KEK-CP workshop DE2008

"Is our Universe really undergoing an accelerated expansion?"

A brief overview of the ideas and issues of the effects of inhomogeneities on cosmic expansion

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KEK Tsukuba Dec. 8th 2008

Introduction

Distant SNe-Ia (at $z \sim 0.5$) appear to be *fainter* than *expected* in *Einstein-de Sitter model*

- The concordance model

Geometry: FLRW Symmetry = Isotropic & *Homogeneous*
Main constituents: Dark Matter & Dark Energy

Still does not have any basis in fundamental physics

The issues of why so small and why now

We might be misinterpreting the cosmological data

- Alternative model?

Geometry: Inhomogeneous
Main constituents: Dark Matter

More economical: no extra energy
Connect two epochs of cosmic acceleration & structure formation
--- may be a solution to "why-now problem"

“ Which is more absurd,

Dark Energy or Inhomogeneous models? ”

Iguchi - Nakamura - Nakao 2002

Purpose of this talk

- Give a brief overview of recent attempts to account for the acceleration of our Universe by the effects of inhomogeneities

- Review basic idea and point out – from theoretical viewpoints— some serious flaws in the idea and raise issues to be addressed

Inhomogeneous cosmology

Models	Ideas	Related talks (perhaps)
Super-horizon perturbations	Effective-stress tensor	AI
Sub-horizon inhomogeneities	spatial-averaging (dressed-parameters)	AI A. Notari D. Wiltshire
Void-model of Swiss-cheese type (Weak lensing)	A fully non-linear GR treatment of inhomogeneous universe	R.A. Vanderveld K. Nakao A. Notari H. Asada K.T. Inoue
Void model of Tomita – type (anti-Copernican)	Live in the center (Large vs Small void)	A. Vanderveld K. Nakao A. Notari A. Romano K. Tomita
General arguments Other ideas	Lines of sight in clumpy universe Confrontation w/ observations	M. Kasai H. Asada A. Notari T. Futamase K.T. Inoue A. Romano D. Wiltshire K. Tomita A.A. Starobinsky

Outline

- Newtonianly perturbed FLRW universe

VS

- *Super*-horizon scale perturbations
- *Sub*-horizon perturbations & averaging
- Anti-Copernican inhomogeneous universe

Newtonianly perturbed FLRW universe

FLRW metric + scalar perturbations

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)\gamma_{ij}dx^i dx^j$$

γ_{ij} homogeneous-isotropic 3-space

Newtonian perturbation $\Psi = \Phi$

$$|\Psi| \ll 1\,,$$

$$\left|\frac{\partial \Psi}{\partial t}\right|^2 \ll \frac{1}{a^2}(D^i\Psi)D_i\Psi\,,$$

$$(D^i\Psi D_i\Psi)^2 \ll (D^i D^j\Psi)D_i D_j\Psi$$

Stress-tensor

Smoothly distributed component

$$T_{ab}^{(s)} \approx \rho^{(s)}(t)dt^2 + P^{(s)}(t)a^2(t)\gamma_{ij}dx^i dx^j$$

e.g., Dark Energy component

Inhomogeneously distributed component

$$T_{ab}^{(m)} \approx \rho^{(m)}(t, x^i)dt^2$$

Einstein equations

$$3\left(\frac{\dot{a}}{a}\right)^2 = \kappa^2\left(\rho^{(s)} + \bar{\rho}^{(m)}\right) - 3\frac{K}{a^2}$$

$$3\frac{\ddot{a}}{a} = -\frac{\kappa^2}{2}\left(\rho^{(s)} + \bar{\rho}^{(m)} + 3P^{(s)}\right)$$

$$\frac{1}{a^2}\Delta_{(3)}\Psi = \frac{\kappa^2}{2}\delta\rho \qquad \left(\delta\rho = \rho^{(m)} - \bar{\rho}^{(m)} \right)$$

Large - scale → FLRW dynamics

Small - scale → Newtonian gravity

- It is commonly stated that when

$$\frac{\delta\rho}{\rho} \gg 1$$

we enter a non-linear regime

This is not the case

Solar system, Galaxies, Clusters of Galaxies

$$\delta\rho/\rho \approx 10^{30}, \approx 10^{5}, \approx 10^{2} \gg 1$$

$$\Psi \approx 10^{-6} \sim 10^{-5} \ll 1$$

Metric perturbations can be of order $\delta\rho/\rho$
in the synchronous gauge

Newtonianly perturbed FLRW metric appears to very accurately describe our universe

on all scales

(except immediate vicinity of BHs and NSs)

If this assertion is correct

⇓

higher order corrections to this metric from inhomogeneities would be negligible

… but we cannot preclude the possibility that other models could also fit all observations

“Backreaction from inhomogeneities” “Fitting problem”

- Ellis 1984 Ellis-Stoeger 1987

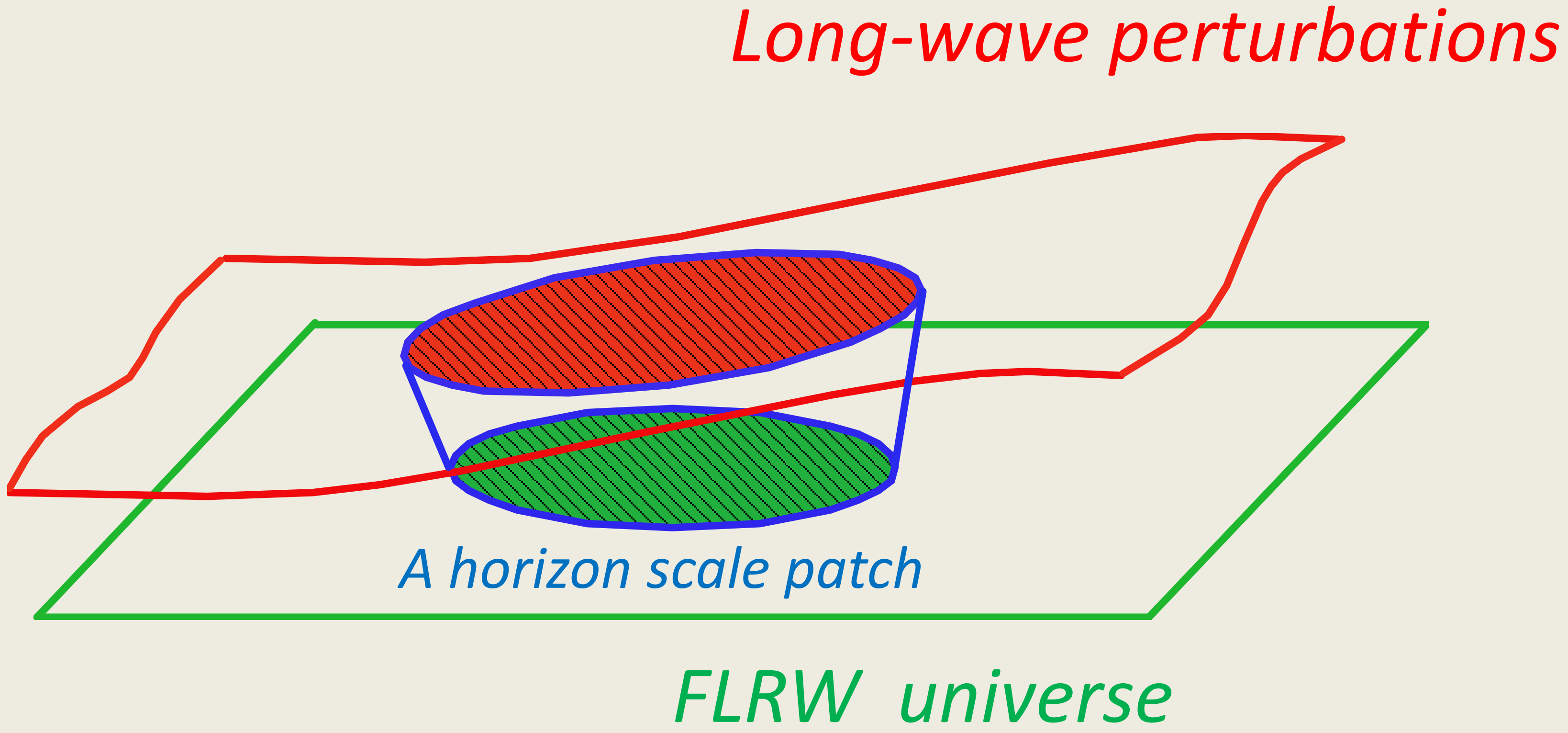
  Futamase 1988

Historical remarks

M. Kasai’s talk

Backreaction from

Super-horizon perturbations


Long-wave perturbations
A horizon scale patch
FLRW universe

2nd-order effective stress-tensor approach

$$g(\alpha) = g^{(0)}_{ab} + \alpha g^{(1)}_{ab} + \alpha^2 g^{(2)}_{ab} + \cdots$$

0th: $G_{ab}[g^{(0)}] = 0$

For vacuum case

1st: $G^{(1)}_{ab}[g^{(1)}] = 0$

2nd: $G^{(1)}_{ab}[g^{(2)}] = -G^{(2)}_{ab}[g^{(1)}]$

View $8\pi G^{(eff)} T_{ab} := -G^{(2)}_{ab}[g^{(1)}]$

and equate as $$G_{ab}[g] = 8\pi G^{(eff)} T_{ab}$$

g : backreacted metric

Brandenberger et al 1997 - 2005

If the effective stress-tensor takes the form

$$^{(eff)}T_{ab} \propto -\Lambda g_{ab}$$

and has the appropriate magnitude

we are done … !?

Martineau – Brandenberger 2005

Serious flaws in this approach

AI & Wald 2006

- "Backreaction equation" is NOT consistently constructed from "perturbation theory"
- 2nd-order effective stress-tensor is gauge-dependent
- If 2nd-order stress tensor has large effects, one can NOT reliably compute backreaction in 2nd-order theory
- Long-wavelength limit corresponds to "other FLRW universe" (e.g., with different initial data)

- C.f. “Shortwave approximation” Brill-Hartle 1964 Isaacson 1968

two small parameter amplitude α

wavelength β

-- can take limit keeping α/β finite, non-zero

Dominant $1/\beta$ $G^{(1)}_{ab}[g^{(0)}] = 0$

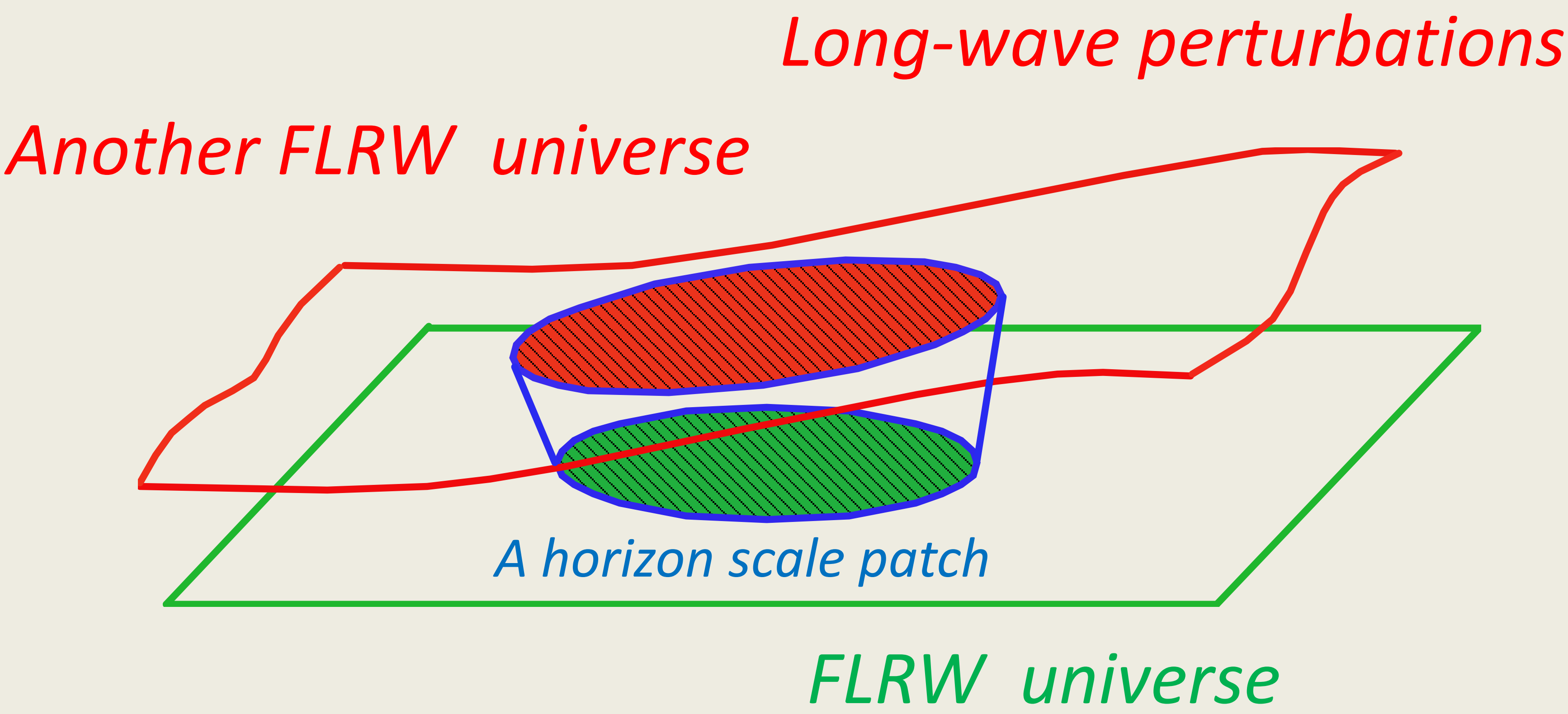
next-order $G_{ab}[g^{(0)}] = -\langle G^{(2)}_{ab}[g^{(1)}]\rangle_{\text{spacetime}}$ $\sim O(\alpha^2/\beta^2)$

Consistent backreaction formalism with large effects

Effective stress-tensor with 4-volume (space&time) average is gauge-invariant

Burnett 1999

Effective stress-tensor for 2nd-order Long-wave perturbations appears to be similar but very different from the shortwave approximation



Other backreaction proposal using super-horizon perturbations

Kolb-Matarrese-Notari-Riotto 2005

and criticisms Flanagan 2005 Hirata-Seljak 2005

Geshnizjani-Chung-Afshordi 2005

Backreaction from

Sub-horizon perturbations

&

spatial averaging

Inhomogeneous metric

$$ds^2 = -\alpha dt^2 + 2\beta_i dt dx^i + q_{ij} dx^i dx^j$$

Raychaudhuri equation: θ : expansion

$$\frac{d}{dt}\theta = -\frac{1}{3}\theta^2 - \sigma^2 - 4\pi G\rho + \omega^2$$

Deceleration unless one has large "*vorticity*" $\omega^2 \neq 0$

"Accelerated" expansion ⇒ need some new mechanism

For simplicity and definiteness we hereafter focus on an inhomogeneous universe with irrotational dust.

Then in the comoving synchronous gauge following Kolb-Matarrese-Riotto 2006

$$ds^2 = -dt^2 + q_{ij}(t, x^m)dx^i dx^j$$

(Different from Newtonian gauge, metric perturbations can be of order $\delta\rho/\rho$ in the synchronous gauge)

Spatial-Averaging

Buchert et al

Definition over Domain : $\langle \phi \rangle_D \equiv \frac{1}{V_D} \int_D \phi d\Sigma$

Depend on the choice of domain

Averaged scale factor: $a_D \equiv (V_D)^{1/3}$

Smoothing out inhomogeneities

→ Effective FLRW universe

Equations for "averaged quantities"

Buchert 2000

$$3\frac{\ddot{a}_D}{a_D} = -\frac{\kappa^2}{2}\langle\rho\rangle_D + Q_D$$

$$3\left(\frac{\dot{a}_D}{a_D}\right)^2 = \kappa^2\langle\rho\rangle_D - \frac{1}{2}\langle\mathcal{R}\rangle_D - \frac{1}{2}Q_D$$

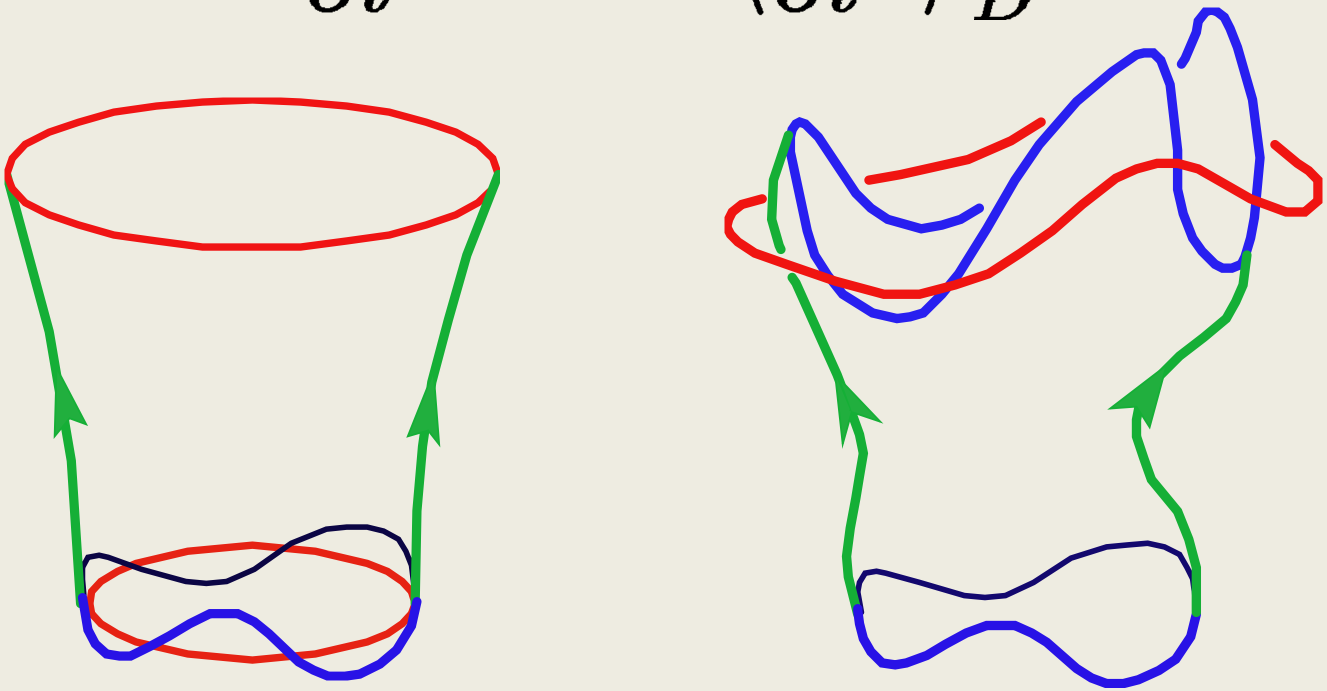
Integrability condition: $\left(a_D^6 Q_D\right)^{\cdot} + a_D^4\left(a_D^2\langle\mathcal{R}\rangle_D\right)^{\cdot} = 0$

Kinematical backreaction: $Q_D \equiv \frac{2}{3}\left(\langle\theta^2\rangle_D - \langle\theta\rangle_D^2\right) - \langle\sigma_{ij}\sigma^{ij}\rangle_D$

If $Q_D > \frac{\kappa^2}{2}\langle\rho\rangle_D$ ⇒ $\ddot{a}_D > 0$ Acceleration

Spatial averaging and time evolution do NOT commute

$$\frac{\partial}{\partial t}\langle\phi\rangle_D \neq \left\langle\frac{\partial}{\partial t}\phi\right\rangle_D$$



The same initial data

Contributions from non-linear sub-horizon perturbations to Q_D and the apparent acceleration of the volume-averaged scale factor have been studied by using *gradient expansion* method

Perturbation series appear to diverge

Kolb-Matarrese-Notari-Riotto 2005
Rasanen 2004

⇨ **Many related supportive work**
as well as criticisms on many grounds

An example of averaged acceleration

Averaging a portion of *expanding open* FLRW universe and a portion of *collapsing closed* FLRW universe exhibits "acceleration" in the averaged scale factor

Nambu & Tanimoto 2005

Even if $\ddot{a}_1 < 0 \quad \ddot{a}_2 < 0$

$$a_D^2 \ddot{a}_D = a_1^2 \ddot{a}_1 + a_2^2 \ddot{a}_2 + \frac{2}{a_D^3} a_1^3 a_2^3 \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right)^2$$

can be positive

This does NOT mean that we can obtain physically observable acceleration by spatial averaging

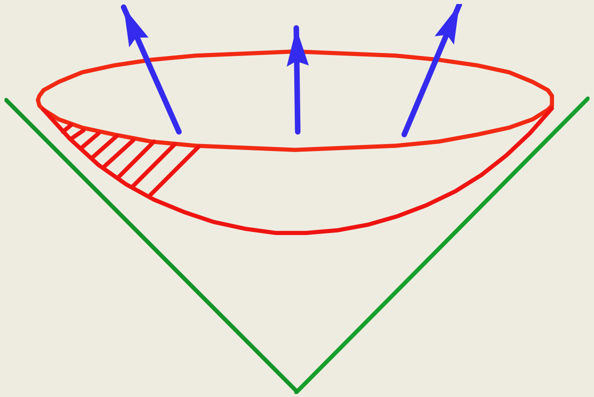
-- rather implies "spurious acceleration"

AI & Wald 2006

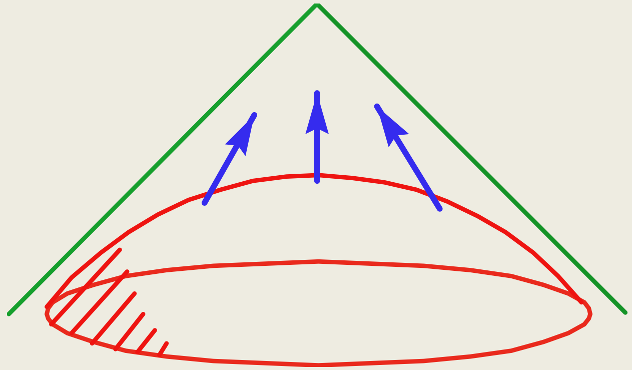
An example of spurious Acceleration

in Minkowski spacetime

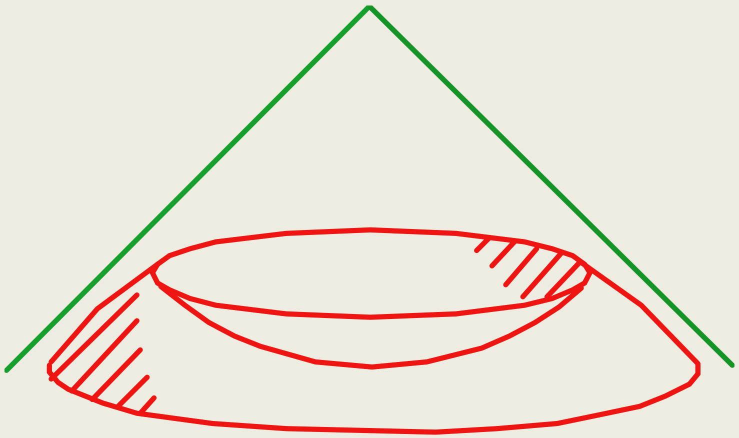
Expanding Hyperboloid



Contracting Hyperboloid



Always possible to take two (portions of) hyperboloids so that



$$\frac{\ddot{a}_D}{a_D} = -\frac{1}{3}\langle \text{Curvature of hyperboloid} \rangle_D = \frac{2}{a_D^2} > 0$$

Lessons

Gauge artifacts due to choice of time-slice & domain

The averaged scale factor displays “acceleration” without there being any physically observable consequence

> No reason to believe that the averaged quantities correspond to any physical effects
>
> The definition of “acceleration” should be given in terms of observables

Small inhomogeneities generate negligible effects

2nd-order analyses Kasai-Asada-Futamase 2006
Vanderveld et al 2007
Behrend et al 2008

How do we deduce Cosmological parameters from observational data?

Calculations of observables in inhomogeneous universe: Luminosity distance - redshift relation

$D_L(z, \theta, \phi)$				
Void-model of Tomita type (Inverse problem in LTB-metric) (+ small-scale clumps)		Celerier Iguchi-Nakao-Nakamura Garfinkle Vanderveld-Flanagan-Wasserman Tanimoto-Nambu Alnes-Amarzguioui-Groen Chung-Romano Yoo-Kai-Nakao		
Void-model of Swiss-cheese type		Biswas-Notari Marra-Kolb-Matarrese (latticed) Vanderveld-Flanagan-Wasserman (randomized)		
FLRW + perturbations		Futamase-Sasaki		
Super-horizon perturbations		Flanagan Hirata-Seljak Kumar-Flanagan		
Sub-horizon perturbations Newtonian Post-Newtonian		 Holz – Wald Asada Kasai Vanderveld-Flanagan-Wasserman		

Anti-Copernican universe

Inhomogeneous (non-perturvative) models

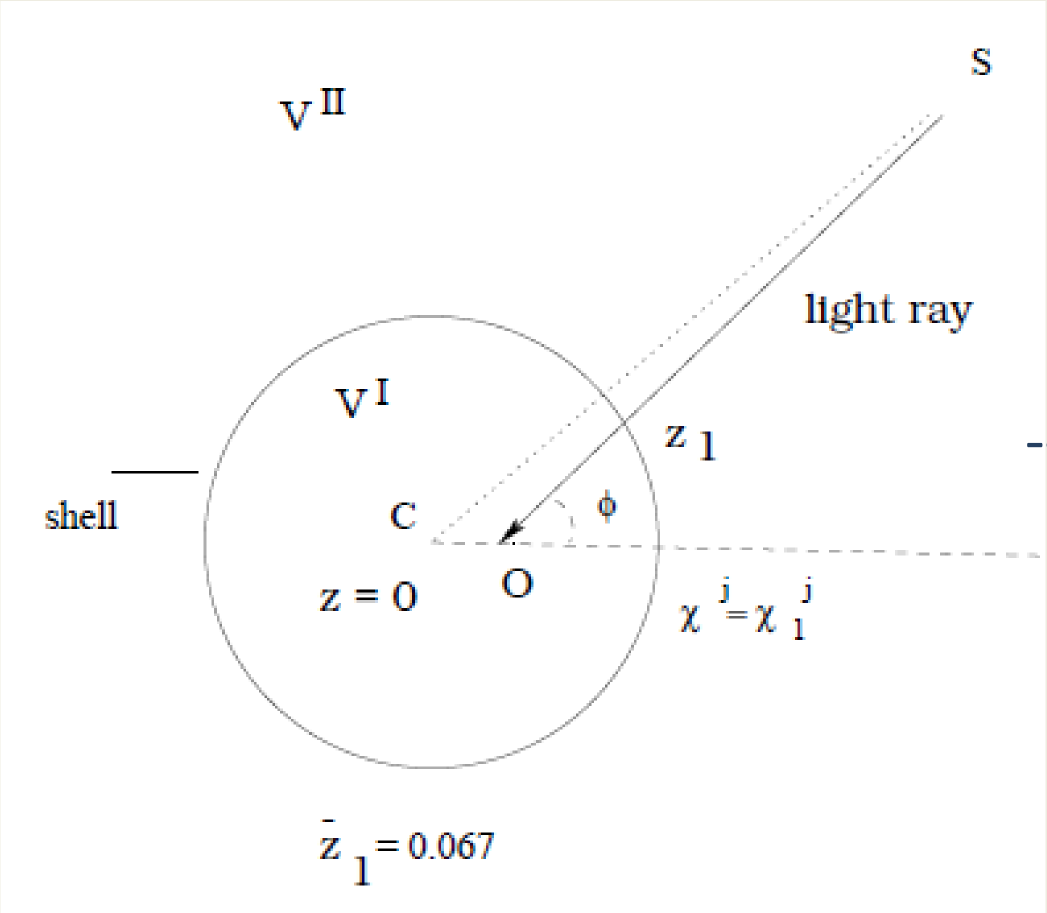
Geometry: *Spherically* Symmetric
Main constituents: Dark Matter

We are living in the center of the void

e.g. Local void of a few hundred Mpc: Tomita 2000

Local void of a few Gpc : Alnes-Amarzguioui-Groen 2006
Garcia-Bellido & Haugboelle 2008

A local void model Tomita 2000



V^I : low-dense region

V^{II} : high-dense outer

-- can be positioned 50Mpc away from the center

The mismatch between the local and global expansion can explain the observed dimming of SN-Ia luminosity

$$H_0^I > H_0^{II}$$

Simplest model: Lemaitre-Tolman-Bondi (LTB) metric

$$ds^2 = -dt^2 + \frac{R'(r,t)^2}{1+2E(r)}dr^2 + R(r,t)^2 d\Omega^2$$

$R(t,r) = ra(t) \quad 2E(r) = -Kr^2$ ⇒ FLRW metric

$$\dot{R}^2 = 2E + \frac{F(r)}{R}, \quad \rho = \frac{F'}{8\pi G R' R}$$

Two arbitrary functions $E(r) \quad F(r)$

Null vector $l_a = dt_a + \frac{R'}{\sqrt{1+2E}}dr_a \quad k^a = (\partial/\partial\lambda)^a = -\omega l^a$

$$1 + z = \omega$$

Luminosity-distance: $d_L = (1+z)^2 R$

The LTB model can fit well the redshift-luminosity relation

Iguchi – Nakamura – Nakao 2002

Garfinkle 2006

$$m = M_B + 5\log(H_0 d_L)$$

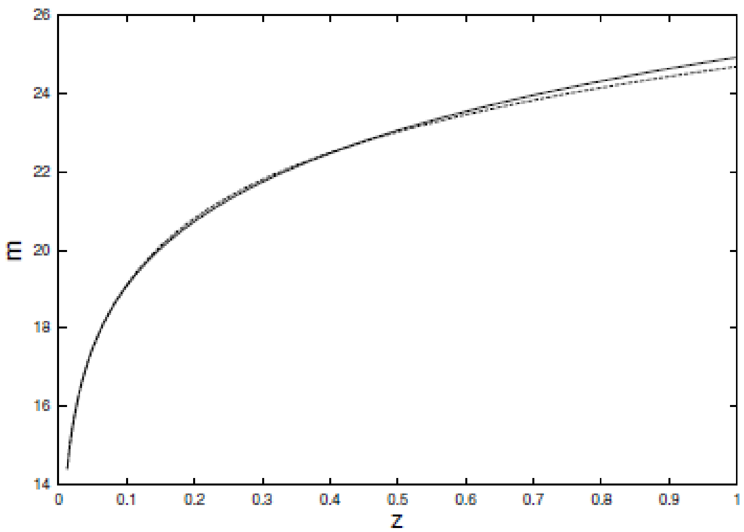


Figure 2. Plot of effective magnitude versus redshift for the standard ΛCDM model (solid and the $\Omega_M = 0.3$ LTB model (dashed curve).

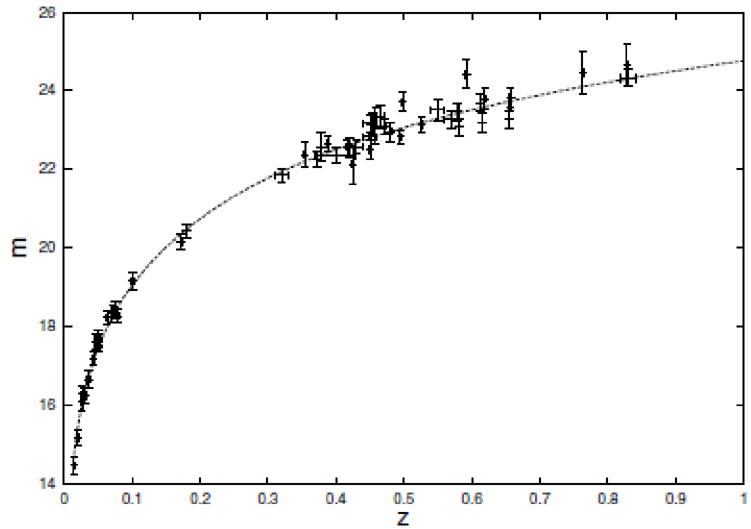


Figure 3. Plot of effective magnitude versus redshift for the $\Omega_M = 0.2$ LTB model (curve) and the supernova data.

However …

- Many LTB models contain a weak singularity at the center

  Vanderveld-Flanagan-Wesserman 2006

- We have more cosmological data than SN-Ia

- How to reconcile large scale structure formation *without* Dark Energy?

  --- density perturbations would have grown too much

- How to confront with CMB spectrum?

  1st-peak of CMB power spectrum can be made to match WMAP observations

  e.g. Alnes-Amarzguioui-Groen 2006

  Garcia-Bellido – Haugboelle 2008

Summary

- Inhomogeneous models *can* mimic an “accelerated expansion” *without Dark Energy*

  --- but depends on the definition of “acceleration”

- Backreacted quantities based on “spatial averaging”

  have gauge ambiguity: Not directly related to observables

  “Acceleration” should be defined in terms of physical observables

- Anti-copernican model has attracted attention
- Seems unlikely that all cosmological data can be explained by inhomogeneous models

- But not yet definitively ruled out:

  still a lot of issues to be addressed

Models \ Issues	Ideas (Appealing prospects)	Problems (Theoretical/Observational)
Super-horizon perturbations	Effective-stress tensor FLRW inside Horizon	Gauge artifacts AI
Sub-horizon inhomogeneities (Non-linear perturbations)	Spatial-Averaging Effective Friedmann equation A. Notari D. Wiltshire	Not directly related to physical observables AI R.A. Vanderveld
Void-model of Swiss-cheese type (Weak lensing)	A fully non-linear GR treatment of inhomogeneous universe K. Nakao A. Notari H. Asada CMB (anomalies) K.T. Inoue	Lines of sight in regular latticed vs randomized R.A. Vanderveld
Void-model of Tomita type (Anti-Copernican)	Live in the center K. Tomita Luminosity distance – redshift K. Nakao M. Kasai A. Notari A. Romano CMB K.Tomita	Why live in the center? Singular center? R.A.Vanderveld Void origin? Other cosmological observations kSZ BAO Structure formation?
General arguments **Develop new techniques** **Other ideas**	M.Kasai A.Romano T.Futamase K.T. Inoue K. Tomita H. Asada D. Wiltshire	K. Tomita A.A. Starobinsky
FLRW-Universe (+Newtonian perturbations)	Copernican principle Simple and successful (so far)	Need “Dark Energy” How to test the homogeneity? A.A. Starobinsky K. Tomita