Phase structure of SU(2) gauge theory with adjoint Wilson fermions

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Motivation

Study of phase structure of SU(N) gauge theories

- Search for conformal window: possible alternative to Standard Model Higgs sector
- Fundamental/adjoint (or higher) representations
- At zero and finite temperature

- SU(2) theories:
  - Conformal behavior is expected with less #flavor
  - Nf=2 adjoint fermions: "Minimal Walking technicolor"

Del Debbio et al. (2008), Bursa et al. (2010)
Hietanen, Rummukainen, Tuominen (2009)
Our approach

Use of overlap fermion
- Exact chiral symmetry
- Epsilon regime to explore chiral symmetry breaking

• For locality of overlap operator,
  - Wilson-Dirac kernel must have gap (mobility edge)
    ⇔ Out of Aoki phase (Golterman and Shamir, 2003)
  — Motivation of this work

Present work:
• SU(2) gauge theories with Nf=2 fundamental and adjoint Wilson fermions (+ twisted mass ghost)
  - Investigation of Aoki phase
  - Preparation to overlap simulations
  - Exercise to probe conformal behaviors
Aoki phase

Flavor-parity broken phase of Wilson-Dirac operator
- Proposed by Aoki, 1984
- Numerical evidence
- Chiral Lagrangian analysis
  (Sharpe and Singleton, 1998)

- As the kernel of overlap operator, *to be in between fingers*

  - Eigenmodes of $H_W$ is local below "mobility edge"
  - Aoki phase is characterized by vanishing mobility edge

To be here
Aoki phase

Results in QCD: Around 1st 'finger',

• 1st order phase transition at high $\beta$
  e.g. Ilgenfritz et al. (2004); Farchioni et al. (2005)

• 1st order transition is also observed at strong coupling
  e.g., JLQCD Collaboration (2005); Nagai et al. (2009)
  – Dynamics may differ from high $\beta$ region

In present work,

• Wide range of bare quark mass of Wilson-Dirac operator is explored

• Not only 1st 'finger', 2nd 'finger' is also investigated
  – Number of light d.o.f. different from 1st finger
**Lattice setup**

**SU(2) Iwasaki gauge + Nf=2 adjoint Wilson fermions (+ twisted mass ghost)**

- Fermions introduced as topology fixing term
  

\[
\det \left( \frac{H_W^2}{H_W^2 + \mu^2} \right) = \int \mathcal{D}\chi^\dagger \mathcal{D}\chi \exp[-S_E]
\]

- Twisted mass ghost cancels high frequency effect:
  — not expected to change low energy dynamics

**Present stage:**

- Lattice: \(8^3 \times 16, \beta = 0.80, 0.90, 1.0, 1.2\) (mainly at 0.90)
- **All results are very preliminary**
- Similar behavior is observed for fundamental fermions
Analysis procedure

Observed quantities:

- **Meson correlators**
  - PS and V meson masses, PCAC quark mass
  - Propagators with twisted mass \( m_1 \rightarrow \text{charged pion mass} \)
  - Linearly extrapolated to \( m_1 = 0 \) with smallest 3 points
  - *Vanishing charged pion mass = Aoki phase*

- **Static quark potential**
  - Fundamental static quark

- **Spectrum of Wilson-Dirac operator (in progress)**
  - Locality

- **Spectrum of overlap-Dirac operator (in progress)**
  - Chiral condensate
  - Comparison with Random Matrix Theory
\( \beta = 0.90 \), around 1st finger

PS meson mass vs \( M_{0V} \) (valence) around 1st 'finger'
- Partially quenched data: sign of slope suddenly changes
- Valence=sea data shows cusp-like structure
- Consistent with 1st order phase transition
\( \beta = 0.90: \) PS meson mass vs PCAC mass

- \( M_0 \leq 1.51: \) Positive PCAC mass
  - Partially quenched data shows \( m_{PS}^2 \propto m_q \)
  - Not enough light
- \( M_0 \geq 1.52: \) Negative PCAC mass
  - \( M_0 = 1.52 \) corresponds to our lightest case
$\beta=0.90$: Vector meson mass

- $M_0$ below transition: QCD-like behavior
- $M_0$ above transition: $m_V/m_{PS}$ seems to be const.
  - Consistent with signature of near-conformal
$\beta=0.90$: Static potential

- Static potential in fundamental repr.
  - $M_0 \leq 1.51$: QCD-like confining potential
    
    Cf: at $M_0=1.40$, $a(r_0)\sim 0.2\text{fm}$  \[r_0=0.5\text{fm}: \text{just a guide}\]
  - At $M_0=1.52$, string tension is consistent with zero
    Consistent with conformal phase
  - At $M_0 \geq 1.55$, tiny string tension
$\beta=0.90$: Plaquette

- **2-state signal**
  - Update for $M_0=1.52$ with hot($M_0=1.51$) and cold initial configs. exhibit different plaquette values
  - **Supports 1st order phase transition**
Discussion

Result at $\beta = 0.90$ around 1st finger:

- **1st order phase transition** (No Aoki phase)
- At $M_0 = 1.52$ ($m_q < 0$, smallest $|m_q|$) near-conformal behavior
  - V/PS meson mass ratio
  - Static potential
  - Increasing $M_0$ would wash out conformal behavior
- In positive $m_q$ region, quark mass is not enough light

*Conjecture*: While light quark mass region is near-conformal, difficult to observe due to 1st order phase transition.

To confirm this scenario,

- At larger $\beta$, small $m_q$ should be explored: conformal-like behavior should be observed
\[ \beta = 0.90: \text{ around 2nd finger} \]

- Light d.o.f is 8 instead of 2 around 1st finger
- PS meson mass vs \( M_{0V} \)
  - Consistent with 2nd order phase transition?
  - Existence of Aoki phase?
- Static potential exhibits no string tension \( 2.3 \leq M_0 \leq 2.6 \)

*More detailed study is in progress*
\( \beta = 1.0 \) and 1.2: around 1st finger

- At \( \beta = 0.80 \), similar result at \( \beta = 0.90 \) is observed
- At \( \beta = 1.0 \): 1\(^{st}\) order transition still remains, but weak
- At \( \beta = 1.2 \): 1\(^{st}\) order transition seems to disappear

More detailed study is in progress
Conclusion and outlook

We are exploring phase structure of SU(2) gauge theories with Nf=2 adjoint Wilson fermions

- Structure around 1st and 2nd fingers
- 1st order transition around 1st finger at $\beta \leq 1.0$
- Conformal-like behavior is observed for small PCAC mass region around 1st finger

Works in progress:

- Extension to larger lattice sizes and other $\beta$ values
- Spectrum of Wilson and overlap Dirac operator
- Fundamental fermions
- Dynamical overlap fermions

Outlook

- Finite temperature