Lecture 3 on String Cosmology:

The wavefunction of the universe in the Landscape

Henry Tye 戴自海

Cornell University
and
Hong Kong University of Science and Technology

Asian Winter School, Japan
January 2012

In the previous 2 lectures :

- We have seen, so far, the failure of finding a single Type IIA meta-stable de-Sitter vacuum.
- We argue that the number of meta-stable de-Sitter vacua in string theory is a lot fewer than naively expected.
- However, we probably still expect a good number of such local minima in the Landscape. We argue that tunneling can be much faster than naively expected.
- In the He-3 superfluid system, experiments have shown that the exponent in the tunneling rate is about 6 orders of magnitude bigger than that predicted by theory.

Eternal Inflation

- Suppose our universe sits in a meta-stable de-Sitter vacuum for time T.

What is eternal inflation ?

$$a(t) \sim e^{Ht}$$

$$H^2 \sim 1/G\Lambda$$

Consider a patch of size $1/H$.
Suppose $T \sim 3/H$.
The universe would have grown by a factor of $e^{3\cdot 3} = e^9$.
If inflation ends in one patch, there are still many other (causally disconnected) patches which continue to inflate.
So inflation never ends.

Eternal Inflation

- A very simple and elegant scenario
- With the original picture of Bousso-Polchinski and KKLT, it seems unavoidable that the wavefunction will sit at some point in the Landscape
- Naively it implies that most of the universe is still undergoing eternal inflation today, so our observable universe is a tiny tiny part of the whole universe
- No matter how suppressed, parts of the universe can tunnel to other vacua, both with higher and lower CC. So the whole Landscape is “populated”.

Eternal Inflation

- We need a strong version of the Anthropic principle to explain our presence.
- If there are say 10^{500} vacua, there'll be many vacua extremely similar to ours. Among these many very similar vacua, we cannot tell which one we live in. In this sense, we'll lose predictive power.
- The question of measure cannot really be resolved since there is no global time.

Can we avoid eternal inflation ?

- Based on what we discussed in the previous 2 lectures, I argue that eternal inflation is not only avoidable, it is unlikely.
- Since we know little about the potential of the Landscape, except that it is very large and complicated, is there much we can say about it ?
- In this lecture, I like to argue “yes”.
- This lecture is partly based on hep-th/0611148 and 0708.4374.

Strategy :

- Treat the landscape as a d-dimensional random potential
- Use the scaling theory developed for random potential (disordered medium) in condensed matter physics
- justify the key points of the above scenario
- do a renormalization group analysis on the mobility of the wavefunction of our universe
- calculate some of the properties of the landscape, e.g., the critical CC
- argue why we should end up in a vacuum with an very small C.C.

Condensed matter physics want to understand when a sample is conducting or insulating.

When conducting, the charge carriers are mobile.
When insulating, the charge carriers are localized.
They study this behavior for a random potential.

For us, the landscape is the random potential.
The wavefunction of a charge carrier becomes the wavefunction of the universe.

A localized or trapped wavefunction will imply eternal inflation.

We like to learn when it is trapped and when it is mobile

We like to see :

Mobility at high CC and trapped at very low CC.

There is a critical CC, which is exponentially small.

At sites with CC larger than this critical value, the wavefunction of the universe is mobile.

At sites with CC smaller than this critical value, the wavefunction of the universe is isolated, with exponentially long lifetime.

The transition at this critical CC is sharp.

Anderson localization transition

- random potential/disorder medium
- insulation-superconductivity transition
- quantum mesoscopic systems
- conductivity-insulation in disordered systems
- percolation
- strongly interacting electronic systems
- high Tc superconductivity
- . . . .

Some references :

- P. W. Anderson, Absence of Diffusion in Certain Random Lattices, Phys. Rev. Lett. 109, 1492 (1958).
- E. Abrahams, P. W. Anderson, D. C. Licciardello and T. V. Ramakrishnan, Scaling Theory of Localization : Absence of Quantum Diffusion in Two Dimensions, Phys. Rev. Lett. 42, 673 (1979).
- B. Shapiro, Renormalization-Group Transformation for Anderson Transition, Phys. Rev. Lett. 48, 823 (1982).
- P. A. Lee and T. V. Ramakrishnan, Disordered electronic systems, Rev. Mod. Phys. 57, 287 (1985).
- M. V. Sadovskii, Superconductivity and Localization," (World Scientific, 2000).
- Les Houches 1994, Mesoscopic Quantum Physics.

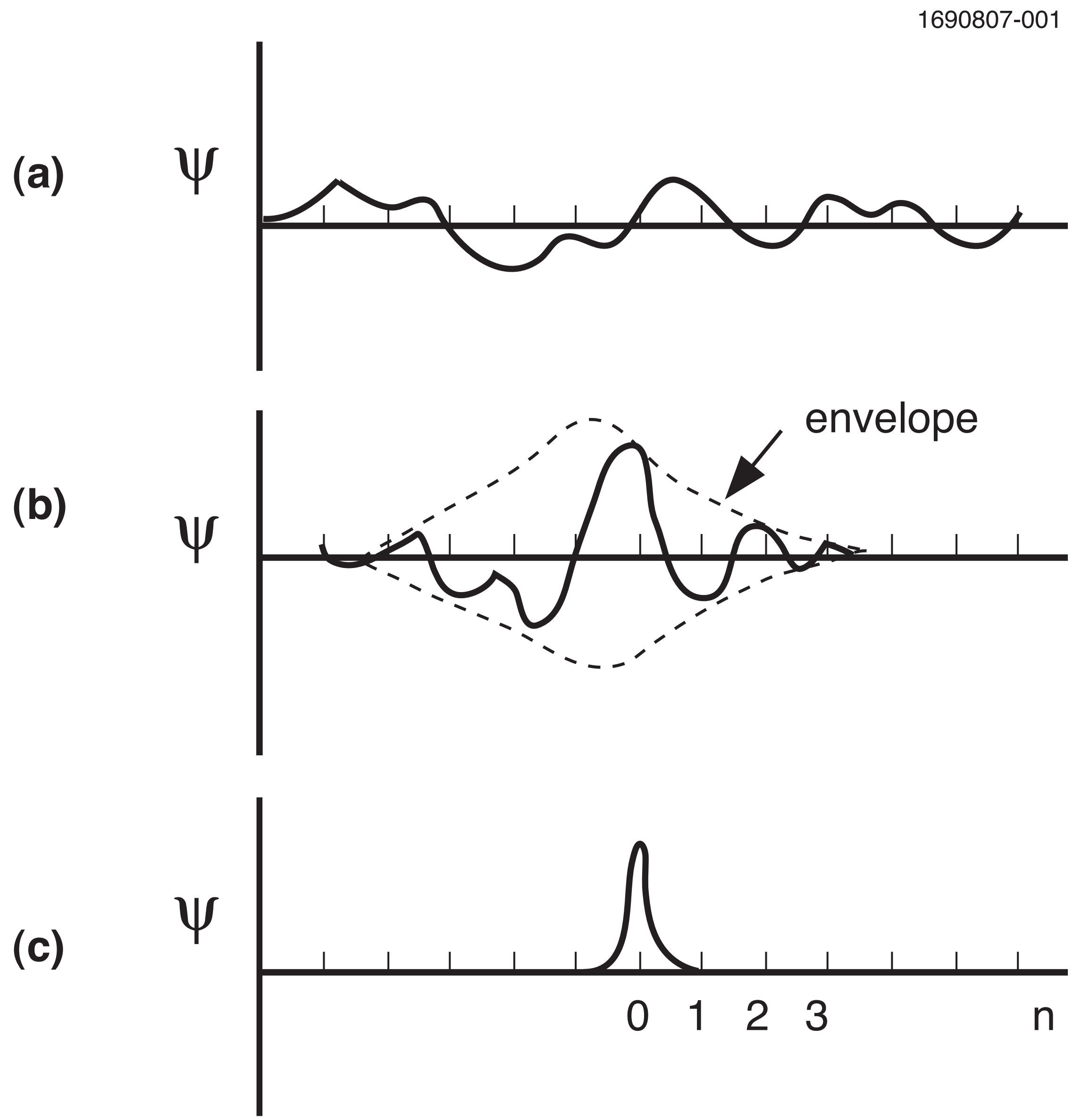
Define a dimensionless conductance g in a d-dim. hypercubic region of macroscopic size L

$$g_d(L) = \sigma L^{d-2}$$

$$d = 3,\ g = \sigma(\text{Area})/L \sim \sigma L$$

$$g(L) \sim (L/a)^{(d-2)}$$

Conducting/mobile (metalic) with finite conductivity



$$|\psi(\mathbf{r})| \sim \exp(-|\mathbf{r} - \mathbf{r}_0|/\xi)$$

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

How does g scale ?
Given g at scale a, what is g at scale L as L becomes large ?

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$g_d(L) \sim e^{-L/\xi}$$

$\downarrow$

Insulating, localized, trapped, eternal inflation

$$g_d(L) = \sigma L^{d-2}$$

$\downarrow$

Conducting, mobile

$$\beta_d(g_d(L)) = \frac{d \ln g_d(L)}{d \ln L}$$

$$g_d(L) \sim e^{-L/\xi}$$

$$g_d(L) = \sigma L^{d-2}$$

↓ ↓

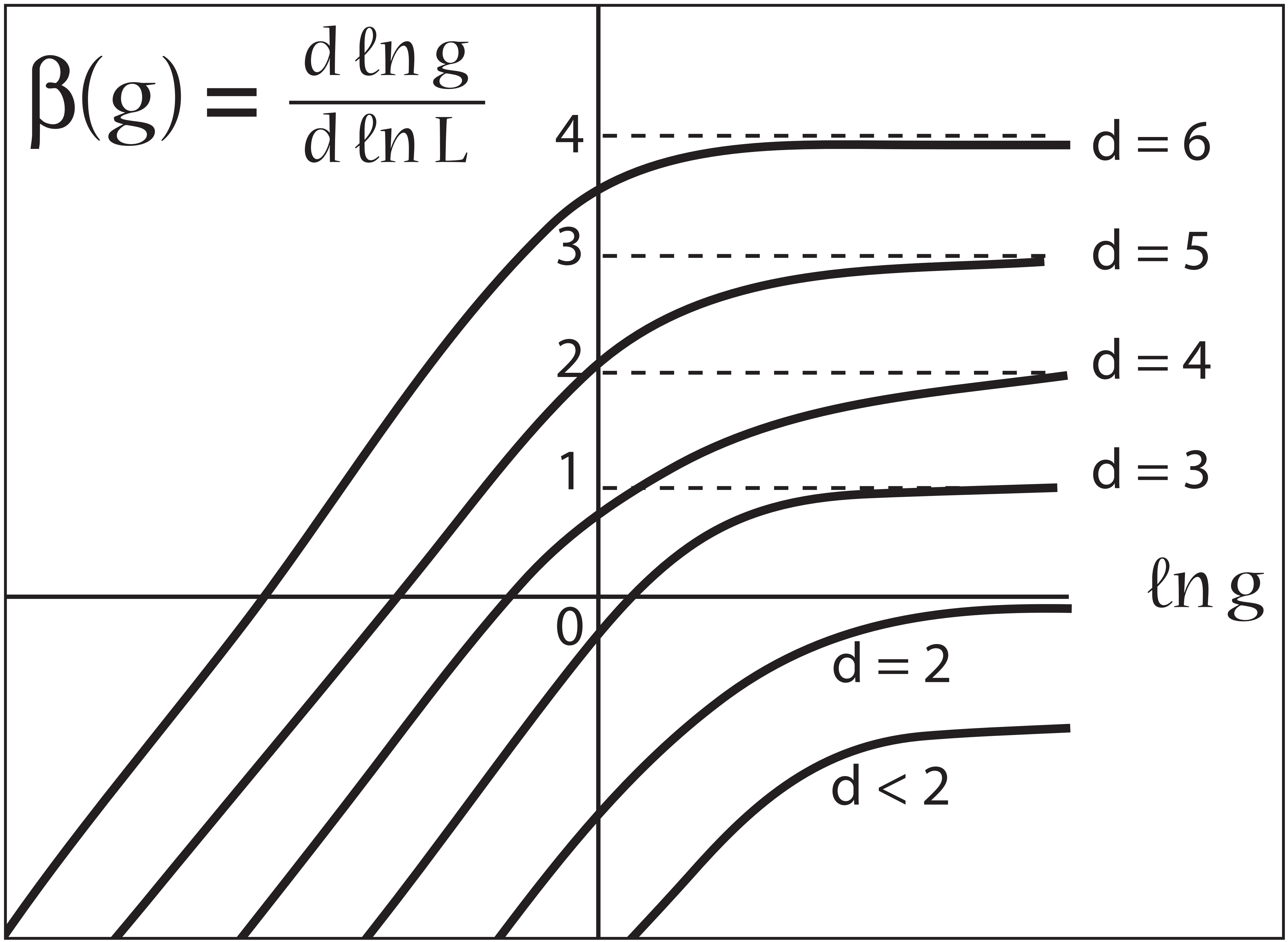
$$\lim_{g \to 0} \beta_d(g) \to \ln \frac{g}{g_c}$$

$$\lim_{g \to \infty} \beta_d(g) \to d - 2$$

For $g \sim e^{-L^n}$

$$\beta(g) \sim n \ln(g/g_c)$$

1690807-002



A simple way to get a feeling of this

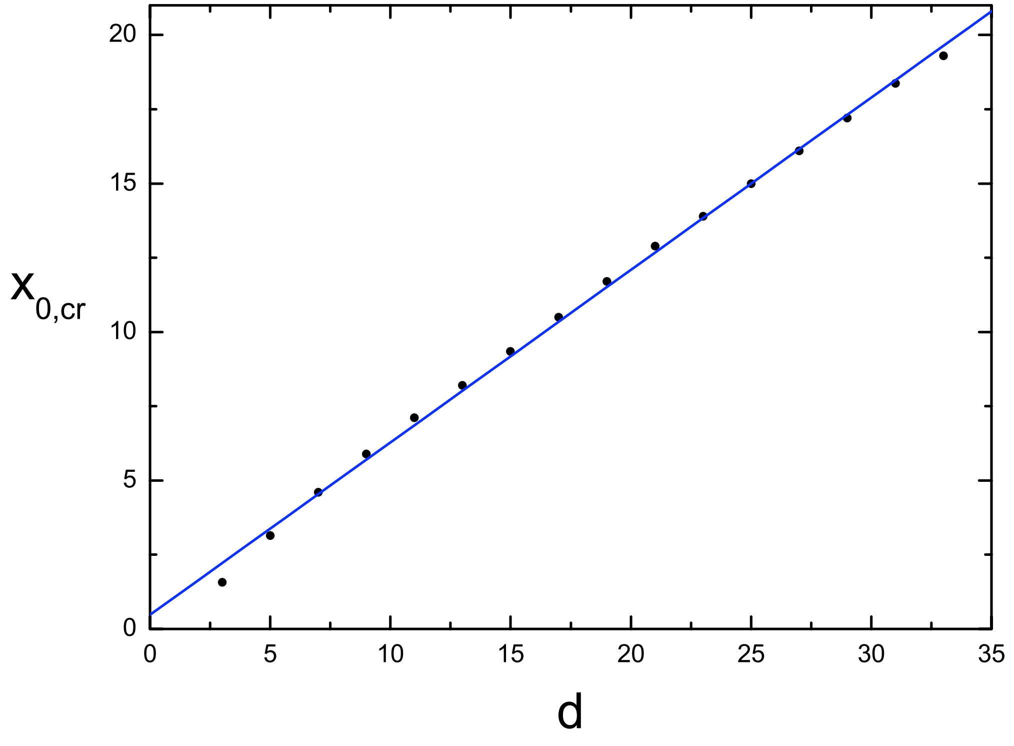
Recall that there is a bound state for a 1-dim attractive delta potential, but no bound state in the 3-dim case.

A spherical square-well attractive potential in QM :

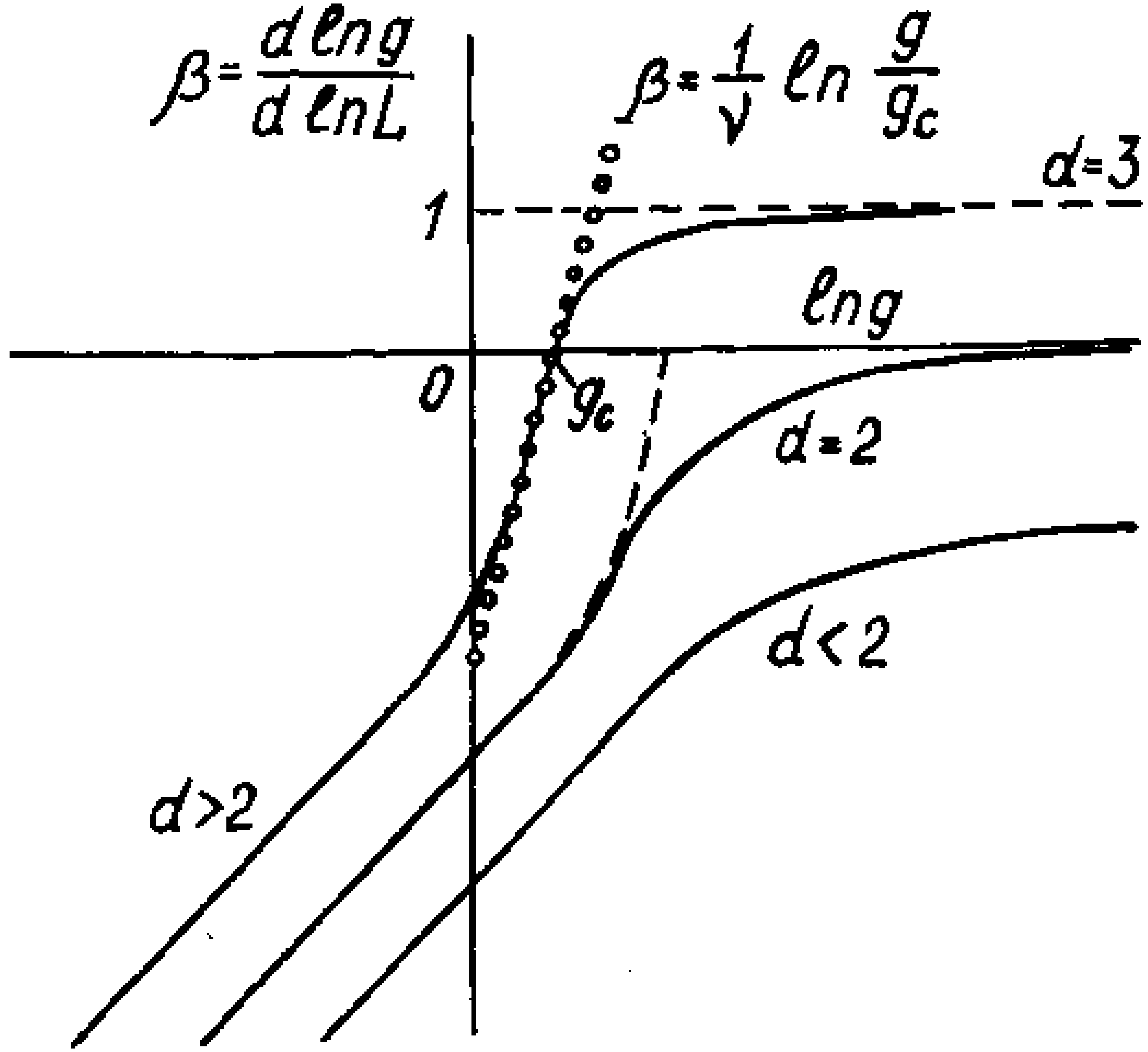
In the $d = 1$ case, there is always at least one bound state. In the $d = 3$ case, there is no bound state if $x_0 < \pi/2$. For large (odd) d, one finds that there is no bound state solution if $x_0 = k_0 R$ is less than a critical value P_c,

Bound State Condition
in d-dim QM

$$k_0 R < P_c \qquad P_c \simeq 0.58d + 0.5$$



GQ Huang



$\beta = \frac{d \ln g}{d \ln L}$
$\beta = \frac{1}{\nu} \ln \frac{g}{g_c}$
d=3
1
ln g
0
g_c
d=2
d<2
d>2

$$\beta_d(g) \approx \frac{1}{\nu} \ln \frac{g}{g_c} \approx \frac{1}{\nu} \frac{g - g_c}{g_c}$$

this zero of $\beta_d(g)$ corresponds to an unstable fixed point

$$g(a) < g_c$$

↓

$$g_d(L) \sim e^{-L/\xi}$$

↓

Insulating, localized, trapped, eternal inflation

$$g(a) > g_c$$

↓

$$g_d(L) = \sigma L^{d-2}$$

↓

Conducting, mobile

What is the critical g_c ?

$$\Delta \ln g_c = \ln g_c(d) - \ln g_c(d+1) = k > 0$$

$$g_c(d) \simeq e^{-(d-3)k} g_c(3)$$

Shapiro :

$$\beta_d(g) = (d-1) - (g+1)\ln(1+1/g)$$

$$g_c = e^{-(d-1)} \qquad \nu \to 1$$

d=1 :

Anderson, Thouless, Abraham, Fisher, Phys. Rev. B 22, 3519 (1980)

A generalization to include percolation

$$\beta_g(g,p) = (d-1)\left(1 + \frac{1-p}{p}\ln(1-p)\right) - (g+1)\ln(1+1/g)$$

$$\beta_p(p) = \frac{\partial p}{\partial \ln L} = p\ln p - (d-1)(1-p)\ln(1-p)$$

$$1 > p_2 > 0$$

stable

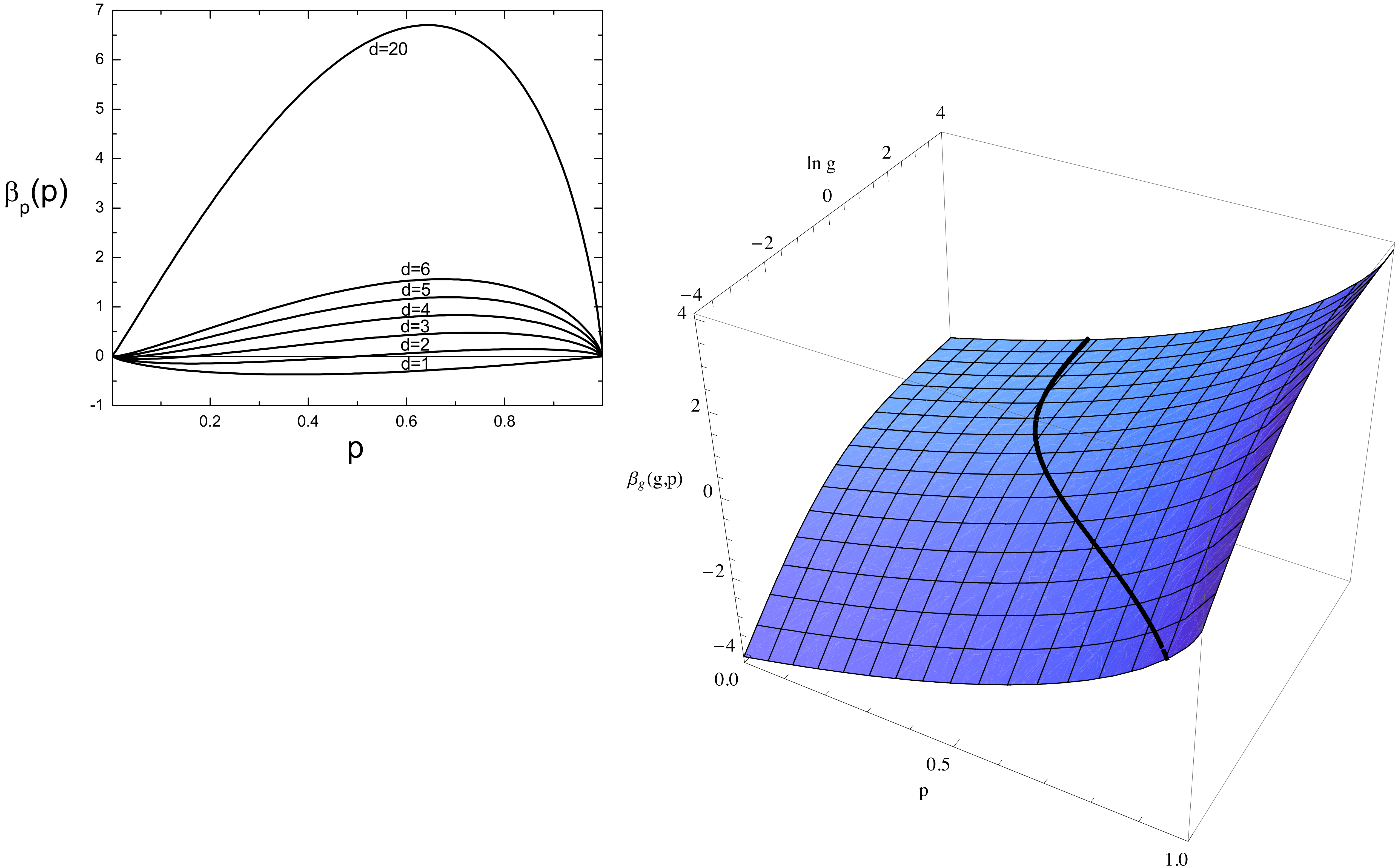
unstable

stable

reduces to the above simple case

Insulating

$$p_2 \simeq e^{-(d-1)}$$



The β-function $\beta_g(g,p)$ with inclusion of percolation for $d = 6$ and the β-function $\beta_p(p)$ for the running of p, for different p and d. The black line corresponds to $\beta_g(g,p) = 0$.

Condition for mobility

$$g_c = e^{-(d-1)}$$

$$g(a) \sim |\psi(a)| \sim e^{-a/\xi}$$

$$d > \frac{a}{\xi} + 1$$

$$\Gamma_0 \sim |\psi(a)|^2 \sim e^{-2a/\xi}$$

$$\Gamma_0 > e^{-2(d-1)}$$

The Quantum Landscape

$\Lambda > \Lambda_c$

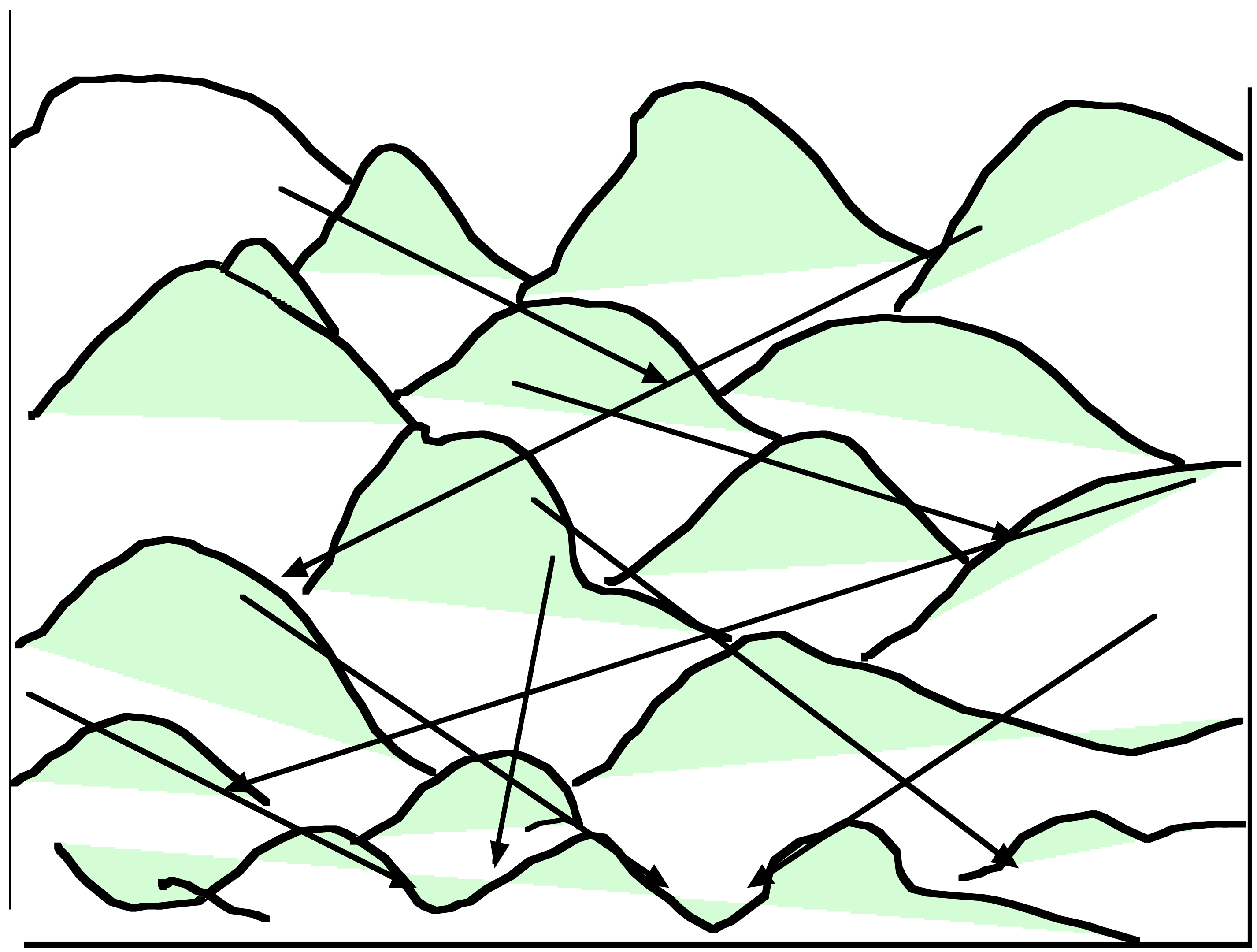
↓

moves quickly
little time to inflate

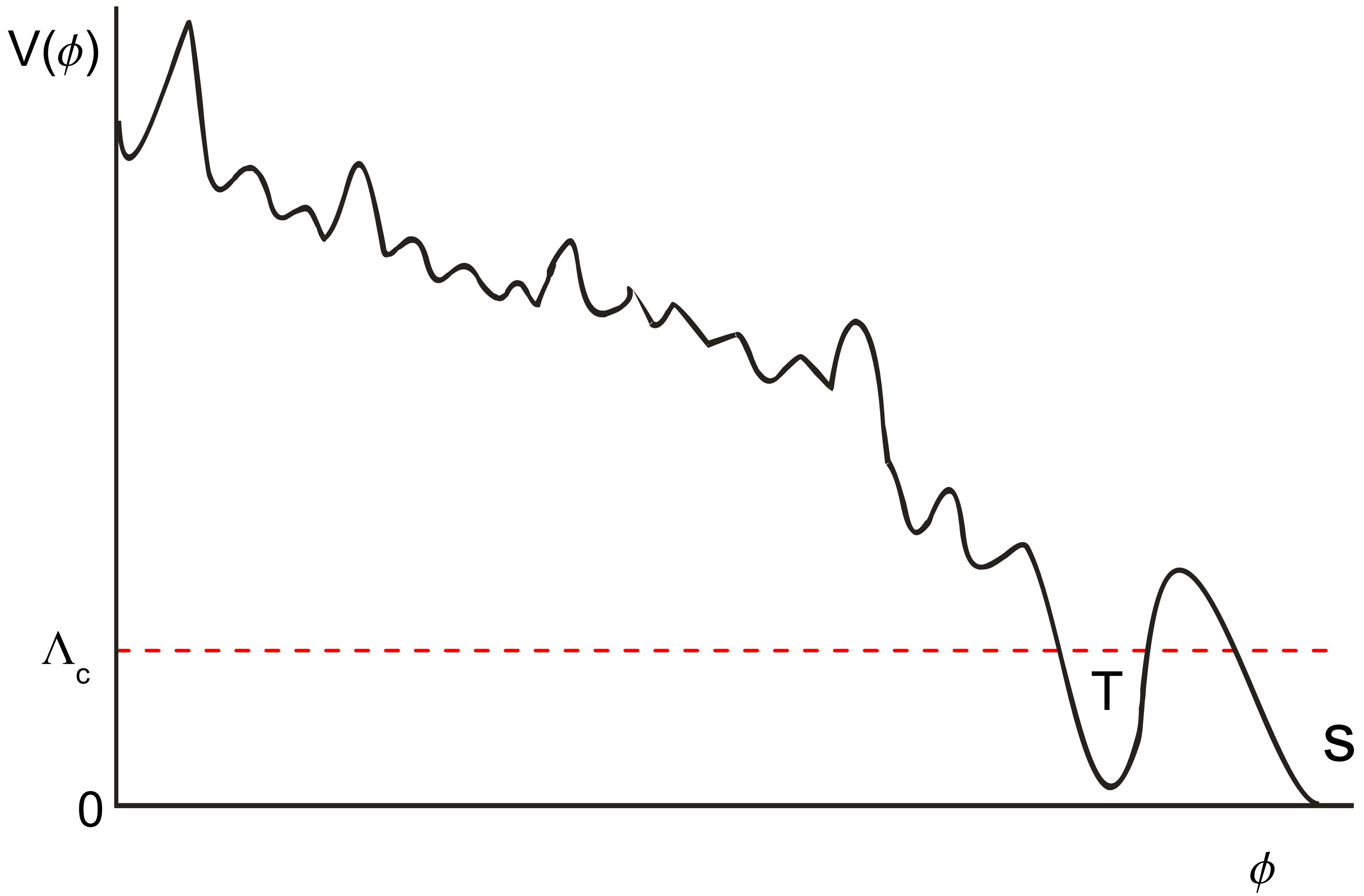
$\Lambda < \Lambda_c$

↓

trapped
very long lifetime



Behavior of the wavefunction



Estimate of critical C.C.

$$d(\Lambda) > \frac{a(\Lambda)}{\xi(\Lambda)} \quad \rightarrow \quad d > \frac{a(\hat{\Lambda})}{\xi}$$

$\Lambda = \hat{\Lambda}^4$ in 4-dimensional spacetime

$$f(\hat{\Lambda}) \approx (\hat{\Lambda}/M_s)^s$$

$$N_T = N(M_s) = \left(\frac{L}{a(M_s)}\right)^d \qquad N(\hat{\Lambda}) = f(\hat{\Lambda}) N_T = \left(\frac{L}{a(\hat{\Lambda})}\right)^d$$

flat distribution :

$$a(\hat{\Lambda}) = a(M_s) f(\hat{\Lambda})^{-1/d}$$

$$\hat{\Lambda}_c \approx d^{-d/s} M_s \qquad \Lambda_c \sim d^{-d} M_s^4$$

Big assumption

- Tunneling or evolving to AdS vacua ?

History of our Universe -
A Speculation Picture

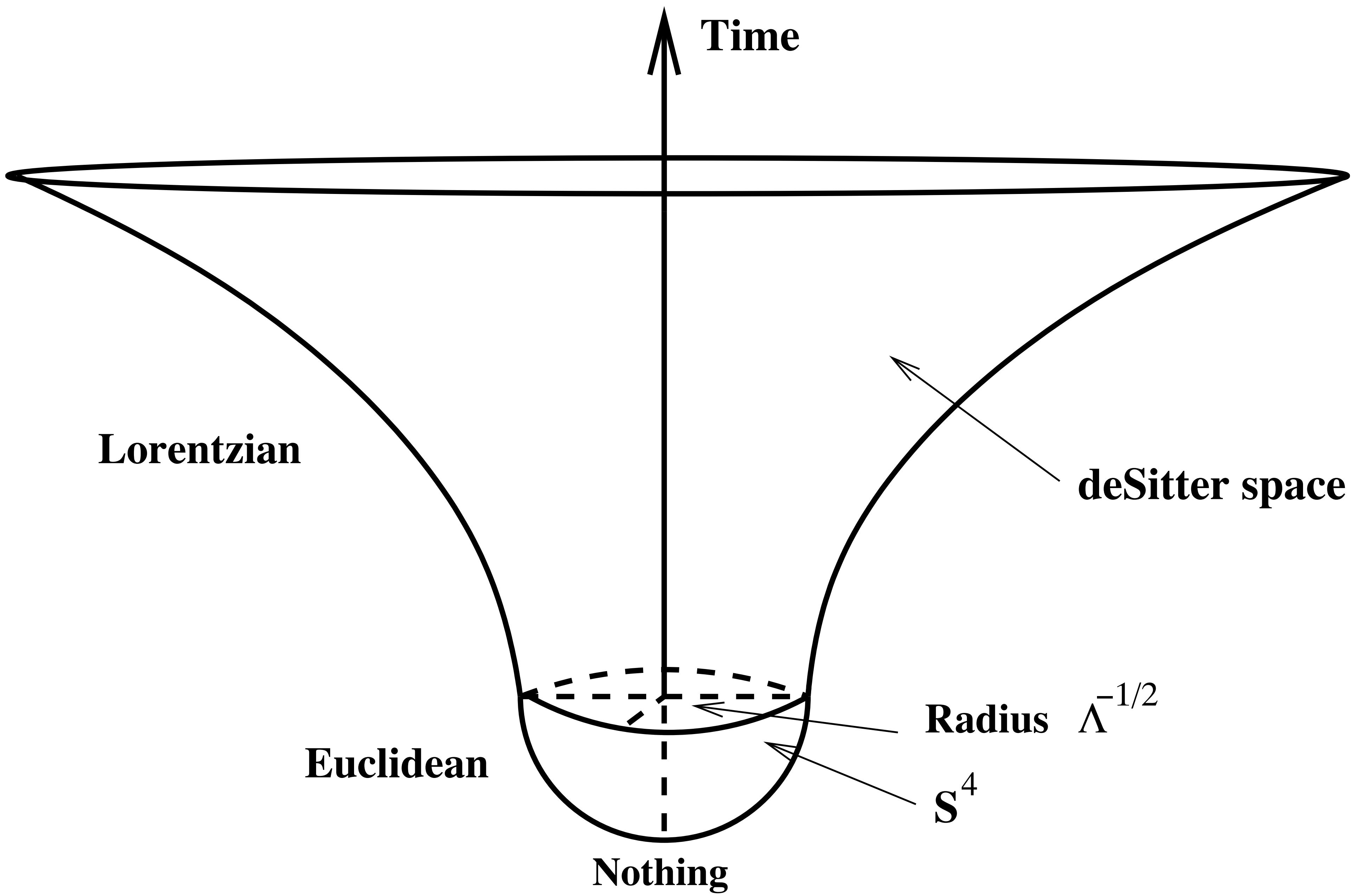
Taking a crude look at the whole

Gell-mann

- Parts can be replaced by better parts later, as our understanding gets better, with more input from data.

Quantum Creation of the Universe

“Tunneling” from Nothing



Vilenkin, 1982, 1983
Hartle and Hawking, 1983

$$P = e^{3\pi/G\Lambda}$$

- This implies that the most likely ones are the ones with extremely small CC. However, the size of the de-Sitter space goes like

$$1/\Lambda^{1/2}$$

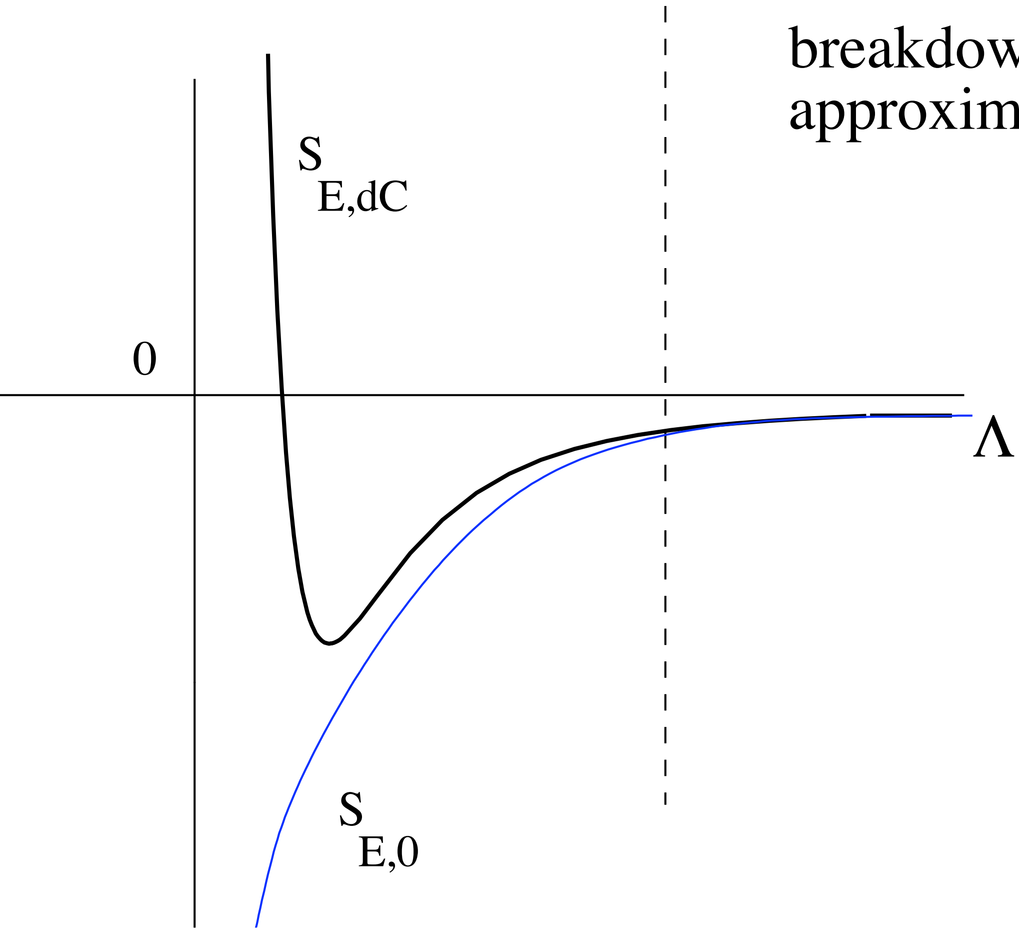
Classical effects must come into play for such a large universe.

That means we should include decoherence effect.

What is wrong ? How to fix it ?

- Tunneling is to a huge super-macroscopic universe, with nothing in it.
- Interacting with the environment suppresses tunneling.
- There must be some gravitational radiation.
- Its interaction with the system (cosmic scale factor a) introduces decoherence.
- Smaller CC means larger universe means more radiation implies more decoherence thus suppressing the tunneling.

The improved Euclidean action
$S_{E,dC}$ vs $S_{E,0}$



Tunneling probability

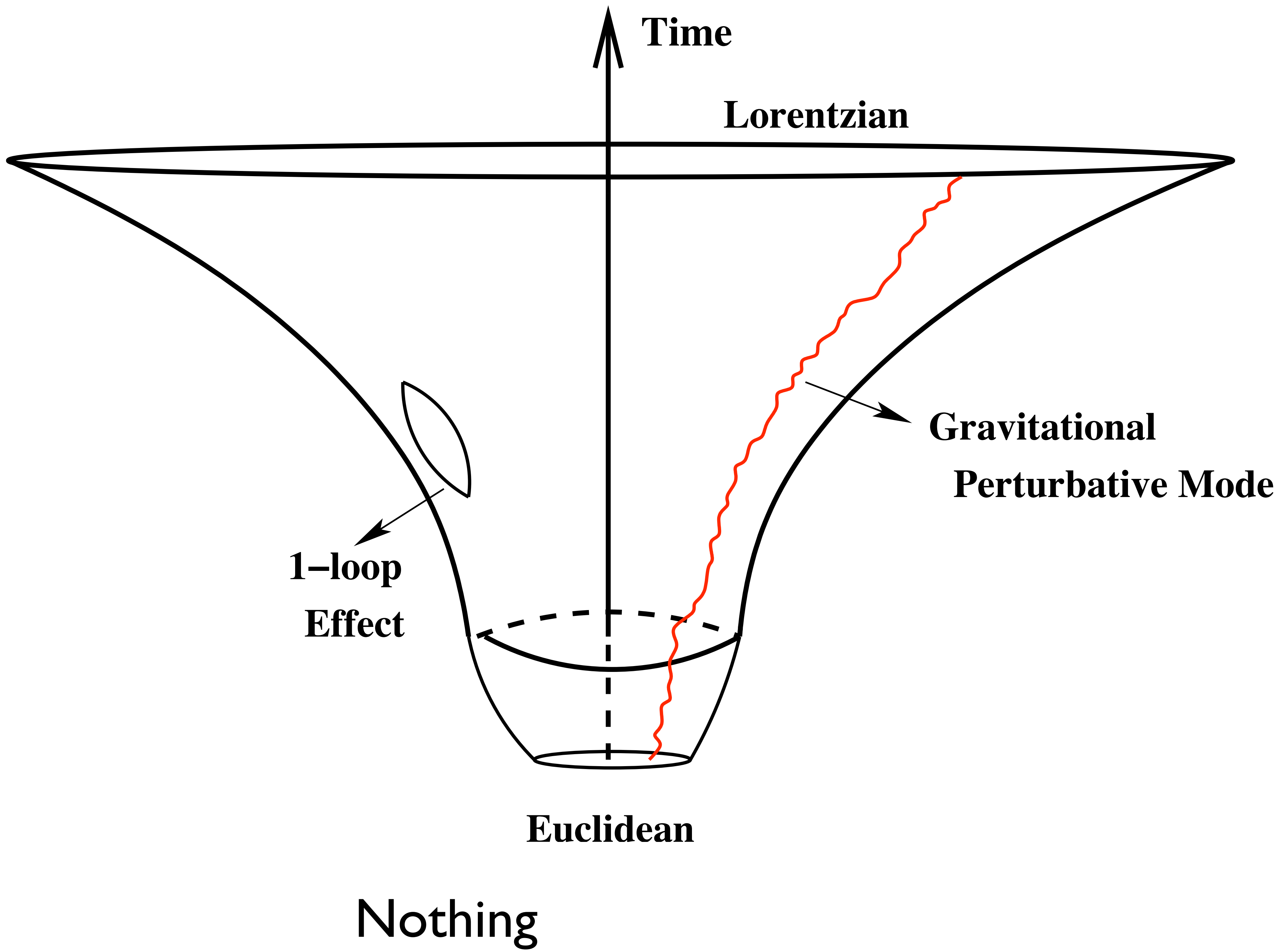
$$P \simeq e^{-S_{E,dC}} = e^{F}$$

$$F = \frac{3\pi}{G\Lambda} - \frac{12 n_d}{\Lambda^2 l_s^4}$$

$S_{E,0}$ is unbounded from below, but the interaction with the environment has made $S_{E,dC} = -F$ bounded from below.

Firouzjahi, Sarangi and H.T., hep-th/0406107, 0505104

Tunneling from Nothing, including decoherence



History ?

- The Universe is a spontaneous creation from NOTHING.
- It starts with a vacuum energy somewhere below the string scale.
- It evolves in the landscape, producing inflation.
- It then reaches a vacuum site in the landscape with a small CC and an exponentially long lifetime.
- This is where we live.

History?

A Quantum Fluctuation from Nothing
(No classical space or time)

↓

Universe moves in the Landscape (and inflates)

↓

Brane Inflation (Branes moving slowly towards each other)
(Universe grew exponentially)

↓

Branes collided to heat up the universe
10^{-30} sec.

↓

Hot Big Bang Epoch

(Nucleosynthesis around 10 sec.)

↓

Matter-dominated Epoch

(Star/galaxy formation begins at 10^{12} sec.)

↓

Today's Universe

size ~ 1 10^{18} sec.