$$S_{M^3} = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) + S_M$$

$$S_{BH} = \frac{\text{Area}}{4G} = \sqrt{16 G^2 M \ell^2 - 2 r_+^2} \ \frac{\pi}{4G}$$

$$R_{\mu\nu} \sim R_{\mu\nu\alpha\beta}$$

$$ds^2 = \left(8GM - \frac{r^2}{\ell^2} \right) d\tau^2$$

$$+ \frac{dr^2}{8GM - r^2/\ell^2} + r^2 d\phi^2$$

$$r_+^2 = 8\ell\sqrt{G^2M^2\ell^2 - G^2J^2}$$

Brown Henneaux

Asymptotical AdS_3 backgrounds

$$H(\xi) = \int_{D^1} d\Sigma^\mu B_{\mu\nu}\,\xi^\nu + \int_{D^2} \xi^\mu C_\mu$$

Impose boundary conditions

$$g_{tt} = -\frac{r^2}{l^2} + \mathcal{O}(1)$$

$$g_{r\phi} \sim \mathcal{O}\left(\frac{1}{r^3}\right)$$

Asymptotic Symmetry group

$$= \frac{\text{Allowed Diffeo.}}{\text{Trivial Diffeo.}}$$

$$\zeta^t = \ell\left(T^+ + T^-\right) - \frac{\ell^3}{2r^2}\left(\partial_+^2 T^+ + \partial_-^2 T^-\right) + \mathcal{O}\left(\frac{1}{r^3}\right)$$

$$T^+ = T^+(t + \ell\phi)$$

$$T^- = T^-(t - \ell\phi)$$

$$T_n^+ = e^{in\left(\frac{t+\ell\phi}{\ell}\right)}$$

$$H(T_n^+) = L_n$$

$$H(T_n^-) = \bar{L}_n$$

$$[L_m, L_n] = (m-n) L_{m+n}$$
$$+ \frac{c}{12}(m^3 - m)\,\delta_{m+n}$$

$$c = \frac{3\ell}{2G}$$

BTZ BH

$$S_{BH} = 2\pi\sqrt{\frac{c L_0}{6}} + 2\pi\sqrt{\frac{c \bar{L}_0}{6}}$$

$$\left(c = \frac{3\ell}{2G},\ L_0 = \frac{1}{2}(M\ell + J),\ \bar{L}_0 = \frac{1}{2}(M\ell - J) \right)$$

$$= \pi\sqrt{\frac{\ell(M\ell+J)}{2G}} + \pi\sqrt{\frac{\ell(M\ell-J)}{2G}}$$

$$= \pi \sqrt{\frac{\ell(M\ell+J)}{2G}} + \pi \sqrt{\frac{\ell(M\ell-J)}{2G}}$$

$$l = 2\pi\alpha' \sqrt{g}\,(Q_1 Q_5)^{\frac{1}{4}}$$

$$\frac{1}{G} = \frac{2(Q_1 Q_5)^{\frac{1}{4}}}{\pi\alpha' \sqrt{g}}$$

$$c = \frac{3l}{2G} = 6 Q_1 Q_5$$

$$A^{\pm\ a}_{\mu\ \ b} = \omega_\mu{}^a{}_b \pm e_{\mu c}\,\epsilon^{ca}{}_b$$

$$S_{cs} = k_L \int \left(A\,dA + \frac{2}{3} A^3 \right) + k_R$$

Guess BCFT constraints

(a) $c = \dfrac{3l}{2G}$

(b) No primary op. $b < \dfrac{c}{24}$

(c) $Z = \mathrm{tr}\, q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{c}{24}}$ $\cdot\cdot$ modular invariant

Assumption

$Z = Z(q) Z(\bar{q})$: holomorophic

$$S = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)$$

$$+ \frac{1}{16\pi G\mu} I_{CS}$$

$$I_{CS} = \frac{1}{2}\int d^3x\sqrt{-g}\ \epsilon^{\lambda\mu\nu}\Gamma^{\rho}{}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu} + \Gamma^{(2)}\right)$$

DJT

$$(h, \bar{h}) = \left(\frac{1}{2}(1+3\mu\ell), \frac{1}{2}(\mu\ell - 1)\right)$$

$$(2,0)\ ,\ (0,2)$$

$$c_L = \frac{3\ell}{2G}\left(1 - \frac{1}{\mu\ell}\right)$$

$$c_R = \frac{3\ell}{2G}\left(1 + \frac{1}{\mu\ell}\right)$$

critical value

$$\mu\ell = 1$$

$$c_L = 0, \quad c_R = \frac{3\ell}{G}$$

$$(\bar{h}, \bar{h}) = (2, 0)$$

$$\ell M(\mu L) = \ell M(\infty) + \frac{J(\infty)}{\mu\ell}$$

$$J(\mu\ell) = J(\infty) + \frac{1}{\mu} M(\infty)$$

$$\ell M(1) = J(\infty)$$ → everthing of theory

become right-moving