Literature

Nastase 0712.0689 [hep-th] → AdS/CFT

- Minwalla hep-th/9712074

Dolan+Osborn 0112251+0209056

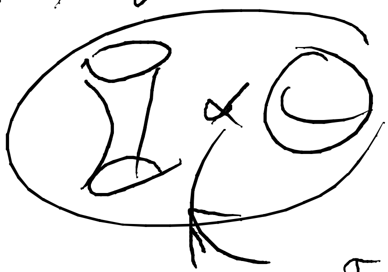
Ph.D. thesis of N. Beisert 0407277

Many old papers by

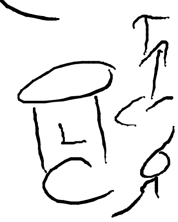
Günaydin, Bars

Classical in String Theory

Symmetries of $\overline{IIB}$ ST on $AdS_5 \times S^5$



- $PSU(2,2|4)$
- Worldsheet diffeomorphism invariance



$m = 1 \ldots 10$

$$\sigma'(\sigma,\tau),\ \tau'(\sigma,\tau) \qquad X^m(\sigma',\tau') = X^m(\sigma,\tau)$$

- WS Weyl invariance

$$\gamma'_{ab}(\sigma,\tau) = e^{2w(\sigma,\tau)}\gamma_{ab}(\sigma,\tau)$$

- Local fermionic 16-symmetry

$$T_{ab} = \frac{\delta S}{\delta \gamma^{ab}} = 0$$

What <u>is</u> a σ-model?

Ex. 1: $O(n)$ σ-model   sigma

$$S = \frac{\sqrt{\lambda}}{4\pi}\int d\tau\, d\sigma\ \partial_a \vec{X} \cdot \partial^a \vec{X}$$

$$\vec{X} = (X_1, \ldots, X_n) \qquad \boxed{\vec{X}^2 = 1}$$

often called "$\vec{\pi}$-field"

$\vec{X} = \vec{\pi}$ $\quad S^{n-1}$

"exactly solved":

- Polyakov + Wiegmann
- Faddeev-Reshetikhin

Ex 2: $SU(m)_L \times SU(m)_R$ $\quad m=2$

principal chiral model PCF

$$S = -\frac{\sqrt{\lambda}}{8\pi}\int d\tau\, d\sigma\ \mathrm{Tr}\, j_a j^a \qquad a=0,1$$

(current $j_a = g^{-1}\partial_a g$

$g \in SU(m)$ $\quad j_a \in su(m)$

$U \in SU(m)$ $g \to Ug$ $\quad V \in SU(m)$ $g \to gV$

Left current $l_a = \partial_a g \cdot g^{-1}$

$m=2$

$$g = \begin{pmatrix} Y_1 + iY_2 & Y_3 + iY_4 \\ -Y_3 + iY_4 & Y_1 - iY_2 \end{pmatrix}$$

$g \in SU(2) \quad gg^{\dagger} = g^{-1}g = \mathbb{1}_2$

$\det g = 1 \quad \text{iff} \quad Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 = 1$

PCF $m=2 \equiv O(4)$-σ model

$n = 4$

$$O(4) \sim SU_L(2) \otimes SU_R(2)$$

<u>Metsaev-Tseytlin σ-model</u>

PCF $g \in \dfrac{PSU(2,2|4)}{SO(4,1) \times SO(5)}$

$M \in su(2,2|4)$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

$A \in u(2,2)$ $D \in u(4)$

$\mathrm{Tr}\, A - \mathrm{Tr}\, D = 0 \to su(2,2|4)$

extra $u(1)$ generator $i\mathbb{I}_8$

$[M_1, M_2\} = M_3 + iC\,\mathbb{I}_8$

central charge $\to C \in \mathbb{R}$

Bosonic subalgebra

$su(2,2) \oplus su(4) \oplus u(1)$

$$psu(2,2|4) = \frac{su(2,2|4)}{u(1)}$$

Crucial feature:

$\mathbb{Z}_4$ - grading

$$M = \overset{20}{M^{(0)}} \oplus \overset{16}{M^{(1)}} \oplus \overset{10+1}{M^{(2)}} \oplus \overset{16}{M^{(3)}}$$

$$\Omega : M \to \Omega(M)$$

$$\Omega(M^{(k)}) = i^k M^{(k)}$$

$$\Omega(M) = \begin{pmatrix} K A^T K & -K C^T K \\ K B^T K & K D^T K \end{pmatrix}$$

$$K = \begin{pmatrix} \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} & O_4 \\ O_4 & \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} \end{pmatrix}$$

$M^{(1)} \sim 16$ fermionic gen.

$M^{(3)} \sim 16$ " " .

The point:

$$M^{(0)} \in so(4,1) \oplus so(5) \sim 20$$

Cool!

$$J_a = -g^{-1}\partial_a g \in psu(2,2|4)$$

$$J_a = J_a^{(0)} + J_a^{(1)} + J_a^{(2)} + J_a^{(3)}$$

$$S = -\frac{\sqrt{\lambda}}{8\pi}\int d\tau\, d\sigma\, \mathrm{STr}\, J_a^{(2)} J^{(2)a}$$

$\Rightarrow$ Global Left-invariance under $PSU(2,2|4)$

Let's put in the fermions:

$$S = -\frac{\sqrt{\lambda}}{8\pi}\int d\tau\, d\sigma\, \gamma^{ab}\, \mathrm{STr}\left[J_a^{(2)} J_b^{(2)} - \frac{k}{4\pi}\varepsilon^{ab} J_a^{(1)} J_b^{(3)}\right]$$

$k = \pm 1$ k symmetry

$\hookrightarrow$ Wess-Zumino-term

need to vary $\frac{\delta S}{\delta \gamma^{ab}}$

→ Virasoro:

$$\mathrm{STr}\Big[\dot{\partial}_a^{(2)}\,\dot{\partial}_b^{(2)} - \frac{1}{2}\,\gamma_{ab}\,\gamma^{cd}\,\dot{\partial}_c^{(2)}\,\dot{\partial}_d^{(2)}\Big] = 0$$