Lecture 2 on String Cosmology : Tunneling in The Landscape : an Experimental Test

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Asian Winter School, Japan

Outline

- Lecture 1: Some classical properties of the cosmic Landscape. There are many fewer meta-stable de-Sitter vacua than naively expected.
- Lecture 2 : Quantum tunneling in the Landscape.
- Lecture 3 : Wavefunction of the universe in the Landscape and some speculations.
- Lecture 4 : An overall picture and some very strong observational tests.

Motivation

- String theory suggests a very rich complicated landscape for the wavefunction of our universe
- Tunneling among multiple minima should be studied.
- Do we understand tunneling when there are more than 2 local minima ?

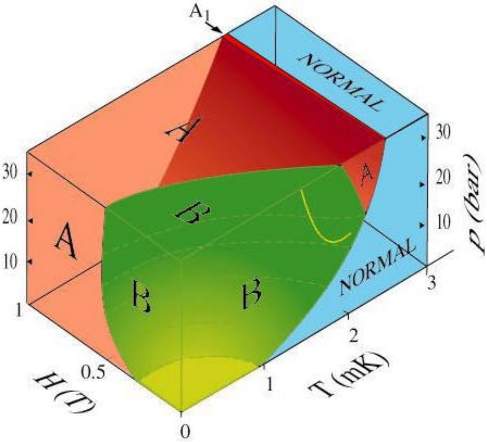
Background

- Fortunately we can compare our understanding with experiments
- An excellent system : He-3 superfluid
- Pairing of 2 He-3 atoms in *p*-wave and Spin 1 state
- Order parameter (Higgs field) is 3×3 complex matrix (i.e., like 18 moduli), with a non-trivial potential (free energy)
- Many possible phases :
  Normal, A_1, planar, polar, α, β, A, B, ...
- defects, like boojums . . . A very rich system

Background

- Observed : normal, A_1, A and B phases, where A and B are degenerate
- With a small magnetic field, the A phase is stable near the superfluid transition temperature
- B phase is stable at zero temperature
- Their degeneracies (A phase has 5 Goldstone modes) are largely lifted by container effect, magnetic field and other higher order corrections
- The A phase is energetically favorable near the container walls

Phase Diagram



Puzzle

- One can calculate as well as measure the domain wall tension and the free energy density difference between the A phase and the B phase (to a few percent accuracy)
- The $A \to B$ transition time due to thermal fluctuation is $\sim 10^{1,470,000}$ years
- Optimistic $A \to B$ transition time due to quantum fluctuations is $\sim 10^{20,000}$ years ($\sim 10^{20,007}$ seconds)

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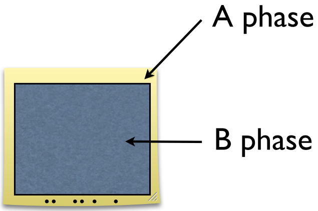
- Optimistic $A \to B$ transition time due to quantum fluctuations is $\sim 10^{20,000}$ years ($\sim 10^{20,007}$ seconds)
- But A to B transition typically occurs in seconds, minutes or at most hours
- The superfluid sample has no impurities



Wall effect stabilizes the A phase even when the bulk condition prefers the B phase.

Possible explanations

- Cosmic Rays hitting the sample (Baked Alaska Model suggested by Leggett)
- Resonant Tunneling (Quantum effect)
- Classical Transitions

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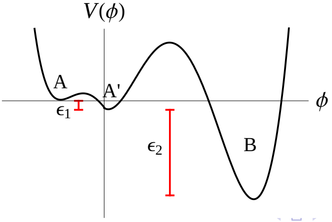
- Big implications to cosmic landscape if not due to external interference
- No correlation between cosmic ray events and the A to B transition

Resonant Tunneling?

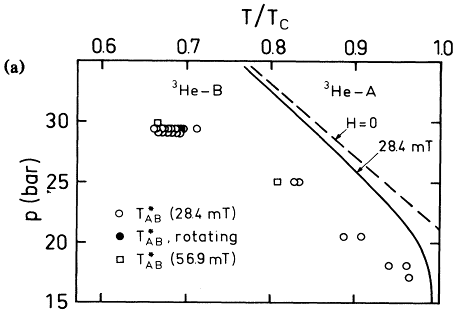
Starting with the initial A^i sub-phase, the tunneling goes via $A^i \to A^j \to B^k$, where A^j is a lower sub-phase of the A phase andB^k is a B sub-phase.

- A^j and B^k are chosen by nature to offer the best chance for the resonant tunneling phenomenon.
- Free energy difference between the A sub-phases are small and may be spatially varying.



Prediction

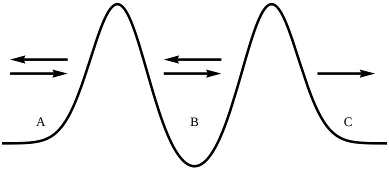
- Resonant tunneling phenomenon happens only under some fine-tuned conditions, say, at certain values of the temperature, pressure and magnetic field. Away from these resonant peaks, the transition simply will not happen.
- Cooling may provide tuning needed to satisfy resonance conditions



Hakonen, Krusius, Salomaa, Simola, PRL 54, 245 (1985).

Outline

Double Barrier



- Tunneling rate for single-barrier tunneling is $\Gamma_{A\to B} = Ae^{-S}$
- Tunneling probability for single-barrier tunneling is $P_{A\to B} = Ke^{-S} \approx e^{-S}$.
- Suppose $P_{B\to C} \approx e^{-S}$ also. What is $P_{A\to C}$?

A Simple Question

Probability $P_{A\to B} \approx \Gamma_{A\to B} \approx e^{-S}$ and time $t_{A\to B} \approx e^{S}$

- $P_{A\to C} \approx P_{A\to B} P_{B\to C} \approx e^{-2S}$

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- $t_{A\to C} = t_{A\to B} + t_{B\to C}$, i.e., $1/\Gamma_{A\to C} = 1/\Gamma_{A\to B} + 1/\Gamma_{B\to C}$, or equivalently
  $P_{A\to C} = P_{A\to B} P_{B\to C} / (P_{A\to B} + P_{B\to C}) \approx e^{-S}$

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- Which is correct ? $P_{A\to C} \approx e^{-2S}$ or $\approx e^{-S}$?

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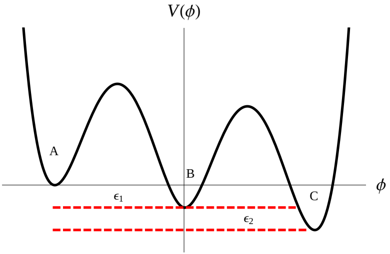
Answer :

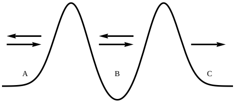
$$P_{A\to C} \approx e^{-S}$$

The Bottom Line

- In QM, $P_{A \to C} \approx e^{-S}$, due to resonant tunneling effect.
- Show existence of similar resonant tunneling effect in QFT.
- New effect "catalyzed tunneling" can enhance single-barrier tunneling.
- This resonant phenomenon may be tested in He-3 superfluid.



WKB Approximation



- Expand a general wavefunction $\Psi(x) = e^{if(x)/\hbar}$ in powers of $\hbar$
- $\psi_{L,R}(x) \approx \frac{1}{\sqrt{k(x)}} \exp\left(\pm i \int dx k(x) \right)$ in classically allowed region, where $k(x) = \sqrt{\frac{2m}{\hbar^2}(E - V(x))}$
- $\psi_{\pm}(x) \approx \frac{1}{\sqrt{\kappa(x)}} \exp\left(\pm \int dx \kappa(x) \right)$ in the classically forbidden region, where $\kappa(x) = \sqrt{\frac{2m}{\hbar^2}(V(x) - E)}$

Matching Conditions

- Complete solution $\psi(x) = \alpha_L \psi_L(x) + \alpha_R \psi_R(x)$ in region A
- $\psi(x) = \alpha_+ \psi_+(x) + \alpha_- \psi_-(x)$ in the classically forbidden region
- $\psi(x) = \beta_L \psi_L(x) + \beta_R \psi_R(x)$ in region B
- $\begin{pmatrix} \alpha_R \\ \alpha_L \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \Theta + \Theta^{-1} & i(\Theta - \Theta^{-1}) \\ -i(\Theta - \Theta^{-1}) & \Theta + \Theta^{-1} \end{pmatrix} \begin{pmatrix} \beta_R \\ \beta_L \end{pmatrix}$
- $\Theta \simeq 2 \exp \left(\frac{1}{\hbar} \int_{x_1}^{x_2} dx \sqrt{2m(V(x) - E)} \right)$
- Tunneling probability $P_{A \to B} = |\frac{\beta_R}{\alpha_R}|^2 = 4 \left(\Theta + \frac{1}{\Theta} \right)^{-2} \simeq \frac{4}{\Theta^2}$

Double-Barrier Tunneling

- Same method of analysis
- $P_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $\Theta \simeq 2\exp\left(\frac{1}{\hbar}\int_{x_1}^{x_2} dx \sqrt{2m(V(x)-E)}\right)$
- $\Phi \simeq 2\exp\left(\frac{1}{\hbar}\int_{x_3}^{x_4} dx \sqrt{2m\left(V(x)-E\right)}\right)$
- $W = \frac{1}{\hbar}\int_{x_2}^{x_3} dx \sqrt{2m(E-V(x))}$

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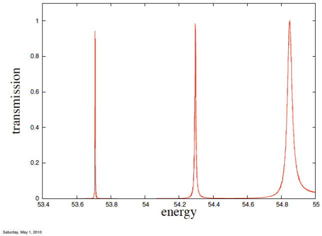
- If $W = (n_B + 1/2)\pi$, then $P_{A\to C} = \frac{4}{(\Theta/\Phi+\Phi/\Theta)^2}$.
  If $\Theta = \Phi$, $P_{A\to C} = 1$

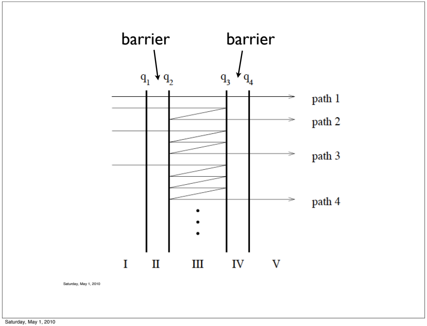
Tunneling Probability versus Energy



- If $\Theta \sim \Phi$, $P_{A \to C} \sim 1$ at resonance energies.
- Away from resonance energies, $P_{A \to C} \sim P_{A \to B} P_{B \to C} \sim e^{-2S}$.
- On average, $P_{A \to C} = P_{A \to B} P_{B \to C} / (P_{A \to B} + P_{B \to C}) \sim e^{-S}$

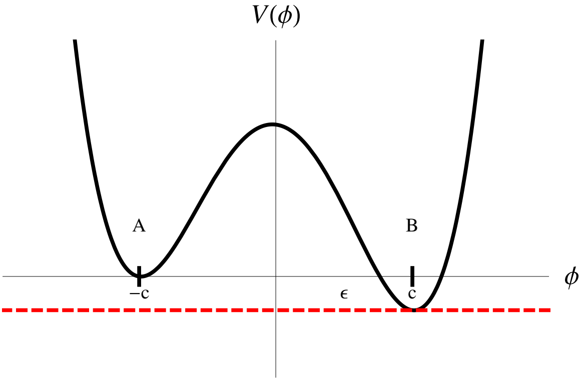
Coherent Sum of Paths

Barriers at $q_1 \to q_2$ and $q_3 \to q_4$



- Comercialized : resonant tunneling diodes

Effective Potential



- $V(\phi) = \frac{1}{4}g(\phi^2 - c^2)^2 - B(\phi + c)$

Thin-Wall Approximation

- Tunneling rate per unit volume [1] is $\Gamma/V = A\exp(-S_E/\hbar)$

[1] Coleman

Thin-Wall Approximation

- Tunneling rate per unit volume [1] is $\Gamma/V = A\exp(-S_E/\hbar)$
- Euclidean EOM is $\left(\frac{\partial^2}{\partial\tau^2} + \nabla^2\right)\phi = V'(\phi)$
- Assume O(4) symmetry
- Solution to Euclidean EOM is $\phi_{DW}(\tau, x, R) = -c\tanh\left(\frac{\mu}{2}(r - R)\right)$
- Inverse thickness of domain wall $\mu = \sqrt{2gc^2}$

[1] Coleman

Euclidean Action

- $S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial\tau}\right)^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$
- $S_E = -\frac{1}{2}\pi^2 R^4 \epsilon + 2\pi^2 R^3 S_1$
- Domain-wall tension is $S_1 = \int_{-c}^{c} d\phi \sqrt{2V(\phi)} = \frac{2}{3}\mu c$

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- $\frac{dS_E}{dR} = 0$ implies $\mathcal{E} = -\frac{4}{3}\pi R^3 \epsilon + 4\pi R^2 S_1 = 0$
- $R = \lambda_c \equiv 3S_1/\epsilon$
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- Bubble grows if $d\mathcal{E}/dR < 0$ or $R > 2\lambda_c/3$

Functional Schrödinger Method

Basic Idea

QFT $\rightarrow$ one-dimensional QM problem

- In semiclassical limit, the vacuum tunneling rate is dominated by a discrete set of classical paths [2] [3] [4]
- Equivalent to Euclidean instanton method for single-barrier tunneling
- Easily generalizes to multiple-barrier tunneling

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[2] Bender, Banks, Wu
[3] Gervais, Sakita
[4] Bitar, Chang

Functional Schrödinger Equation

- $H = \int d^3x \left(\frac{\dot{\phi}^2}{2} + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right)$
- Quantize using $[\dot{\phi}(x), \phi(x')] = i\hbar\delta^3(x - x')$
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- Make ansatz $\Psi(\phi) = A\exp(-\frac{i}{\hbar}S(\phi))$
- $H\Psi(\phi(x)) = E\Psi(\phi(x))$

Semiclassical Expansion

- $S(\phi) = S_{(0)}(\phi) + \hbar S_{(1)}(\phi) + \ldots$
- $\int d^3x \left[\frac{1}{2} \left(\frac{\delta S_{(0)}(\phi)}{\delta \phi} \right)^2 + \frac{1}{2}(\nabla \phi)^2 + V(\phi) \right] = E$
- $\int d^3x \left[-i \frac{\delta^2 S_{(0)}(\phi)}{\delta \phi^2} + 2 \frac{\delta S_{(0)}(\phi)}{\delta \phi} \frac{\delta S_{(1)}(\phi)}{\delta \phi} \right] = 0$
- Solve $S_{(0)}$, then $S_{(1)}$, and so on.
- Here, we ignore higher-order terms.

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Goal

Determine value of $S_{(0)}$ that gives dominant contribution to the tunneling probability.

Determining $S_{(0)}$

- Effective tunneling potential
  $U(\lambda) = U(\phi(x,\lambda)) = \int d^3x \left(\frac{1}{2}(\nabla\phi(x,\lambda))^2 + V(\phi(x,\lambda))\right)$
- Path length $(ds)^2 = \int d^3x (d\phi(x))^2 =$
  $(d\lambda)^2 \int d^3x \left(\frac{\partial\phi(x,\lambda)}{\partial\lambda}\right)^2 = (d\lambda)^2 m(\phi(x,\lambda))$

Zeroth-Order Solution (Classically Forbidden Region)

$$S_{(0)} = i \int_0^s ds' \sqrt{2[U(\phi(x,s')) - E]} =$$
$$i \int_{\lambda_1}^{\lambda_2} d\lambda \left(\frac{ds}{d\lambda}\right) \sqrt{2[U(\phi(x,\lambda)) - E]}$$

Approach

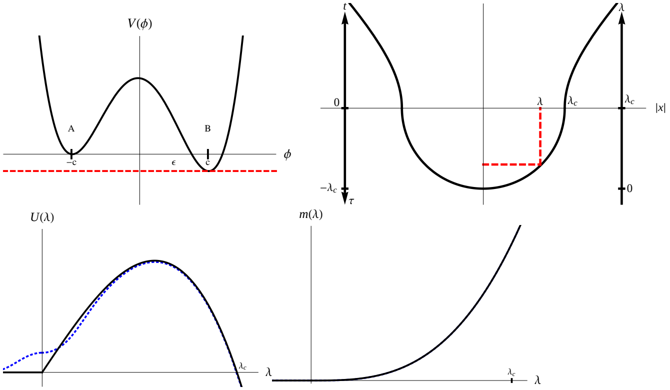
Reduction

Choose MPEP or ansatz : $\phi_0(x,\lambda)$

- Position-dependent mass $m(\lambda) \equiv \int d^3x \left(\frac{\partial \phi_0(x,\lambda)}{\partial \lambda} \right)^2$
- Effective tunneling potential $U(\lambda)$:
  $U(\lambda) = \int d^3x \left(\frac{1}{2}(\nabla \phi_0(x,\lambda))^2 + V(\phi_0(x,\lambda)) \right)$
- Now have a one-dimensional time-independent QM problem, with potential $V(\lambda) = m(\lambda)U(\lambda)$ and $E = 0$:

$$\left(-\frac{\hbar^2}{2}\frac{d^2}{d\lambda^2} + m(\lambda)U(\lambda) \right)\Psi_0(\lambda) = 0$$

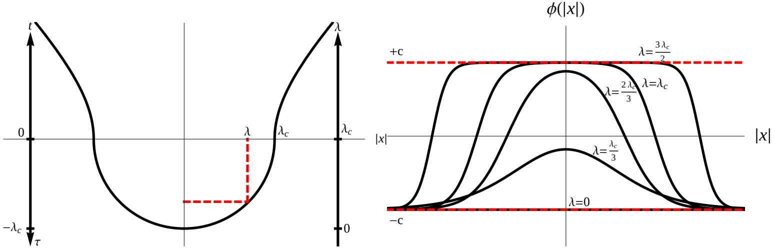
Single Barrier Case



Advantages of Functional Schrödinger Method

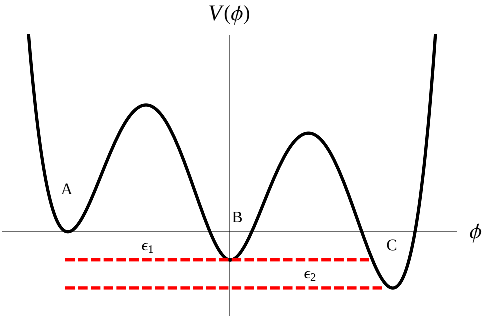
- Same arguments lead to $S_{(0)}(\phi(x,\lambda)) = \int d\lambda \sqrt{2m((\phi(x,\lambda))[-U((\phi(x,\lambda))]}$ and Lorentzian EOM in classically allowed regions
- If we choose $\lambda = \sqrt{\lambda_c^2 + t^2}$ as parameter, MPEP takes form $\phi_0(x,\lambda) = -c\tanh\left(\frac{\mu}{2}(|x| - \lambda)\frac{\lambda}{\lambda_c}\right) = -c\tanh\left(\frac{\mu}{2}\frac{(|x|-\lambda)}{\sqrt{1-\dot{\lambda}^2}}\right)$
- Single real parameter λ describes entire system

Advantages of Functional Schrödinger Method II



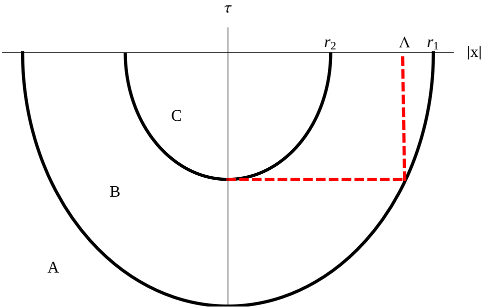
- $\lambda = \sqrt{\lambda_c^2 - \tau^2} \leq \lambda_c$ for $\tau \leq 0$ in classically forbidden region.
- $\lambda = \sqrt{\lambda_c^2 + t^2} \geq \lambda_c$ for $t \geq 0$ in classically allowed region.
- Single real parameter λ describes both.

Effective Potential with Double Barriers



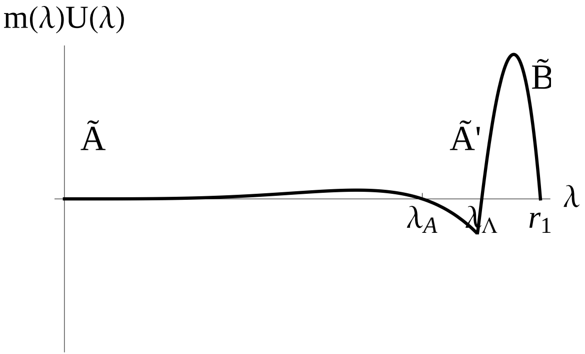
- $$V(\phi) = \begin{cases} \frac{1}{4}g_1((\phi + c_1)^2 - c_1^2)^2 - B_1\phi - 2B_1c_1 & \phi < 0 \\ \frac{1}{4}g_2((\phi - c_2)^2 - c_2^2)^2 - B_2\phi - 2B_1c_1 & \phi > 0 \end{cases}$$

MPEP



- $\phi_0(|x|,\lambda) = -c_1 \tanh\left(\frac{\mu_1}{2}\frac{\lambda}{r_1}(|x|-\lambda)\right) - \Theta\left(\frac{\lambda}{\Lambda} - 1\right) c_2 \tanh\left(\frac{\mu_2}{2}\frac{\lambda'}{r_2}(|x|-\lambda')\right) + c_2 - c_1$ solves both Euclidean

Effective Tunneling Potential



Consistency Conditions

- We require zero total energy at nucleation
- $\mathcal{E}_{(2)} = 4\pi(S_1^{(1)} - \frac{1}{3}r_1\epsilon_1)r_1^2 + 4\pi(S_1^{(2)} - \frac{1}{3}r_2\epsilon_2)r_2^2 = 0$
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- Equivalent to condition that action is stationary

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- Must also ensure existence of a classically allowed region $U(\lambda) < 0$ for $\Lambda > \lambda > \lambda_B$

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- We do not demand that the energy of each bubble vanishes individually
- Equivalent to condition that action is stationary
- Must also ensure existence of a classically allowed region $U(\lambda) < 0$ for $\Lambda > \lambda > \lambda_B$
- Also require the existence of a second classically forbidden region

Resonant Tunneling or Catalyzed Tunneling

Resonant Tunneling

If the inside bubble is large enough $\lambda_{2c} > r_2 > 2\lambda_{2c}/3$ tunneling from A to C will complete.

Resonant Tunneling or Catalyzed Tunneling

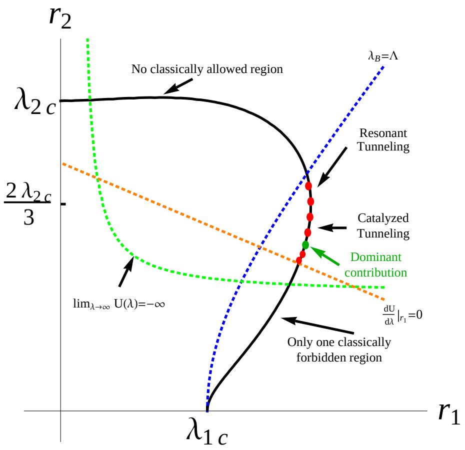
Resonant Tunneling

If the inside bubble is large enough $\lambda_{2c} > r_2 > 2\lambda_{2c}/3$ tunneling from A to C will complete.

Catalyzed Tunneling

- If the inside bubble is too small $0 < r_2 < 2\lambda_{2c}/3$, inside bubble will collapse after nucleation
- Effect will be tunneling from A to B
- Tunneling rate is exponentially enhanced by presence of C

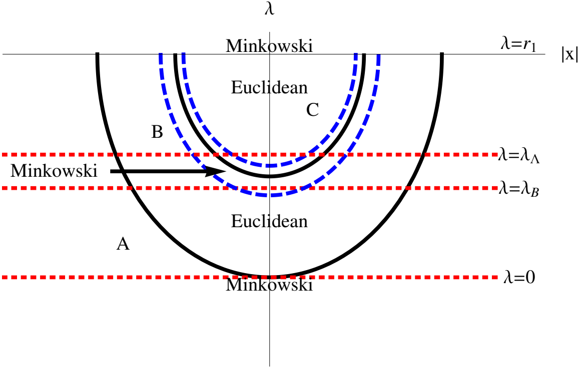
Consistency Conditions



Resonant Tunneling in QFT

- $P_{A\to C} = 4\left(\left(\Theta\Phi + \frac{1}{\Theta\Phi}\right)^2 \cos^2 W + \left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)^2 \sin^2 W\right)^{-1}$
- $W = \int_{\lambda_2}^{\lambda_3} d\lambda \sqrt{2m(\lambda)(-U(\lambda))}$
- $W = \frac{S_1^{(1)}\lambda_\Lambda}{\lambda_B}\sqrt{\lambda_\Lambda^2 - \lambda_B^2} - S_1^{(1)}\lambda_B \log\left[\frac{\lambda_\Lambda + \sqrt{\lambda_\Lambda^2 - \lambda_B^2}}{\lambda_B}\right]$
- Resonance condition is $W = (n + \frac{1}{2})\pi$.
- $P_{A\to C}$ is smaller of $P_{A\to B}/P_{B\to C}$, $P_{B\to C}/P_{A\to B}$. So preferred choice is when $\Gamma_{A\to B}$ is closest to $\Gamma_{B\to C}$.
- The figure uses $S_1^{(1)} = 1$ and $S_1^{(1)} = 5$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.3$, $P_{A\to B} = e^{-10^5} \to e^{-10^4}$

Euclidean vs Minkowski Description



This is why it is difficult to extend Coleman's approach to resonant tunneling case.

Probability of Hitting Resonance

- Treat tunneling probability as function of λ_Λ
- Expand around resonance at $\lambda_\Lambda = \lambda_R$ of width Γ_{λ_Λ}
- $\Gamma_{\lambda_\Lambda} = \frac{2}{\Theta\Phi(\frac{\partial W}{\partial \lambda_\Lambda})}\left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)$

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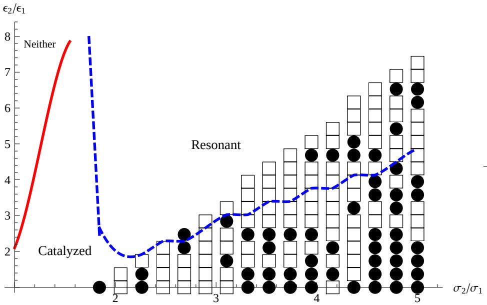
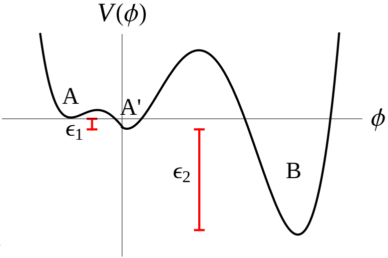
- $\Gamma_{\lambda_\Lambda} = \frac{2}{\Theta\Phi(\frac{\partial W}{\partial \lambda_\Lambda})}\left(\frac{\Theta}{\Phi} + \frac{\Phi}{\Theta}\right)$
- Separation between resonances $\Delta\lambda \simeq \frac{\pi}{(\frac{\partial W}{\partial \lambda_\Lambda})}$
- Probability of hitting resonance $p(A \to C) = \frac{\Gamma_\Lambda}{\Delta\Lambda} \simeq \frac{2}{\pi\Theta\Phi} = \frac{1}{2\pi}\left(P_{A\to B} + P_{B\to C}\right)$ is the larger of two decay probabilities
- Average tunneling probability $< P_{A\to C} >= p(A \to C)P_{A\to C} \sim \frac{P_{A\to B}P_{B\to C}}{P_{A\to B}+P_{B\to C}}$ given by smaller of two tunneling probabilities

Enhancement of Tunneling



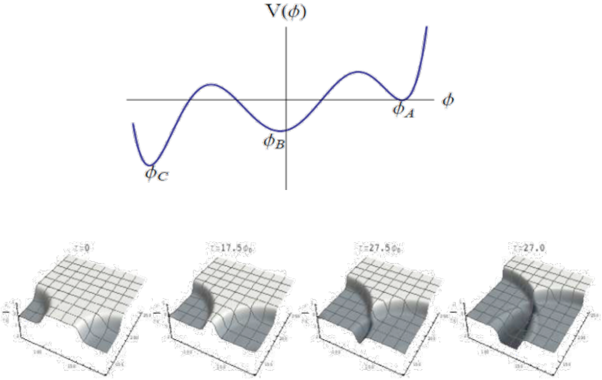


An alternative to resonant tunneling is classical transition

- Based on work with I-Shing Yang and Ben Shlaer (1110.2045)

Classical transitions

Classical Transition



Physics behind :

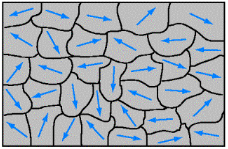
$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = -\frac{\partial V(\phi)}{\partial\phi}$$

$$\gamma^2 = 1/(1-v^2) \qquad \frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} \sim 0$$

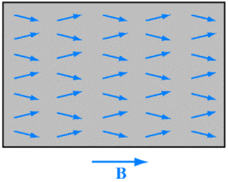
$$\begin{aligned}\phi(x,t) &= \phi_M + f_L\left(\frac{-x+vt}{\sqrt{1-v^2}}\right) + f_R\left(\frac{x+vt}{\sqrt{1-v^2}}\right) \\ &\approx \phi_M + (\phi_L - \phi_M)\Theta(-x+t) + (\phi_R - \phi_M)\Theta(x+t)\end{aligned}$$

$$\phi(0, t<0) = \phi_M$$

$$\phi(0, t>0) = \phi_M + (\phi_L - \phi_M) + (\phi_R - \phi_M) = \phi_L + \phi_R - \phi_M$$



(a) Unmagnetized domains



(b) Magnetized domains

Conclusions

- Fast phase transition may be due to resonant tunneling or classical transition.
- This phenomenon can be tested by simple experiments on He-3 superfluid. We may have explained a 40 year old puzzle.
- The complicated potential for He-3 order parameter mimics the cosmic landscape. If confirmed in He-3, tunneling in the cosmic landscape can be much more efficient than naively expected.