Inhomogeneities as alternatives to Dark Energy

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Outline

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Dark energy problems

- **Supernovae Ia data** interpreted in the framework of **FRLW** solution strongly disfavor a matter dominated universe and strongly support a **dominant dark energy** component
- Cosmological constant problem
- What is dark energy ?
- Dark energy dominance coincides with epoch of nonlinear structure formation

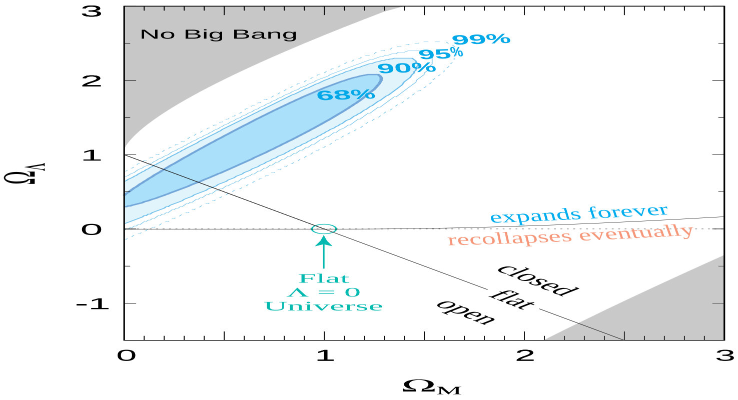

Figure: Best-fit confidence regions in the Ω_M–Ω_λ plane. The 68%, 90%, 95%, and 99% statistical confidence regions are shown.From Perlmutter,1998np

Different approaches to mimic dark energy

- ## Spatial averaging
- Attempts to obtain a positive **averaged** acceleration
- It is not related to **direct** observables, because light **“feels” real geometry**
- It can lead to the definition of **unobservable** quantities because of **causality violation**

- ## Inversion method
- It tries to reproduce **direct** observables such as $D_L(z)$
- Numerical **instability** and **degenerancy**
- Need of **extra** constraints $\Rightarrow$ **RSSE** , CMB

Outline

LTB solution

- $SO(3)$ invariant, $P = 0$, perfect fluid
- is not a perturbative deviation from FRLW

$$ds^2 = -dt^2 + \frac{(R_{,r})^2}{1+2E(r)}dr^2 + R^2 d\Omega_2^2,$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{2E(r)}{R^2} + \frac{2M(r)}{R^3},$$

$$T^{\mu}_{\nu} = [\rho = \frac{M_{pl}^2}{4\pi}\frac{M'(r)}{R^2 R_{,r}}, -P = 0]$$

- $E(r)$ and $M(r)$ are arbitrary functions of r
- $R(t,r)$

- Bang function $t_b(r)$ gives initial condition for R

$$R(t_b(r),r) = 0$$

- Introducing the following variables

$$a(t,r) = \frac{R(t,r)}{r}, \quad k(r) = -\frac{2E(r)}{r^2}, \quad \rho_0(r) = \frac{6M(r)}{r^3}$$

$$ds^2 = -dt^2 + a^2\left[\left(1+\frac{a_{,r}r}{a}\right)^2 \frac{dr^2}{1-k(r)r^2} + r^2 d\Omega_2^2\right]$$

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k(r)}{a^2} + \frac{\rho_0(r)}{3a^3}$$

Spatial averaging

- Following the standard spatial averaging procedure we define the **volume** for a **spherical domain**,$0 < r < r_D$

$$V_D = 4\pi \int_0^{r_D} \frac{R^2 R_{,r}}{\sqrt{1+2E(r)}} dr$$

- The length associated to the domain is

$$L_D = V_D^{1/3}$$

- The deceleration parameter q_D and the **average acceleration** a_D are defined as:

$$q_D = -\ddot{L}_D L_D / \dot{L}_D^2$$

$$a_D = \dot{L}/L$$

Some examples

- Chuang, Gu and Hwang (2008) studied this type of LTB models:

$$t_b(r) = -\frac{h_{tb}(r/r_t)^{n_t}}{1+(r/r_t)^{n_t}}$$

$$k(r) = -\frac{(h_k+1)(r/r_k)^{n_k}}{1+(r/r_k)^{n_k}} + 1$$

$$\rho_0(r) = \text{constant}$$

Table: Three examples of the domain acceleration.

	t	r_D	ρ_0	r_k	n_k	h_k	r_t	n_t	h_{tb}	q_D
1	0.1	1	1	0.6	20	10	0.6	20	10	-0.0108
2	0.1	1.1	10^5	0.9	40	40	0.9	40	10	-1.08
3	10^{-8}	1	10^{10}	0.77	100	100	0.92	100	50	-6.35

Plot of $k(r)$ and of the bang function $t_b(r)$

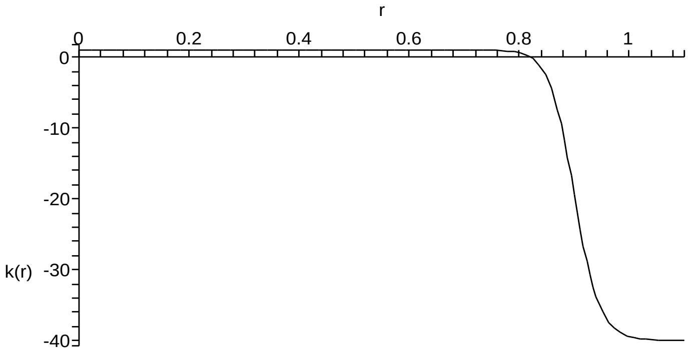

Figure: $k(r)$ is plotted for the model corresponding to row 2 of Table I .

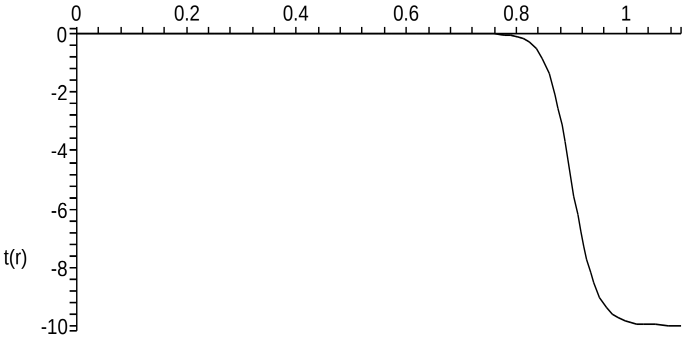

Figure: $t_b(r)$ is plotted for the model corresponding to row 2 of Table I

Relation between a_D and physical observations

- Defining t_q as the time at which $q(t_q) = q_D$, we solve the null geodesic equation assuming the following initial conditions :

$$\frac{dT(r)}{dr} = -\frac{R'(r,T(r))}{\sqrt{1+2E(r)}}.$$

$$T(r=0) = t_q$$

$$R(t_b(r), r) = 0$$

- This is the natural way to **map** these models into **luminosity distance** observations for a central observer which should receive the light rays at the time t_q at which the **averaged acceleration** is positive.

Averaging scale is larger than the event horizon

- We then evolved the differential equation to find :

$$T(r_{Hor}) = t_b(r_{Hor})$$

- For one model we obtain

$$r_{Hor} < r_D$$

r_D is the upper limit of the volume averaging integral. (Romano, 2007)

- In this context r_{Hor} can be thought of as the **co-moving horizon**, the **maximum radial** coordinate from which photons can **reach** the central observer O_c at time t_q.

Singularity along the geodesic

- As it can be seen in figure at $r = r_{Hor}$ there is a **singularity** along the geodesic, corresponding to the fact that **light cannot reach** the central observer at time t_q from points at radial coordinate $r > r_{Hor}$.

$$R(T(r_{Hor}), r_{Hor}) = R(t_b(r_{Hor}), r_{Hor}) = 0.$$

- Since regions of the universe at radial coordinates greater than r_{Hor} have **never** been in **causal contact** with O_c at time t_q, the scale at which q_D is defined is beyond the region causally connected to O_c. Therefore q_D cannot be detected from **local observation** of the luminosity distance which is used to define a^{FRLW}.

Plot of the geodesics

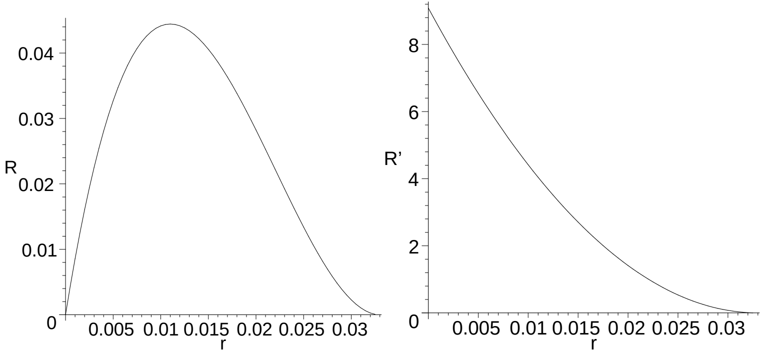

Figure: $R(T(r), r)$ and $R'(T(r), r)$ are plotted for model 2 .At about r_{Hor}=0.03298 there is a singularity.

What did we learn?

- This example shows how a_D may **not** even be **causally related** to the **local observation** of $D_L(z)$, and gives a **reverse example** of the results obtained by Enqvist where it was studied a **LTB model fitting the observed luminosity distance**, consistent with a positive a^{FRLW}, but **without positive averaged acceleration** a_D.
- Our results **do not rule out LTB** models as alternatives to dark energy since, since the **inversion** method allows to obtain the observed luminosity distance **without any averaging**.

- We can conclude that the luminosity distance $D_L(z)$ contains **more information than** the spatially averaged acceleration a_D because the first is **sensitive** to the causal structure of the **entire space-time** while the second is the result of averaging only the **spatial** part of the geometry, making the relation between them in general **not one-to-one**.
- The study of the constraints on the **local observability** of **averaged quantities** will be addressed in a more general way, not only in the context of LTB models, in a future work.

Outline

Inversion method

$$\{E(r), M(r), R_0(r)\} \Rightarrow D_L(z) = R(1+z)^2$$

$$\Downarrow$$

$$\{E(r), D_L(z), R_0(r)\} \Rightarrow M(r)$$

Results

- If $\{E(r) = R_0(r) = 0\}$ we can reproduce FRLW with $\Omega_M = 1$ and $D_L(z)$ is **independent** of $M(r)$
- Making an ansatz for $E(r)$ we find a $M(r)$ which **corresponds** to the observed luminosity distance best fitted by a $\Omega_\Lambda = 0.7$, $\Omega_M = 0.3$ FRLW model.

Geodesic equations

- The radial coordinate as a function of time along null geodesic satisfies the equations :

$$\frac{dr}{dz} = \frac{s\sqrt{1+2E(r(z))}\sqrt{2E(r)+\frac{2M}{R(t,r)}}}{(1+z)[E'(r)+M'/R - M\partial_r R/R^2]}$$

$$\frac{dt}{dz} = \frac{-s|\partial_r R(t(z),r(z))|\sqrt{2E(r)+\frac{2M}{R(t,r)}}}{(1+z)[E'(r)+M'/R - M\partial_r R/R^2]}.$$

- The physics is that if one knows a single radial **geodesic history** of a photon which was emitted at an event (t_1, r_1) and observed at (t_2, r_2), one knows the full **space-time geometry** in the region $(\mathbf{t_1} < \mathbf{t} < \mathbf{t_2}, \mathbf{r_1} < \mathbf{r} < \mathbf{r_2})$ of the LTB solution owing to its spherical symmetry.

Another equation

- **three unknown** functions $\{M_1(z), r(z), t(z)\} \Rightarrow$
- **another independent** equation.
  This is provided by dR/dz through the chain rule:

$$\frac{d}{dz}R = s\sqrt{2E + \frac{2M_1}{R}}\frac{dt}{dz} + \partial_r R \frac{dr}{dz}$$

- Finally we can solve these 3 equations to find $M(r)$

  $\{E(r), D_L(z), R_0(r)\} \Rightarrow \{t(z), r(z), M_1(z)\}$

- $$M(r) = M_1(z(r))$$

- As long as $\mathbf{M}/\mathbf{R} \gg \mathbf{E}$, the luminosity distance curve no longer accurately probes the geometry of the LTB model since **different geometries** lead to approximately the **same** $\mathbf{R}(\mathbf{z}) = \mathbf{D_L}(\mathbf{z})/(\mathbf{1}+\mathbf{z})^{\mathbf{2}}$.
- The **inversion** method will necessarily be **unstable** once the curvature term E can be neglected Schematically, we will have

$$\frac{dr}{dz} \sim \frac{ER}{M_1}F + \frac{\sqrt{2M_1R}}{(1+z)[\frac{2}{3}\frac{d}{dz}M_1]}\frac{dr}{dz},$$

when ER/M_1 becomes small and $F \sim \mathcal{O}(\frac{dr}{dz})$.
Since $\frac{\sqrt{2M_1R}}{(1+z)[\frac{2}{3}\frac{d}{dz}M_1]} \sim 1$ in the limit that $ER/M_1 \to 0$,

Outline

How to consistently slice redshift space in spherical shells?

- $t(z)$ and $r(z)$ are both functions of redshift z and **dipend** on the **cosmological model**
- It is important to respect the observed object evolution **time scale**

$$\Delta t = t(z) - t(z + \Delta Z)$$

$$\Delta Z(z) = t^{-1}[t(z) - \Delta t] - z$$

Determining Δt

- The value of Δt depends on the particular **type** of **astrophysical objects** considered, and requires a good understanding of their evolution.
- In order to provide an example comparable with **previous homogeneity studies**, we will use the value implied by the redshift range $0.2 < z < 0.35$ studied by Hogg, assuming a flat FRLW model with $\{\Omega_M = 0.3, \Omega_\Lambda = 0.7\}$:

$$t(z)_{FRLW} = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{\left[\Omega_M x^{-1} + \Omega_\Lambda x^2\right]^{1/2}}$$

$$\Delta t = t_{FRLW}(0.2) - t_{FRLW}(0.35) \approx 1.4 Gyr$$

$$\Delta Z_{FRLW}(0.2) = 0.35 - 0.2 = 0.15$$

Plot of $\Delta Z(z)$

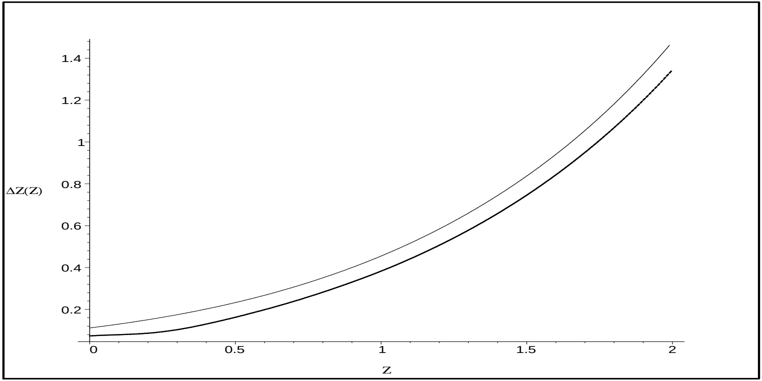

Figure: $\Delta Z(z)$ is plotted as a function of the redshift for the LTB model considered (thick line) and for a FRLW flat Universe with $\Omega_\Lambda = 0.7, \Omega_M = 0.3$ (thin line).

Redshift spherical shell energy

- Using the volume element

$$dV(r,t) = dv(r,t)dr = 4\pi \frac{R^2 R_{,r}}{\sqrt{1+2E(r)}} dr \, .$$

we can now define (**RSSE**) (Romano, 2007) $\rho_{SS}(z)$ as:

$$\rho_{SS}(z) = \int_{z}^{z+\Delta Z(z)} \rho(z')dV(z') =$$

$$\int_{z}^{z+\Delta Z(z)} 4\pi \frac{M_{,r}(r(z))}{\sqrt{1+2E(r(z))}} \frac{dr}{dz} dz$$

$$\rho(z) = \rho(r(z), t(z))$$

Check : FRLW case near z=0

- In the case of a FRLW Universe we get :

$$r(z)_{FRLW} = H_0^{-1} \int_{(1+z)^{-1}}^{1} \frac{dx}{\left[\Omega_M x^1 + \Omega_\Lambda x^4\right]^{1/2}}$$

$$\rho_{SS}(z)_{FRLW} = \int_{z}^{z+\Delta Z(z)} \frac{\rho_0}{a(z)^3} a(z)^3 4\pi r(z)^2 d(r(z)) =$$

$$\frac{4\pi\rho_0}{3}[r_{FRLW}(z+\Delta Z(z))^3 - r_{FRLW}(z)^3]$$

$$\rho_{SS}(0)_{FRLW} \propto r_{FRLW}(\Delta Z(0))^3$$

- As expected, $\rho_{SS}(0)$ **scales** as the **third power of the co-moving distance** r_{FRLW}(Scaramella).

Examples

- We considered **two models** which both **fit** successfully the observed high redshift supernovae **luminosity distance** $D_L(z)$
- **Flat FRLW** model with $\{\Omega_\Lambda = 0.7, \Omega_M = 0.3\}$
- **LTB** model proposed by Alnes corresponding to:

$$E(r) = \frac{1}{2}H_{\perp,0}^2 r^2(\beta_0 - \frac{\Delta\beta}{2}[1 - \tanh\frac{r - r_0}{2\Delta r}])$$

$$M(r) = \frac{1}{2}H_{\perp,0}^2 r^3(\alpha_0 - \frac{\Delta\alpha}{2}[1 - \tanh\frac{r - r_0}{2\Delta r}])$$

$$\alpha_0 = 1, \beta_0 = 0, \Delta\beta = -\Delta\alpha = -0.9, \Delta r = 0.4r_0$$

$$r_0 \approx \frac{1}{5H_0}, H_{\perp,0} \approx H_0$$

where $H_0 \approx 50\mathrm{km/s/Mpc}$.

Plot of RSSE for the two models

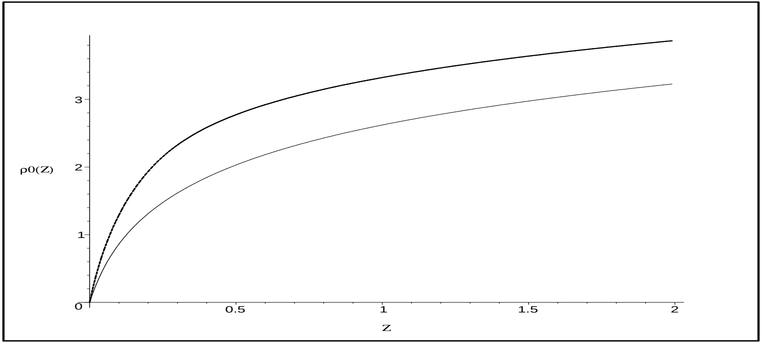

Figure: $\rho_0(z) = Log_{10}[\rho_{SS}(z)/\rho_{SS}(0)]$ is plotted as a function of the redshift for the LTB model considered (thick line) and for a FRLW flat Universe with $\Omega_\Lambda = 0.7, \Omega_M = 0.3$ (thin line).

Relation of RSSE to astrophysical observations

- The quantity we have introduced to describe the **radial energy distribution** of isotropic universes, could be used to **distinguish** between different **models only if** we can relate it to some **astrophysical observable**.
- A natural candidate, which has been used in previous homogeneity tests, would be the **galaxy number count** $n(z)$, but its relation to $\rho(z)$ can be rather complicated, due to different factors which should be taken into consideration.

- In general we can write :

$$n(z) = F(z)\rho(z)$$

  where $F(z)$ is function of the redshift which depends on different effects such as K-correction, **distance selection effect**, the **bias** between **barionic** and **dark matter**, and the **mass light ratio** relation.
- **Astrophysical evolution** could in fact play a very important role when comparing shells at **very different redshift**, introducing a **time dependency** which is **not included** in the **cosmological** model, and could make more difficult to distinguish between one model and another.

- An **alternative** application of the proposed method could consist in the calculation of RSSE for **shells centered at different points** in space time.
- It would also be important to asses if the **same RSSE** could correspond to different **homogeneous** and **inhomogeneous models**, a degeneracy which could be possible as it has been shown for other cosmological observables such as the **luminosity distance** for example.

Final remarks

- **FRLW cosmic acceleration** is **not necessarily** related to **LTB spatially averaged acceleration**
- **Spatial averaging can lead** to the construction of **unobservable quantitie**s because of causality violation
- **"Cosmological acceleration"** is inferred **indirectly assuming homogeneity**, so the real focus should be the **direct observations** which lead to its estimation such as $D_L(z)$**, CMB, RSSE**.
- Recent work by Scott put **stronger** constraints on LTB models using both $D_L(z)$ and CMB
- Even if **inhomogeneities cannot completely** explain the observational data, they could still play an **important role** and compete with **Dark Energy**.