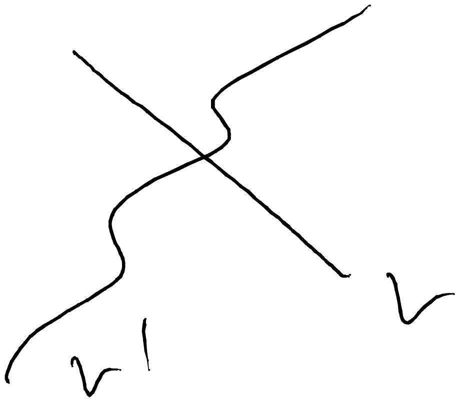
$$= \bar{\Omega}$$

$$\mathrm{Tr}\,\bar{\Omega} = \bar{T}(u)$$

$$[\bar{T}(u), \bar{T}(u')] = 0$$

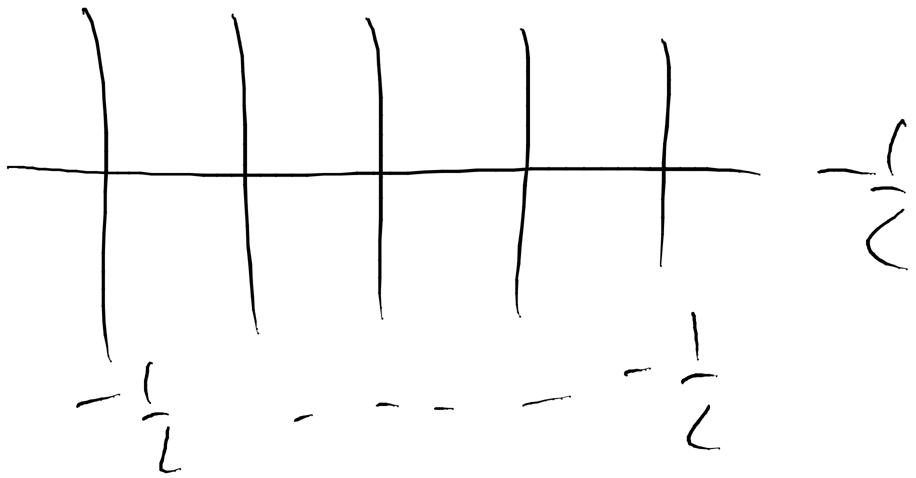
$[\bar{T}(u), \bar{T}(u')] = 0$

Universal R-matrix



$su(2)$

$\sim \Gamma$-fnctn

$\Omega =$ 

$T(u) = \mathrm{Tr}\,\Omega$

$[T(u), T(u')] = 0$

$\psi(x) = \frac{d}{dx} \log \Gamma(x)$

$$\psi(x) = \frac{d}{dx} \log \Gamma(x)$$

$$H \cdot (D^{s_1} Z)(D^{s_2} Z) =$$

$$= \left[\psi(s_1+1) + \psi(s_2+1) - 2\psi(1)\right] (D^{s_1} Z)(D^{s_2} Z)$$

$$- \sum_{n'} \frac{1}{n'} (D^{s_1 - n'} Z)(D^{s_2 + n'} Z)$$

$$[T(u), T(u')] = 0$$

$$\left(\frac{u_k + s i}{u_k - s i} \right)^L = \prod_{\substack{j=1 \\ j \neq k}}^{K} \frac{u_k - u_j + i}{u_k - u_j - i}$$

XXX_s

Two Articles of L.D.
Faddeev:

hep-th / 9410032
9605187

Book by Bill
Sutherland:

"Beautiful Models"

Nepomechie

hep-th/9810032