Equation of state and magnetic monopoles in SU(2) gluon plasma

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Outline

- Thermodynamics of Yang-Mills theory
- Models of color confinement at $T < Tc$

  - Abelian monopoles
- In deconfinement (gluon plasma at $T > Tc$)

  - Are they (still) alive as real object?
- Contribution to (trace of) energy-momentum tensor from Abelian monopoles

Thermodynamics

- Free Energy (T is temperature and V is *spatial* volume)

$$F = -T \log Z(T,V)$$

- Pressure

$$p = \frac{T}{V}\frac{\partial \log Z(T,V)}{\partial \log V} = -\frac{F}{V} = \frac{T}{V}\log Z(T,V)$$

- Energy density

$$\varepsilon = \frac{T}{V}\frac{\partial \log Z(T,V)}{\partial \log T}$$

- Entropy density

$$s(T) = \frac{\varepsilon + p}{T} = \frac{\partial p(T)}{\partial T}$$

Thermodynamics: Trace Anomaly

- Trace anomaly of the energy-momentum tensor $T_{\mu\nu}$

$$\theta(T) = \langle T^{\mu}_{\mu} \rangle \equiv \varepsilon - 3p = T^5 \frac{\partial}{\partial T} \frac{p(T)}{T^4}$$

- Trace anomaly is vanishing,
  - if excitations are massive, $M \gg T \quad \varepsilon \sim p \sim \exp\{-M/T\}$
  - if particles are massless and non-interacting, $\varepsilon = 3p$
- Pressure via trace anomaly

$$p(T) = T^4 \int^{T} \frac{dT_1}{T_1} \frac{\theta(T_1)}{T_1^4}$$

- Energy density via trace anomaly

$$\varepsilon(T) = 3\,T^4 \int^{T} \frac{dT_1}{T_1} \frac{\theta(T_1)}{T_1^4} + \theta(T)$$

- Trace anomaly is a key quantity

Trace Anomaly for SU(2) pure gluons

- Partition Function

$$Z(T,V) = \int DU \; \mathrm{exp}\{-\beta \sum_P S_P[U]\},\; S_P[U] = (1 - \frac{1}{2} TrU_P)$$

- Trace Anomaly

$$\theta(T) = T^5 \frac{\partial}{\partial T} \frac{\log Z(T,V)}{T^3 V}$$

- Asymmetric $N_s^3 N_t$ lattice:

$$T = 1/(N_t a)\,, \quad V = (N_s a)^3$$

- Trace anomaly on the lattice

$$\frac{\theta(T)}{T^4} = 6\, N_t^4 \left(\frac{\partial \beta(a)}{\partial \log a} \right) \cdot (\langle S_P \rangle_T - \langle S_P \rangle_0)$$

Mechanisms of color confinement

- Dual superconductor picture

  ['t Hooft, Mandelstam, Nambu, '74-'76]

  - Based on existence of special gluonic configurations, called ``magnetic monopoles''
  - Monopoles are classified with respect to the Cartan subgroup $[U(1)]^{N-1}$ of the $SU(N)$ gauge group
  - Confinement is due to monopole condensation
  - Monopole dominance for various quantities
    - String tension
    - Polyakov loop behaviors
    - Critical exponents

Confinement (T<Tc) and plasma (T>Tc)

- The monopoles are percolating and condensed in confining vacuum
- The percolating monopole cluster disappears and monopole condensate vanishes in deconfinement phase
- Must emerge as a real (thermal) component of deconfinement plasma

  [V.I.Zakharov, M.N.Chernodub,'07] *[Liao and Shuryak, '06-'07]*

- Similar to electrically neutral electron-positron plasma:
  - individual particles exist at high temperatures in a heat bath
  - annihilate at low temperatures, but still present in the vacuum
- Check: if the suggestion is true, then the monopoles must contribute to the equation of state of the gluon plasma

Trace Anomaly from monopoles

- Gauge fixing (MA gauge): maximize

$$R = \sum_{s,\mu} Tr[\sigma_3 U_\mu(s) \sigma_3 U_\mu^\dagger(s)]$$

[A.S.Kronfeld, M.L.Laursen, G.Schierholz, U.J.Wiese '87]

- Define particular singular gluon objects (monopoles) *[T.A.DeGrand, D.Toussaint '80]*

$$k_\mu(s) = \epsilon_{\mu\nu\rho\sigma} \partial_\nu n_{\rho\sigma}(s + \hat{\mu})/2$$

- Extract the plaquettes around the monopole
- Decompose the trace anomaly into two parts
  - The contribution around the monopole and the rest

[for center vortex ; M.N.Chernodub, A.Nakamura, V.I.Zakharov '08]

Trace Anomaly from monopoles

- Action density:

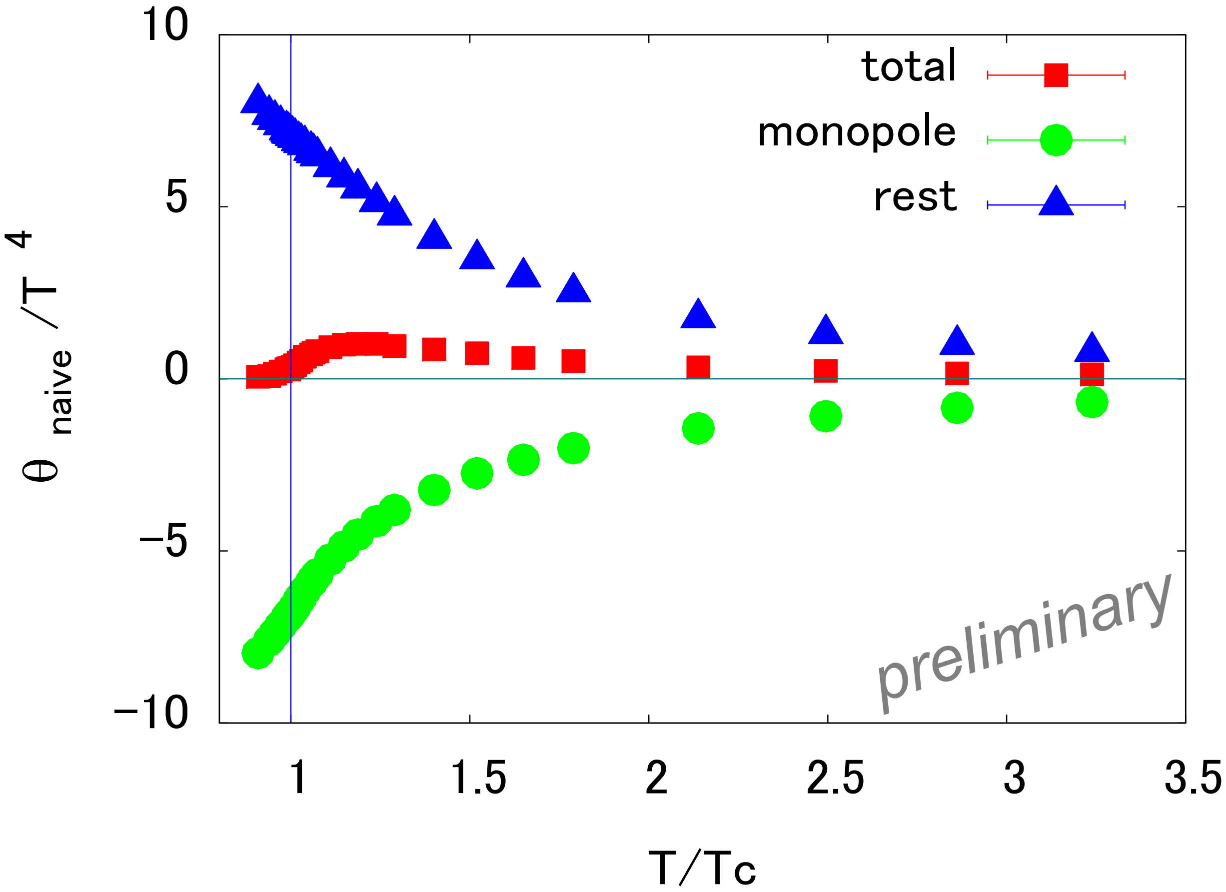
$$\langle S_P \rangle = \langle S_P \rangle^{\text{mon}} + \langle S_P \rangle^{\text{rest}}$$
$$= \frac{1}{6N_s^3 N_t}\left[\langle \sum_P \rho_P S_P \rangle + \langle \sum_P (1-\rho_P) S_P \rangle\right]$$
$$\rho_P = 1 \ (P \in \Sigma) \text{ or } 0 \ (P \notin \Sigma)$$
$$\Sigma : \text{plaquettes around monopoles}$$

- Trace anomaly (naive regularization):

$$\frac{\theta^{\text{mon}}_{\text{naive}}}{T^4} = 6N_t^4 \left(\frac{\partial \beta}{\partial \log a}\right)\left[\langle S_P \rangle_T^{\text{mon}} - \rho(T)\langle S_P \rangle_0\right]$$
$$\frac{\theta^{\text{rest}}_{\text{naive}}}{T^4} = 6N_t^4 \left(\frac{\partial \beta}{\partial \log a}\right)\left[\langle S_P \rangle_T^{\text{rest}} - (1-\rho(T))\langle S_P \rangle_0\right]$$

Trace Anomaly from monopoles

- Trace anomaly (naive regularization):



$T = 0$:
16^4 lattice,
1000 conf.

$T > 0$:
$16^3 \times 4$ lattice,
5000 conf.

Calculated by
RICC at RIKEN

Not sensitive to the phase transition
⇒ regularization not appropriate

Trace Anomaly from monopoles

- Specific action density:

$$\langle s_P \rangle^{\text{mon}} = \frac{\langle S_P \rangle^{\text{mon}}}{\rho} = \frac{\langle \sum_P \rho_P S_P \rangle^{\text{mon}}}{\langle \sum_P \rho_P \rangle}$$

$$\langle s_P \rangle^{\text{rest}} = \frac{\langle S_P \rangle^{\text{rest}}}{1-\rho} = \frac{\langle \sum_P (1-\rho_P) S_P \rangle^{\text{rest}}}{\langle \sum_P (1-\rho_P) \rangle}$$

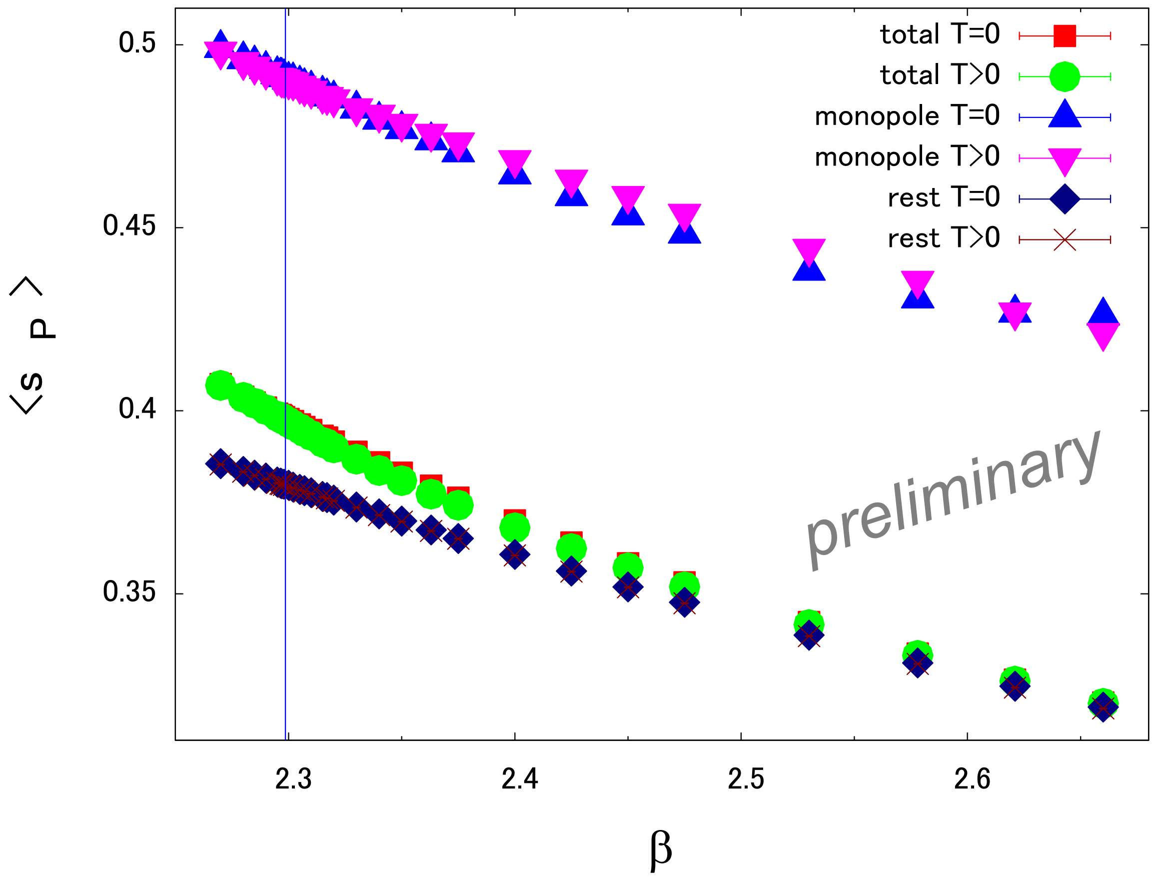
(action density per an elementary plaquette)

- Regularized trace anomaly :

$$\frac{\theta^{\text{mon}}_{\text{reg}}}{T^4} = 6 N_t^4 \left(\frac{\partial \beta}{\partial \log a} \right) \rho(T) \left[\langle s_P \rangle^{\text{mon}}_T - \langle s_P \rangle^{\text{mon}}_0 \right]$$

Trace Anomaly from monopoles

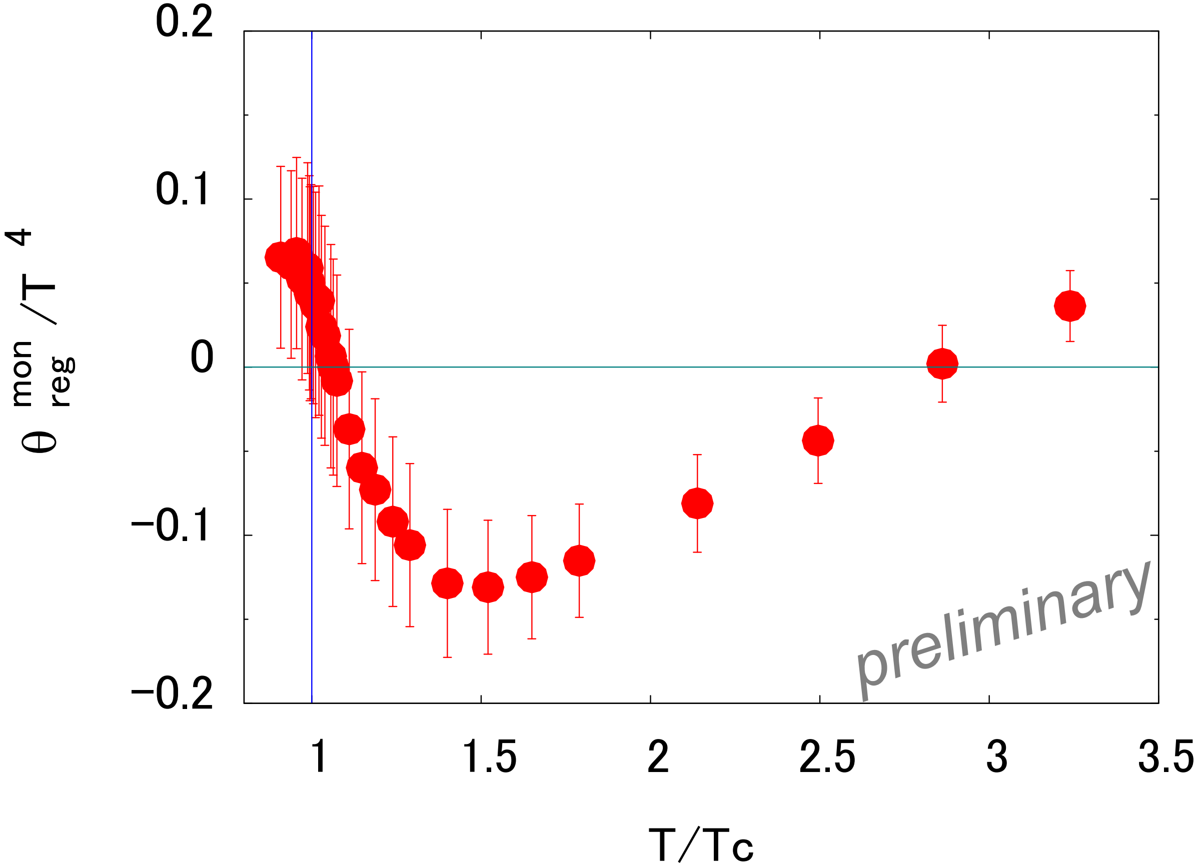
- Specific action density :



The difference between $\langle s_P \rangle^{\mathrm{mon}}$ and $\langle s_P \rangle^{\mathrm{rest}}$ is seen clearly.

Trace Anomaly from monopoles

- Regularized trace anomaly :



•Sensitive to the phase transition.

•The behavior is similar to the case of center vortex. [M.N.Chernodub et.al. '08]

Conclusion and future works

- Conclusion
  - Found: strong contributions from the plaquettes around Abelian monopoles to the trace anomaly, and, consequently, to the pressure and to the energy density of the gluon plasma.
  - Gluonic configurations around the Abelian monopoles are similar to the worldsheets of the center vortex.
- Future works
  - Check of scaling for trace anomaly (wrapped monopole ($T>0$) and the largest monopole cluster ($T=0$))
  - What is the correct regularization scheme?