QCD and Monte Carlo Event Generators

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Work with Zoltán Nagy.

Also work with Michael Krämer, Stephen Mrenna

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Some references


A critique of pure perturbation theory

Consider $e^+ + e^- \rightarrow jets$.

- Calculation of the three jet fraction $f_3$ at NLO is fine.
- But consider distribution of masses $M$ of the three jets.
- Standard NLO calculation of $df_3/dM$ gives nonsense.
Adding showering gives a better answer

- Calculate $df_3/dM$ with combined NLO + PS + Had
- Compare to Pythia and to the pure NLO result. 

![Graph](image)
Status of NLO + showers

- Krämer, Mrenna and DES, NLO $e^+e^- \rightarrow 3$ jets with Pythia. (As on previous slide)

- Frixione, Nason and Webber, NLO hadron + hadron $\rightarrow$ heavy objects with Herwig, “MC@NLO.”

- Nagy and DES, theory on integrating NLO + showers with matching scheme for different numbers of jets.

We now turn to showers based on lowest order matrix elements.
Showers from the inside out

Think of shower branching as developing in a “time” that goes from most virtual to least virtual.

Thus shower time proceeds backward in physical time for initial state radiation.

This was an important idea in the creation of shower algorithms. (Gottschalk & Sjöstrand.)
Shower development in MC time

Real time picture.

Shower time picture.
What does soft radiation from a quark-antiquark-gluon system look like? To leading order in $1/N_c^2$, there are two $3\bar{3}$ dipoles that radiate independently.
Dipole radiation pattern

The wide angle dipole gives a wide angle pattern, with \( \frac{dE}{d\Omega} \propto N_c/2 \).

The narrow angle dipole gives a narrow angle pattern, with \( \frac{dE}{d\Omega} \propto N_c/2 \).
Angular ordering

At wide angles, soft gluon sees only color sum of sister partons.

Herwig solution is to let wide angle soft emission come first in MC time. (Webber, Marchesini, R.K. Ellis.)

Otherwise (as in this talk) the angular distribution of soft gluon emissions is determined from dipole angles.
Showers based on dipoles

- Rather than use angle as the shower ordering variable, one can base the emission probabilities on the dipole picture.

- Lönnblad (Ariadne) has been a pioneer in this.

- Skands and Sjöstrand have based a new version of Pythia on this.

- Nagy and I have found the Catani-Seymour dipole formalism good for this purpose.
Some notation for showers

For a generic description of shower MCs, use a notation adapted to \textit{classical} statistical mechanics.

- State with $m$ final state partons, $|\{p, f, c\}_m\rangle$.
  - Includes momentum fractions for two initial state partons.

- General state, $|A\rangle$.

- Probability for $|A\rangle$ to have specified quantum numbers, $(\{p, f, c\}_m|A\rangle)$.

- Completeness relation
  
  \[ 1 = \sum_m \int [d\{p, f, c\}_m] \{p, f, c\}_m \langle \{p, f, c\}_m | 1 = \sum_m \int [d\{p, f, c\}_m] \{p, f, c\}_m \langle \{p, f, c\}_m | 
  \]
Normalization to unit probability

\[
\sum_m \int \left[ \frac{d\{p, f, c\}_m}{m} \right] \left( \{p, f, c\}_m | A \right) = 1.
\]

That is

\[
\left( 1 | A \right) = 1,
\]

where \( 1 \) is defined by

\[
\left( \{p, f, c\}_m | 1 \right) = 1.
\]
Development in MC time

The evolving shower is represented by a state $|A(t)\rangle$ that begins with an initial state $|A(0)\rangle$. The evolution is given by a linear operator $U(t', t)$, with

$$|A(t)\rangle = U(t, 0)|A(0)\rangle .$$

- Group composition property:

$$U(t_3, t_2) U(t_2, t_1) = U(t_3, t_1) .$$

- Evolution preserves probabilities:

$$\left(1|U(t', t)|A\right) = \left(1|A\right) .$$
Evolution equation

\[ U(t_3, t_1) = N(t_3, t_1) + \int_{t_1}^{t_3} dt_2 \, U(t_3, t_2) \, \mathcal{H}(t_2) \, N(t_2, t_1) . \]

- \( \mathcal{H}(t) \) is *splitting* operator, \( m \to m + 1 \) for simple shower.

- \( N(t', t) \) is *no splitting* operator, diagonal in the \( \{p, f, c\}_m \) basis

\[ N(t', t)\{p, f, c\}_m = \Delta(\{p, f, c\}_m; t', t)\{p, f, c\}_m . \]
Probability conservation

Use

\[ (1|U(t', t)|A) = (1|A) \]

to derive

\[ \Delta(\{p, f, c\}_m; t_2, t_1) = \exp \left( - \int_{t_1}^{t_2} dt \, (1|\mathcal{H}(t)|\{p, f, c\}_m) \right) \]

• \( \Delta(\{p, f, c\}_m; t_2, t_1) \) is **Sudakov exponential**.
Review

\[ U(t_3, t_1) = N(t_3, t_1) + \int_{t_1}^{t_3} dt_2 \ U(t_3, t_2) \mathcal{H}(t_2) \ N(t_2, t_1) . \]

- Red dots are splitting operators \( \mathcal{H} \).
- Yellow ovals are Sudakov exponentials.
Shower cross section

Start with $2 \rightarrow 2$ cross section and iterate the evolution equation, say, twice:
Deficiency of the standard shower

- The shower approximation relies on small $P_T$ splitting.
- Maybe the exact matrix element would be better.
- But that lacks the Sudakov exponentials.
An improved version

\[ \times \left( \frac{\text{image of green and yellow shapes}}{\text{image of green shape}} \right) \equiv \left( \text{image of green shape} \right) \]

- This is the essential idea of Catani, Krauss, Kuhn, and Webber.

- CKKW use the $k_T$ jet algorithm to define the ratio.
Two step calculation

- CKKW break evolution into $0 < t < t_{\text{ini}}$ and $t_{\text{ini}} < t < t_f$. 
CKKW use improved weighting for $0 < t < t_{ini}$

- $N$th term has $N + 1$ jets at scale $t_{ini}$.
- Last term has $> 4$ jets at scale $t_{ini}$, so is pretty small.
**CKKW & partial cross sections**

- Consider cross section for an observable $F$, $\sigma[F]$.

- CKKW break this into a sum of contributions $\sigma_m[F]$ from final states with $m$ jets at resolution scale $t_{ini}$.

- $\sigma_m[F]$ is evaluated with parton splitting and Sudakov factors.

- If $F$ is an infrared safe observable, CKKW method gets $\sigma_m[F]$ correct to leading perturbative order, $\alpha_s^m$.

- Nagy and I showed how to get $\sigma_m[F]$ correct to next-to-leading perturbative order, $\alpha_s^{m+1}$, for an infrared safe observable $F$. 
Return to lowest order showers

Recall the approach based on $t_{ini}$. 

\[
\begin{align*}
&+ t_{ini} \\
&+ t_{ini} \\
&+ t_{ini} \\
&+ \\
&+ t_{ini}
\end{align*}
\]
Why not get rid of $t_{\text{ini}}$?

Nagy and I suggest to modify the CKKW idea a little bit:

- $N$th term has $N + 1$ jets at scale $t_f \gg 1$.
- Thus the last term is the main one.
Future directions

Leading order

• Perhaps the Catani-Seymour dipole formalism will help.

• It would be nice to get beyond the leading $1/N_c^2$ approximation.

• Perhaps Sudakov reweighting of $|M|^2$ can be improved.

• We might do Sudakov reweighted $|M|^2$ up to a fixed number of partons instead of a fixed $k_T$ scale.

Beyond leading order

• All of the above should be integrated with an NLO approach.

• We would be better off with an NLO version of $\mathcal{H}$. 