

Quantum Hall states: lessons from curved space

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Motivation

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- This talk is not be related to AdS/CFT,

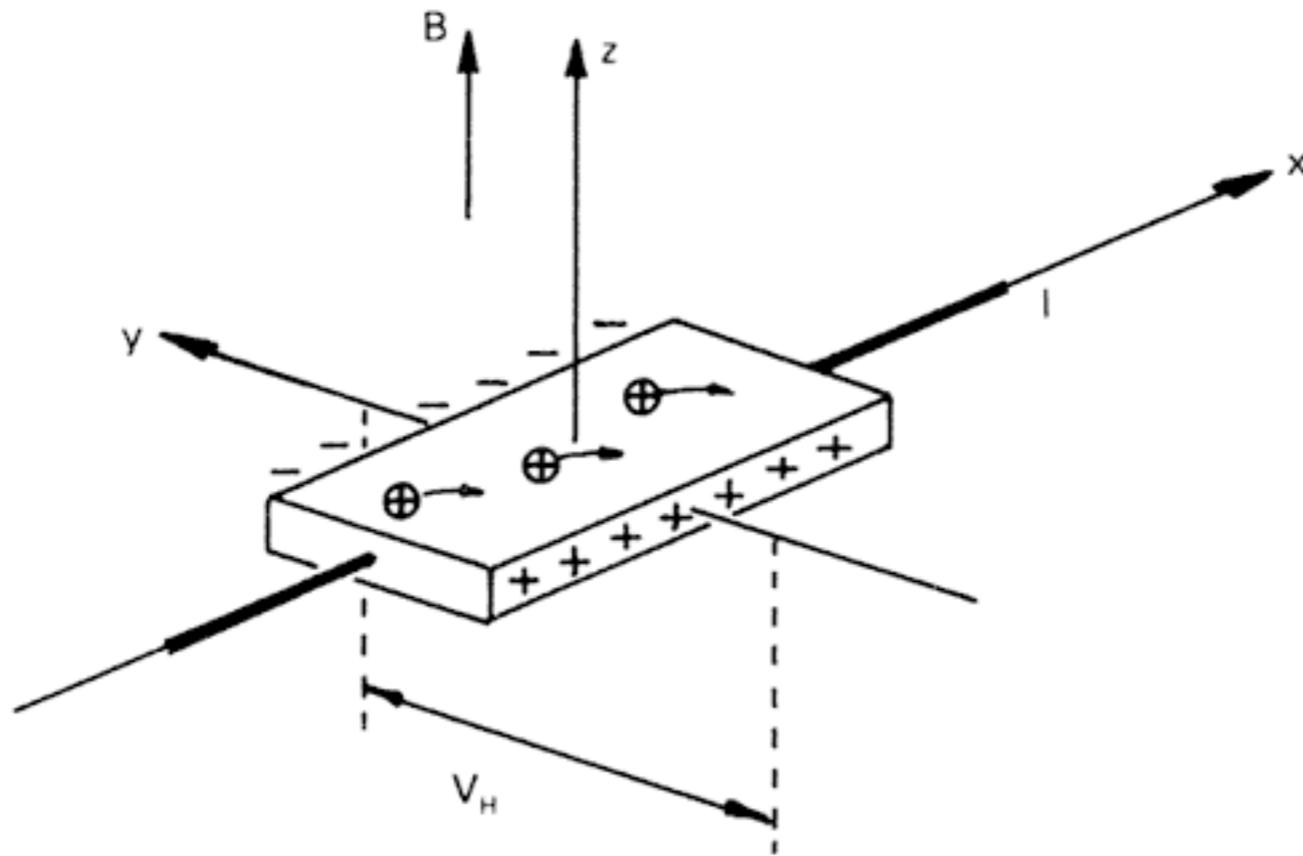
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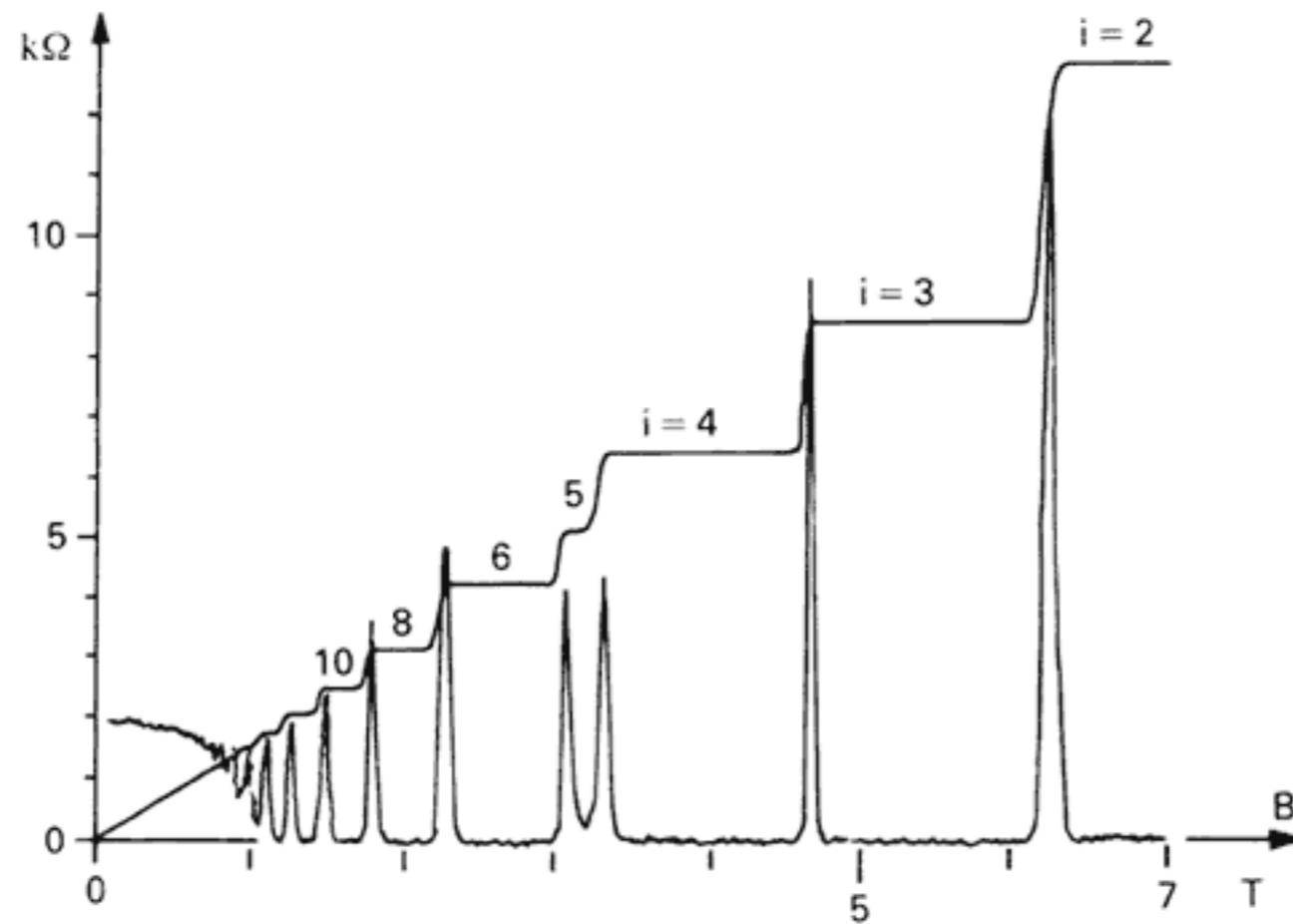
- This talk is not be related to AdS/CFT,
- but we will see how thinking about curve space helps us understand flat-space physics
- Namely, we will see how general symmetry principles help constrain the physics of the quantum Hall state, even in the nonperturbative regime (FQHE)

Hall effect



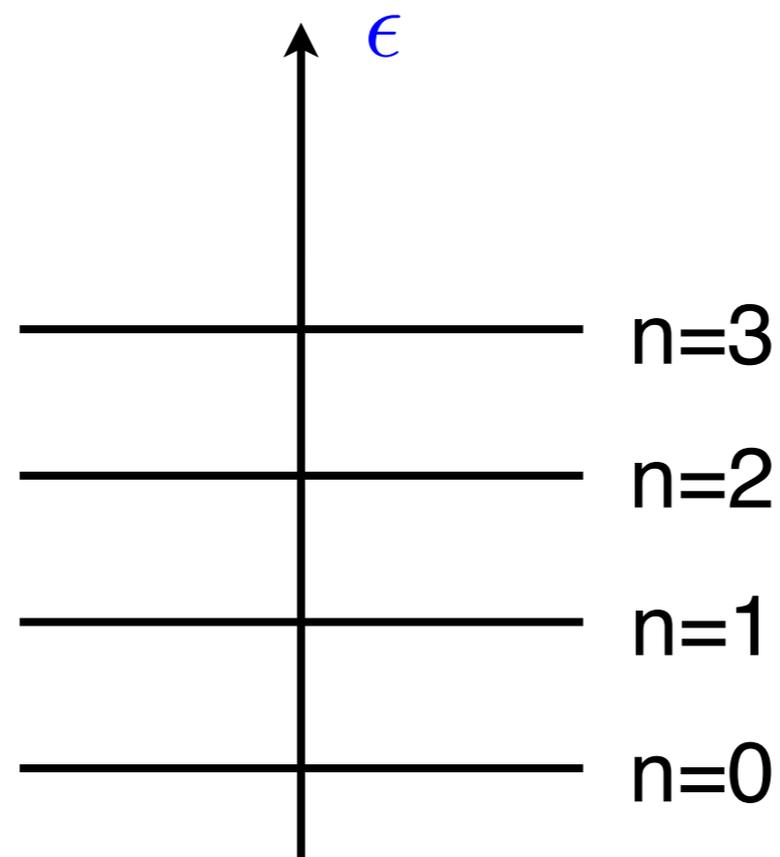
$$R_{xy} = \frac{V_y}{I_x}$$

Integer Quantum Hall effect



Integer quantum Hall state

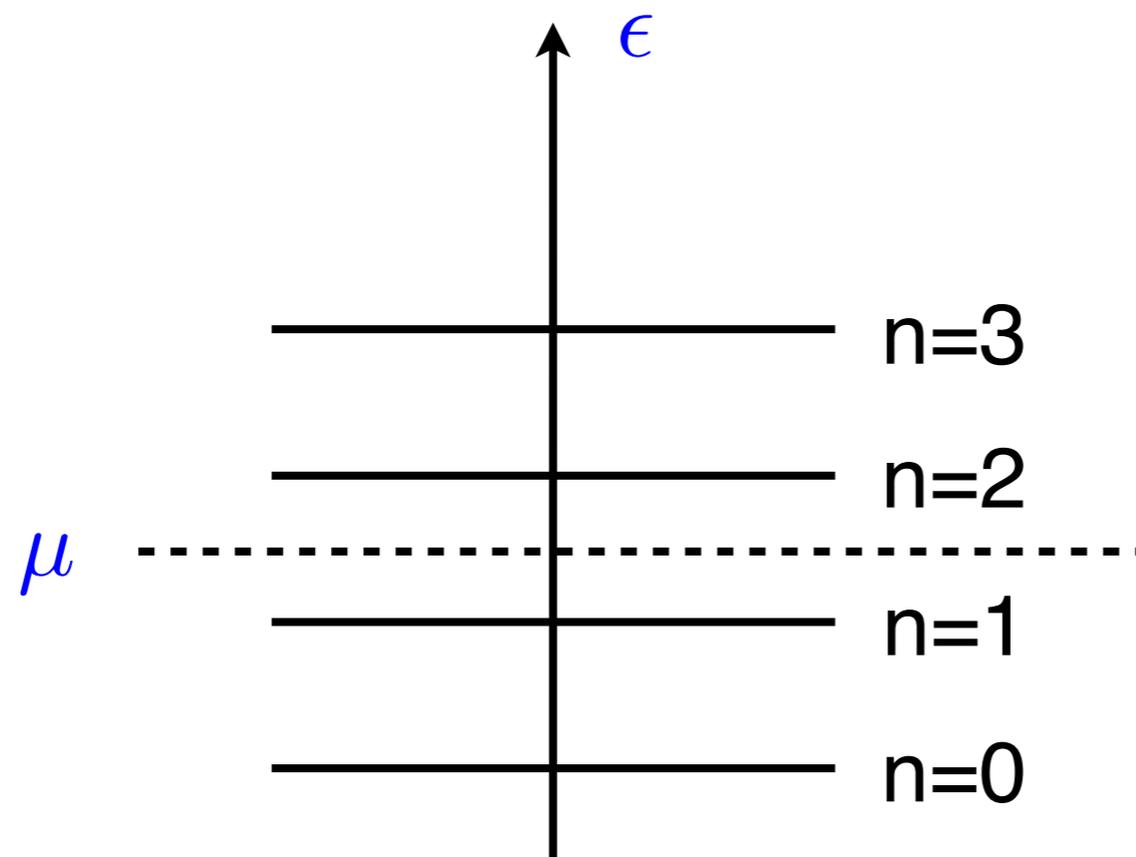
- noninteracting electrons filling n Landau levels



$$\epsilon_n = B \left(n + \frac{1}{2} \right)$$

Integer quantum Hall state

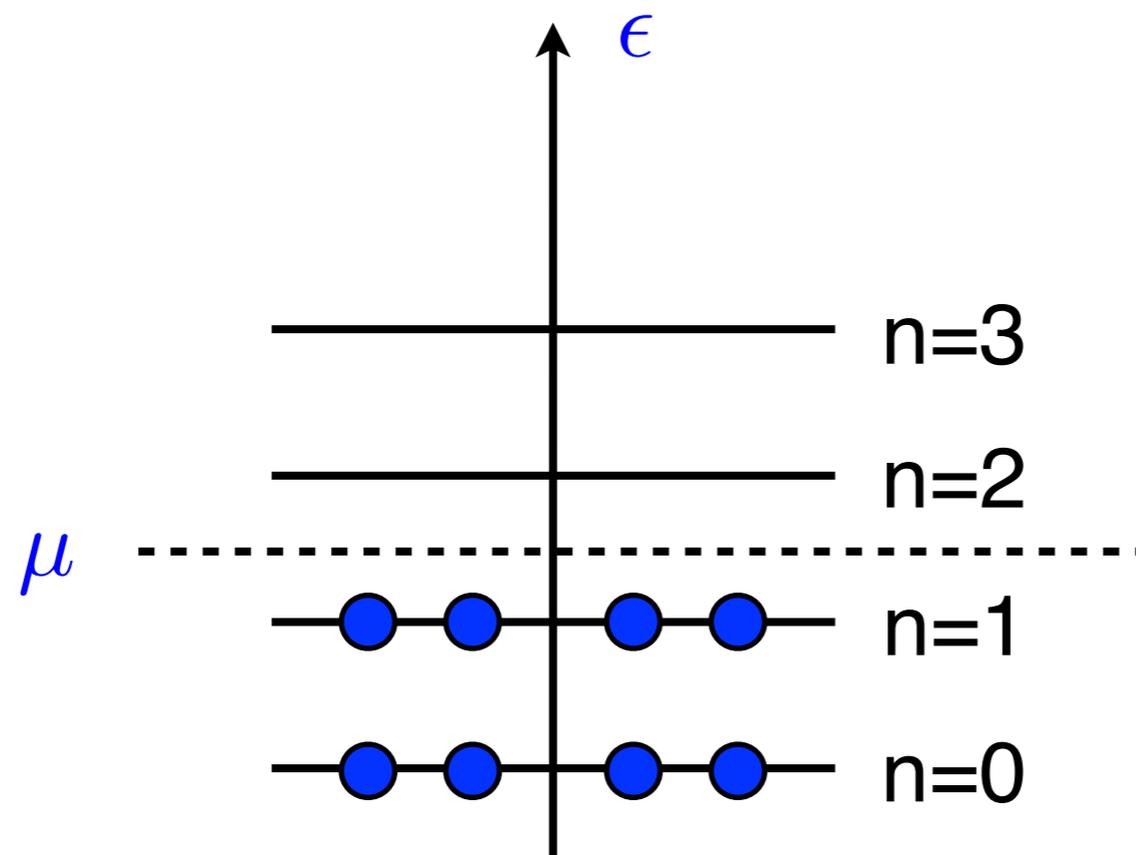
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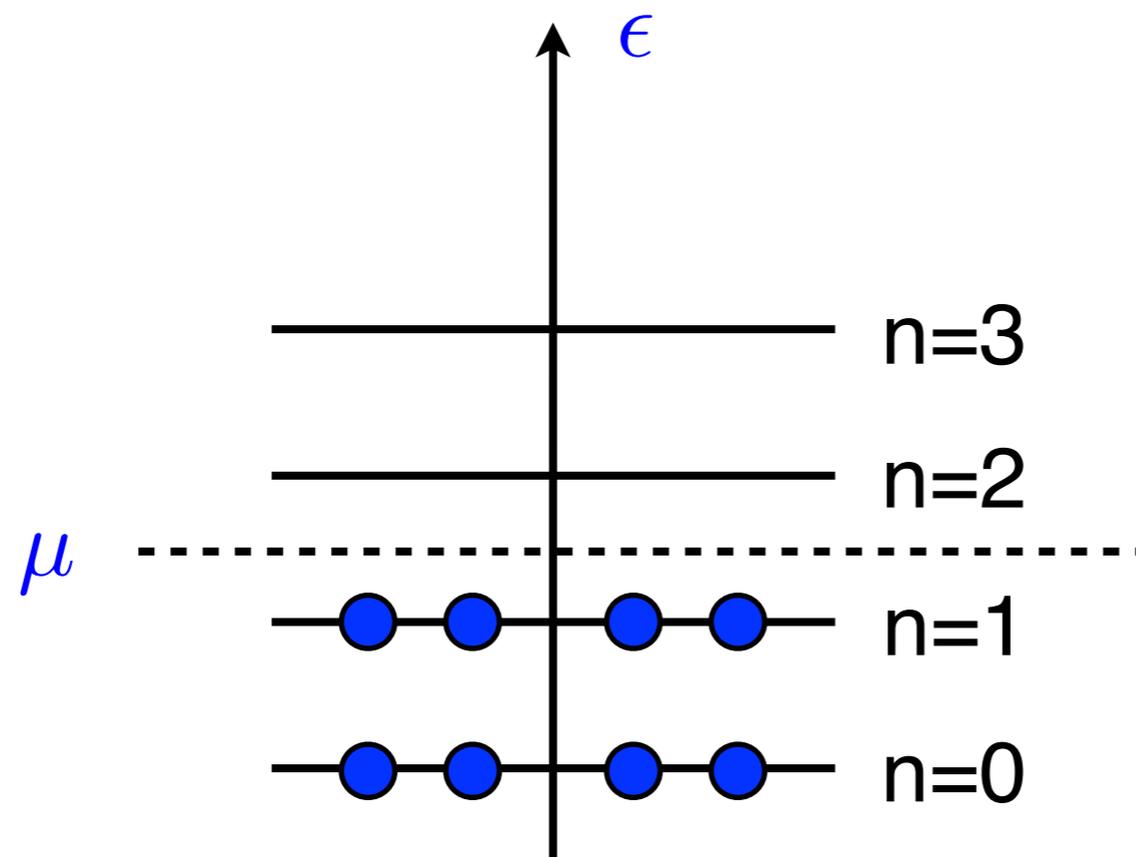
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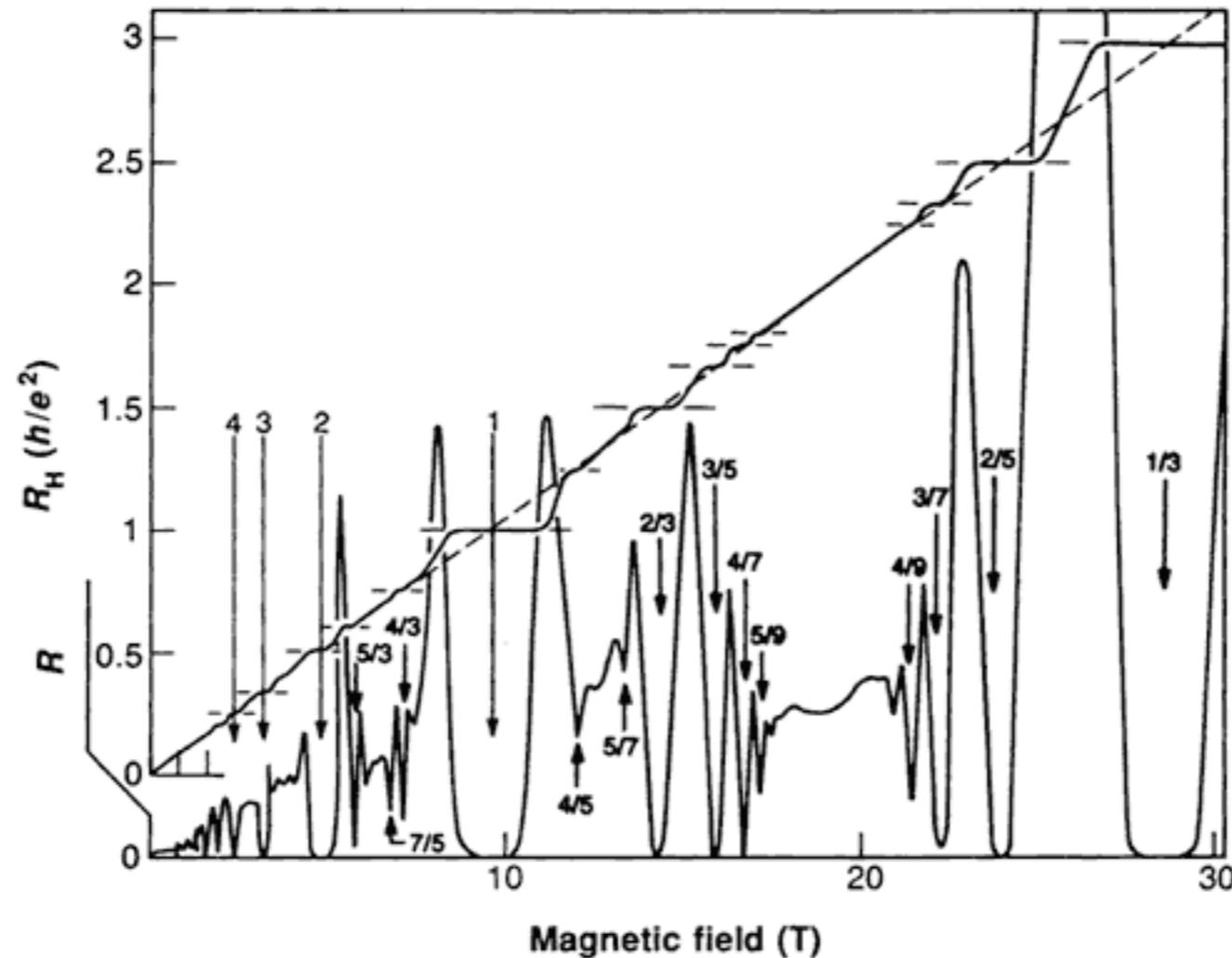


$$\epsilon_n = B \left(n + \frac{1}{2} \right)$$

Integer filling factor:

$$\nu = \frac{\rho}{B/2\pi}$$

Fractional QHE



Cannot be understood from free particles.
Theory: Laughlin's wave function

Effective field theory

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 - there are none! system completely gapped
- Effective action can be expanded in polynomials of external fields

Chern-Simons action

$$S = \frac{\nu}{4\pi} \int d^3x \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda$$

$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{\nu}{2\pi} \epsilon^{\mu\nu\lambda} F_{\nu\lambda}$$

filling factor

$$j^x = \frac{\nu}{2\pi} E_y$$

||

$$\sigma_{xy}$$

What is missing

- CS action does not involve metric
- Stress-energy tensor = 0
- It is not how real quantum Hall system behaves

Hall viscosity

Avron et al 1995

- Turn on $h_{xy}(t)$ metric perturbations
- observe $T_{xx} = -T_{yy} \sim h'_{xy}(t)$
- there must be a term proportional first derivative of metric in the effective Lagrangian
- How? curvature \sim 2nd derivative

Wen-Zee term

- Hall viscosity: described by Wen-Zee term
(W.Goldberger & N.Read unpublished; N.Read 2009 KITP talk)
- Introduce spatial vielbein (viel=2) $g_{ij}=e^a_i e^a_j$
- We can now define the spin connection

$$\omega_i = \frac{1}{2} \epsilon^{ab} e^{aj} \nabla_i e^{bj} \qquad \omega_0 = \frac{1}{2} \epsilon^{ab} e^{aj} \partial_0 e^{bj}$$

Vielbein defined up to a local O(2) rotation

$$e_i^a \rightarrow e_i^a + \lambda \epsilon^{ab} e_i^b$$

$$\omega_\mu \rightarrow \omega_\mu - \partial_\mu \lambda$$

like an abelian gauge field

Vielbein and curvature

$$\partial_1 \omega_2 - \partial_2 \omega_1 = \frac{1}{2} \sqrt{g} R$$

Wen-Zee terms

in addition to the Chern-Simons term

$$\frac{1}{2\pi} \epsilon^{\mu\nu\lambda} (\kappa A_\mu \partial_\nu \omega_\lambda + \kappa' \omega_\mu \partial_\nu \omega_\lambda)$$

will not be important for
further discussions



The first term gives rise to

- Wen-Zee shift
- Hall viscosity

Wen-Zee shift

$$\frac{\kappa}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu \omega_\lambda = \frac{\kappa}{4\pi} \sqrt{g} A_0 R + \dots$$

Total particle number:

$$Q = \int d^2x \sqrt{g} j^0 = \int d^2x \sqrt{g} \left(\frac{\nu}{2\pi} B + \frac{\kappa}{4\pi} R \right) = \nu N_\phi + \kappa \chi$$

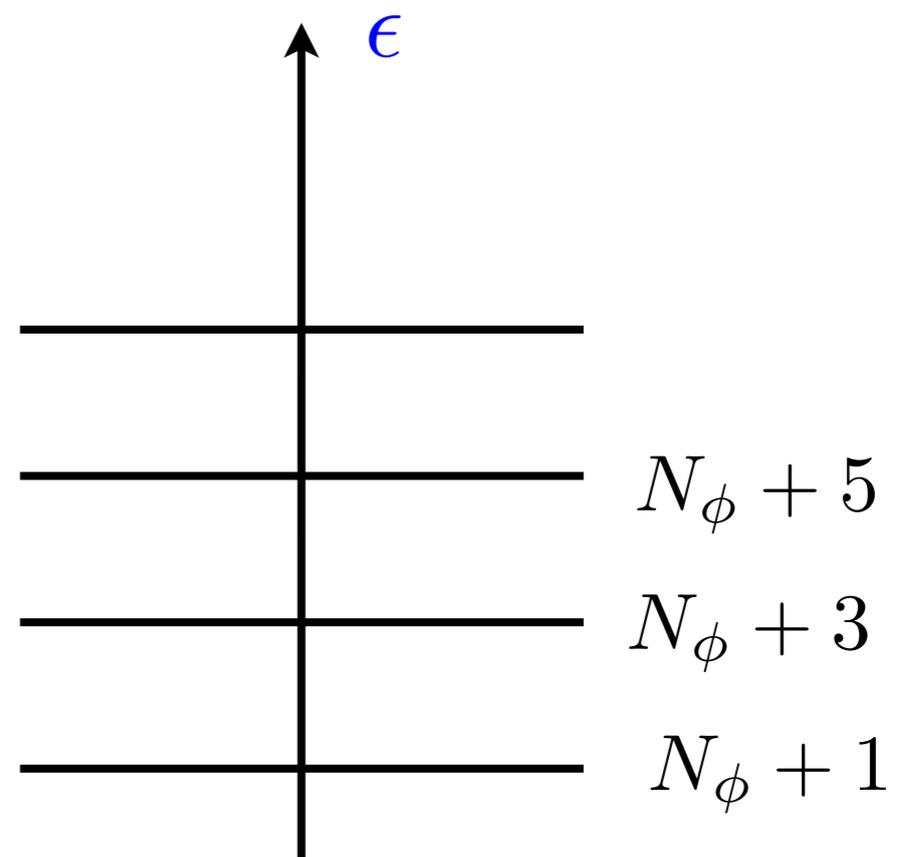
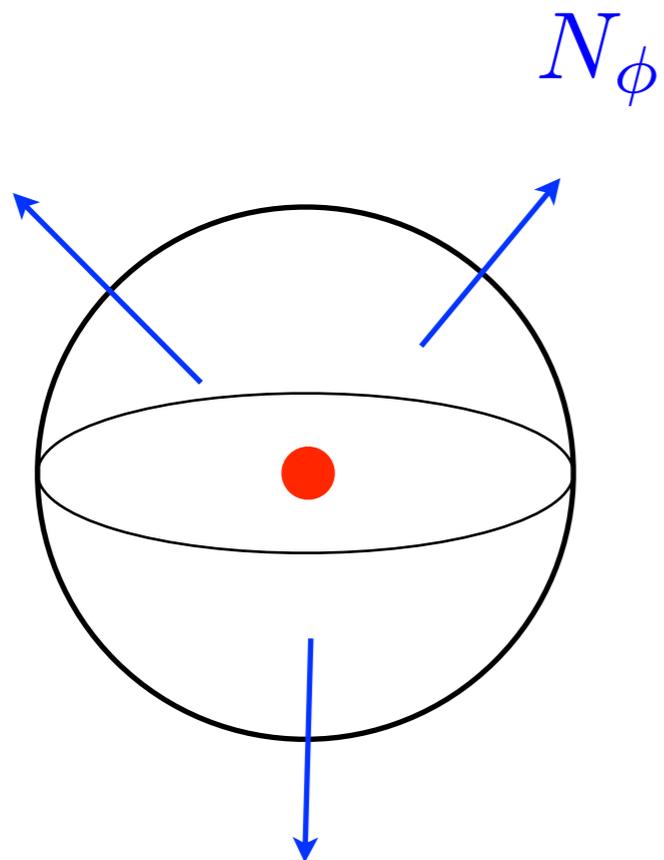
of magnetic fluxes
↓
Euler

On a sphere: $Q = \nu(N_\phi + \mathcal{S}), \quad \mathcal{S} = \frac{2\kappa}{\nu}$

‘shift’

κ can be determined by counting the number of particles in the QH state

Particle on a sphere



Examples

IQH states: $\nu=n$, $\kappa=n^2/2$

Laughlin's states: $\nu=1/n$, $\kappa=1/2$

Hall viscosity from WZ term

$$S_{\text{WZ}} = -\frac{\kappa B}{16\pi} \epsilon^{ij} h_{ik} \partial_t h_{jk} + \dots$$

stress \sim time derivative of metric

$$\eta^a = \frac{\kappa B}{4\pi} = \frac{1}{4} S n$$

derived by N.Read
previously



Flat space physics

- But is this Wen-Zee term be important for physics in flat space?
- In this talk we will argue that it is
- Reason: nonrelativistic diffeomorphism
- For a nonrelativistic system of particles with the same charge/mass ratio, there is a nonrelativistic principle of equivalence
 - accelerated frame \sim electric field
 - rotating frame \sim magnetic field (Coriolis force \sim Lorentz force)
 - nonrelativistic diffeomorphism mixes metric and EM field

Symmetries of NR theory

DTS, M.Wingate 2006

Microscopic theory

$$S_0 = \int dt d^2x \sqrt{g} \left[\frac{i}{2} \psi^\dagger \overleftrightarrow{D}_t \psi - \frac{g^{ij}}{2m} D_i \psi^\dagger D_j \psi \right] \quad D_\mu \psi \equiv (\partial_\mu - iA_\mu) \psi$$

Gauge invariance: $\psi \rightarrow e^{i\alpha} \psi \quad A_\mu \rightarrow A_\mu + \partial_\mu \alpha$

General coordinate invariance: $\xi = \xi(\mathbf{x})$

$$\delta \psi = -\xi^k \partial_k \psi \equiv \mathcal{L}_\xi \psi$$

$$\delta A_0 = \xi^k \partial_k A_0 \equiv \mathcal{L}_\xi A_0$$

$$\delta A_i = -\xi^k \partial_k A_i - A_k \partial_i \xi^k \equiv \mathcal{L}_\xi A_i$$

$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{kj} \partial_i \xi^k - g_{ik} \partial_j \xi^k \equiv \mathcal{L}_\xi g_{ij}$$

NR diffeomorphism

- These transformations can be generalized to be time-dependent: $\xi = \xi(t, \mathbf{x})$

$$\delta\psi = -\mathcal{L}_\xi\psi$$

$$\delta A_0 = -\mathcal{L}_\xi A_0 - A_k \dot{\xi}^k$$

$$\delta A_i = -\mathcal{L}_\xi A_i - m g_{ik} \dot{\xi}^k$$

$$\delta g_{ij} = -\mathcal{L}_\xi g_{ij}$$

Time dependent diffeomorphisms mix metric and gauge field

Galilean transformations: special case $\xi^i = v^i t$

Where does it come from

Start with complex scalar field

$$S = - \int dx \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \phi^* \phi)$$

Take nonrelativistic limit:

$$g_{\mu\nu} = \begin{pmatrix} -1 + \frac{2A_0}{mc^2} & \frac{A_i}{mc} \\ \frac{A_i}{mc} & g_{ij} \end{pmatrix}$$

$$\phi = e^{-imcx^0} \frac{\psi}{\sqrt{2mc}}$$

$$S = \int dt d\mathbf{x} \sqrt{g} \left[\frac{i}{2} \psi^\dagger \overleftrightarrow{\partial}_t \psi + A_0 \psi^\dagger \psi - \frac{g^{ij}}{2m} (\partial_i \psi^\dagger + iA_i \psi^\dagger) (\partial_j \psi - iA_j \psi) \right].$$

Relativistic diffeomorphism

$$x^\mu \rightarrow x^\mu + \xi^\mu$$

$\mu = 0$: gauge transform

$$\phi = e^{-imcx^0} \frac{\psi}{\sqrt{2mc}}$$

$\mu = i$: general coordinate transformations

Interactions

- Interactions can be introduced that preserve nonrelativistic diffeomorphism
- interactions mediated by fields
- For example, Coulomb interactions: mediated by photon propagating in 3+1 dimensions

$$S = S_0 + \int dt d^2x \sqrt{g} a_0 (\psi^\dagger \psi - n_0) + \frac{2\pi\epsilon}{e^2} \int dt d^2x dz \sqrt{g} [g^{ij} \partial_i a_0 \partial_j a_0 + (\partial_z a_0)^2]$$

$$\delta a_0 = -\xi^k \partial_k a_0$$

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$$\delta S_{\text{CS}} = \frac{\nu m}{2\pi} \int dt d^2x \epsilon^{ij} E_i g_{jk} \dot{\xi}^k$$

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But this cannot be achieved by local terms

Resolution

- Higher order terms contain inverse powers of B

$$\varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \frac{m}{B} g^{ij} E_i E_j + \dots$$

- Quantum Hall state with diff. invariance does not exist at zero magnetic field

Diff invariant terms

$$\mathcal{L}_1 = \frac{\nu}{4\pi} \left(\varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda + \frac{m}{B} g^{ij} E_i E_j \right)$$

Kohn's theorem ~ 1960

$$\mathcal{L}_2 = \frac{\kappa}{2\pi} \left(\varepsilon^{\mu\nu\lambda} \omega_\mu \partial_\nu A_\lambda + \frac{1}{2B} g^{ij} \partial_i B E_j \right)$$

$$\mathcal{L}_3 = -\epsilon(B) - \frac{m}{B} \epsilon''(B) g^{ij} \partial_i B E_j$$

$\sigma_{xy}(\mathbf{q})$

↑
ground state
energy density

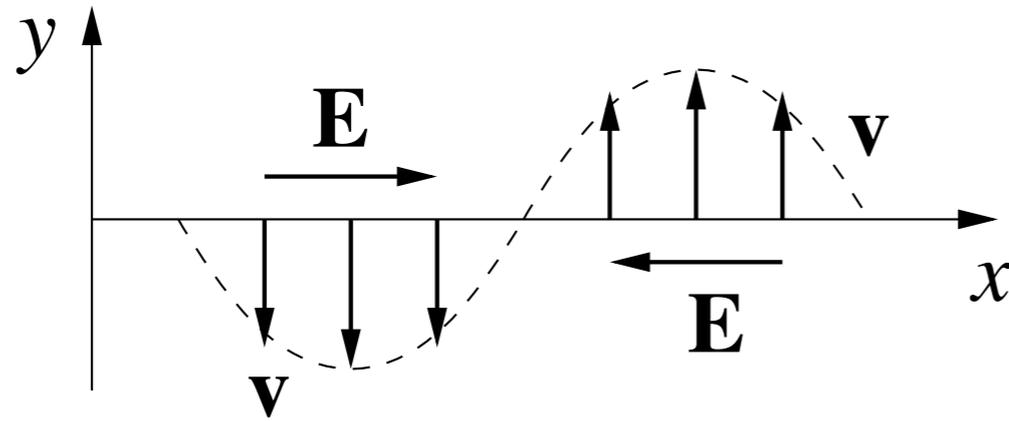
black: leading

blue: subleading

Kohn's theorem

- Response of the system on uniform electric field does not depend on interactions
- Effective action captures first order in ω corrections to conductivities at $q=0$

$\sigma_{xy}(q)$: new prediction



$$E_x = E e^{iqx}$$

$$j_y = \sigma_{xy}(q) E_x$$

From effective field theory

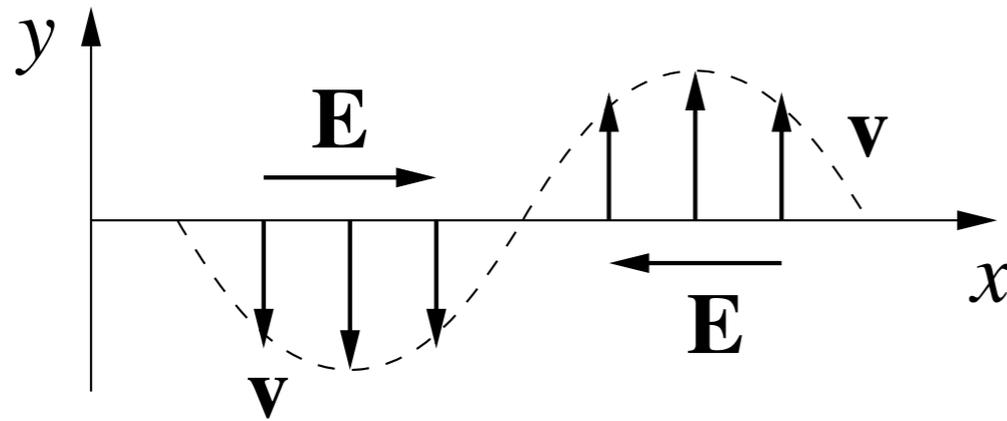
$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 + C_2 (ql)^2 + \mathcal{O}(q^4 l^4)$$

$$C_2 = \frac{\eta^a}{\hbar n} - \frac{2\pi}{\nu} \frac{\ell^2}{\hbar \omega_c} B^2 \epsilon''(B)$$

||
 $S/4$

Physical interpretation

- First term: Hall viscosity



$$\cancel{\partial_x v_y + \partial_y v_x \neq 0}$$

$$T_{xx} = T_{xx}(x) \neq 0$$

additional force $F_x \sim \partial_x T_{xx}$

Hall effect: additional contribution to v_y

Physical interpretation (II)

- 2nd term: more complicated interpretation

Fluid has nonzero angular velocity

$$\Omega(x) = \frac{1}{2} \partial_x v_y = -\frac{cE'_x(x)}{2B} \qquad \delta B = 2mc\Omega/e$$

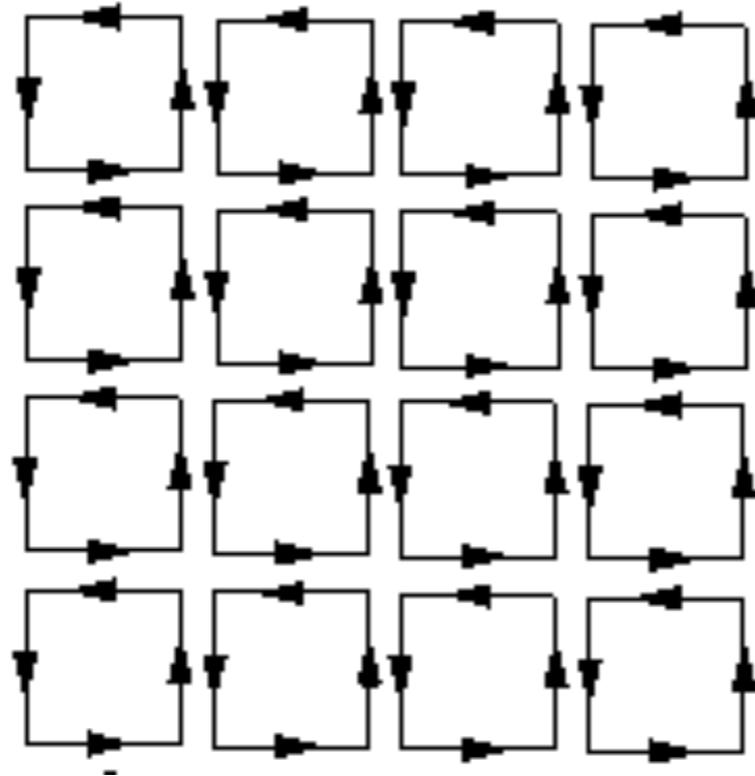
Coriolis=Lorentz

Hall fluid is diamagnetic: $d\epsilon = -MdB$

M is spatially dependent $M=M(x)$

Extra contribution to current $\mathbf{j} = c\hat{\mathbf{z}} \times \nabla M$

Current \sim gradient of magnetization



$$\mathbf{j} = c \hat{\mathbf{z}} \times \nabla M$$

High B limit

- In the limit of high magnetic field: $\epsilon(B)$ known: free fermions
 - n Landau levels for IQH states
 - first Landau level for FQH states with $\nu < 1$
- Wen-Zee shift is known

$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 - \frac{3n}{4}(q\ell)^2 + \mathcal{O}(q^4\ell^4) \quad \nu = n$$

$$\frac{\sigma_{xy}(q)}{\sigma_{xy}(0)} = 1 + \frac{2n-3}{4}(q\ell)^2 + \mathcal{O}(q^4\ell^4), \quad \nu = \frac{1}{2n+1}$$

exact nonperturbative results!

Conclusions

- Thinking about the curved space is productive in nonrelativistic physics
- Reason: NR principle of equivalence
- NR diffeomorphism mixes metric and EM field
- Nontrivial consequences in quantum Hall physics
- Wen-Zee term in the action leads to one contribution to the Hall conductivity at finite q