Semiclassics near the QCD phase transition

Falk Bruckmann
(Univ. Regensburg)

Japanese-German Seminar, Mishima, Nov. 2010

SFB/
TR 55

Motivation

towards understanding the crossover

- semiclassical approach to path integral at $T = 0$
  - instantons + fluctuations (one-loop determinant) + quark zero modes (index theorem)

✓ chiral condensate

✓ axial anomaly

? confinement

- at $T > 0$
  - different instanton solutions: calorons

    new mechanisms

(✓) confinement and deconfinement

✓ chiral condensate at diff. bc.s

lattice for semianalytic studies or cross-check [all results are quenched]

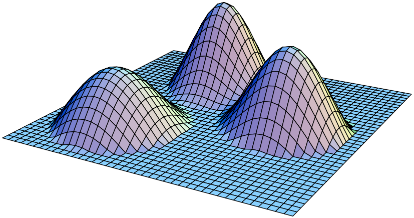
Calorons

calorons $\equiv$ selfdual solutions of Yang-Mills eqn.s at finite temperature

i.e. instantons over $S^1_{\beta=1/kT} \times R^3$ — Harrington, Shepard 78

general solutions [ADHM-Nahm formalism]: — Kraan, van Baal; Lee, Lu 98

FB, Nogradi, van Baal 02, 04



topological (action) density for total charge $Q = 1$ in $SU(3)$

substructure: N_c constituents (instead of 1 lump) when well separated

- magnetic monopoles
- selfduality $\vec{E} = \vec{B}$: dyons — cf. Bornyakov, Schierholz 96

confirmed by cooling lattice config.s at finite T — Ilgenfritz et al. 02, FB et al. 04

Dimensional reduction

monopoles/dyons are 3D objects

temporal extension β 'small': $\beta \ll$ instanton size $\sim$ dyon distance

- well separated dyons[1]: static
- near dyons: instanton-like

dyon locations: free parameters (moduli)

size: fixed to $O(\beta)$

$A_0 \Rightarrow$ scalar field in the adjoint representation: Higgs?

[1] replace large instantons, for which the size distribution is not under control

Higgs effect

asymptotic Polyakov loop:

$$\mathcal{P}_\infty \equiv \lim_{|\vec{x}|\to\infty} \mathcal{P}(\vec{x}) \ldots \text{holonomy}$$

direction independent [finite action], 'environment' for solutions

eigenvalues of $\mathcal{P}$: superselection parameters

aspects of a Higgs field:

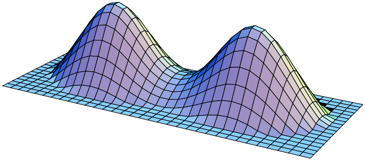
- generic symm. breaking: $SU(N_c) \to U(1)^{N_c-1}$
- at monopole core residual symmetry enlarged, 'false vacuum':
  local Polyakov loop has degenerate eigenvalues
  $SU(2): \ \mathcal{P}(\text{dyon}_1) = 1_2 \ [A_0 = 0], \ \mathcal{P}(\text{dyon}_2) = -1_2$
- abelian direction in $\mathcal{P}_\infty$:
  $A_\mu^{||}$ decay algebraically: 'photons'
  $A_\mu^{\perp}$ decay exponentially: 'W-bosons'

- vev ⇒ masses:

  fractional dyon masses (integrated action, adding up to 1) are given by eigenvalue differences of $\mathcal{P}_\infty$

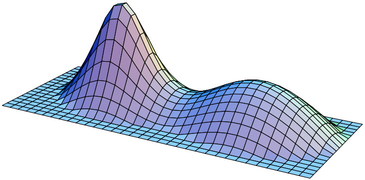
  in $SU(2)$ with 2 dyons:

$$P_\infty = \exp\begin{pmatrix} 2\pi i(-\frac{1}{4}) & \\ & 2\pi i(\frac{1}{4}) \end{pmatrix} \qquad P_\infty = \exp\begin{pmatrix} 2\pi i(-\epsilon) & \\ & 2\pi i(\epsilon) \end{pmatrix}$$



equal mass constituent dyons

$\mathrm{tr}\, P_\infty = 0$



heavy + light constituent dyon

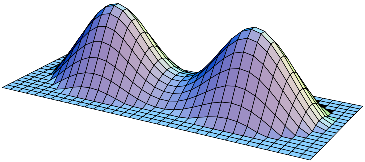
$\mathrm{tr}\, P_\infty \simeq 1$

- vev $\Rightarrow$ masses:

  fractional dyon masses (integrated action, adding up to 1) are given by eigenvalue differences of $\mathcal{P}_\infty$

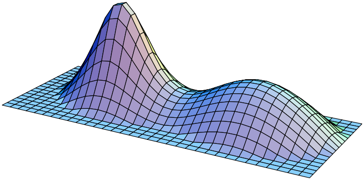
  in $SU(2)$ with 2 dyons:

$$P_\infty = \exp\begin{pmatrix} 2\pi i(-\frac{1}{4}) & \\ & 2\pi i(\frac{1}{4}) \end{pmatrix} \qquad P_\infty = \exp\begin{pmatrix} 2\pi i(-\epsilon) & \\ & 2\pi i(\epsilon) \end{pmatrix}$$



equal mass constituent dyons

$\mathrm{tr}\, P_\infty = 0$ as in conf. phase



heavy + light constituent dyon

$\mathrm{tr}\, P_\infty \simeq 1$ as in deconf. phase

conjecture: holonomy $\mathrm{tr}\, P_\infty \leftrightarrows$ order parameter $\langle\, \mathrm{tr}\, P \,\rangle$

$\Rightarrow$ dyons sensitive to the phase of QCD

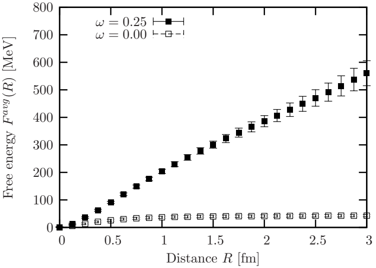
Caloron superpositions & Confinement

superpose caloron gauge fields [not simple]

measure Polyakov loop correlators:

$$-\frac{1}{\beta}\log\langle \mathrm{tr}\mathcal{P}(\vec{x})\mathrm{tr}\mathcal{P}(\vec{y})\rangle \overset{?}{\sim} \sigma|\vec{x}-\vec{y}|$$

- confinement from ensembles of calorons



Gerhold, Ilgenfritz, Mueller-Preussker 06

SU(2)

trivial holonomy $\frac{1}{2}\,\mathrm{tr}\,\mathcal{P}_\infty = 1$ ('$\omega = 0$') vs.

max. nontrivial hol. $\mathrm{tr}\,\mathcal{P}_\infty = 0$ ('$\omega = 1/4$')

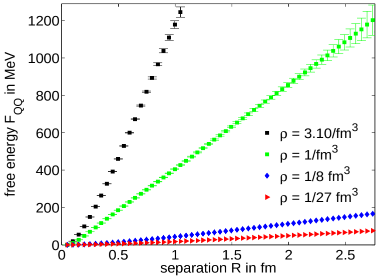
all other parameters fixed

confinement just if holonomy changed to the confining $\mathrm{tr}\,\mathcal{P} = 0$

$\Rightarrow$ different gauge structure compared to e.g. $T = 0$ instantons

- confinement from ensemble of dyons

FB, Dinter, Ilgenfritz, Mueller-Preussker, Wagner 09



SU(2)

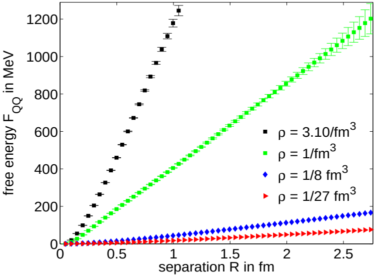
nontrivial holonomy $\mathrm{tr}\,\mathcal{P}_\infty = 0$

different densities of dyons

confinement already from randomly placed dyons

- confinement from ensemble of dyons

FB, Dinter, Ilgenfritz, Mueller-Preussker, Wagner 09



SU(2)

nontrivial holonomy tr $\mathcal{P}_\infty = 0$

different densities of dyons

confinement already from randomly placed dyons

- an important technicality

FB, Dinter, Ilgenfritz, Maier, Mueller-Preussker, Wagner in progress

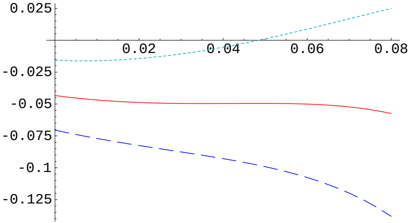
long range fields $A^{||}$ $\rightarrow$ large finite volume effects when restricting to a finite number of dyons

'Ewald sum' from plasma physics

Quantum weight & Confinement

one-loop determinant of small A_μ oscillations around class. solutions
= part of semiclassical path integral 't Hooft 76

- calorons with well-separated dyons Diakonov et al. 04



modified eff. potential for A_0 incl.
perturbative part Weiss 81

different temperatures $T \downarrow$

($A_0 = 0$ means trivial $P = 1_2$)

trivial Polyakov loop as preferred by perturbation theory at high T
unstable at low T due to the nonperturbative caloron contribution

onset of confinement

Moduli space metric & Confinement

Jacobian from A_μ to classical moduli = another part of semiclassical path integral

- known for indiv. dyons and calorons   Gibbons,Manton 95, Kraan,van Baal 98
- caloron ensemble with well-separated dyons   Diakonov, Petrov 07

  det(metric): fermionisation
  Coulombic terms therein: scalar field theory   Polyakov 77
  $\Rightarrow$ exactly solvable ['Toda lattice'] due to selfduality of dyons

  confining $\mathrm{tr}\,\mathcal{P} = 0$ preferred
  (high T: perturbative contribution drives deconfinement again)

  string tension in Polyakov loop correlators

  area law for spatial Wilson loops

  large N_c easily calculable

but:

- including anti-dyons ($Q < 0$ like anti-instantons) as necessary for CP-invariance removes selfdual structures $\rightarrow$ guesswork
- at realistic densities diluteness assumption breaks down

  $\Rightarrow$ det(metric)$<$ 0: unphysical

FB, Dinter, Ilgenfritz, Mueller-Preussker, Wagner 09

complete interactions: non-factorization of action, quantum weight and moduli space metric

Connection to vortices & Confinement

instantons $\rightarrow$ monopoles/dyons ✓ what about center vortices?!

mechanism: percolating vortices confine Wilson loops by ‘piercing’

Connection to vortices & Confinement

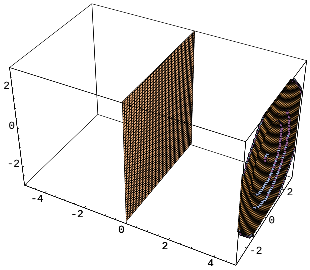
instantons → monopoles/dyons ✓ what about center vortices?!

mechanism: percolating vortices confine Wilson loops by 'piercing'

- vortices in caloron background — FB, Ilgenfritz, Martemyanov, Zhang 09

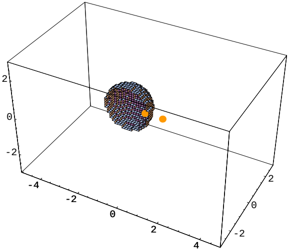
  single caloron, purely spatial vortex:

$\mathrm{tr}\,\mathcal{P} = 0$ as in conf. phase [equal mass dyons]



mid-plane between dyons [+artefact]

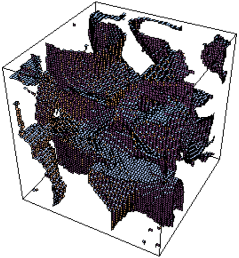
$\mathrm{tr}\,\mathcal{P} \to 1$ as in deconf. phase [heavy + light dyon]



'bubble' around a dyon

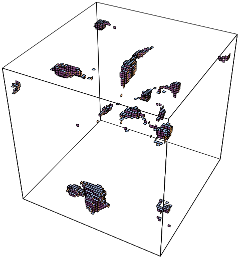
spatial vortices for a caloron gas $\approx$ superpositon of indiv. vortices:

tr $\mathcal{P} = 0$ as in conf. phase [equal mass dyons]



percolation = confinement of Polyakov loop correlators

tr $\mathcal{P} \to 1$ as in deconf. phase [heavy + light dyon]



no percolation = deconfinement

✓ (de)confinement mechanism of vortices

$\rightleftarrows$ no drastic change for *spatial* Wilson loops pierced by ▸ space-time vortices

Index Theorem and localisation

- caloron: $Q = 1 \Rightarrow 1$ chiral zero mode Nye, Singer 00

  localisation to which constituent dyon?

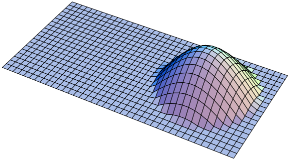
Index Theorem and localisation

- caloron: $Q = 1 \Rightarrow 1$ chiral zero mode — Nye, Singer 00

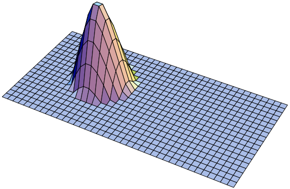
  localisation to which constituent dyon?

  to diff. dyons depending on twisted bc.s — Garcia-Perez et al. 99

$$\psi(x_0 + \beta) = e^{i\varphi}\psi(x_0)$$



periodic, $\varphi \simeq 0$: at light dyon



antiperiodic, $\varphi \simeq \pi$: at heavy dyon
[physical bc.: fermion!]

- indiv. dyons: zero mode only in certain φ-intervals — Callias 78
- hopping also seen in thermalised configurations — Gattringer, Schaefer 03

Condensates

mechanism known from $T = 0$ instantons:

exact zero modes ⇒ near zero modes ⇒ condensates $\langle\bar{\psi}\psi\rangle \sim \rho(\lambda = 0)$

- mechanism above T_c FB 09

  heavy dyons suppressed

  ⇒ top. susceptibility suppressed ✓ (light dyons=smaller top. units)

  ⇒ $\langle\bar{\psi}\psi\rangle_{\text{periodic}}$ suppressed: physical bc., chiral transition ✓

  ⇒ $\langle\bar{\psi}\psi\rangle_{\text{antiperiodic}}$ stays: unphysical bc., confirmed on the lattice

  lattice: Bornyakov et al. 09 and others

  needed since 'dual condensate' Bilgici, FB, Gattringer, Hagen 08
  proportional to φ-variation of $\langle\bar{\psi}\psi\rangle$ should be finite above T_c in order to break center symmetry

FB @ Tsukuba

Summary

$T > 0$: calorons relate instantons and dyons (monopoles) and vortices

$\Rightarrow$ new 'environment': holonomy $\mathrm{tr}\,\mathcal{P}_\infty \overset{\text{conjecture}}{\longleftrightarrow}$ order parameter $\langle \mathrm{tr}\,\mathcal{P} \rangle$

confinement and deconfinement from

- plain superpositions
- quantum weight
- metric on moduli space
- vortex percolation

chiral condensate incl. twisted bc.s

$\Rightarrow$ different gauge structure (than e.g. $T = 0$ instantons)

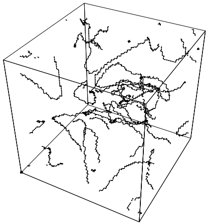
outlook:

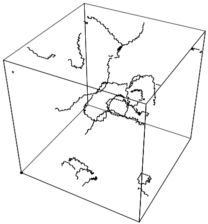
- complete model
- reconsider $T = 0$
- finite chem. potential?!

Space-time vortices in caloron background

space-time vortices for a caloron gas:

tr $\mathcal{P} = 0$ as in conf. phase [equal mass dyons]	tr $\mathcal{P} \to 1$ as in deconf. phase [heavy + light dyon]





no drastic change in space-time vortices confining spatial Wilson loops

✓ no phase transition for spatial Wilson loops