Quarkonium mass splittings in three-flavor lattice QCD

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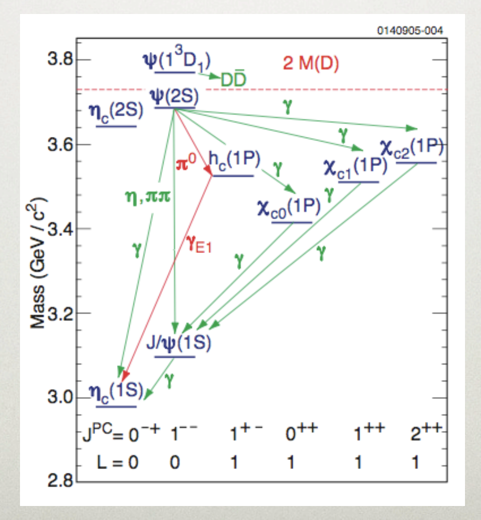
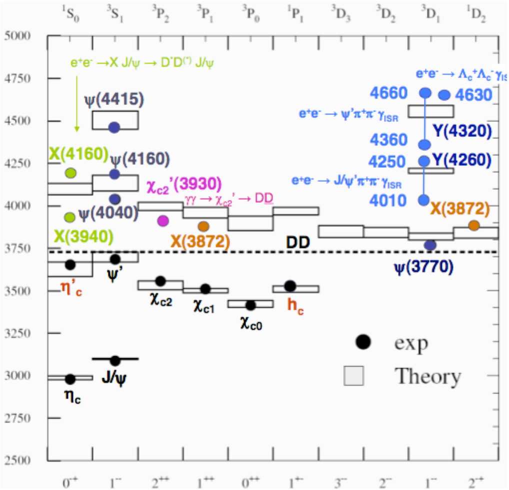
Outline

1. Objectives
2. Challenges
3. Results
4. Conclusions

Charmonium spectroscopy before the B-factories



Charmonium spectroscopy after the B-factories



J^{PC}

The message

- We want to guide the discovery and classification of excited quarkonium states.
- There has been significant recent progress in methods and results for "gold-plated" levels (*i.e.* ground states).
- These successes point the way to improvements in all quantities.

Lattice challenges for heavy quarks

- Reducing lattice cutoff $1/a$ effects, especially for heavy quarks. $\mathcal{O}(Ma)$ errors are bad for charm and bottom when, typically $1/a \approx 1.8 - 3$ GeV.
  - Nonrelativistic QCD: expansion in p/M slow for charm, good for bottom.
  - Fermilab quarks good for both. Further improvements under study [Oktay, Kronfeld]
  - Highly Improved Staggered Quarks (HISQ) errors first at $\mathcal{O}(\alpha_s^2(aM_c)^2)$: good for charm, not so good for bottom with today's lattices.
- Excited states. Ground state properties are easiest. Excited states more difficult.
- Multihadronic states, *e.g.*, open charm are complicated. We are just beginning to learn how to treat them.

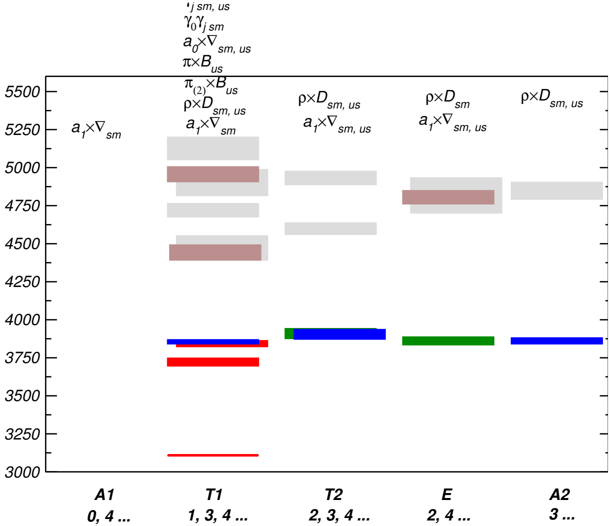
Quarkonium spectroscopy

- Hadron masses are determined from propagators

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j(0)|0\rangle \to z_i^* z_j \exp(-mt) \quad \text{for large } t \quad ,$$

  where $\mathcal{O}_i$'s are suitable interpolating operators. For example, for the J/ψ we could use $\mathcal{O} = \bar{q}\gamma_\mu q$.
- Operators are classified according to lattice symmetries.
- The cubic group replaces the rotation group. A_1 replaces $J = 0$, T_1 replaces $J = 1$, etc.
- Ambiguities: A_1 sees $J = 0$, 4, 6; T_1 could be 1, 3, 4, etc. With a little effort we can often resolve them.
- If we use a large basis set $\mathcal{O}_j$ for the same quantum numbers, we get a correlation matrix. The eigenvalues contain information about the ground and excited states. Several groups now use this method successfully.
- In the next slides I show a sample of results obtained by Dudek *et al.* (2007) using this method.

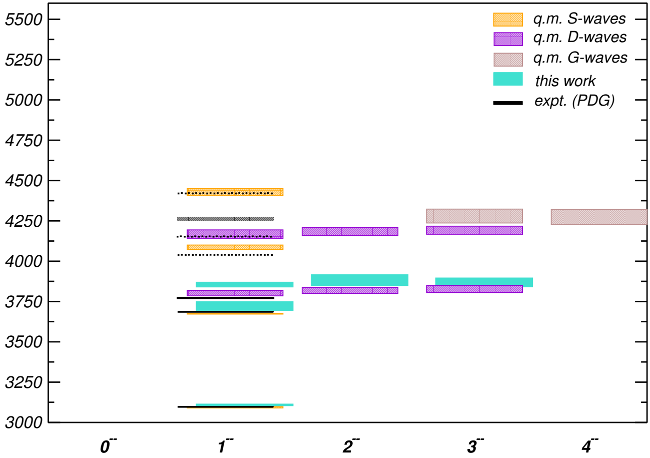
JLab quarkonium: J^{--} states



$J=1$, $J=2$, $J=3$, Gray, brown: undetermined J.

[Dudek, Edwards, Mathur, and Richards, arXiv:0707.4162]

JLab quarkonium: J^{--} states



- Comparison with experimental masses and quark potential model masses. Tabulated masses in MeV.

[Dudek, Edwards, Mathur, and Richards, *op cit.*]

JLab quarkonium J^{--} states

- How well does this work? Look at the T_1^{--} channel, where the ground state J/ψ has a very clean signal.

state	predict	PDG	difference
J/ψ	3109(2)	3097	12
$\psi(2S)$	3722(24)	3686	36
$\psi(3S)$	3855(12)	3773	82
$\psi(4040)$	3843(18)?	4039	-196
$\psi(4160)$	4472(79)?	4153	319
$\psi(4415)$	4442(48)?	4421	21

- Here ? means the authors did not make any assignment.
- Higher excitations: more difficult to assign J.
- Errors grow with excitation, as expected.
- When the ground state signal is not so clean, less can be extracted.

Hadron Spectrum Collaboration quarkonium future

These results are pioneering and impressive, but ...

- No sea quarks (quenched approximation).
- Only one lattice spacing, so no continuum limit.
- No open charm states (none, anyway, in quenched approximation.)

The collaboration is currently remedying these shortcomings.
[Ryan, Lattice 2010]

Open charm mixing?

Causes level shifts. Is it significant?

- For static quarks, string breaking studies on the lattice suggest mixing is weak.
- For dynamical quarks, very little is known from lattice studies.
- Bali and Ehmann (Lattice 2009) studied mixing between S-wave charmonium and a $D\bar{D}$ "molecule" using a variational method.

state	$(c\bar{c})_l$	$(c\bar{c})_n$	$(c\bar{u}\bar{c}u)_l$	$(c\bar{u}\bar{c}u)_n$
η_c	0.54(3)	-0.02(1)	-0.1(1)	-0.31(5)
$D_1\bar{D}^*$	0.07(1)	0.01(1)	-0.46(8)	0.14(2)
J/ψ	0.51(4)	-0.03(1)	0.09(1)	0.21(6)
$D_1\bar{D}$	0.08(6)	0.04(1)	-0.18(1)	0.53(4)
χ_{c1}	0.39(5)	0.69(3)	-0.22(3)	-0.49(4)
$D\bar{D}^*$	0.63(4)	-0.23(3)	-0.73(4)	0.12(3)

- ("l" and "n" refer to different basis wave functions.)
- In some cases such as the χ_{c1} the mixing appears to be large, but a more thorough study is now needed.

[Bali, Ehmann, arXiv:0911.1238]

MILC Collaboration gauge field ensembles

- Ensembles to help us reach the physical point and continuum.
- Parameters of a publicly available archive of gauge configurations based on u, d, and s sea quarks.

ensemble	a (approx) (fm)	sea quark ratio m_{ud}/m_s
Extra coarse	0.18	0.6, 0.4, 0.2, 0.1
Medium coarse	0.15	0.6, 0.4, 0.2, 0.1
Coarse	0.12	0.6, 0.4, 0.2, 0.15, 0.1
Fine	0.09	0.4, 0.2, 0.1, 0.05
Superfine	0.06	0.4, 0.2, 0.1
Ultrafine	0.045	0.2

- With these one can carry out an extrapolation to physical m_{ud} ($\approx 0.037 m_s$) and $a = 0$ (continuum).

[MILC, Rev Mod Phys **82**, 1349 (2010)]

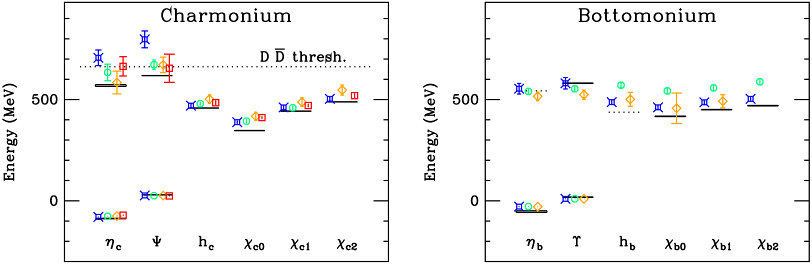
Progress in lattice actions and analysis campaigns

Both campaigns use MILC lattices to extrapolate to the physical point.

Study	Charm	Bottom	Ops	States	Comment
Old FNAL/MILC	FNAL	FNAL	2	low S, P	$a \geq 0.09$ fm
New FNAL/MILC	FNAL	–	JLab	many	$a \geq 0.06$ fm
HPQCD	HISQ	NRQCD	2	low S, P	$a \geq 0.06$ fm
Future FNAL/MILC	HISQ	FNAL	JLab	many	$a \geq 0.06$ fm

FNAL/MILC Overview

- Charmonium levels constructed from splittings from the spin-averaged $\overline{1S}$ level.

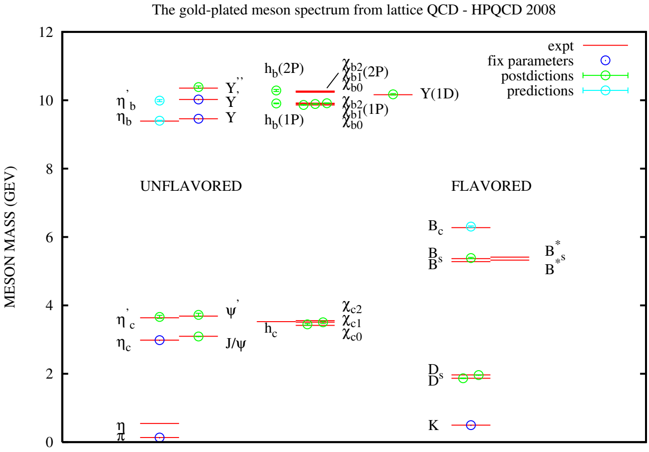


  - Lattice spacings are 0.18 fm, 0.15 fm, 0.12 fm, 0.09 fm.

[MILC/FNAL, PRD **81**, 034508 (2010)]

HPQCD Overview

- Gold-plated meson spectrum based on a subset of the MILC ensembles.

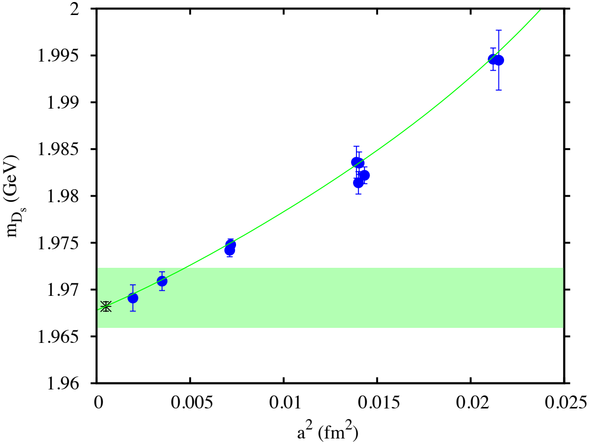


- Five states are used to get the four quark masses and lattice spacing.
- Three states were predictions.

[HPQCD-PoS **LATTICE2008**, 118]

HPQCD D_s

Example of achievable precision: D_s: 3 MeV!

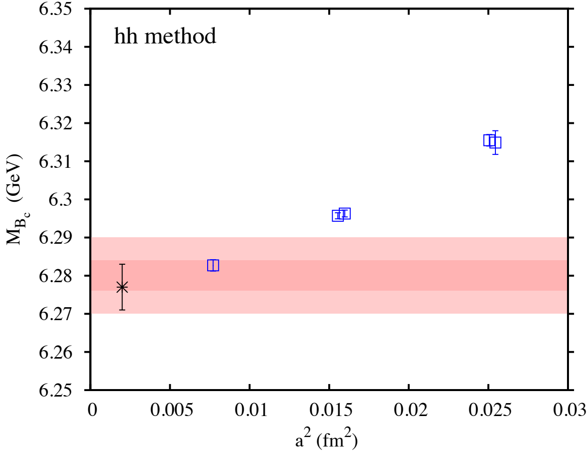


- Based on splitting $M(c\bar{s}) - \frac{1}{2}M(\eta_c)$
- PACS-CS Lattice 2009 result using their relativistic heavy quark action: 1.972(2) GeV vs expt 1.968. (Error is statistical only.)

[HPQCD, arXiv:1008.4018; PACS-CS arXiv:0911.5362]

HPQCD B_c

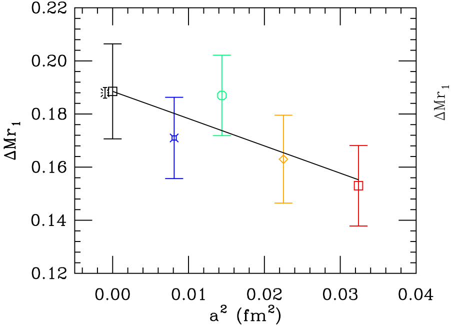
Example of achievable precision: B_c: 10 MeV!



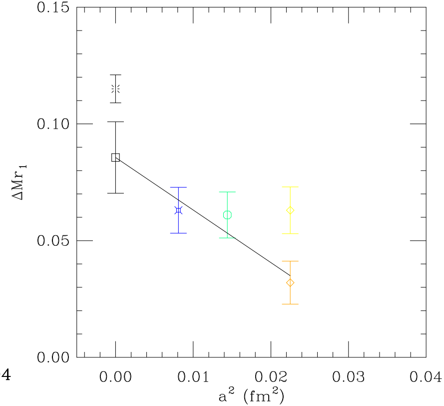
- Based on splitting $M(B_c) - M(b\bar{b})/2 - M(\eta_c)/2$.
- Light shaded band includes all errors.

[HPQCD, arXiv:1010.3848]

MILC/FNAL 1S hyperfine splitting



Charmonium

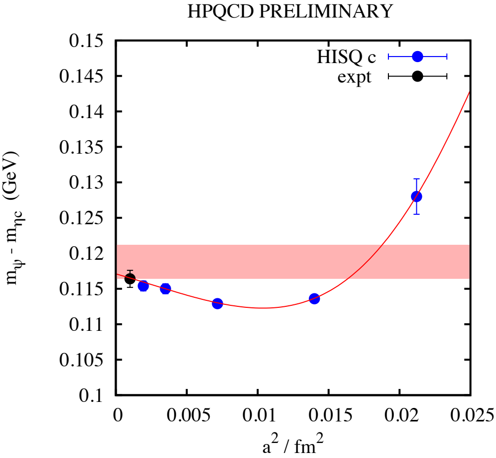


Bottomonium

- Charm result 117(11) MeV.
- Bottom result 53(9) MeV.
- Annihilation effects are ignored here.

[FNAL/MILC PRD **81**, 034508 (2010)]

HPQCD 1S hyperfine splitting

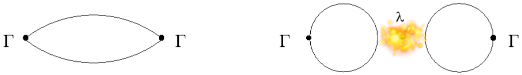


- Annihilation effects are estimated from perturbation theory. They and other corrections shift the result up to the pink band, which indicates all errors: 2 MeV

[HPQCD, private communication, 2010]

Annihilation contribution to charm HFS

Calculated from the lattice rather than perturbation theory.



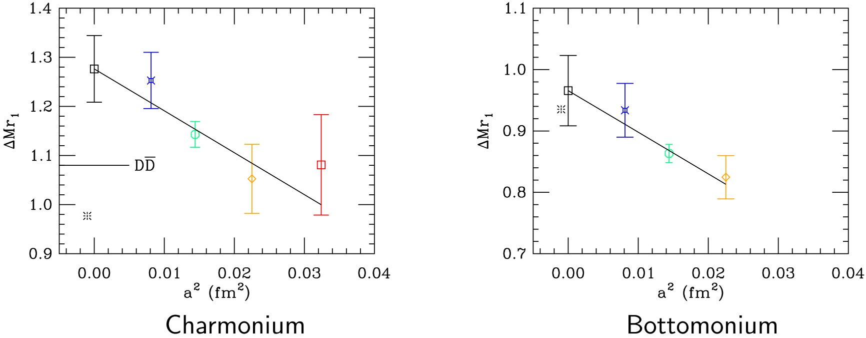
Connected and disconnected diagrams

- Decreases the HFS splitting - by about 2 MeV.
- Sign contrary to perturbation theory (partly due to axial anomaly).

[Levkova and DeTar, Lattice 2010 and forthcoming paper.]

MILC/FNAL $\overline{2S} - \overline{1S}$ splitting

Where we don't do so well.



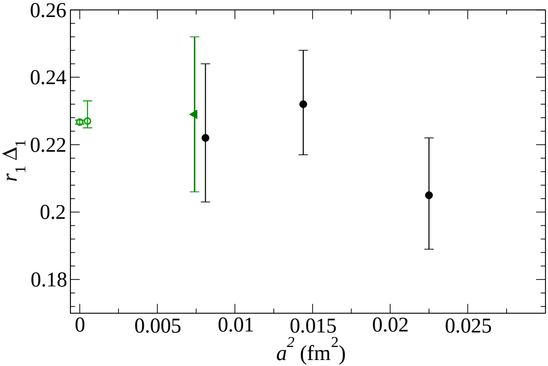
Charmonium

Bottomonium

- The calculation does not treat the open charm threshold.
- Note the open bottom threshold is safely off scale here.
- Is the disgreement in charmonium caused by open charm?

[FNAL/MILC PRD **81**, 034508 (2010)]

MILC/FNAL $D_s^* - D_s$

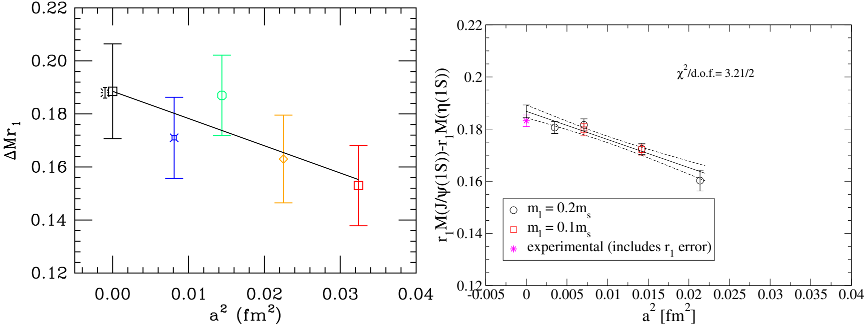


- Based on our "old" data.
- Splittings are in $r_1 = 0.311$ fm units here. $(1/r_1 = 635$ MeV$)$.
- The green point includes discretization errors. Accuracy: 4 MeV.
- PACS-CS Lattice 2009 result with their relativistic heavy quark action: 135(3) MeV vs expt 144. (Statistical only.)

[FNAL/MILC arXiv:1003.1937, PACS-CS arXiv:0911.5362]

Fermilab-Lattice/MILC quarkonium preview

Charmonium 1S hyperfine splitting



old data 2009

new data 2010

[FNAL/MILC 2010 PRELIMINARY]

Toward precision lattice charmonium results

Through tackling "gold-plated" quantities, we have learned what is required to do good lattice charm physics for all quantities:

- A heavy quark action with small discretization errors and an accurate tuning of the heavy quark masses.
- An accurate determination of the lattice scale.
- A full treatment of sea quarks. Simulate at the physical light quark masses or do a controlled extrapolation.
- A careful extrapolation to zero lattice spacing.
- Good interpolating operators and an adequate data sample.

Conclusions

- Improved lattice charm quark formulations yield high precision for gold-plated quantities.
- This experience is teaching us how to do good charm physics on the lattice.
- Gauge field ensembles at smaller lattice spacing enable significant reductions in errors for all quantities.
- Expect improvements in excited state and exotic predictions from the Hadron Spectrum and MILC/Fermilab Lattice collaborations.
- Treating two-hadron mixing (*e.g.* open charm) remains a challenge.

Purple Mangosteen = die Mangostane =マンゴスチン

The Purple Mangosteen (*Garcinia mangostana*), colloquially known simply as ``the mangosteen'', is a tropical evergreen tree believed to have originated in the Sunda Islands and the Moluccas of Indonesia. Botanically an aril, the fragrant edible flesh can be described as sweet and tangy, citrusy with peach flavor and texture. (Wikipedia)