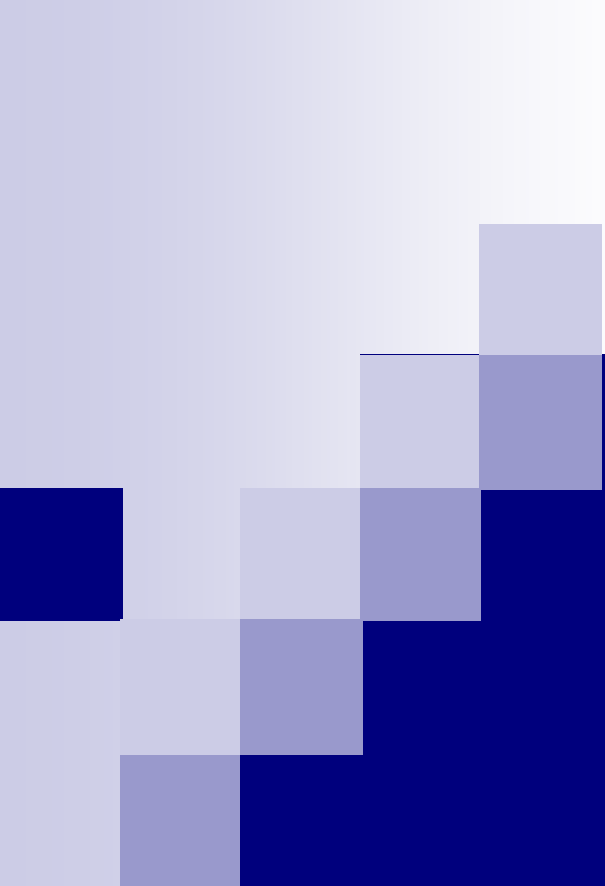




Matrix model formulations of superstring theory

6th Asian Winter School on strings, particles and
cosmology

2012 1/10-1/20, Kusatsu, Japan

Jun Nishimura (KEK, Sokendai)

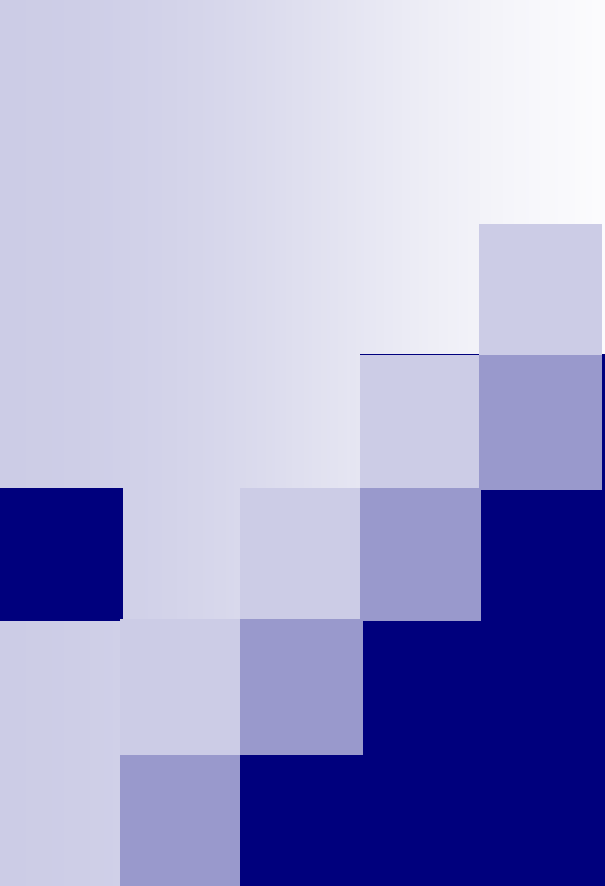
Plan of the lectures

I. Superstring theory and matrix models
(1/11 10:45-12:00)

II. Birth of our universe
(1/12 10:45-12:00)

III. Confirmation of gauge/gravity duality
(1/14 10:45-12:00)

Rem.) I will be here until 1/14 morning.
Please ask me questions before I leave.



II. Birth of our Universe



Plan of the 2nd lecture:

Birth of our universe

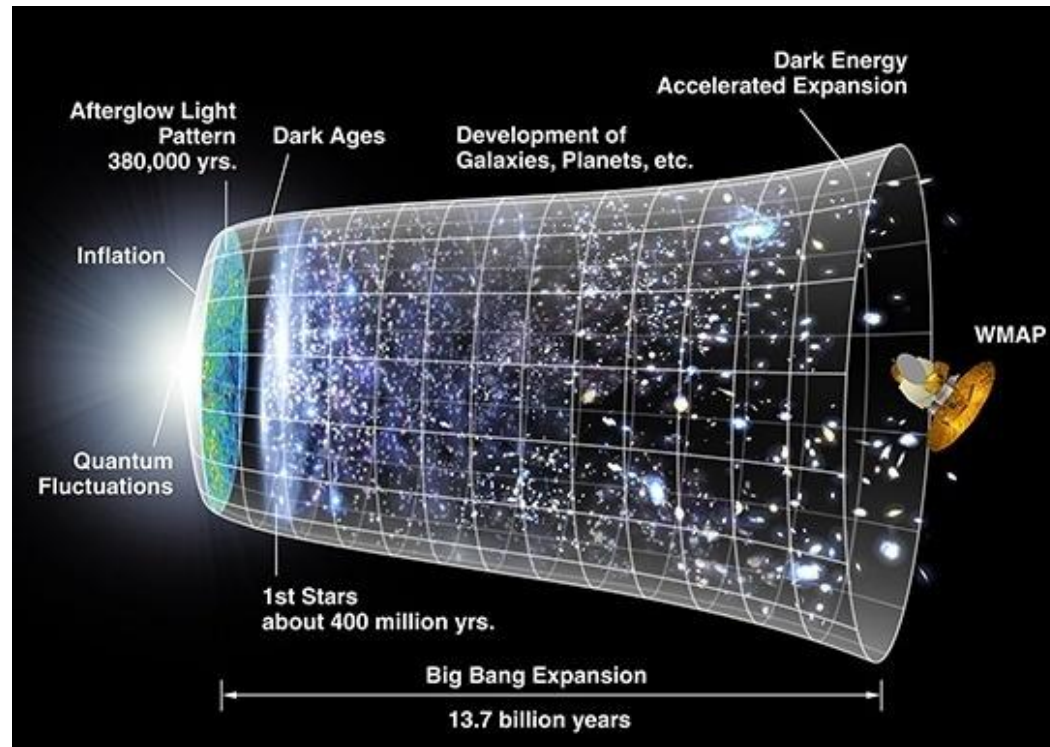
1. Review of previous works
2. Lorentzian matrix model
3. Summary



II-1 Review of previous works

1. Review of previous works

■ Big Bang cosmology



Our universe is expanding since it was born as an invisibly tiny point 13.7 billion years ago.

3 evidences for standard Big Bang cosmology

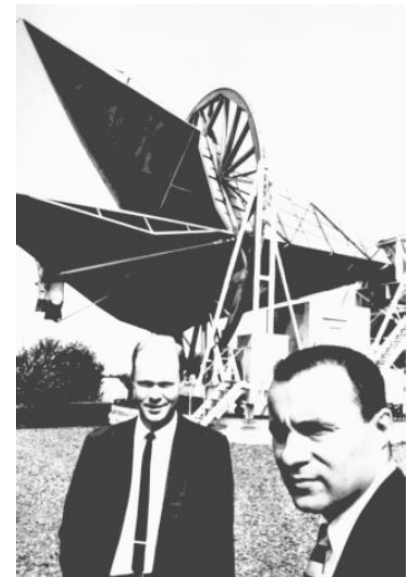
- Discovery of cosmic expansion (Hubble 1929)
- Theory of nucleosynthesis (Alpher-Bethe-Gamov 1948)
- Discovery of Cosmic Microwave Background
(Penzias-Wilson 1965)



Hubble

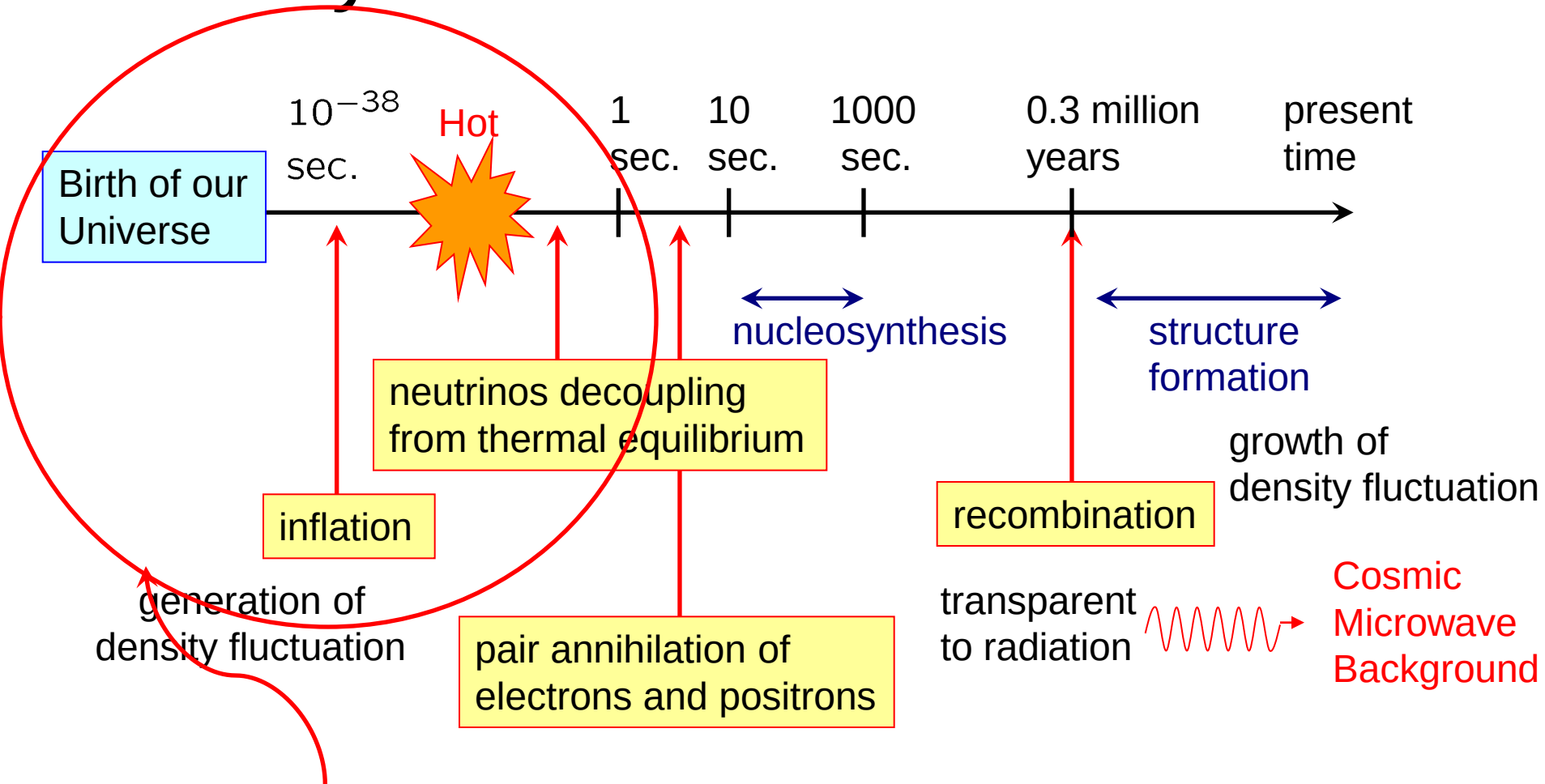


Gamov



Penzias, Wilson

History of our Universe

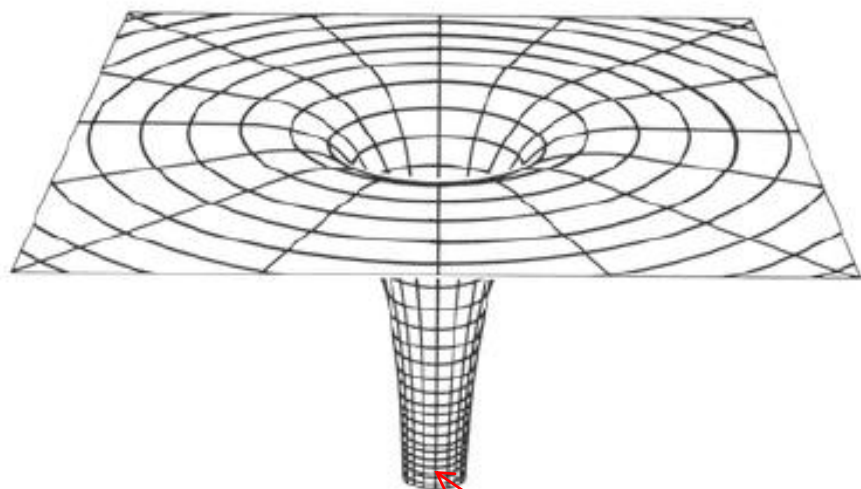


Unknown Region !

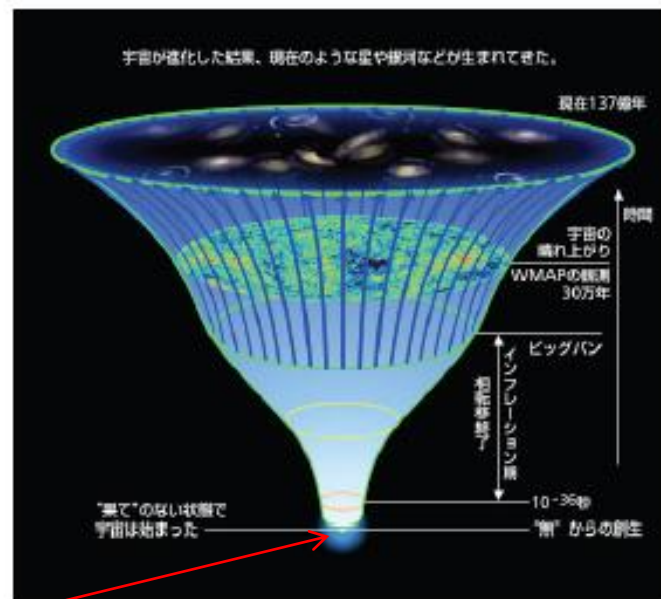
established by standard cosmological model

Singularities (space-time curvature diverges)

Black hole



Big bang



singularity (curvature diverges)

General Relativity becomes invalid!

(Quantum effects become non-negligible.)

We need to go beyond Einstein !

Quantum Cosmology in 1980s

- “Creation of Universes from nothing”

Vilenkin ('82,'84)

tunneling effects discussed with imaginary time

- “Wave function of the Universe”

Hartle-Hawking ('83)

“no boundary” proposal in path-integral formulation

The problem of UV divergence ignored
by restriction to a uniform isotropic universe.

More recent works

■ Causal Dynamical Triangulation

Ambjorn-Jurkiewicz-Loll ('05)

Path-integral over the metric

$\implies$ summation over simplicial manifolds

Lorentzian nature implemented
by introducing causal structure.

- Smooth (3+1)-dimensional space-time emerges dynamically.
- Restoration of full Lorentz symmetry is nontrivial. (c.f., Horava gravity)



II-2 Lorentzian matrix model

Kim-J.N.-Tsuchiya
PRL 108 (2012) 011601
[arXiv:1108.1540]

What we do here:

- Study the **Lorentzian matrix model** **nonperturbative** formulation of superstring theory in (9+1) dimensions.
- **Highly non-trivial time-evolution** of space obtained from dominant matrix configurations.
- **SO(9)** symmetry of space is broken spontaneously to **SO(3)** at some point in time, strongly suggesting **the birth of our Universe**.
- **No free parameter** in the theory except for 1 scale parameter (c.f., Λ_{QCD})
In particular, there is **no “initial condition” problem**.

Wick rotation is not obvious at all in gravitational theory !

- quantum field theory in flat space,

Wick rotation can be justified by [analytic continuation](#) for Green's functions.

- That's not the case when we have gravity.

- Amjorn, Jurkiewicz, Loll ('05)

- Within [dynamical triangulation approach](#),

- Lorentzian gravity is quite different from Euclidean gravity.

- Kawai-Okada ('10)

- [Coleman's multiverse proposal](#) for the cosmological constant problem revisited using Lorentzian gravity.

Matrix model with $SO(9,1)$ symmetry

$$S_b = -\frac{1}{4g^2} \text{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$

$$S_f = -\frac{1}{2g^2} \text{tr}(\Psi_\alpha (\mathcal{C} \Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta])$$

$N \times N$ Hermitian matrices

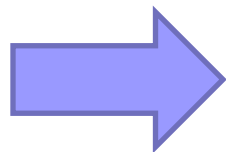
A_μ ($\mu = 0, \dots, 9$) Lorentz vector

Ψ_α ($\alpha = 1, \dots, 16$) Majorana-Weyl spinor

raised and lowered by the metric

$$\eta = \text{diag}(-1, 1, \dots, 1)$$

Wick rotation ($A_0 = -iA_{10}$, $\Gamma^0 = i\Gamma_{10}$)



Euclidean model with $SO(10)$ symmetry

Difference between Euclidean and Lorentzian

(I) bosonic action

■ Euclidean model

$$S_b \propto \text{tr} (F_{\mu\nu})^2 \qquad F_{\mu\nu} = -i[A_\mu, A_\nu]$$

positive definite

Classical flat direction is lifted up by quantum effects.

➡ The model is **well defined without any cutoff**.

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)

■ Lorentzian model

$$S_b \propto \text{tr} (F_{\mu\nu} F^{\mu\nu}) = -2 \underbrace{\text{tr} (F_{0i})^2}_{\text{opposite sign !}} + \text{tr} (F_{ij})^2$$

opposite sign !

Looks extremely **unstable** ! ➡

Hence, no one ever dared to study this model seriously!

Difference between Euclidean and Lorentzian

(II) Pfaffian (obtained by integrating out fermions)

$$\int d\Psi e^{iS_f} = \text{Pf}\mathcal{M}(A)$$

■ Euclidean model $\text{Pf}\mathcal{M}(A) \in \mathbb{C}$

The phase plays a crucial role in **SSB of SO(10)**,
But it makes **Monte Carlo studies extremely difficult**.

J.N.-Vernizzi ('00), Anagnostopoulos-J.N.('02)

■ Lorentzian model $\text{Pf}\mathcal{M}(A) \in \mathbb{R}$

Good news for Monte Carlo studies, but
we **lose a source of SSB**.

The definition of the partition function

■ Euclidean model

$$Z = \int dA d\psi e^{-S} = \int dA e^{-S_b} \text{Pf} \mathcal{M}(A)$$

■ Lorentzian model

$$Z = \int dA d\psi e^{iS} = \int dA e^{iS_b} \text{Pf} \mathcal{M}(A)$$

connection to the worldsheet theory

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4} \{X^\mu, X^\nu\}^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu \{X^\mu, \Psi\} \right)$$

$$\xi_0 \equiv -i\xi_2 \quad (\text{We need to Wick rotate the worldsheet coordinate, too.})$$

Regularizing the Lorentzian model

- We need to introduce **IR cutoffs** in both temporal and spatial directions.

$$\begin{aligned}\frac{1}{N} \text{tr} (A_0)^2 &\leq \kappa \\ \frac{1}{N} \text{tr} (A_i)^2 &\leq L^2\end{aligned}$$

- It turns out that we can **remove them** as we take the **large-N** limit. (highly nontrivial dynamical property)
- **Lorentz symmetry and supersymmetry** is broken explicitly, but this explicit breaking is expected to disappear in the large-N limit.
(need to be checked by studying **Ward identities**.)

How to deal with the phase factor e^{iS_b} in

$$Z = \int dA d\Psi e^{iS} = \int dA e^{iS_b} \text{Pf} \mathcal{M}(A)$$

Under the scale transformation $A_\mu \mapsto \rho A_\mu$

$$S_b \mapsto \rho^4 S_b$$

$$dA \mapsto \rho^{10(N^2-1)} dA$$

$$\text{Pf} \mathcal{M}(A) \mapsto \rho^{8(N^2-1)} \text{Pf} \mathcal{M}(A)$$

Integrating over the scale factor first, we get $\delta(S_b)$

and $\frac{1}{N} \text{tr}(A_i)^2 = L^2$

The model we put on a computer

$$Z = \int dA \delta \left(\frac{1}{N} \text{tr} (F_{\mu\nu} F^{\mu\nu}) \right) \text{Pf} \mathcal{M}(A) \\ \times \delta \left(\frac{1}{N} \text{tr} (A_i)^2 - 1 \right) \theta \left(\kappa - \frac{1}{N} \text{tr} (A_0)^2 \right)$$

Monte Carlo simulation : **Rational Hybrid Monte Carlo algorithm**
no sign problem unlike in the Euclidean model

c.f.) Yoneya's model ('97)

$$Z = \int dA \delta \left(\frac{1}{N} \text{tr} (F_{\mu\nu} F^{\mu\nu} - C) \right) \text{Pf} \mathcal{M}(A)$$

$C < 0$: a model motivated by space-time uncertainty principle

Extracting the time-evolution from matrices

- eigenvalue distribution of A_0

extends smoothly as $\kappa \rightarrow \infty$
thanks to supersymmetry

c.f.) bosonic model

eigenvalues of A_0 attract each other

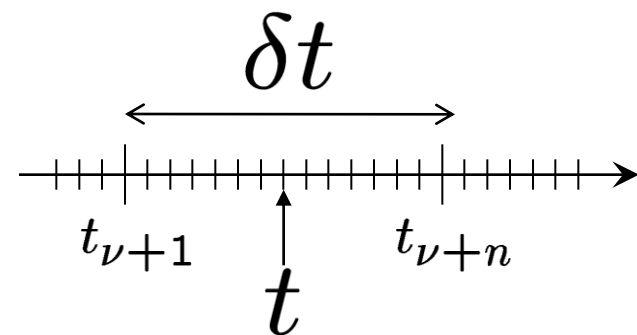


The distribution has finite extent
even in the $\kappa \rightarrow \infty$ limit

SUSY plays a crucial role in generating the time !

The diagram shows a large matrix A_0 enclosed in large parentheses. The diagonal elements are labeled $t_1, t_2, \dots, t_N$. A submatrix of size $n \times n$ is highlighted with a blue border. The top-left element of this submatrix is labeled $t_{\nu+1}$, and the bottom-right element is labeled $t_{\nu+n}$. An arrow points from the text "average t " to the center of the submatrix.

$$A_i = \left[\begin{array}{c|c} & \text{small} \\ \hline & n \\ \hline \text{small} & \bar{A}_i(t) \end{array} \right]$$



$$\nu = 0, 1, \dots, N - n$$

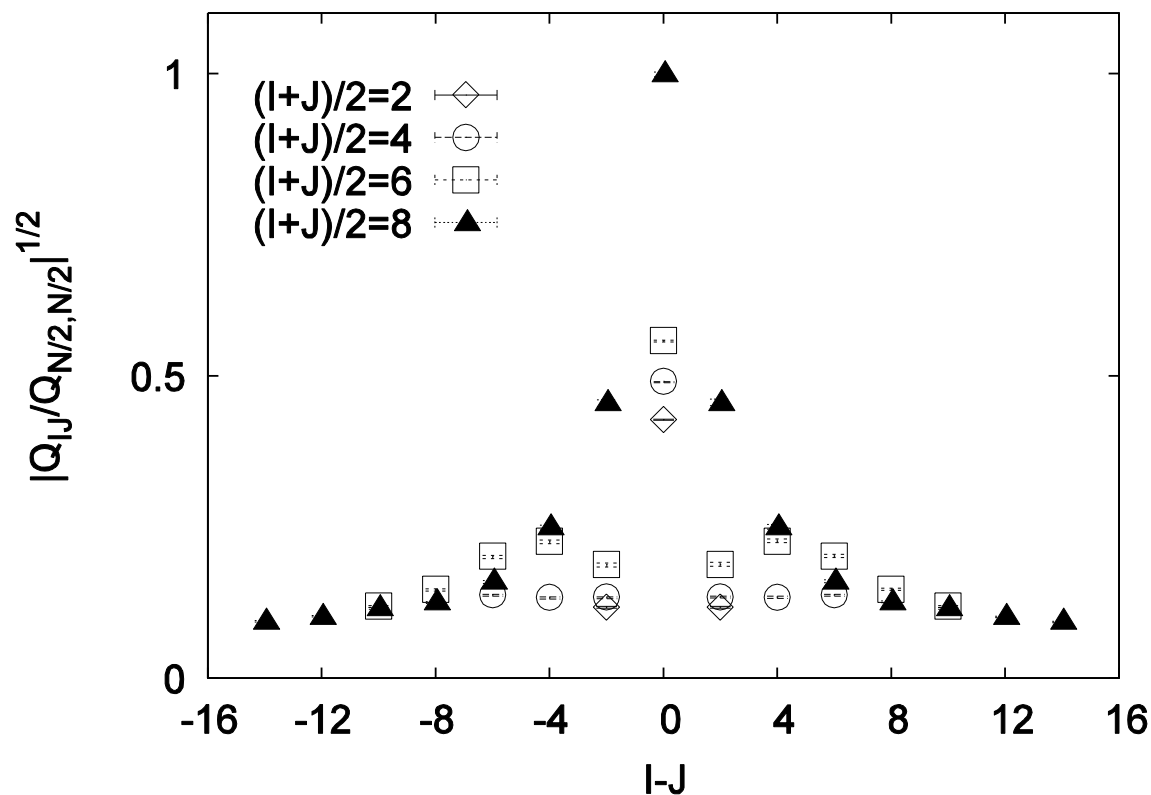
$$t = \frac{1}{n} \sum_{a=1}^n t_{\nu+a}$$

We observe band-diagonal structure

$\bar{A}_i(t)$ represents space structure at fixed time t

Determination of the block size

$N = 16$



$$(A_i)^2 = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

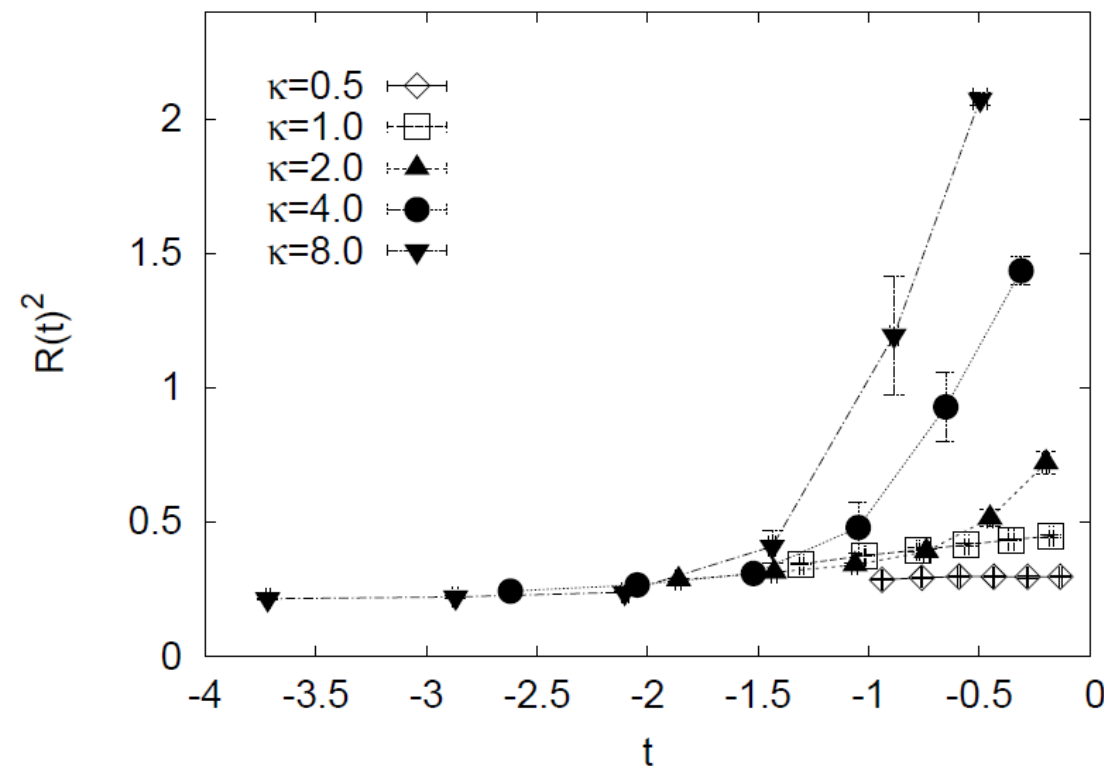
Diagonal elements are labeled:

- $(I+J)/2=2$
- $(I+J)/2=4$
- $(I+J)/2=6$
- $(I+J)/2=8$

We take $n = 4$

The size of the space v.s. time

$$R(t)^2 \equiv \frac{1}{n} \text{tr} \bar{A}_i(t)^2 \quad \text{peak at } t = 0 \text{ starts to grow for } \kappa > \kappa_{\text{cr}}$$



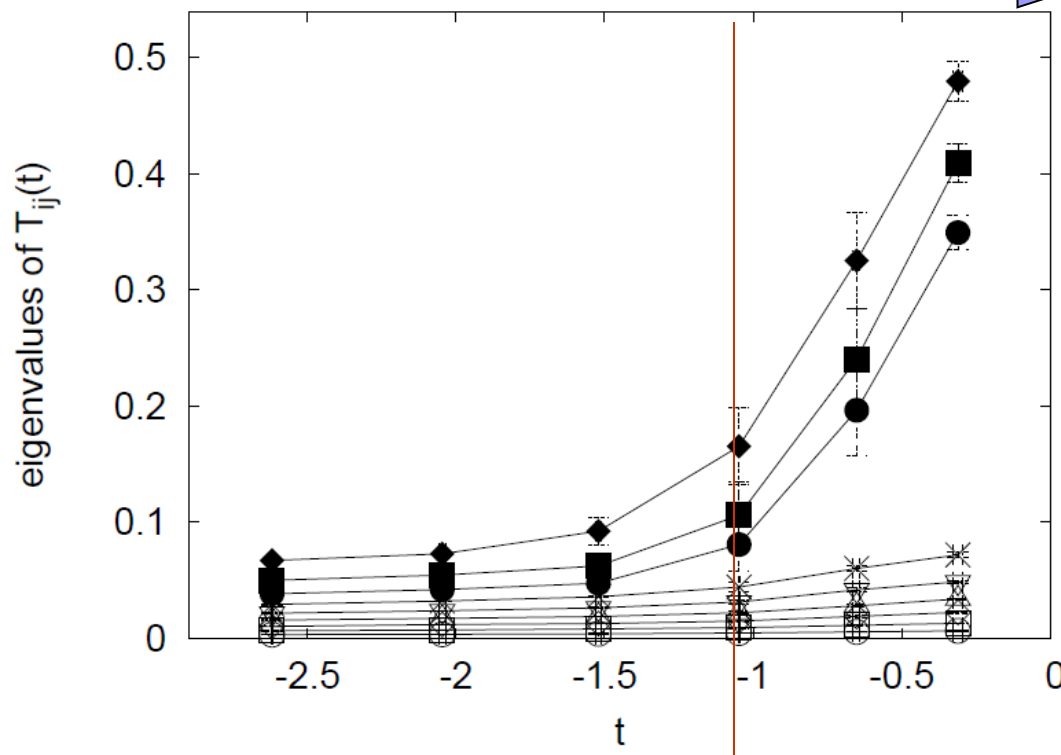
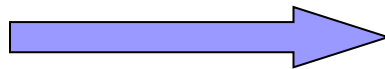
$$N = 16, \quad n = 4$$

Symmetric under $t \rightarrow -t$
We only show $t < 0$

Spontaneous breaking of SO(9)

$$T_{ij}(t) = \frac{1}{n} \text{tr} \{ \bar{A}_i(t) \bar{A}_j(t) \}$$

$\text{SO}(9) \xrightarrow{\text{SSB}} \text{SO}(3)$



$N = 16$, $\kappa = 4.0$

Mechanism of SSB

$$\text{tr}(F_{\mu\nu}F^{\mu\nu}) = 0 \iff 2\text{tr}(F_{01})^2 = \text{tr}(F_{ij})^2$$

$$F_{\mu\nu} = -i[A_\mu, A_\nu]$$

large $\kappa \implies \text{tr}(F_{0i})^2$ become large, and so does $\text{tr}(F_{ij})^2$

$\frac{1}{N}\text{tr}(A_i)^2 = 1 \implies$ It is more efficient to maximize $\text{tr}(F_{ij})^2$ at some fixed time

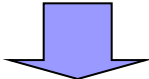
Middle point $t = 0$ is chosen, because the eigenvalue distribution of A_0 is denser around $t = 0$ so that enhancement to $\text{tr}(F_{0i})^2$ is the least.

Peak of $R(t)^2 = \frac{1}{n}\text{tr}(\bar{A}_i)^2$ at $t = 0$ grows as κ increases

Mechanism of SSB (cont'd)

$$\begin{aligned} &\text{maximize } \frac{1}{N} \text{tr} (F_{ij})^2 \\ &\text{with fixed } \frac{1}{N} \text{tr} (A_i)^2 = 1 \end{aligned}$$

$$\text{maximize } G = \text{tr} (F_{ij})^2 - \lambda \text{tr} (A_i)^2$$



 Lagrange multiplier

$$2 [A_j, [A_j, A_i]] - \lambda A_i = 0$$

solution :

$$\begin{aligned} A_i &= \chi L_i \quad \text{for } i \leq d \\ A_i &= 0 \quad \text{for } d < i \leq 9 \end{aligned}$$



representation matrices of a compact semi-simple Lie algebra
with d generators

 $L_i = \begin{pmatrix} \frac{1}{2} \sigma^i \\ 0 \end{pmatrix}$



d=3 !

An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.

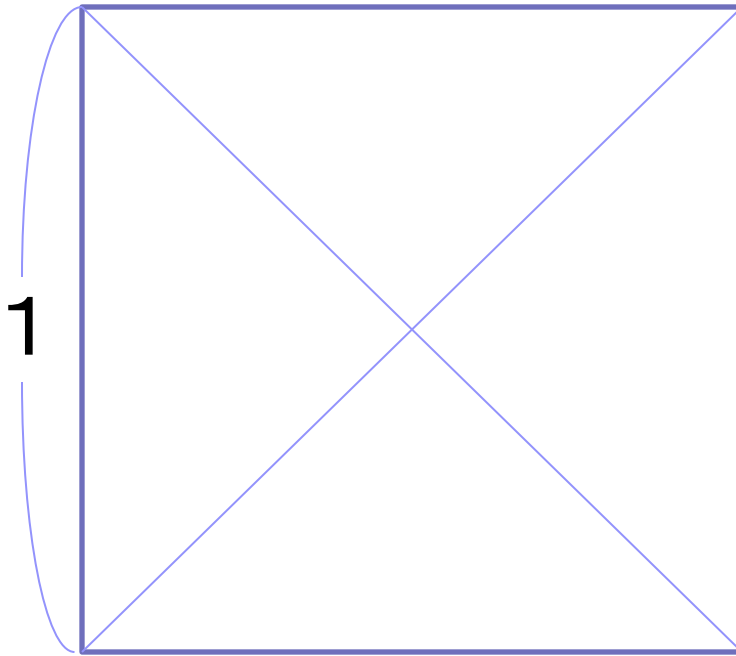


The set-up is invariant under rotation by 90°

This problem has rotational symmetry.

An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.



invariant under rotation

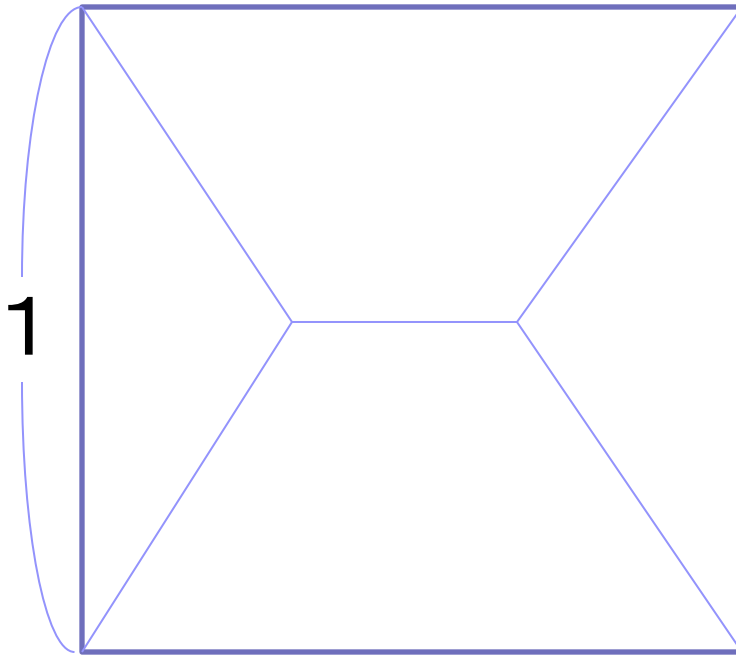
preserves rotational symmetry

$$\begin{aligned}\ell &= 2\sqrt{2} \\ &\sim 2.828\end{aligned}$$

Wrong !

An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.



Becomes a different figure if we rotate it by 90 degrees.

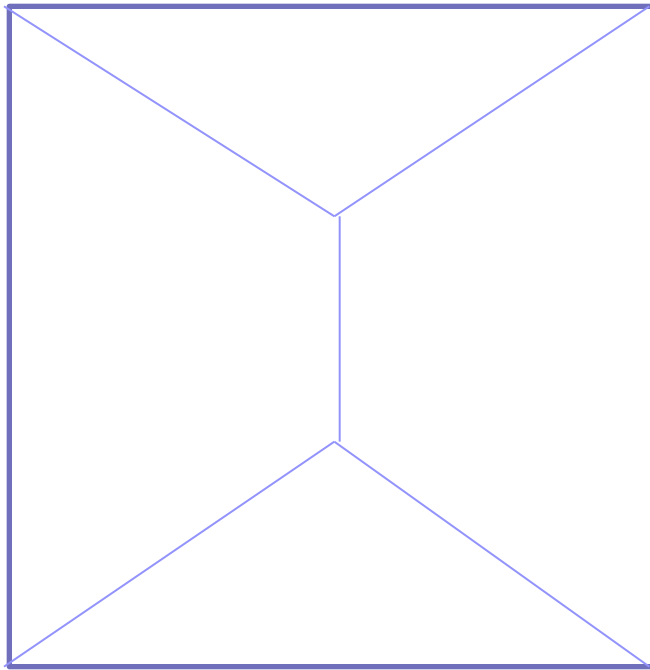
Breaks rotational symmetry

$$\begin{aligned}\ell &= 1 + \sqrt{3} \\ &\sim 2.732\end{aligned}$$

Correct !

An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.



This can be obtained by rotating the previous figure.

Correct, as well !

Examples of spontaneous symmetry breaking in physics

- Bardeen-Cooper-Schrieffer theory of superconductor formation of **Cooper pairs** breaks $U(1)$ symmetry
- Nambu:
the origin of hadron mass in QCD
formation of **chiral condensate** breaks chiral symmetry
pions can be considered as **Nambu-Goldstone bosons**.
- Higgs mechanism:
the origin of mass in Standard Model

our conjecture:

The SSB of $SO(9,1)$ in the Lorentzian matrix model:
the origin of our Universe

Removing the two cutoffs κ and L
in the $N \rightarrow \infty$ limit

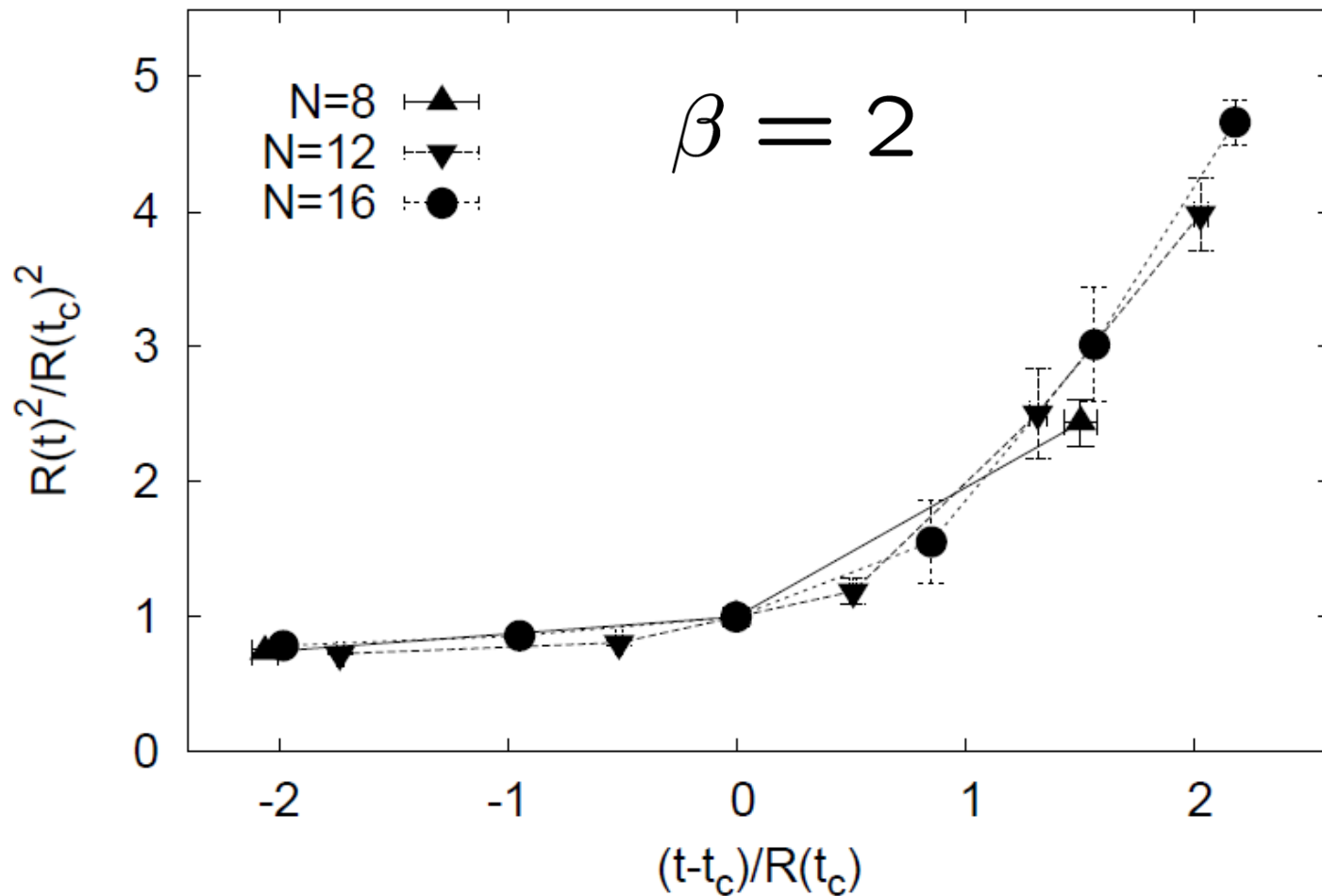
$$\left\{ \begin{array}{ll} 1) \ \kappa \rightarrow \infty \text{ with } N \rightarrow \infty & \text{(continuum limit)} \\ \kappa = \beta N^{1/4} & \\ 2) \ L \rightarrow \infty \text{ with } \beta \rightarrow \infty & \text{(infinite volume limit)} \\ \text{fix the scale by } R(t_c) & \end{array} \right.$$

The theory thus obtained has
no parameters other than one scale parameter !

[A property that nonperturbative superstring theory
is expected to have !]

Large-N scaling

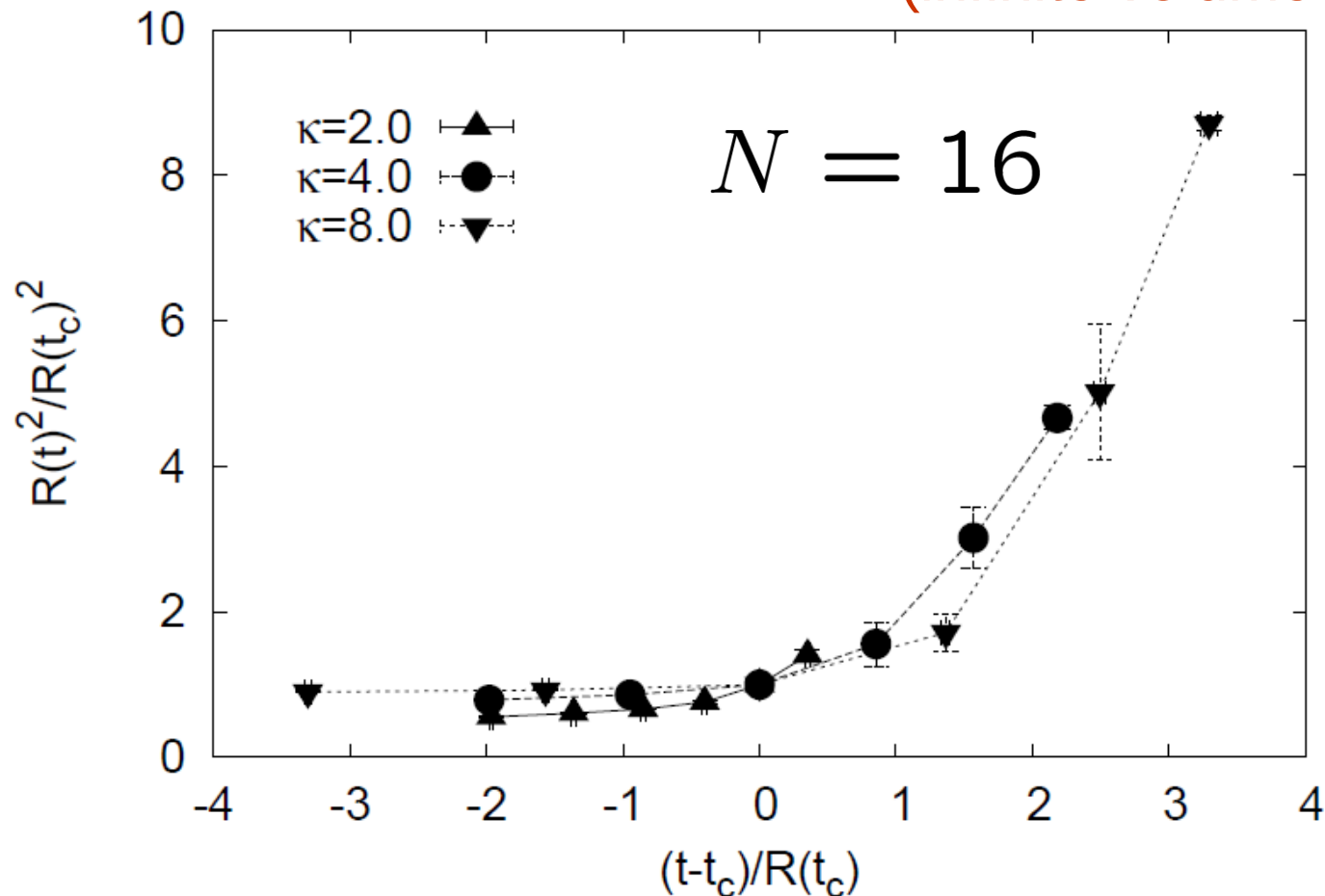
Clear large-N scaling behavior observed with $\kappa = \beta N^{1/4}$
(continuum limit)



Infinite volume limit

The extent of time increases and
the size of the universe becomes very large at later time.

(infinite volume limit)





I-3 Summary

Summary of the 2nd lecture

- A **new proposal** for the nonperturbative formulation of type IIB superstring theory in ten dimensions.

instead of making Wick rotation,

we introduce the **IR cutoffs**

for both temporal and spatial directions

(κ)

(L)

- The two cutoffs can be removed in the large-N limit.
- The theory thus obtained has **no parameters** other than one scale parameter.

- 
- Integrating over the scale factor first,
we obtain a model **without sign problem**.

c.f.) Monte Carlo studies of Euclidean model
difficult due to sign problem

- Monte Carlo simulation revealed
SSB of $SO(9)$ down to $SO(3)$ at some critical time.
mechanism is totally different from
that for the SSB in the Euclidean model

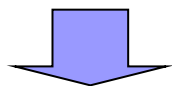
the size of the 3d space increases with time

- **Cosmological singularity** is naturally avoided
due to noncommutativity.

Crucial role played by SUSY

c.f.) bosonic model

eigenvalues of A_0 attract each other



the distribution does not extend as $\kappa \rightarrow \infty$

no expansion and no SSB !

Complementary studies based on classical equations of motion

Kim-J.N.-Tsuchiya arXiv:1110.4803

A class of $SO(3)$ symmetric solutions

- the time-dependence compatible with the expanding universe
- noncommutativity of space time : OK

Discussion

- It is likely that we are seeing the birth of our Universe in the Monte Carlo results.
- **We need larger N to see late times.**
- The mechanism of SSB $SO(9) \rightarrow SO(3)$ relies crucially on noncommutative nature of space.
- Does commutative space-time, we observe now, appear at late time?

Classical solutions

- At later times, we naively expect a **classical solution** to dominate because the action gets larger due to expansion.
- As a **complementary approach**, we therefore study classical solutions.
- It turns out that there are **infinitely many solutions** in the large- N limit (reminiscent of the **Landscape**).
- We need to find **which one is connected to Monte Carlo result**.
- We find some interesting examples of solutions which represent expanding universe with **commutative space-time**.

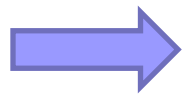
Classical solutions (cont'd)

EOM
$$\frac{\delta}{\delta A_\mu} \left(-\frac{1}{4} \text{tr}([A_\mu, A_\nu]^2) - \frac{\lambda}{2} \text{tr}(A_i)^2 - \frac{\tilde{\lambda}}{2} \text{tr}(A_0)^2 \right) = 0$$

Example
$$A_0 = \sqrt{-\lambda} T_0, \quad A_1 = c_1 \sqrt{-\tilde{\lambda}} T_1, \quad A_2 = c_2 \sqrt{-\tilde{\lambda}} T_1, \quad A_3 = c_3 \sqrt{-\tilde{\lambda}} T_1$$

$SL(2, R)$ algebra commutative space-time

$$[T_0, T_1] = iT_2 \quad [T_0, T_2] = -iT_1 \quad [T_1, T_2] = -iT_0$$

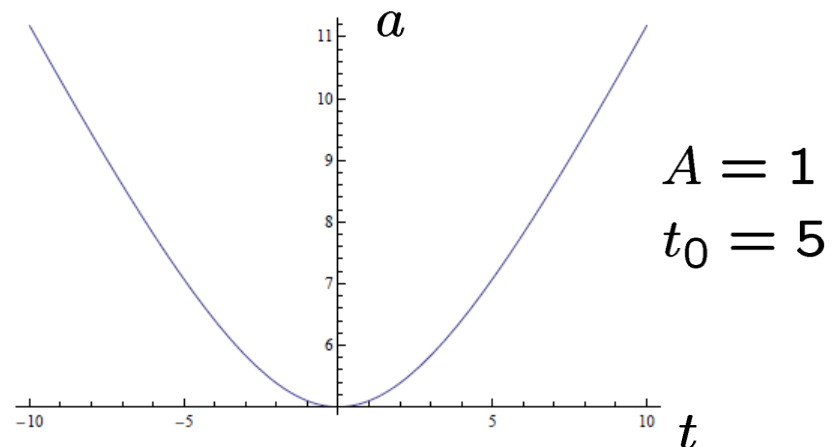


$$a(t) = A \sqrt{t^2 + t_0^2}$$

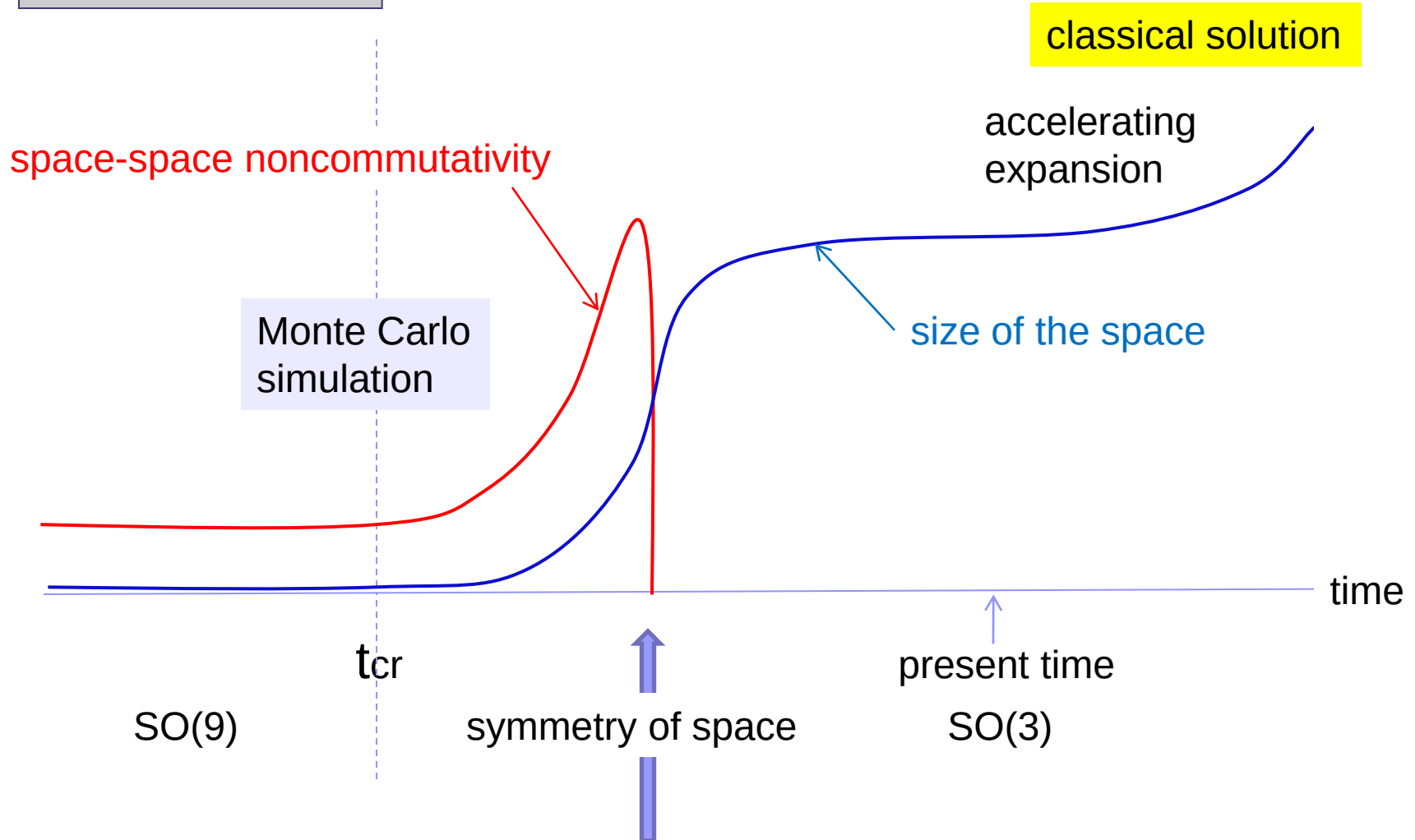
$$H = \frac{\dot{a}}{a} \sim a^{-\frac{3}{2}(1+w)} \quad w = -\frac{1}{3} \left(\frac{2t_0^2}{t^2} + 1 \right)$$

$$t = t_0 \longrightarrow w = -1$$

$$t \rightarrow \infty \longrightarrow w = -\frac{1}{3}$$



Speculations



Space-space NC disappears for some dynamical reason.

My main message:

- The birth of our Universe might be considered as an issue that goes far beyond the reach of science.
- Intriguingly, the issue might be the first “real physics” that can be addressed by superstring theory.
- This is not so surprising since superstring theory is supposed to be a fundamental theory at the Planck scale.
- Reproducing Standard Model at the TeV scale may be more challenging, but it is totally well-defined.
That will prove that superstring theory/matrix model is indeed describing our Nature.

Future prospect (I)

Inflation (1981)

A rapid accelerating expansion believed to have occurred just after the birth of our iverse

Solves various puzzles in the Big Bang cosmology naturally.

Explains detailed properties of the Cosmic Microwave Background



Katsuhiko Sato



Alan Guth

Can we clarify the mechanism of inflation ?

Future prospects (II)

Accelerating expansion
in the present epoch

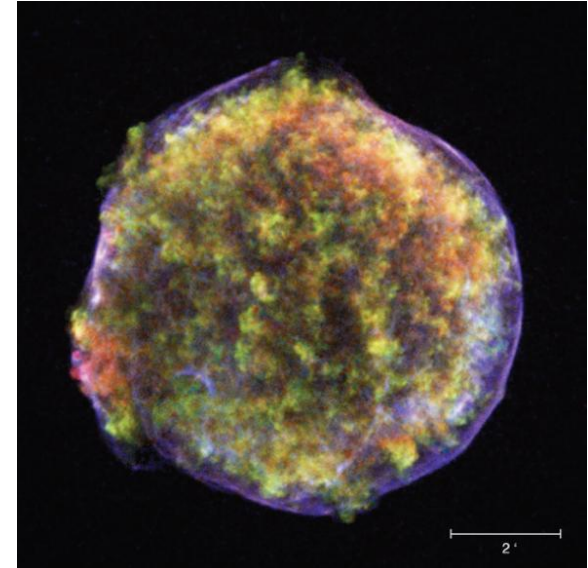
Observation of **type Ia supernovae** →

Nobel Prize in Physics, 2011

Precise measurement of CMB

(WMAP etc.)

etc.



➡ The existence of **dark energy**

Mysterious energy that does not dilute as the universe expands
Occupies more than 70% of the total energy of the Universe

Can we explain it by quantum gravitational effects
in superstring theory ?

Future prospects (III)

Remaining puzzles in particle physics

- dark matter

Strongly suggested by cosmic observations

No good candidate in Standard Model of particle physics
(possibly) superpartners, excited modes in extra dimensions

- Higgs particle

The only particle in Standard Model that is still undiscovered.

Introduced for a mechanism to give mass to particles.

Unnatural from the viewpoint of particle physics including gravity

Needs clarification from both experimental and theoretical sides.

Can we clarify all these puzzles using superstring theory ?

Important issues to be addressed in Lorentzian matrix model

- Does a **local** field theory on a **commutative** space-time appear at later time ?
- How do **4 fundamental interactions** and the **matter fields** appear at later time ?



Monte Carlo simulation AND
Studies of classical solutions (+ quantum corrections)
Some approach like “renormalization group”

We hope the Lorentzian matrix model
provides a new perspective on

- [particle physics beyond the standard model
- [cosmological models for inflation, modified gravity, etc..



BACKUP SLIDES

Regularizing the Lorentzian model

In order to separate space and time,
we “gauge fix” the boost invariance.

the gauge-fixed config. $\{\tilde{A}_\mu\}$ with $\tilde{A}_\mu = O_{\mu\nu}A_\nu$
is chosen such that $\frac{1}{N}\text{tr}(\tilde{A}_0)^2$ is minimized
with respect to $O \in \text{SO}(9, 1)$

$\text{SO}(9)$ symmetry is still manifest.

(1) IR cutoff in the temporal direction

$$\frac{1}{N}\text{tr}(A_0)^2 \leq \kappa \frac{1}{N}\text{tr}(A_i)^2$$

(invariant under $A_\mu \rightarrow \rho A_\mu$)

Regularizing the Lorentzian model (cont'd)

Regularizing oscillating functions

$$Z = \int dA e^{iS_b} \underbrace{e^{-\epsilon|S_b|}}_{\text{convergence factor}} \text{Pf} \mathcal{M}(A)$$

$\epsilon \rightarrow 0$ limit after path integral

$$Z = \int dA \underbrace{\int_0^\infty dr \delta\left(\frac{1}{N} \text{tr}(A_i)^2 - r\right)}_{\text{inserting unity}} e^{iS_b - \epsilon|S_b|} \text{Pf} \mathcal{M}$$

rescaling variables $A_\mu \rightarrow r^{1/2} A_\mu$

$$\int_0^\infty dr r^{\frac{18}{2}(N^2-1)-1} e^{r^2(iS_b - \epsilon|S_b|)} \propto \frac{1}{|S_b|^{\frac{18}{4}(N^2-1)}}$$

diverges at $S_b = 0$

Regularizing the Lorentzian model (cont'd)

This divergence comes from
integrating over the scale factor r



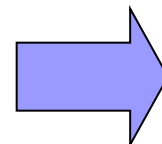
Cure this divergence by imposing :

(2) IR cutoff in the spatial direction

$$\frac{1}{N} \text{tr} (A_i)^2 \leq L^2$$

$$\int_0^{L^2} dr r^{\frac{18}{2}(N^2-1)-1} e^{r^2(iS_b - \epsilon|S_b|)} \propto \left(\frac{1}{L^4} + c|S_b| \right)^{-\frac{18}{4}(N^2-1)}$$

for sufficiently large L and N



$\delta(S_b)$

Thus we arrive at

$$Z = \int dA \delta \left(\frac{1}{N} \text{tr} (F_{\mu\nu} F^{\mu\nu}) - C \right) \text{Pf} \mathcal{M}(A) \\ \times \delta \left(\frac{1}{N} \text{tr} (A_i)^2 - 1 \right) \theta \left(\kappa - \frac{1}{N} \text{tr} (A_0)^2 \right)$$

$C = 0$, according to our derivation

$\left[C < 0 : \text{a model motivated by space-time un-} \right.$
 $\left. \text{certainty principle} \qquad \qquad \text{Yoneya ('97)} \right]$

Monte Carlo simulation : **Rational Hybrid Monte Carlo algorithm**
no sign problem unlike in the Euclidean model