Recent developments in perturbative QCD

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LPTHE, Universities of Paris VI and VII and CNRS

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20 April 2006
Concentrate on the effort to ‘get QCD in shape for LHC era’:

- Predicting multi-jet final-states:
  [because new-particle signatures involve many jets]
  - New tree level techniques
  - NLO and progress in 1-loop calculations

- Aiming for accuracy
  [because NLO theory is often far behind HERA/LEP precision]
  - NNLO jets: status & progress report
  - What NNLO is teaching us about QCD itself

- Other developments (mostly ’all-order’ QCD)

Some recently very active fields, not covered:

- Small-\(x\) saturation
- Generalised parton distributions – a field in its own right
  - talk by Diehl, + hep-ph/0512201
<table>
<thead>
<tr>
<th>Multi-jets</th>
<th>Tree level</th>
<th>NLO</th>
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<tr>
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Other results:
- MC
- Resummation
Heavy objects: multi-jet final-states

- Need to understand QCD multi-jet production (background)
- Max # jets: tree level $\leq 8$ jets

MadEvent, AlpGen, Helac/Phegas
CompHEP, Grace, Amegic

<table>
<thead>
<tr>
<th># jets</th>
<th># events for 10 fb$^{-1}$</th>
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<tr>
<td>3</td>
<td>$9 \cdot 10^8$</td>
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<tr>
<td>4</td>
<td>$7 \cdot 10^7$</td>
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<tr>
<td>5</td>
<td>$6 \cdot 10^6$</td>
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<td>6</td>
<td>$3 \cdot 10^5$</td>
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<tr>
<td>7</td>
<td>$2 \cdot 10^4$</td>
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<tr>
<td>8</td>
<td>$2 \cdot 10^3$</td>
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$p_t(jet) > 60$ GeV, $\theta_{ij} > 30$ deg, $|y_{ij}| < 3$

Draggiotis, Kleiss & Papadopoulos '02

**Multi-jet final-states**

- Motivation, state of the art

---

**Development in pQCD (G. Salam, LPTHE)**
\[ A^{\text{tree}}(1, 2, \ldots, n) = g^{n-2} \sum_{\text{perms}} \text{Tr}(T_1 T_2 \ldots T_n) \]

\( A^{\text{tree}}(1, 2, \ldots, n) \)

colour ordered amp.
\[ A^{\text{tree}}(1, 2, \ldots, n) = g^{n-2} \sum_{\text{perms}} \hfill \underbrace{\text{Tr}(T_1 T_2 \ldots T_n)}_{\text{colour struct.}} \hfill \overbrace{A^{\text{tree}}(1, 2, \ldots, n)}_{\text{colour ordered amp.}} \]

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<td>10</td>
<td>10525900</td>
<td>7335</td>
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non-planar | planar
\[ A^{\text{tree}}(1, 2, \ldots, n) = g^{n-2} \sum_{\text{perms}} \text{Tr}(T_1 T_2 \ldots T_n) A^{\text{tree}}(1, 2, \ldots, n) \]

**Tree level history**

**Maximal Helicity Violating (MHV)**

Helicity amplitude: simplifies!

\[ A^{\text{tree}}(- - + + \ldots) = \frac{i \langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle} \]

Parke & Taylor, Kunszt '85
Berends & Giele '88
\[ A_{\text{tree}}(1, 2, \ldots, n) = g^{n-2} \sum_{\text{perms}} \frac{\text{Tr}(T_1 T_2 \ldots T_n)}{\text{colour struct.}} \frac{A_{\text{tree}} (1, 2, \ldots, n)}{\text{colour ordered amp.}} \]

Next to Maximal Helicity Violating (NMHV)

Kosower, '90
\[
\mathcal{A}^{\text{tree}}(1, 2, \ldots, n) = g^{n-2} \sum_{\text{perms}} \text{Tr}(T_1 T_2 \cdots T_n) \mathcal{A}^{\text{tree}}(1, 2, \ldots, n)
\]

Helicity amplitude: simplifies!

\[
\mathcal{A}^{\text{tree}}(+-+-+\ldots) = \frac{1}{F_{3,1}} \sum_{j=4}^{n-1} \langle 1| P_{2,j} P_{j+1,2} |3\rangle \frac{1}{P_{2,j}^2 P_{j+1,2}^2} \times \frac{\langle j+1 | j \rangle}{[2| P_{2,j} | j+1 \rangle \langle j | P_{j+1,2} | 2\rangle}
\]

Britto et al., hep-th/0503198

NEXT to Maximal Helicity Violating (NMHV) Just one of vast array of results obtained with new recursion (Twistor) techniques.
Build multi-leg amplitudes by joining sub-amplitudes.

Berends Giele (1988): Join smaller off-shell amplitudes through a (colour-stripped) three or four-gluon vertex:

\[
\Sigma_{\Sigma} = \sum_{j} \cdot \cdot \cdot + \sum_{j,k} \cdot \cdot \cdot + \cdot \cdot \cdot
\]

This is basis of many tree-level multi-particle Monte Carlo programs.

Why powerful?

Sub-amplitudes can be simplified before joining them together. Feynman diagrams, in contrast, can only be simplified after full calculation.
New recursion relations (twistors)

Britto-Cachazo-Feng (BCF): Join smaller sub-amplitudes by a propagator. Sub-amplitudes made on-shell by analytic continuation ($\pm z_j$) of two reference momenta:

Britto, Cachazo & Feng hep-th/0412265; idem. + Witten hep-th/0501052

Earlier (related) rules: Cachazo, Svrcek & Witten hep-th/0403047

Proof based on analytic structure of tree-graphs (they are a sum of poles in complex plane) — very general.

Simplicity lies in on-shellness of sub-amplitudes and the need for just a scalar propagator to join them.


Recursion tree-level highlights

Very active field: 200 articles in 2 years (∼50 by ‘QCD people’)

Tree level

- Specific compact results, including NNMHV
  - Hints of yet deeper simplifications
  - Kosower ’04; Roiban et al ’04
  - Luo & Wen ’05; Britto et al ’05

- Efficient (recursive) formulations
  - Bena, Bern, Kosower ’04
  - NB: recall ∃ ‘standard’ numerical methods for tree-level calculations:
    - Berends-Giele (’88); ‘Alpha’ (’95)

- Massless quarks, gluinos
  - Georgiou, Glover & Khoze ’04; Wu & Zhu ’04

- External Higgs boson
  - Dixon/Badger, Glover & Khoze ’04

- External weak boson (& fermions)
  - Bern, Forde, Kosower & Mastrolia ’04

- Collinear limits
  - Birthwright et al ’05

- Massive quarks, scalars
  - Forde & Kosower ’05; Schwinn & Weinzierl ’06
  - Ferrario, Rodrigo & Talavera ’06; Ozeren & Stirling ’06

Amazing progress in short time...
Recursion (Twistor) Papers

- "Twistor" papers
- " in hep-ph or by "Pheno" authors

Papers / month

Jan 04  Jul 04  Jan 05  Jul 05  Jan 06

Jan 04 to Jul 05: Increase in papers
Jan 06 to Jul 05: Stabilization of papers
Jan 06: Decrease in papers

Developments in pQCD (G. Salam, LPTHE) p.9/29
- Multi-jets
- Improving tree-level field theory
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**Other results**
- MC Resummation
Currently available

NLOJET++, MCFM, PHOX, ...

http://www.cedar.ac.uk/hepcode/

Experimenters’ priorities

1. $pp \to WW + \text{jet}$  Les Houches ’05
2. $pp \to H + 2 \text{ jets}$
   - Background to VBF Higgs production
3. $pp \to t\bar{t}b\bar{b}$
4. $pp \to t\bar{t} + 2 \text{ jets}$
   - Background to $t\bar{t}H$
5. $pp \to WW b\bar{b}$
6. $pp \to VV + 2 \text{ jets}$
   - Background to $WW \to H \to WW$
7. $pp \to V + 3 \text{ jets}$
   - General background to new physics
8. $pp \to VVV + \text{jet}$
   - Background to SUSY trilepton
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</tr>
<tr>
<td>4. pp → t\bar{t} + 2 jets</td>
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<tr>
<td>5. pp → WW b\bar{b}</td>
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<td>▶ Background to</td>
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<td>W W → H → W W</td>
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<td>7. pp → V + 3 jets</td>
<td>▶ General background to new physics</td>
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NLOJET++, MCFM, PHOX, ...
http://www.cedar.ac.uk/hepcode/

Theorist’s list (G. Heinrich)

- 2 → 3 (OK for a good student!)
  - pp → WW + jet
  - pp → VVV
  - pp → H + 2 jets
- 2 → 4 (Beyond today’s means)
  - pp → 4 jets
  - pp → t\bar{t} + 2 jets
  - pp → t\bar{t}b\bar{b}
  - pp → V + 3 jets
  - pp → VV + 2 jets
  - pp → VVV + jet
  - pp → WW bb

Experimenters’ priorities

1. pp → WW + jet  Les Houches ’05
2. pp → H + 2 jets
   - Background to VBF Higgs production
3. pp → t\bar{t}b\bar{b}
4. pp → t\bar{t} + 2 jets
   - Background to t\bar{t}H
5. pp → WW b\bar{b}
6. pp → VV + 2 jets
   - Background to $W W \rightarrow H \rightarrow W W$
7. pp → V + 3 jets
   - General background to new physics
8. pp → VVV + jet
   - Background to SUSY trilepton
Developments in pQCD (G. Salam, LPTHE) p.12/29

— Multi-jets
— New methods @ 1-loop

**NLO bottleneck: loop calc.**

\[ 2 \to 3 \text{ @ NLO} \sim \begin{array}{c}
\text{2 } \to \text{ 4 @ Tree} \\
\text{2 } \to \text{ 3 @ 1-loop}
\end{array} + \text{ Tricks to cancel divergences}
\]

(dipole subtraction)

Traditionally: 1-loop for 2→3 proc. takes 1–2 years

Two ways of doing this more efficiently:

- Understand field theory better
- Get a computer to do most of the work for you

Enormous progress on this in past two years: ~ 200 articles

First full 2→4 (6-leg) result obtained this way
Multi-jets

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Multi-jets

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\[ 2 \to 4 @ \text{NLO} \sim 2 \to 5 @ \text{Tree} \]

\[ 2 \to 4 @ 1\text{-loop} \]

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Very hard!

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Would like a relation that avoids need for loop integrations. *Various kinds of recursion possible*

Technically: loop diagrams have more complex analytic properties than trees (*cuts* as well as poles), so BCFW does not apply.

Complex problem, much progress made, many people involved.

Bedford, Bena, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Cachazo, Del Duca, Dixon, Dunbar, Feng, Forde, Ita, Kosower, McNamara, Mastrolia, Perkins, Roiban, Spence, Travaglini, [...]

Recursion for loops?
Would like a relation that avoids need for loop integrations. Various kinds of recursion possible

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One ingredient of one of the “priority processes” \((pp \rightarrow 4 \text{ jets})\) is the 6-gluon 1-loop amplitude:

\[
\mathcal{A}_g = (A_g + 4A_f + 3A_s) - 4(A_f + A_s) + A_s
\]

\(\mathcal{N} = 4\) SUSY \quad \mathcal{N} = 1\) chiral SUSY \quad \text{scalar}

<table>
<thead>
<tr>
<th>(\mathcal{N} = 4)</th>
<th>(\mathcal{N} = 1)</th>
<th>(S (c, d, e))</th>
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Table adapted from hep-ph/0603187; NB: many results go beyond 6 gluons

Promising + much progress made! But QCD loops are still far from having simplicity of the tree-level results...
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Loops from Feynman diagrams

Automation of loop calculations with Feynman diagram techniques:

- Get expressions for all Feynman graphs (QGRAF, FeynArts). This gives answer in terms of a set of loop integrals
- Use recursion relations to reexpress each loop integral in terms of a basis set of known standard integrals. NB: recursion for integrals, not amplitudes!
- Analytically with algebraic manipulation programs (Binoth, Guillet, Heinrich, Pilon, Schubert '05; + others)
- Semi-numerically, “on the fly”, (Ellis, Giele, Glover, Zanderighi '04-05)
- Results unstable at special phase-space points (e.g. co-planar momenta): use dedicated strategies there.

- Alternative integration techniques: e.g. subtract out divergences before integrating, do rest numerically.

Nagy, Soper '03
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  Nagy, Soper ’03
Automated loops: applications

- **Full 6-gluon 1-loop amplitude!**
  - Ellis, Giele, Zanderighi ’06
  - Only fully known 2 → 4 1-loop amplitude in QCD

- **pp → H + 2 jets:** amplitudes done, implementation into MCFM in progress
  - Ellis, Campbell, Giele, Zanderighi, ’05-06

- **gg → WW via quark loop**
  - Binoth, Ciccolini, Kauer, Krämer ’05

- Similar techniques in EW: e^+e^- → 4 fermions
  - Denner, Dittmaier, Roth, Wieders ’05

Automated techniques have advantage of flexibility
But: speed can be issue in numerical variants.

NB: more ‘traditional’ NLO methods still important, talk by Oleari
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Other results:
- MC
- Resummation
Processes **with two QCD partons** @ LO are mostly done

- $e^+e^- \rightarrow \text{hadrons, } \tau \rightarrow \nu \pm \text{hadrons}$
- DIS coeff. fns., sum rules
- $pp \rightarrow W, Z, \gamma^*, H, WH, ZH$ (many including spin correl.)

**Next in line:** $e^+e^- \rightarrow 3$ jets?

- simplest!
- $\alpha_s$ & other measurements at LEP are theory limited
  
  **theory uncertainty** $\sim 3 - 4 \times \text{exp. error}$

- useful for studying perturbative/ non-perturbative interface.

Then DIS $\rightarrow 2 + 1$ and $pp \rightarrow 2$ jets…
NNLO bottleneck

$1 \to 3 \text{ @ NNLO} \sim \begin{array}{c}
1 \to 5 \text{ @ Tree} \\
1 \to 4 \text{ @ 1-loop} \\
1 \to 3 \text{ @ 2-loop}
\end{array} + \text{Tricks to cancel divergences}$
NNLO bottleneck

4 + 2\epsilon \text{ dim: } \int d\Phi_5 J(p_{1..5}) + \int d\Phi_4 \epsilon^{-2} J(p_{1..4}) + \int d\Phi_3 \epsilon^{-4} J(p_{1..3})

J is observable

1 \rightarrow 3 @ NNLO

1 \rightarrow 5 @ Tree

1 \rightarrow 4 @ 1-loop

1 \rightarrow 3 @ 2-loop

Tricks to cancel divergences
Developments in pQCD (G. Salam, LPTHE) p.19/29

NNLO

NNLO jets

4 + 2ε \text{ dim:} \quad J(\Phi^5) \quad J(\Phi^4) \quad J(\Phi^3)

$$\int d\Phi_5 \ J(p_{1..5}) \quad \int d\Phi_4 \ \varepsilon^{-2} J(p_{1..4}) \quad \int d\Phi_3 \ \varepsilon^{-4} J(p_{1..3})$$

Tricks to cancel divergences

Bottleneck
NNLO bottleneck

4 + 2\varepsilon \text{ dim:} \quad \int d\Phi_5 \, J(p_{1..5}) \quad \int d\Phi_4 \, \varepsilon^{-2} J(p_{1..4}) \quad \int d\Phi_3 \, \varepsilon^{-4} J(p_{1..3})

$1 \rightarrow 3$ @ NNLO \quad ~ \quad 1 \rightarrow 5$ @ Tree \quad 1 \rightarrow 4$ @ 1-loop \quad 1 \rightarrow 3$ @ 2-loop

"You have to do the integral, but you don’t know the integrand”

Anastasiou (KITP LoopFest III)

- **Subtraction:**
  - find an integrable function with same divergences as amplitudes
  - subtract it from real
  - add integrated version to virtuals.

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J is observable

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Developments in pQCD (G. Salam, LPTHE) p.19/29

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- **Tricks to cancel divergences**
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  - Binoth, Heinrich '00

Catani, Seymour '96 + earlier authors
Subtraction

- Applied to $C_F^3$ colour part of $e^+e^- \rightarrow 3$ jets
  $$(\alpha_s C_F / 2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4$$
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Cone-jets: misuse

(N)NLO is useless if

- Jet-algo is not IR safe
  - CDF has modified midpoint cone
  - New ‘search-cone step’ IR unsafe
    [discovered by Wobisch]

- Theory and experiment use different algorithms
  - $R_{sep}$ in NLO theory, but not data

- NB: ‘NNLO-NLL’ – rough approx. of NNLO, ignorant of jet-algo

Good news:

- CDF also has $k_t$-algo result
- Progress in making $k_t$-algo faster/friendlier

Cacciari [talk] & GPS '05–06
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CDF hep-ex/0512020

Midpoint

$Q = 6.1 \text{M}, (\mu = p_T^{\text{jet}}/2)$
$E_{\text{merge}} = 0.75, R_{\text{sep}} = 1.3$

Total systematic uncertainty
- Data corrected to parton level
- NLO pQCD

CDF Run II

$\int L = 385 \text{ pb}^{-1}$

$k_t$-algo

Cacciari [talk] & GPS '05–06
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Other results
- MC
- Resummation


Various unexpected structures in MVV results. E.g. at large $x$, can write

$$P_{ij}(x) = \frac{A}{(1-x)^+} + B \delta(1-x) + C \ln(1-x) + \mathcal{O}(1), \quad A = \sum A_n(\alpha_s/4\pi)^n, \text{ etc.}$$

Remarkably, different coefficients seem to be interrelated:

- $C_2 = A_1^2$
- $C_3 = 2A_1A_2$

There is a proposal that there is a more fundamental evolution equation with a universal splitting function

$$\partial_{\ln Q^2} D(x, Q^2) = \int_0^1 \frac{dz}{z} \mathcal{P}(x, \alpha_s(Q^2/z)) D(\frac{x}{z}, z^\sigma Q^2)$$

$$\sigma = 1: \text{ time-like} \quad \sigma = -1: \text{ space-like}$$

Postulate new universal splitting function $\mathcal{P}$ to be classical at large $x \Rightarrow C = A^2$ at all orders; get most of NNLL $\mathcal{O}(1)$ term too!
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Dokshitzer, Marchesini, GPS '05
Original aim of Dokshitzer was to understand difference between time-like ($\sigma = +1$) and space-like ($\sigma = -1$) splitting functions.

i.e. fragmentation function and splitting function evolution

Normally related at order $n$ via:

$$P_{\sigma = +1}(z) \iff P_{\sigma = -1}(1/z)$$

Curci, Furmanski, Petronzio '80
Stratmann & Vogelsang '97

New universality: get difference at order $n$ from result at order $n - 1$

$$P_{\sigma = \pm 1}(z) \implies P_{\sigma = +1}(z) - P_{\sigma = -1}(z)$$

For non-singlet NNLO: both approaches give same prediction for time-like case

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Mitov, Moch, Vogt '06
Many other “goodies” in the MVV papers (even more in supersymmetric limit)...

In $\mathcal{N} = 4$ SUSY Yang-Mills amplitudes, planar $n$-loop seems to be reducible just to powers of 1-loop:

$$M_{n-\text{leg}}^{(2-\text{loop})} = \frac{1}{2}(M_{n}^{(1)})^2 + f(\epsilon)M_{n}^{(1)}(2\epsilon) - \frac{\pi^4}{72} + \mathcal{O}((\epsilon))$$

4-legs: Anastasiou, Bern, Dixon, Koswoer '03

5-legs: Cachazo, Spradlin, Volovich '06; Bern et al '06

NB: numerical loop calcs: Anastasiou & Daleo [talk] '05 ; Czakon '05

$$M_{n-\text{leg}}^{(3-\text{loop})} = -\frac{1}{3}(M_{n}^{(1)})^3 + M_{n}^{(1)}(\epsilon)M_{n}^{(2)}(\epsilon) + f^{(3)}(\epsilon)M_{n}^{(1)}(3\epsilon) + C^{(3)} + \mathcal{O}(\epsilon)$$

4-legs: Bern, Dixon, Smirnov '05

In large-angle soft-colour resummation (‘fifth form factor’) for $2 \rightarrow 2$ scattering, symmetry in exch. of kinematic variables and # of colours:

$$\ln \frac{s^2}{ut} - 2\pi i \frac{\ln \frac{u}{t}}{\ln \frac{u}{t}} \iff NC$$

Dokshitzer & Marchesini '05 (see also Seymour '05)
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Fixed order calculations

- 4-loop decoupling relations for $\alpha_s$ (i.e. heavy-quark thresholds)
  
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- IR safety for jet flavour
  
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MC calculations

- Herwig++ → adolescence ($pp \rightarrow DY$), Ariadne++
  
  **Lönnblad’s talk**

- Steady progress in matching MC & NLO
  
  **MC@NLO: Frixione’s talk**

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- Using NNLL and NNLO for reweighting of event generators
  
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Analytical resummations:

- Collinear region (and threshold): $\text{MVV} \Rightarrow \alpha_s^n L^{n-2}$

- Generic large angle region, even $\alpha_s^n L^n$ much less well understood
  
  - Gaps-between-jets phenomenology
    
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Twistors / amplitude-recursion: major theory advance — starting to give very non-trivial results, especially for loops

Many string theorists now thinking about QCD
Some phenomenologists diverted into strings

Automated 1-loop calculations are important complementary development. More flexible; crucial for cross-checks

$e^+ e^- \rightarrow 3$ jets at NNLO is on final stretch

How much longer before DIS 2+1 and pp 2 $\rightarrow$ 2?

Once NNLO is available, comparison to data is not the only thing to be done with it. Learn about structure in QCD

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Thanks to: Bern, Butterworth, P. Ciafaloni, Comelli, Dokshitzer, R.K. Ellis, Kosower, Lönnblad, Marchesini, Moretti, Seymour, Vogt, Webber
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EXTRA SLIDES
CPU time in seconds for the computation of the $n$ gluon amplitude on a standard PC (2 GHz Pentium IV), summed over all helicities.

<table>
<thead>
<tr>
<th>$n$</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
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<tr>
<td>Berends-Giele</td>
<td>0.00005</td>
<td>0.00023</td>
<td>0.0009</td>
<td>0.003</td>
<td>0.011</td>
<td><strong>0.030</strong></td>
<td><strong>0.09</strong></td>
<td><strong>0.27</strong></td>
<td><strong>0.7</strong></td>
</tr>
<tr>
<td>CSW</td>
<td><strong>0.00001</strong></td>
<td>0.00040</td>
<td>0.0042</td>
<td>0.033</td>
<td>0.24</td>
<td>1.77</td>
<td>13</td>
<td>81</td>
<td>—</td>
</tr>
<tr>
<td>BCF</td>
<td><strong>0.00001</strong></td>
<td>0.00007</td>
<td>0.0003</td>
<td>0.001</td>
<td>0.006</td>
<td>0.037</td>
<td>0.19</td>
<td>0.97</td>
<td>5.5</td>
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Dinsdale, Ternick & Weinzierl ’06

Gain a factor of $\sim 4$ for moderate $n$ — useful, not overwhelming.

Slowly making it into phenomenological work

NB: trees in MadEvent, ALPGEN, HELAC/PHEGAS, CompHEP, GRACE, Amegic

But: real progress here is in discovery of new analytical structures in field theory (helpful also elsewhere, e.g. loops).
Supersymmetric decomposition (allow gluons, fermions and scalars in loops)

\[ \mathcal{A}_g = (\mathcal{A}_g + 4\mathcal{A}_f + 3\mathcal{A}_s) - \frac{4(\mathcal{A}_f + \mathcal{A}_s)}{\mathcal{N}} + \mathcal{A}_s \]

\( \mathcal{N} = 4 \) SUSY  \( \mathcal{N} = 1 \) chiral SUSY  scalar

SUSY gives many cancellations. Most difficult piece is \textit{scalar}.

Analytical structure involves coefficients \((c, d, e)\) of standard boxes \((l_4)\), triangles \((l_3)\) and bubbles \((l_2)\), and rational terms \((R)\):

\[ \mathcal{A}_s = \sum_i c_i l_4^i + \sum_i d_i l_3^i + \sum_i e_i l_2^i + R \]

- coefficients \((c, d, e)\) can be (i) read off by merging trees (cut constructibility) (ii) obtained recursively (à la BCFW)
- rational parts can be obtained recursively
Example of Giele-Glover method

\[ \int \frac{d^D \ell \ell^{\mu_1} \ell^{\mu_2}}{(\ell + q_1)^2(\ell + q_2)^2(\ell + q_3)^2(\ell + q_4)^2} = \frac{1}{2} g^{\mu_1 \mu_2} I(D + 2; 1, 1, 1, 1) + 2q_1^{\mu_1} 2q_1^{\mu_2} l_4(D + 4; 3, 1, 1, 1) + \ldots \]

Then

\[ 2l_4(8; 3, 1, 1, 1) = -2 \left( \sum_i S_{1i}^{-1} \right) l_4(8; 2, 1, 1, 1) - S_{11}^{-1} l_4(6; 1, 1, 1, 1) - S_{12}^{-1} l_3(6; 1, 0, 1, 1) - S_{13}^{-1} l_3(6; 1, 1, 0, 1) - S_{14}^{-1} l_3(6; 1, 1, 1, 0) \]

The \( l_n(D; 1, 1, 1, 1) \) etc. are the basis integrals. \( S_{ij} \) is kinematical matrix, \( S_{ij} = (q_i - q_j)^2 \).

Reduction procedure done numerically for each kinematic configuration.
Cancelling NNLO divergences

How to get cancellations?

1. **Subtraction** method:

\[
\int d^D \Phi_5 M_5 J(p_{1..5}) + \int d^D \Phi_4 e^{-2} J(p_{1..4}) + \int d^D \Phi_3 e^{-4} J(p_{1..3}) + \ldots
\]

Applied to \(e^+e^- \to 2\) jets and \(C_F^3\) colour part of \(e^+e^- \to 3\) jets:

\[
(\alpha_s C_F/2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4
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Gehrmann-De Ridder, Gehrmann & Glover '04

In principle all \(e^+e^- \to 3\) jet ‘antenna’ subtraction pieces are ready — ‘just’ need to be coded!

*idem.* '05
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1 \rightarrow 3 @ NNLO \sim 1 \rightarrow 5 @ Tree + 1 \rightarrow 4 @ 1\text{-loop} + 1 \rightarrow 3 @ 2\text{-loop}

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Applied to $e^+ e^- \rightarrow 2$ jets and $C_F^3$ colour part of $e^+ e^- \rightarrow 3$ jets:

\[ (\alpha_s C_F / 2\pi)^3 \text{ piece of } \langle 1 - T \rangle = -20.4 \pm 4 \]

Gehrmann-De Ridder, Gehrmann & Glover '04

In principle all $e^+ e^- \rightarrow 3$ jet ‘antenna’ subtraction pieces are ready — ‘just’ need to be coded!

idem. '05
Cancelling NNLO divergences

How to get cancellations?

2. Sector decomposition for isolating divergences

\[
\int d^D\Phi_5 M_5 J(p_{1..5}) = \varepsilon^{-4} \int d^4\Phi_5 f_{-4} M_5 J(p_{1..5}) + \cdots + \int d^4\Phi_5 f_0 M_5 J(p_{1..5})
\]

The \( f_{-i} \) involve plus-distributions of kinematic invariants. Each integral finite.

Applied to
- \( e^+ e^- \rightarrow 2 \) jets
- \( e^+ e^- \rightarrow 3 \) jets (partial)
- \( pp \rightarrow W, Z, H \) (fully exclusive)
Cancelling NNLO divergences

4 + 2ε \dim:
\int d\Phi_5 J(p_{1..5}) \quad \int d\Phi_4 \varepsilon^{-2} J(p_{1..4}) \quad \int d\Phi_3 \varepsilon^{-4} J(p_{1..3})

J is observable

1 \rightarrow 3 @ NNLO \sim \quad 1 \rightarrow 4 @ 1\text{-}loop \quad 1 \rightarrow 5 @ Tree \quad 1 \rightarrow 3 @ 2\text{-}loop

Tricks to cancel divergences

How to get cancellations?

2. Sector decomposition for isolating divergences \hspace{1cm} \text{Binoth & Heinrich '00}

\int d^D\Phi_5 M_5 J(p_{1..5}) = \varepsilon^{-4} \int d^4\Phi_5 f_{-4} M_5 J(p_{1..5}) + \cdots + \int d^4\Phi_5 f_0 M_5 J(p_{1..5})

The \( f_{-i} \) involve plus-distributions of kinematic invariants. Each integral finite.

Applied to

\begin{itemize}
\item \( e^+ e^- \rightarrow 2 \) jets \hspace{1cm} \text{Binoth & Heinrich '04; Anastasiou, Melnikov & Petriello '04}
\item \( e^+ e^- \rightarrow 3 \) jets (partial) \hspace{1cm} \text{Heinrich '06}
\item \( pp \rightarrow W, Z, H \) (fully exclusive) \hspace{1cm} \text{Anastasiou, Dixon, Melnikov & Petriello '04-06}
\end{itemize}
EW is not so weak

Widely discussed for ILC. How about $pp$?

e.g. NLO EW corrections to $pp \rightarrow Z + \text{jet}$

These are significant (even NNLO $\sim$ few %)

Maina Moretti Ross '04
Kulesza et al '04

QED effects $\lesssim 1\%$

Martin et al.
Glosser et al
Fortran

- Matching to multi-parton LO matrix elements now widespread (CKKW)
- New, better shower in Pythia ($k_\perp$ ordered)
- Underlying event models much improved / more practical
- Reaching end of line soon!

C++

<table>
<thead>
<tr>
<th>based on ThePEG</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Herwig++ ready for $e^+e^-$, $pp \rightarrow DY$ ready</td>
<td>Pythia 8 being coded Sherpa ready for $e^+e^-$ and $pp$</td>
</tr>
<tr>
<td>cancelled</td>
<td>see talk by Lönnblad</td>
</tr>
</tbody>
</table>

Includes new, improved angular-ordered shower

New player! Dresden group
## Resummation ingredients summary

<table>
<thead>
<tr>
<th>order</th>
<th>Soft + collinear hadr.</th>
<th>Hard Collinear hadr.</th>
<th>Soft large angle global NG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s^n L^{n+1}$</td>
<td>✓ [✓]+BSZ04</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\alpha_s^n L^n$</td>
<td>✓ [✓]+BSZ04</td>
<td>✓ ✓</td>
<td>✓ [✓] $[N_c \to \infty]$</td>
</tr>
<tr>
<td>$\alpha_s^n L^{n-1}$</td>
<td>MVV04 [FG04]</td>
<td>BCFG03 [FG04]</td>
<td>CGF+HK01 — —</td>
</tr>
<tr>
<td>$\alpha_s^n L^{n-2}$</td>
<td>— —</td>
<td>MVV05 —</td>
<td>MVV05 —</td>
</tr>
</tbody>
</table>

### Large angle global

2 → 2 BKOS89–98; generic: BCMN03; 2 → 3 [partial] KS05

### Large angle NG

hemisph./patch: DS01–02; $k_t$ algo: AS02, BD05

✓≡ historical results/techniques (< ’01)  [...] ≡ only for special cases

hadr. ≡ anything measuring hadrons  NG ≡ non-global

Best accuracies (NNLL) for most inclusive observables

Higgs transverse-mom. distribution.

*First differential NNLL resummation*

Resums \( L \sim \ln \frac{M_H}{Q_t} \) (for \( gg \rightarrow H \))

\[
\exp[\alpha_s^n L^{n+1} + \alpha_s^n L^n + \alpha_s^n L^{n-1}]
\]

Bozzi et al '03

- NNLL uncertainty \( \sim 7\% \)
  \( (\sim \text{NLL}/2) \)
- Shape quite different from plain parton showering (Pythia)
  — relevant for Higgs searches
  \( (gg \rightarrow H \rightarrow WW \rightarrow \ell\nu\ell\nu) \)?

Davatz et al '04
How rare are gaps in $pp \rightarrow 2$ jets with big $\Delta Y$?

Answer needs advanced tools

Non-global logarithms

- Appear for measurements of *part* of phase space
  
  Also e.g. dijet properties, Banfi & Dasgupta ’03

- Only in large-$N_c$ limit! Not automated!
  
  Connections to BFKL: Marchesini-Mueller ’03; Weigert ’03

Multi-jet structure

- Stony Brook soft-colour evolution

- Breakdown of ‘probabilistic radiation’

Are Monte Carlos up to the job? **Unknown...**