A method to calculate meson spectral functions with a variational method in lattice QCD

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Abstract

We propose a new method to calculate meson spectral functions (SPFs) on the lattice using a variational method. First, we confirm that our method can extract signals for several low-lying states in the free quark case. Then we calculate SPFs for S and P-wave charmonia in quenched QCD at zero temperature and compare the results with those obtained by the conventional maximum entropy method (MEM). We find that our results for the location and the height for the ground states are consistent with the location and the area of the first peak of SPF by MEM. Moreover the signals corresponding to the first excited states can be improved by increasing the number of basis functions. Finally we investigate the temperature dependence of SPFs for S-wave charmonia. There is no clear evidence of dissociation of J/Ψ and η_c up to $1.4T_c$.

1. Introduction

- **Meson spectral functions (SPFs)** at finite temperature
  - important to investigate the behavior of mesons in medium
  - e.g. charmonium SPFs → studied to understand the J/Ψ suppression

[T. Matsui and H. Satz (1986)]

- one of the important signals of QGP formation in heavy ion collision experiments (RHIC, LHC)
- Ψ' → J/Ψ 10%, χ_c → J/Ψ 30% → **the sequential J/Ψ suppression**

[S. Digal, P. Petreczky and H. Satz (2001)]

- **Dissociation of excited and P-wave charmonia is also important.**

- Current lattice QCD studies: [e.g. A. Jackovac et al (2007)]
  - calculate the charmonium SPFs with the maximum entropy method (MEM)

[M. Asakawa, T. Htsuda and Y. Nakahara (2001)]

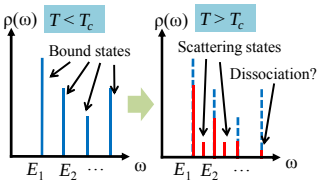
- S-wave charmonia (J/Ψ, η_c) : **survive up to $1.5T_c$?**
- P-wave charmonia (χ_c) : **dissolve just above T_c**
- **excited charmonia has NOT been investigated well yet**

- MEM
  - needs a proper default model which shares as many properties as possible with SPFs
    → **There is ambiguity due to the choice of default model**
  - provides continuous SPFs
    → **However, on a finite volume lattice, there are discrete spectra only**

It is important to check the conclusions drawn from MEM by other methods

- **instead of reproducing the continuous form of SPFs**
- **directly extract such discrete signals**

- Our approach



- **Investigate temperature dependence of SPFs**
  - not whole shape of $\rho(\omega)$ but just $\rho(E_i)$ is needed
  - find modification of $\rho(E_i)$ corresponding to the dissociation
- **Variational method**
  - can extract the properties of some low-lying states.
  - is well-suited for discrete spectra.

2. Meson SPFs with the variational method

- Smeared meson operator

$$\mathcal{O}_\Gamma(\vec{x},t)_i \equiv \sum_{\vec{y},\vec{z}} \omega_i(\vec{y})\omega_i(\vec{z})\bar{q}(\vec{x}+\vec{y},t)\Gamma q(\vec{x}+\vec{z},t)$$

- Gaussian smearing function

$$\omega_i(\vec{x}) \equiv e^{-A_i||\vec{x}||^2} \quad i=1,2,\cdots,n$$

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
point operator →	∞	0.25	0.20	0.15	0.10	0.05	0.02

- Meson correlator matrix

$$\mathbf{C}_\Gamma(t) \equiv \left[C_\Gamma(t)_{ij} = \sum_{\vec{x}} \langle \mathcal{O}_\Gamma(\vec{x},t)_i \, \mathcal{O}^\dagger_\Gamma(\vec{0},0)_j \rangle \right] \quad i,j=1,2,\cdots,n$$

- Generalized eigenvalue problem

$$\mathbf{C}_\Gamma(t)\,\mathbf{v}^{(k)} = \lambda_k(t;t_0)\,\mathbf{C}_\Gamma(t_0)\,\mathbf{v}^{(k)} \quad k=1,2,\cdots,n$$

- **Effective mass**

$$\lambda_k(t;t0) = \frac{\cosh[m_k(t;t_0)(t-N_t/2)]}{\cosh[m_k(t;t_0)(t_0-N_t/2)]}$$

$$\Lambda(t;t_0) = \mathrm{diag}\{\lambda_1(t;t_0),\lambda_2(t;t_0),\cdots,\lambda_n(t;t_0)\}$$
$$\mathbf{V} = [\mathbf{v}^{(1)}\cdots\mathbf{v}^{(n)}]$$
$$\mathbf{C}_\Gamma(t) = \mathbf{C}_\Gamma(t_0)\mathbf{V}\Lambda\mathbf{V}^{-1}$$

- point-point component

$$C_\Gamma(t)_{11} = \sum_k \rho_\Gamma(m_k(t;t_0)) \frac{\cosh[m_k(t;t_0)(t-N_t/2)]}{\sinh[m_k(t;t_0)N_t/2]}$$

- **Meson SPF**

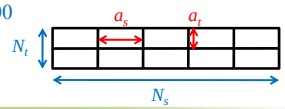
$$\rho_\Gamma(m_k(t;t_0)) = (\mathbf{C}_\Gamma(t_0)\mathbf{V})_{1k}\,(\mathbf{V}^{-1})_{k1}\,\frac{\sinh[m_k(t;t_0)\,N_t/2]}{\cosh[m_k(t;t_0)\,(t_0-N_t/2)]}$$

3. Lattice setup

- Action
  - Standard plaquette gauge action
  - $O(a)$-improved Wilson fermion action
  - Quenched approximation
- Lattice
  - Anisotropic lattice : anisotropy $a_s/a_t = 4$
  - $a_s = 0.0970(5)$ fm ($a^{-1}_s = 2.030(13)$ GeV)
  - $N_s = 20$
  - $N_t = 160$ (zero temperature), 32 ($0.88T_c$), 26 ($1.1T_c$), 20 ($1.4T_c$)

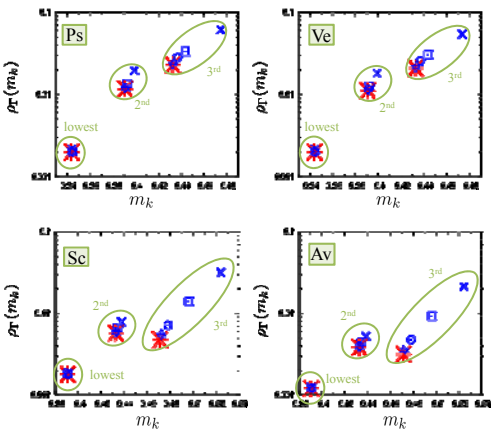
T is varied by changing N_t (fixed-scale approach)

- Number of gauge configurations
  - for zero temperature : 299
  - for finite temperature : 800
- Gauge fixing
  - Coulomb gauge



4. Numerical results

- Test in free quark case **$20^3 \times 128$ lattice**

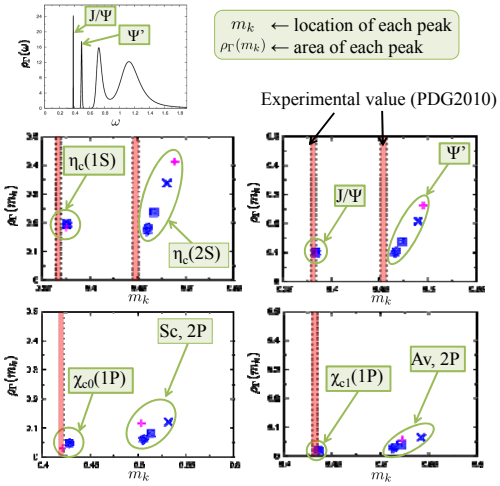


Analytic solution for Wilson quarks: ✳
Variational method: × $n=3$ □ $n=4$ ○ $n=5$ △ $n=6$ ▽ $n=7$

Lowest state → well consistent with analytic solution for all n
2nd, 3rd lowest state → **improved as n increases**

- At $T=0$

Comparison with MEM

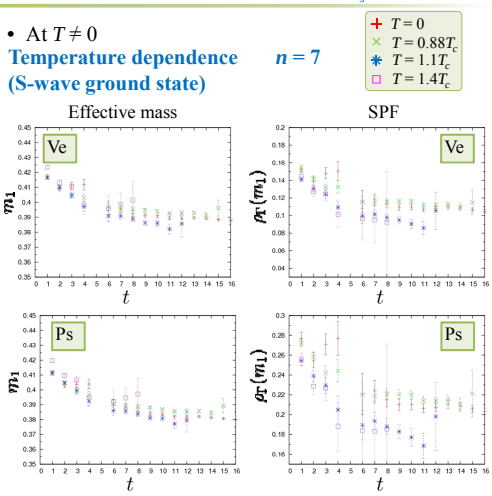


MEM: +
Variational method: × $n=3$ □ $n=4$ ○ $n=5$ △ $n=6$ ▽ $n=7$

Ground state → all data almost consistent with experimental value
1st excited state → there is difference between variational method data and MEM one
→ variational method data get closer to experimental value as n increases
Variational method can improve data accuracy for excited states.

- At $T \neq 0$

Temperature dependence (S-wave ground state) **$n=7$**



No clear temperature dependence for the effective masse.
The value of SPF may change but the modification is quite small.
There is no clear evidence of dissociation up to $1.4T_c$

5. Conclusions

- Meson SPFs are calculated with the variational method.
- At zero temperature,
  - ground state → well extracted
  - excited state → improved by increasing the number of basis op.
- At finite temperature,
  - S-wave ground state charmonia (J/Ψ, η_c)
  - up to $1.4T_c$
  - no clear temperature dependence for the effective masses
  - value of SPF may change but the modification is quite small
  - **no clear evidence of dissociation**