Matrix model formulations of superstring theory

6th Asian Winter School on strings, particles and cosmology
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Plan of the lectures

I. Superstring theory and matrix models
(1/11 10:45-12:00)

II. The birth of our universe
(1/12 10:45-12:00)

III. Confirmation of gauge/gravity duality
(1/14 10:45-12:00)

Rem.) I will be here until 1/14 morning.
Please ask me questions before I leave.

I. Superstring theory and matrix models

Plan of the 1st lecture:
Superstring theory and matrix models

1. What is superstring theory ?
2. Matrix model for superstring theory
3. Summary

I-1 What is superstring theory ?

1. What is superstring theory ?

- Why superstring theory ?

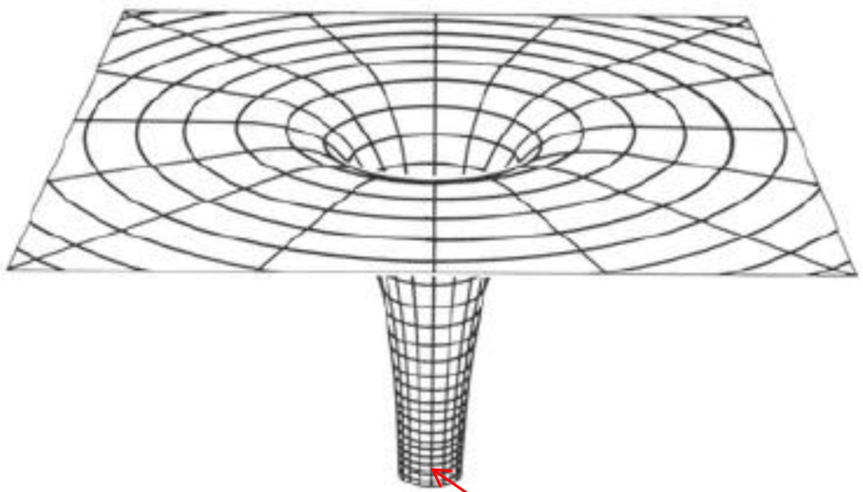
A: To go beyond Einstein's theory of general relativity.

- What's wrong with Einstein's theory of general relativity ?

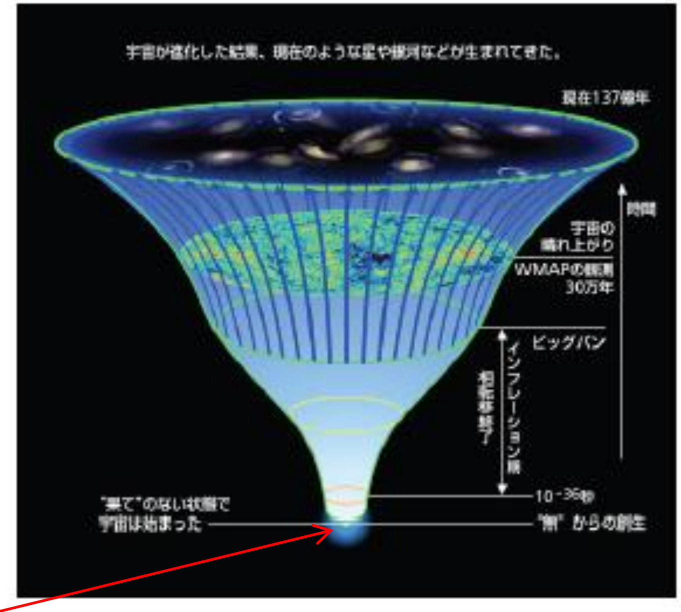
A: Singularities appear
at the center of a black hole
at the beginning of the Universe

Singularities
(space-time curvature diverges)

Black hole



Big bang



singularity (curvature diverges)

General Relativity becomes invalid!

（Quantum effects become non-negligible.）

The scale at which quantum effects of gravity become non-negligible

3 fundamental constants of physics

h	(Planck constant)	quantum mechanics
c	(speed of light)	relativity
G	(gravitational constant)	gravity

written in terms of units of length, time, mass

Planck length $$L_{\rm pl} = \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-33}\text{cm}$$

When the curvature radius of space-time becomes Planck length, one cannot use Einstein's theory of general relativity !

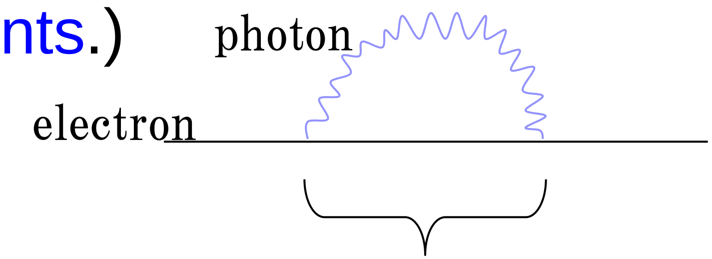
Why strings ?

- The case of other 3 forces

Electromagnectism
Weak interaction
Strong interaction

Quantum Field Theory

(Particles are treated as points.)



can be infinitely short time

(Heisenberg's uncertainty principle)

Intermediate states with infinite energy (momentum) contribute

UV divergence

Renormalization theory

(Tomonaga, Feynman, Schwinger)

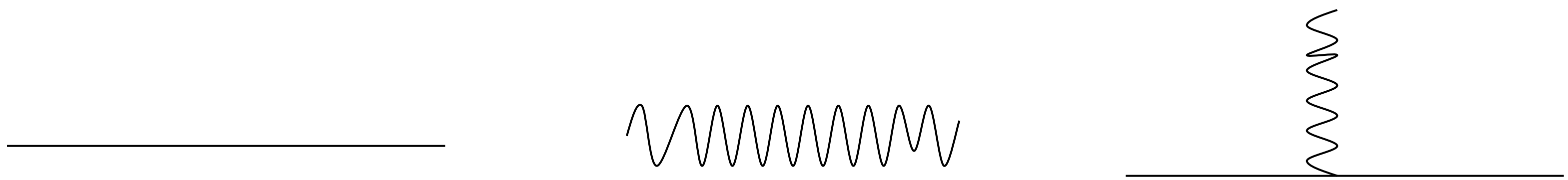
- The results for physical quantities

  (mass spectrum, scattering amplitudes, etc.)

  can be made finite

  by redefining the parameters

  that describe elementary processes



Coupling constant

- Dimensionless in the case of gauge theory

"renormalizable theory"

- Newton's gravitational constant

$$[G] \sim M^{-2}$$

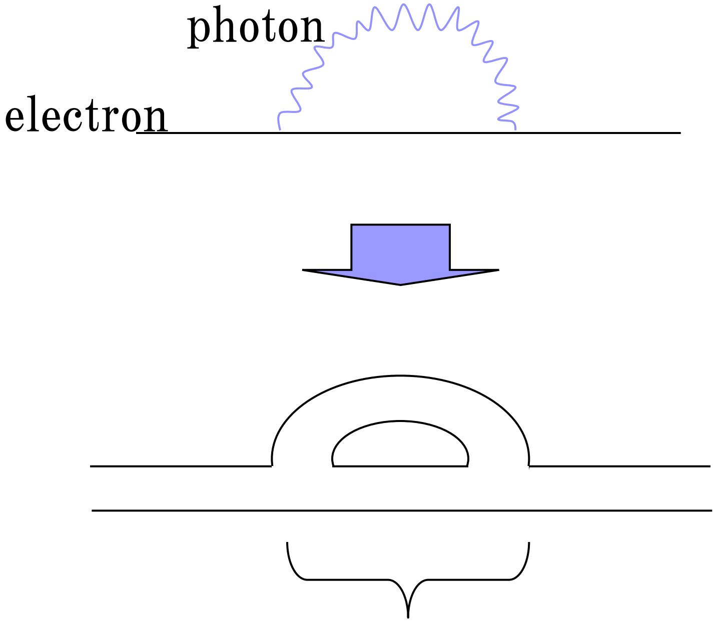
$$F_{\text{Newton}} = G\frac{m_1 m_2}{r^2}$$

$$F_{\text{Coulomb}} = \alpha\frac{q_1 q_2}{r^2}$$

In naïve quantum extension of Einstein's theory :

UV divergence becomes worse at higher orders in the expansion w.r.t. the coupling constant !

String theories do not have UV divergence



Cannot be closer than the string scale

No UV divergences !

Crucial differences from particle theory based on quantum field theory

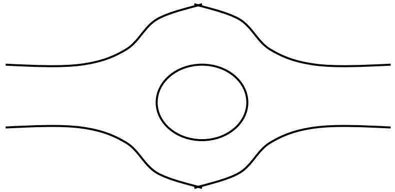
- propagation

  strings can vibrate !

  - A single string can represent various particles. (Fermions as well as bosons appear from superstrings.)

- interactions

  joining and splitting of strings



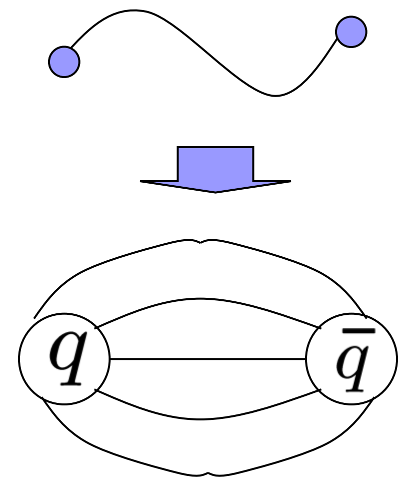
  - There is no particular "interaction point" (hence no UV div.).
  - There is no freedom to introduce ad hoc interactions.

Historical remark

- Closed string inevitably includes
  massless spin-2 particle

  This was a crucial defect as a theory for
  hadrons (Nambu's idea)
  since there is no such states,

  but was turned into a virtue ("graviton")
  in the context of quantum gravity
  (1974 Sherk-Schwarz, Yoneya)



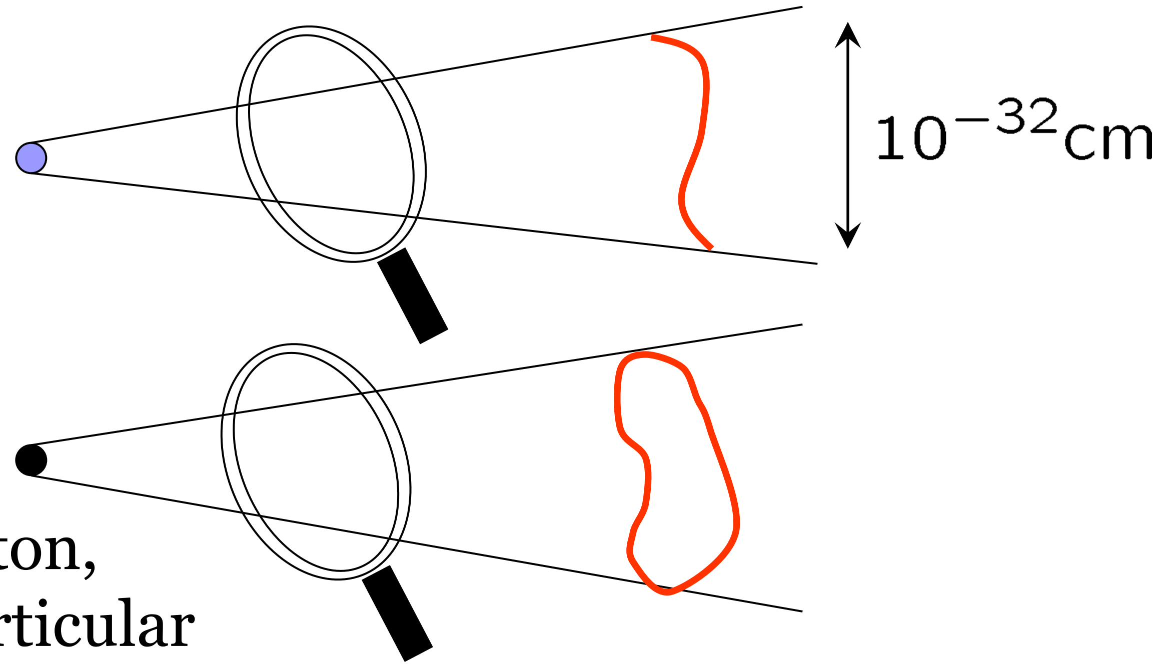
- The scale of the theory had to be changed from
  the scale of hadrons to the Planck scale, though.
  (a few 100 GeV) (10^{19}GeV)

Superstring theory

1974 Sherk-Schwarz, Yoneya
1984 Green-Schwarz

Various vibration modes correspond to various particles.

photon
gluon
$W^{\pm}$, Z etc.



graviton,
in particular

Unified description of 4 forces including gravity

The goals of superstring theory

particle physics

- space-time dimensionality puzzle

  critical dimension is (9+1), but we live in (3+1)d

- particle contents

  gauge group : $SU(3) \times SU(2) \times U(1)$

  matter contents : 3 generations $(q \text{ and } \ell)$ + Higgs(?)

- coupling constants in the Standard Model

cosmology

- the birth of our Universe and “inflation”
- the fate of our Universe

  (dark energy, cosmological constant problem)

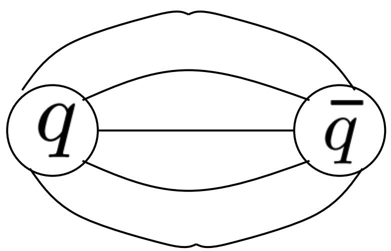
- the interior structure of a black hole

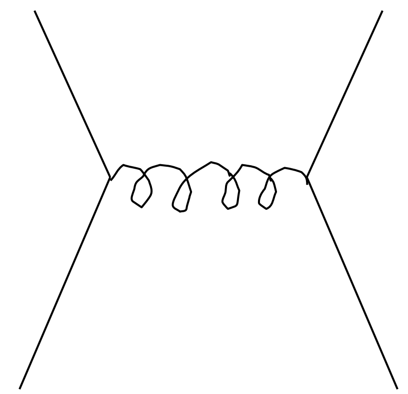
A big obstacle:

non-perturbative definition is not yet established !

- Comparison : QCD

quark confinement





$$V(r) \propto r$$

$$V(r) \propto -\frac{1}{r}$$

nonperturbative
calculations

lattice gauge theory
(Wilson, 1974)

perturbative calculations

Compactification

- Superstring theory is naturally defined in (9+1)dim.

  Unitarity + Lorentz invariance

- (3+1)-dimensional space-time is expected to appear due to some nonperturbative dynamics.

  not known, at least, until recently.

- Search for perturbative vacua with compactified 6d.

  Good : One can obtain SM-like models.

  Bad : Too many vacua. ("Landscape")

Understanding the nonperturbative dynamics of compactification is crucial to understand our real world !

“Landscape”

- Tremendously many vacua

  we are living in one of them

  due to statistical reasons

  or just because of “anthropic principle”

- Pessimism that appeared from studies based on perturbative string theory + D-branes

  ↑ solitons in string theory
  (non-perturbative objects)

- It remains to be seen what happens if
  full nonperturbative effects
  are taken into account !

I-2 Matrix model for superstring theory

Matrix model as a nonperturbative formulation of string theory

- 't Hooft (1974)

Feynman diagrams in U(N) gauge theory	⟺	(discretized) string worldsheet
large-N limit with $\lambda = g_{\mathrm{YM}}^2 N$ fixed		tree diagrams (classical limit)
$\frac{1}{N}$-expansion		perturbative expansion

Note : gauge theory is well-defined for finite N.

One may hope to obtain a nonperturbative formulation of string theory by using matrix degrees of freedom !

An explicit example of nonperturbative string theory

- Brezin-Kazakov, Douglas-Shenker, Gross-Migdal (1990)

$$
\begin{aligned}
Z &= \int d\phi \, e^{-S} \\
S &= N \, \mathrm{tr} \left(\frac{1}{2}\phi^2 - \frac{\kappa}{3}\phi^3 \right)
\end{aligned}
$$

$\kappa \to \kappa_{\mathrm{cr}}$ as $N \to \infty$

with $|\kappa - \kappa_{\mathrm{cr}}|^p N$ fixed

(double scaling limit)

All the diagrams of higher orders equally contribute.

nonperbative formulation of string theory
with 0d target space

Matrix model for superstring theory in 10d

- IKKT model (1996) Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

$$S_{\mathrm{b}} = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_{\mathrm{f}} = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

$N \times N$ Hermitian matrices

A_μ $(\mu = 0, \cdots, 9)$ Lorentz vector

Ψ_α $(\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

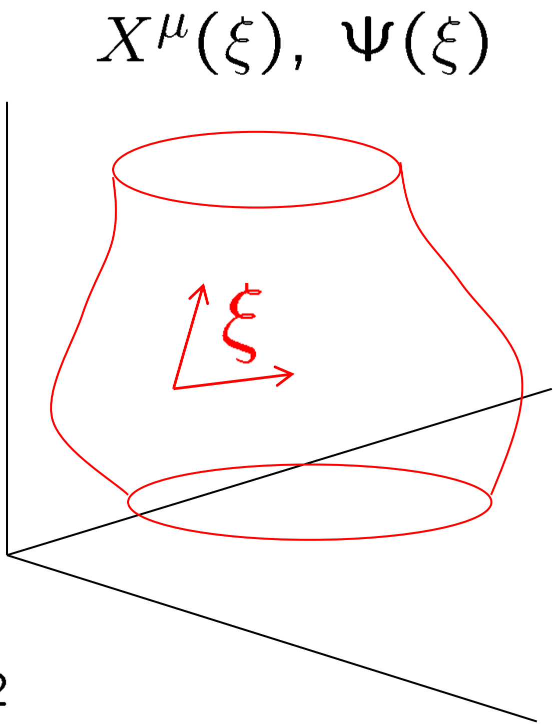
raised and lowered by the metric

$$\eta = \mathrm{diag}(-1, 1, \cdots, 1)$$

The action has manifest SO(9,1) symmetry.

Connection to the worldsheet formulation

- worldsheet action



$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$$\{X, Y\} \equiv \frac{1}{\sqrt{g}} \epsilon^{ab} \frac{\partial X}{\partial \xi^a} \frac{\partial Y}{\partial \xi^b}$$

Poisson bracket (regarding ξ_1 and ξ_2 as p and q in Hamilton dynamics)

quantization $\Longrightarrow$ IKKT $\quad (\hbar \sim \frac{1}{N})$

$$\{X^\mu(\xi), X^\nu(\xi)\} \mapsto -i[A^\mu, A^\nu]$$

$\mathcal{N}=2$ supersymmetry

$$\begin{cases} \delta^{(1)} A_\mu &= i\bar{\epsilon}_1 \Gamma_\mu \psi \\ \delta^{(1)} \psi &= \frac{i}{2} \Gamma^{\mu\nu} [A_\mu, A_\nu] \epsilon_1 \end{cases} \qquad \begin{cases} \delta^{(2)} A_\mu &= 0 \\ \delta^{(2)} \psi &= \epsilon_2 \end{cases}$$

Take a linear combination :

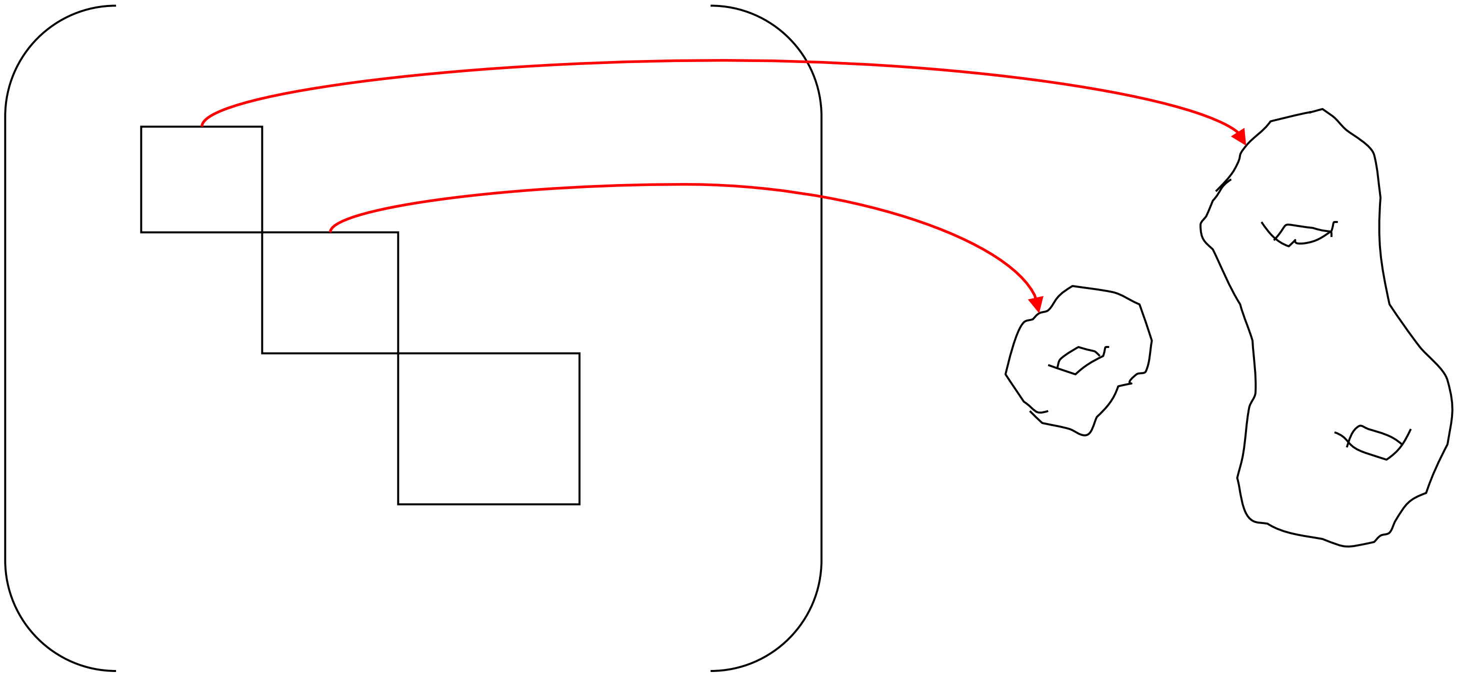
$$\tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)}$$
$$\tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)})$$

$$[\bar{\epsilon}_1 \tilde{Q}^{(i)}, \bar{\epsilon}_2 \tilde{Q}^{(j)}]\, A_\mu = -2\delta^{ij} \bar{\epsilon}_1 \Gamma_\mu \epsilon_2 \mathbf{1}_{N\times N}$$

“translation” is realized by $\delta A_\mu = c_\mu \mathbf{1}_{N\times N}$

⇒ consistent with identification of A_μ as "coordinates"

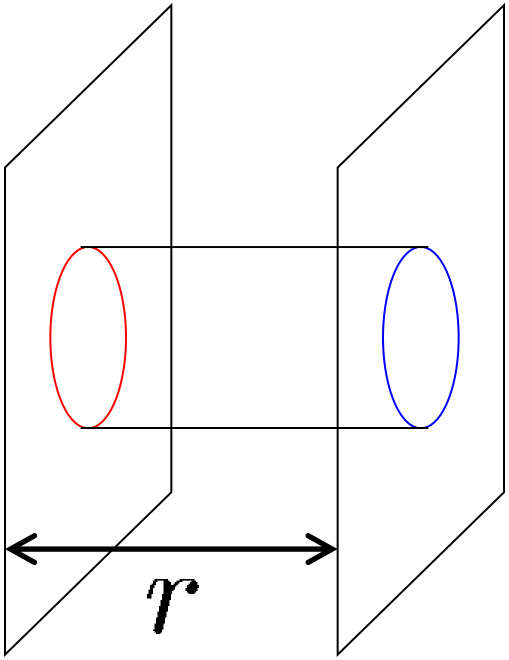
Natural realization of 2nd quantization



Each of these blocks ⟺ disconnected worldsheet

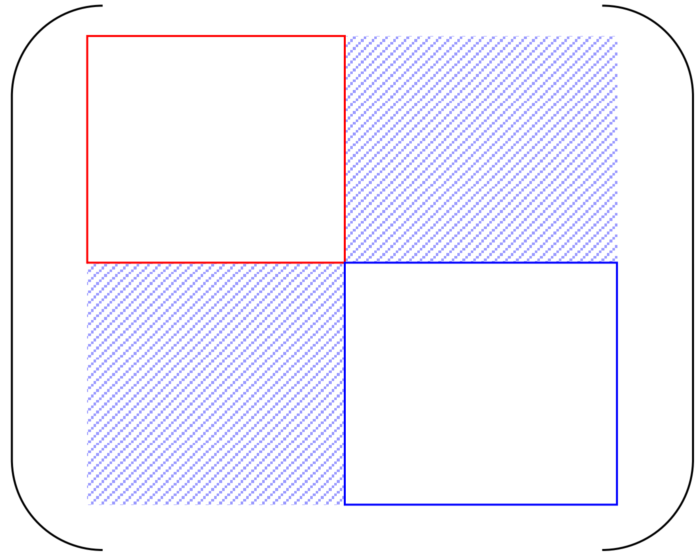
Many-body states of strings are naturally included !

Emergence of gravitons



D-brane D-brane

Propagation of
- graviton
- dilaton
- rank-2 anti-sym. tensor



Integrate out the off-diagonal elements to obtain the effective action

$$\Rightarrow \quad \sim \frac{1}{r^8}$$

Dynamical generation of Euclidean space-time

Wick rotation

$A_0 = iA_{10} \qquad \Gamma^0 = -i\Gamma_{10}$

Euclidean model

Finite without cutoff

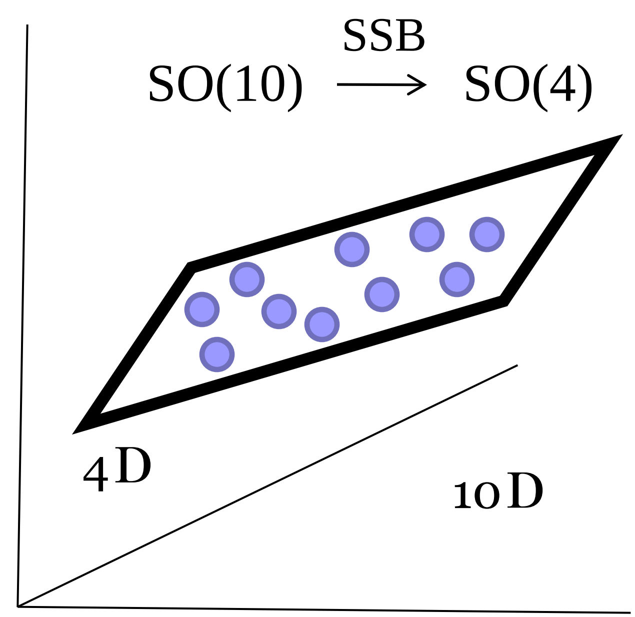
Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

Does our 4-dimensional space-time appear ?

$10\ N \times N$ Hermitian matrices

$$(A_\mu)_{ij} = (x_i)_\mu \delta_{ij} + (a_\mu)_{ij}$$

$$\mu = 1, \cdots, 10$$
$$i, j = 1, \cdots, N$$



Dynamical generation of Euclidean space-time (cont'd)

- Derivation of low-energy effective theory
  branched-polymer-like system   Aoki-Iso-Kawai-Kitazawa-Tada ('99)

- Explicit calculations by the Gaussian expansion method to study SSB of SO(10)

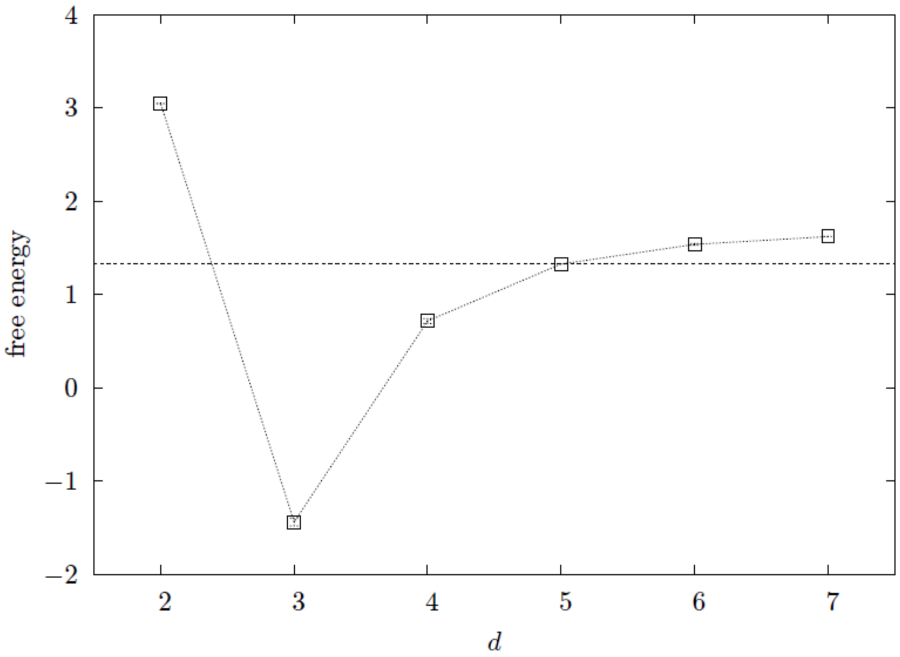
  Nishimura-Sugino ('02), Nishimura-Okubo-Sugino,
  Kawai, Kawamoto, Kuroki, Matsuo, Shinohara, Aoyama, Shibusa,…

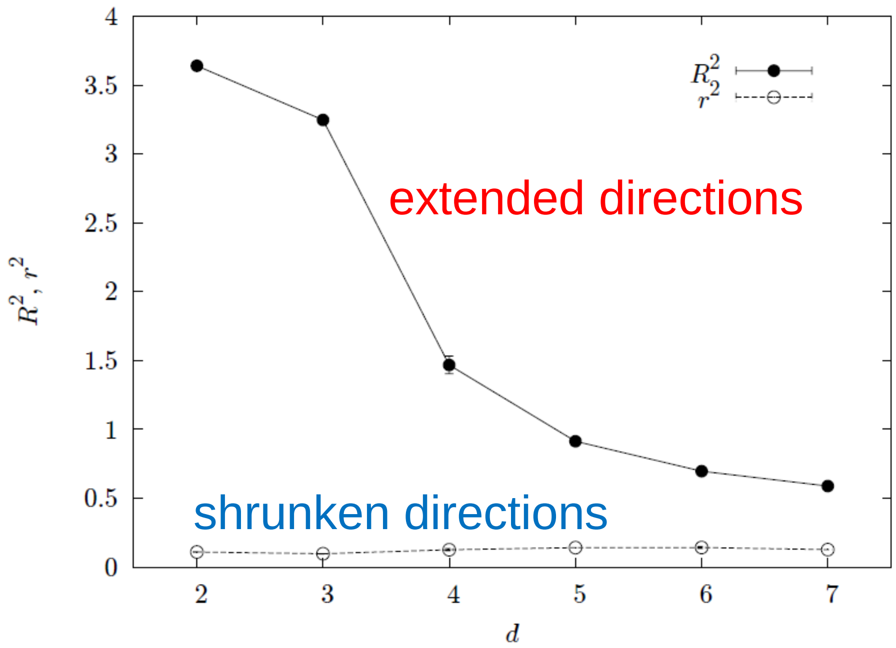
- Recent observation   Nishimura-Okubo-Sugino('11)

  1. free energy of SO(d) symmetric vacua (d=2,3,4,5,6,7)
     minimum at d=3
  2. extent of space-time   finite in all directions

Results of the Gaussian expansion method

J.N.-Okubo-Sugino (arXiv:1108.1293)





Minimum of the free energy occurs at d=3

Extent of space-time finite in all directions

SSB of SO(10) : interesting dynamical property of the Euclidean model, but is it really related to the real world ?

I-3 Summary

Summary of the 1st lecture

- Superstring theory
  - severeness of UV divergence in quantum gravity
    naturally hints at extended objects
  - unified theory of all particles (both forces and matters)
  - however, too many vacua ("landscape")
    due to variety of compactifications from 10d to 4d
  - fully nonperturbative formulation is crucial

- Matrix models
  - analogous to lattice gauge theory for QCD
  - IKKT model : nonperturbative formulation of superstrings
  - the Euclidean version has interesting dynamics
    but not quite realistic… (motivates Lorentzian version)