General consideration

$$G_3 = F_3 - \Phi H_3$$

$$\tilde{F}_5 = (1 + *_{10})\, d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

$$ds_{10}^2 = e^{2A(y)} \sum \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \sum \tilde{g}_{mn} dy^m dy^n$$

(1)

$$R_{MN} = \cdots$$

$M = \mu$, $N = \nu$.

$$\tilde{\nabla}^2 A = e^{-2A} \left(\frac{G_{mnp} \bar{G}^{mnp}}{48\, \mathrm{Im}\, \Phi} + \cdots \right)$$

$\cdots$ $\boxed{2-1}$

② EOM for $\tilde{F}_5$:

$$d\tilde{F}_5 = H_3 \wedge F_3 + \text{localised source} \qquad \text{L2-3}$$

$$\tilde{\nabla}^2 \alpha = \frac{i e^{2A} G_{mnp} *_6 \bar{G}^{mnp}}{12\, \mathrm{Im}\, \Phi} + e^{-6A} \partial_m \alpha \partial^m e^{4A}$$

$$+ 2\kappa_{10}^2 e^{2A} T_3 \rho_3^{loc} \qquad \cdots \text{L2-2}$$

$$(*_6 G)_{mnp} = \epsilon_{mnpqrs} G^{qrs}$$

L2-1, L2-2 $\Rightarrow$

$$\hat{\nabla}^2\left(e^{4A}-\alpha\right) = \frac{e^{2A}}{6\,\mathrm{Im}\,\Phi}\left|iG_3 - *_6 G_3\right|^2 + 2e^{-6A}\left|\partial\left(e^{4A}-\alpha\right)\right|^2 + \underbrace{2\kappa_{10}^2 e^{2A}\left[\frac{1}{4}\left(T^m{}_m - T^\mu{}_\mu\right) - T_3\rho_3^{loc}\right]}_{\text{vanishes for D7|O7 \& D3|O3 planes}}$$

$$\left(\begin{array}{l}\text{for D3|O3} \\ -T^\mu{}_\mu{}^{loc} + T^m{}_m{}^{loc} = T_3 \cdot 4\,\delta(\Sigma)\end{array}\right)$$

Conclude

$iG_3 = *_6 G_3$ $\longrightarrow$ Imaginary self dual (L2-5)

$e^{4A} = \alpha$

$(*_6)^2 = -1$

Other EOM :

$$dF_3 = d*F_3 = 0$$

① $F_{(3)}$ & $H_{(3)}$ Harmonic, closed ⋯ L2-6

② $\tilde{g}_{mn}(y)$, $\Phi(y)$: solve EOM without flux

(This means CY_3 $(-O_3^+)$ are solutions

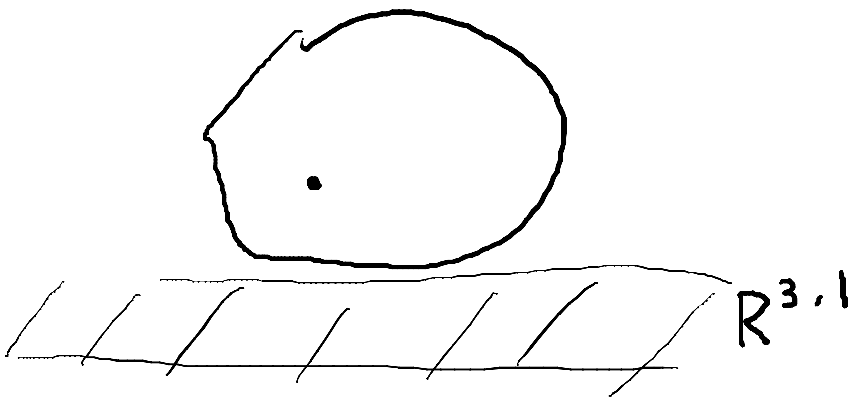
with $\phi(y) = \text{const.}$)

L2-3

$$\int_{\hat{M}} \hat{\tilde{F}}_5 = \int H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 (N_{D3} - N_{\bar{D}3})$$

$$\int_{\hat{M}} H_3 \wedge F_3 + 2\kappa_{10}^2 T_3 N_{D3} = 2\kappa_{10}^2 T_3 N_{\bar{D}3}$$

$\cdots$ L2-4



$H^3(M)$: 3rd cohomology group of $\widetilde{M}$

L2-6 F_3 & $H_3 \in H^3(\widetilde{M})$

D string $\widetilde{F}$ string

F_3 & H_3 : Dirac quantisation

$\alpha^i \in H^3(M, \mathbb{Z})$, $\gamma_j \in H_3(M)$

$\int_{\gamma_j} \alpha^i = p^i{}_j$, $p^i{}_j \in \mathbb{Z}$

$$\frac{1}{(2\pi)^2\alpha'} F_3 = \sum m_i \alpha^i$$

$$\frac{1}{(2\pi)^2\alpha'} H_3 = \sum n_i \alpha^i$$

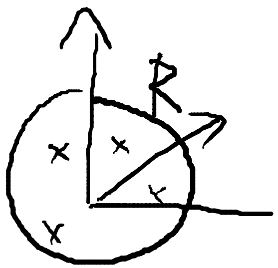
$$m_i , n_i \in \mathbb{Z}$$

$$\dim(H^3(M)) = b_3$$

Total number of choices for F_3, H_3

$$\sim \left(\sqrt{|N_{D\bar{3}}|}\right)^{2b_3}$$

Douglas



$$\sim \frac{R^{2b_3}}{a^{2b_3}}$$

Number of diff. fluxes grows like $\left(|N_{O_3}|\right)^{b_3}$

$b_3 \sim 10^2 - 10^3$ (Typical) can be large

F-theory

$N_{O_3} \longrightarrow \frac{\chi(X_4)}{24}$

Moduli stabilisation : $G_3 = F_3 - \tilde{\Phi} H_3$

$$*_6 G_{(3)} = i\, G_{(3)}$$

$$\underset{\lambda^3}{\sqrt{\tilde{g}}}\, [m_1 m_2 m_3 m_4 m_5 m_6]\, \underset{1/\lambda}{\tilde{g}^{m_4 n_4}}\, \underset{1/\lambda}{\tilde{g}^{m_5 n_5}}\, \underset{1/\lambda}{\tilde{g}^{m_6 n_6}}\, (\tilde{G}_3)_{n_4 n_5 n_6}$$

$$= i\, (G_3)_{m_1 m_2 m_3}$$

$\tilde{g}_{mn}, \ \tilde{\Phi}$

$$\tilde{g}_{mn} \longrightarrow \lambda\, \tilde{g}_{mn}$$

Moduli scalar field arise from $\hat{g}_{mn}$

some of these fixed.