Staudacher No. 6

Classical Integrability

The current j_a is flat :

$$\partial_a j_b - \partial_b j_a - [j_a, j_b] = 0$$

Reduce $\mathbb{R} \times S^3$

$$S_{\text{red}} - \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \left[\frac{1}{2} \operatorname{Tr} j_a j^a + \partial_a X_0 \partial^a X_0 \right]$$

$$j_a = SU(2) \quad , \quad S^{n-1} = \frac{SO(n)}{SO(n-1)} \quad \rightarrow \quad S^3 = \frac{SO(4)}{SO(3)}$$

Virasoro

$$\frac{1}{2} \operatorname{Tr} j_\pm{}^2 + (\partial_\pm X^0)^2 = 0$$

$$\partial_\pm = \partial_\tau \pm \partial_\sigma \quad , \quad j_\pm = j_\tau \pm j_\sigma$$

$$\partial_+ \partial_- X_0 = 0 \implies X^0 = \varepsilon \tau$$

$$\implies \frac{1}{2} \operatorname{Tr} j_\pm{}^2 + \varepsilon^2 = 0$$

$$X^0 \longrightarrow X^0 + \varepsilon$$

$$E = \Delta = \frac{\sqrt{\lambda}}{2\pi} \int_0^{2\pi} d\sigma \, \partial_\tau X_0 = \sqrt{\lambda} \, \varepsilon$$

EOM for j

$$\partial_+ j_- + \partial_- j_+ = 0$$

flatness

$$\partial_+ j_- - \partial_- j_+ + [j_+, j_-] = 0$$

Key idea of integrability

$$j_a \longrightarrow A_a$$

"X" and "x" are same.

Introduce $A_{\pm}(x) = \dfrac{j_{\pm}}{1 \mp x}$

↑ spectral parameter

$\Rightarrow \partial_{+} A_{-} - \partial_{-} A_{+} + [A_{+}, A_{-}] = 0$

2×2 monodromy matrix

$\sim$ closed Wilson line

$$\Omega(x) = P\exp\left(-\int_0^{2\pi} d\sigma\, A_\sigma\right)$$

$$= P\exp\left(\int_0^{2\pi} d\sigma\, \frac{1}{2}\left[\frac{j_+}{x-1} + \frac{j_-}{x+1}\right]\right)$$

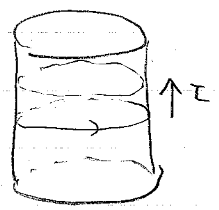
Ω is $\in SU(2)$ $\rightarrow$ can diagonalize Ω

$$\Omega = V^{-1}\begin{pmatrix} e^{iP(x)} & 0 \\ 0 & e^{-iP(x)} \end{pmatrix} V$$

P = quasi momentum

Transfer matrix

$T(x) = \mathrm{Tr}\,\Omega(x) = 2\cos P(x)$



$\Rightarrow \dfrac{\partial P(x)}{\partial \tau} = 0$: conserved

Let's expand $P(x)$ around $x = \pm 1$

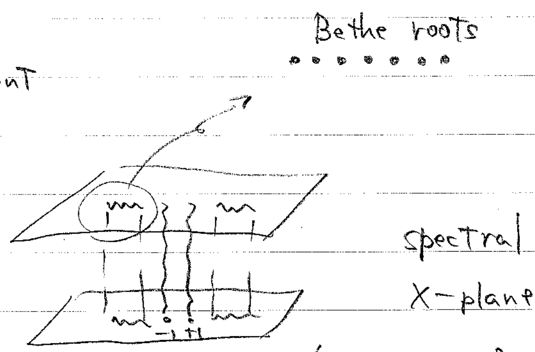
$P(x) = -\dfrac{\pi\varepsilon}{x\pm 1} + \cdots \quad x \to \mp 1$

KMMZ!

resolvent

$P(x) = -\dfrac{\pi\varepsilon}{x\pm 1} + G(x)$

$G(x) = \displaystyle\int dy\, \frac{\sigma(y)}{x-y}$



$\rightarrow$ algebraic curve (degree 2)

$u = x + \frac{1}{x}$ classical

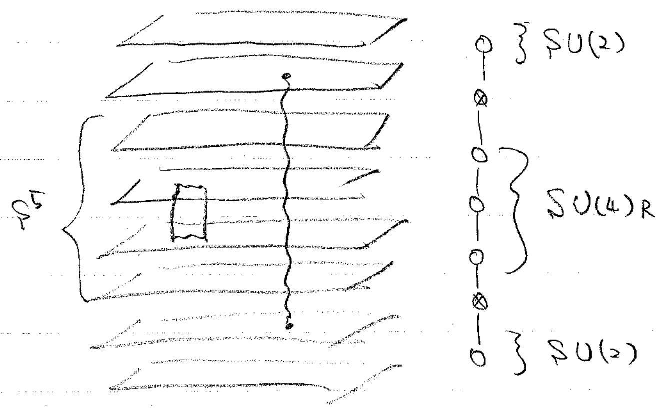
$$\boxed{u^{\pm} = u \pm \frac{i}{2g} = x^{\pm} + \frac{1}{x^{\pm}}}$$ quantum

Bena – Polchinski – Roiban

$$A_a = j_a^{(0)} + \frac{1}{2}\left(\frac{1}{z^2} + z^2\right) j_a^{(2)} + \frac{1}{2}\left(\frac{1}{z^2} - z^2\right) \gamma^{ab} \epsilon^{bc} j_c^{(2)} \quad \leftarrow \text{bosonic}$$

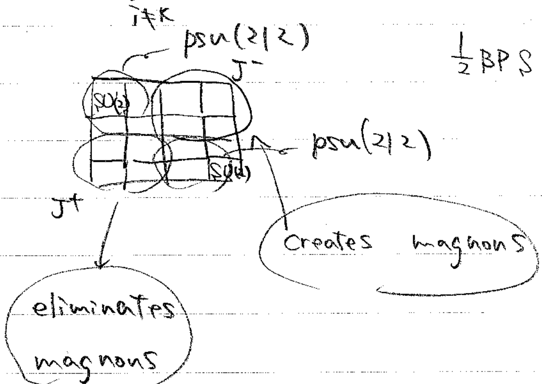
$$+ \frac{1}{z} j_a^{(1)} + z j_a^{(3)} \quad \cancel{\leftarrow \text{fermionic}}$$

AdS_5 part ⋯ 4
S^5 part ⋯ 4
eigenvalues ⟶ 8 Riemann sheets



fermion によって 8つのsheet が閉じている.

$$e^{ip_k L}|\psi\rangle = \prod_{\substack{j=1 \\ j\neq k}}^{M} S(p_j, p_k)\,|\psi\rangle$$



$\frac{1}{2}$BPS $\mathrm{Tr}\, Z^J$

$\Delta = J$

acting with J^-, create $X, Y, \bar{Y}, \bar{X}$

D_μ, $\mu = 0, \cdots 3$

generate 8 ψ's

? $\bar{Z}$, F, $(D^8 Z)$

reduced symmetry

$PSU(2,2) \oplus PSU(2|2) \subset PSU(2,2|4)$

$$S_{12} = \left(\tilde{S}_{12}^{PSU(2|2)_L} \otimes \tilde{S}_{12}^{PSU(2|2)_R} \right) \cdot \sigma_{12}^2$$

$[J_a, S_{12}] = 0$ $\qquad U^{\pm} = X^{\pm} + \frac{1}{X^{\pm}}$

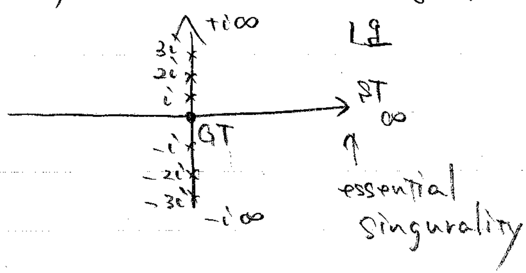
Unitarity : $S_{12}\, S_{21} = 1$

YBE :

$$S_{12} S_{13} S_{23} = S_{23} S_{13} S_{12}$$

Crossing symmetry -- Janik : $S_{12} S_{\bar{1}2} = f_{12}^2$

Analytic structure in g-plane ($\sqrt{\lambda}$ - plane)



GT : gauge theory

ST : string theory

$\mathrm{Tr}\, Z D^S Z$

$\Delta \sim f(g) \cdot \log S \qquad S \to \infty$

$$\left(\frac{U_k + \frac{i}{2}}{U_k - \frac{i}{2}} \right)^L = \prod_{\substack{j=1 \\ j \neq M}}^{S} \frac{U_k - U_j + i}{U_k - U_j - i}$$

$$U^{\pm} = X^{\pm} \pm \frac{g^2}{X^{\mp}}$$

$$\left(\frac{X_k^+}{X_k^-} \right)^L = \prod_{\substack{j=1 \\ j \neq M}}^{S} \frac{X_k^- - X_j^+}{X_k^+ - X_j^-} \; \frac{1 - \frac{g^2}{X_M^+ X_j^-}}{1 - \frac{g^2}{X_k^- X_i^+}} \times \sigma_{kj}^2$$

. . . . . . 1 hour

length \ loop order	1	2	3	4	5	
6	X	V	V	V	V	V
5	X	V	V	V	V	V
4	X	V	V	V	V	?
3	X	V	V	V	?	?
2	X	V	V	?	?	?

integral equation

$$\hat{\sigma}(t) - \hat{\sigma}(0) = f(g)$$

4-loop
Wrapping Problem
Konishi field

$$\mathcal{O} = \mathrm{Tr}(X\bar{X} + Y\bar{Y} + Z\bar{Z}) \sim \mathrm{Tr}\, \varphi_i \varphi_i$$

- Keeler - Mann
- Santambrogio, Zanon