New ideas for $g-2$

Lattice QCD confronts experiments
- Japanese-German Seminar 2010 -

4 - 6 November 2010, Mishima, Japan

Andreas Jüttner
CERN Theory Division

Collaboration

Michele Della Morte, Benni Jäger, Hartmut Wittig
Johannes-Gutenberg Universität, Mainz

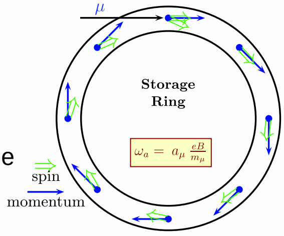
Simulations based on configuration ensembles jointly generated by **C**oordinated **L**attice **S**imulations:

- Berlin
- CERN
- DESY-Zeuthen
- Madrid
- Mainz
- Rome
- Valencia

simulation code developed at CERN (DD-HMC) and at Mainz (qcd-measure)

The muon - a christal ball - experimentally

- monitor the spin's motion in a circular orbit in a homegeneous magnetic field (Lamor precession).
  The muons are polarized by their production process: pions from $p \to$ target and then $\pi^+ \to \mu^+ + \nu_\mu$
- π^+ is spin-0 and ν_μ is left-handed, μ^+ must be left-handed
- the μ^+ decays after a few rounds in the ring and the resulting e^+ keeps the helicity



Experiment	Year	Polarity	$a_\mu \times 10^{10}$	Pre. (ppm)	References
CERN I	1961	μ^+	11 450 000(220000)	4300	[101]
CERN II	1962–1968	μ^+	11 661 600(3100)	270	[102]
CERN III	1974–1976	μ^+	11 659 100(110)	10	[91]
CERN III	1975–1976	μ^-	11 659 360(120)	10	[91]
BNL	1997	μ^+	11 659 251(150)	13	[12]
BNL	1998	μ^+	11 659 191(59)	5	[13]
BNL	1999	μ^+	11 659 202(15)	1.3	[14]
BNL	2000	μ^+	11 659 204(9)	0.73	[15]
BNL	2001	μ^-	11 659 214(9)	0.72	[16]
	Average		11 659 208.0(6.3)	0.54	[92]

Jegerlehner, Nyffeler, Physics Reports 477(2009)1-110

The muon - a christal ball - theoretically

$(\times 10^{-11})$

Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

Jegerlehner, Nyffeler, Physics Reports 477(2009)1-110

The muon - a christal ball - theoretically

$(\times 10^{-11})$

Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

Jegerlehner, Nyffeler, Physics Reports 477(2009)1-110

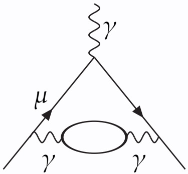
The muon - a christal ball - theoretically

$(\times 10^{-11})$

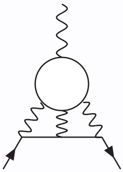
Contribution	Value	Error
QED incl. 4-loops + LO 5-loops	116 584 718.1	0.2
Leading hadronic vacuum polarization	6 903.0	52.6
Subleading hadronic vacuum polarization	−100.3	1.1
Hadronic light-by-light	116.0	39.0
Weak incl. 2-loops	153.2	1.8
Theory	116 591 790.0	64.6
Experiment	116 592 080.0	63.0
Exp. - The. 3.2 standard deviations	290.0	90.3

Jegerlehner, Nyffeler, Physics Reports 477(2009)1-110

vacuum polarisation



light-by-light



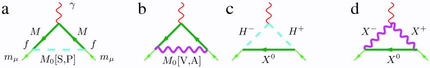
The muon - a christal ball - signs for SM-extensions?

new physics would modify a_μ e.g. through virtual loop contributions which can be computed (magnitude/sign)

experimental results allow to exclude/constrain SM-extensions

possible new physics contributions:

- heavy A, P, S, V states



  e.g. neutral exchange: A, P yield wrong sign, V too small, S possible
- extra dimensions
- super symmetry
- ...

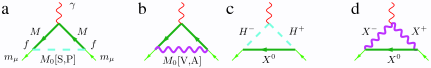
The muon - a christal ball - signs for SM-extensions?

new physics would modify a_μ e.g. through virtual loop contributions which can be computed (magnitude/sign)

experimental results allow to exclude/constrain SM-extensions

possible new physics contributions:

- heavy A, P, S, V states



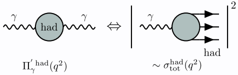
  e.g. neutral exchange: A, P yield wrong sign, V too small, S possible

- extra dimensions
- super symmetry
- ...

first of all - get SM-prediction right ...

Leading hadronic VP from $e^+e^- \to$ hadrons

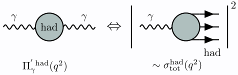
- for leptons VP can be computed in PT - for quarks PT breaks down at small energies
- current prediction for a_μ^{LHV} from experimental measurement of e^+e^--annihilation



illustrations: F. Jegerlehner, A. Nyffeler, Physics Reports 477(2009)1-110

Leading hadronic VP from $e^+e^- \to$ hadrons

- for leptons VP can be computed in PT - for quarks PT breaks down at small energies
- current prediction for a_μ^{LHV} from experimental measurement of e^+e^--annihilation

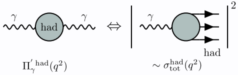


illustrations: F. Jegerlehner, A. Nyffeler, Physics Reports 477(2009)1-110

- there are issues with the current determination of a_μ^{LHV} via e^+e^-/τ
- note: bridging the gap by increasing the had. cross section can lead to decreased upper bound for Higgs mass *Passera, Marciano, Sirlin PRD 78, 013009 (2008)*

Leading hadronic VP from $e^+e^- \to$ hadrons

- for leptons VP can be computed in PT - for quarks PT breaks down at small energies
- current prediction for a_μ^{LHV} from experimental measurement of e^+e^--annihilation

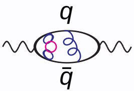


illustrations: F. Jegerlehner, A. Nyffeler, Physics Reports 477(2009)1-110

- there are issues with the current determination of a_μ^{LHV} via e^+e^-/τ
- note: bridging the gap by increasing the had. cross section can lead to decreased upper bound for Higgs mass *Passera, Marciano, Sirlin PRD 78, 013009 (2008)*

- independent and pure theory prediction desireable
- would provide a classical test of the SM

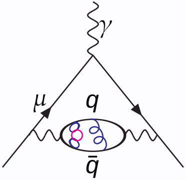
a_μ^{LHV} on the lattice



- vacuum polarisation tensor

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$
$$= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

a_μ^{LHV} on the lattice

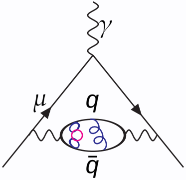


- vacuum polarisation tensor

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

- $$a_\mu^{\mathrm{LHV}} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2) \left(\Pi(q^2) - \Pi(0)\right)$$

$a_\mu^{\rm LHV}$ on the lattice



- vacuum polarisation tensor

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$
$$= (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

- $$a_\mu^{\rm LHV} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2) \left(\Pi(q^2) - \Pi(0)\right)$$

- leading hadronic VP previous lattice efforts compared to $e^+e^- \to$ hadrons

F. Jegerlehner, A. Nyffeler, Physics Reports 477(2009)1-110 — $690.3(5.3) \times 10^{-10}$

lattice QCD:

QCDSF Collaboration NPB 688 (2004) 135164 — $446(23) \times 10^{-10}$

T. Blum, C. Aubin, PRD 75, 114502 (2007) — $713(15) \times 10^{-10}$ and $748(21) \times 10^{-10}$

D. Renner and X. Feng, arXiv:0902.2796 — not yet

Leading hadronic contribution to muon $g-2$

Why is there no better lattice result?

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$

$$a_\mu^{\rm LHV} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2) \Big(\Pi(q^2) - \Pi(0)\Big)$$

usual lattice systematics (a, m_q, L)

Leading hadronic contribution to muon $g-2$

Why is there no better lattice result?

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$

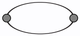
$$a_\mu^{\rm LHV} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2) \Big(\Pi(q^2) - \Pi(0)\Big)$$

usual lattice systematics (a, m_q, L)

but one will not get very far without new ideas:

- contributions of quark-disconnected diagrams neglected so far

$$\left\langle j_\mu^{qq} j_\nu^{qq} \right\rangle = \left\langle \bar{q}\gamma_\mu q\, \bar{q}\gamma_\nu q \right\rangle = \left\langle {\rm Tr}\left\{ S_q \gamma_\mu S_q \gamma_\nu \right\} \right\rangle + \left\langle {\rm Tr}\left\{ S_q \gamma_\mu \right\} {\rm Tr}\left\{ S_q \gamma_\nu \right\} \right\rangle$$



Leading hadronic contribution to muon $g-2$

Why is there no better lattice result?

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq(x-y)} \langle j_\mu^{EM}(y) j_\nu^{EM}(x) \rangle$$

$$a_\mu^{\mathrm{LHV}} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2) \Big(\Pi(q^2) - \Pi(0)\Big)$$

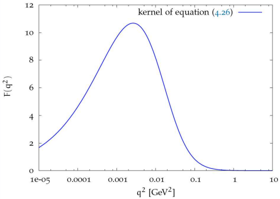
usual lattice systematics (a, m_q, L)

but one will not get very far without new ideas:

- contributions of quark-disconnected diagrams neglected so far

$$\left\langle j_\mu^{qq} j_\nu^{qq} \right\rangle = \left\langle \bar{q}\gamma_\mu q\, \bar{q}\gamma_\nu q \right\rangle = \left\langle \mathrm{Tr}\left\{S_q\gamma_\mu S_q\gamma_\nu\right\}\right\rangle + \left\langle \mathrm{Tr}\left\{S_q\gamma_\mu\right\}\mathrm{Tr}\left\{S_q\gamma_\nu\right\}\right\rangle$$

- momenta restricted: $1/L < Q < 1/a$
  - use lattice for small momenta and PT for large momentum transfers
  - need to fix region of very small momenta – but $\frac{2\pi}{L} \approx 400\,\mathrm{MeV}$



Outline:

- analytically predicting quark-disconnected diagrams
- improving the momentum resolution
- preliminary numerical results

Outline:

- analytically predicting quark-disconnected diagrams
    - computationally expensive
    - problem in many lattice QCD calculations
    - analytical predictions would be of great help
- improving the momentum resolution
- preliminary numerical results

A closer look - the vector 2pt function

EM current $j_\mu(x) \equiv \frac{2}{3} j_\mu^{uu}(x) - \frac{1}{3} j_\mu^{dd}(x) - \frac{1}{3} j_\mu^{ss}(x)$

$\Pi_{\mu\nu}(q) =$ 

$$= \frac{1}{(2\pi)^2} \int d^4x\, e^{-iqx} \langle j_\mu(x) j_\nu(0) \rangle$$

A closer look - the vector 2pt function

EM current $j_\mu(x) \equiv \frac{2}{3} j_\mu^{uu}(x) - \frac{1}{3} j_\mu^{dd}(x) - \frac{1}{3} j_\mu^{ss}(x)$

$$\Pi_{\mu\nu}(q) \quad = \quad$$

$$= \frac{1}{(2\pi)^2} \int d^4x \, e^{-iqx} \langle j_\mu(x) j_\nu(0) \rangle$$

$$\stackrel{\text{iso-spin}}{=} \frac{1}{(2\pi)^2} \int d^4x \, e^{-iqx} \left\{ \frac{5}{9} \langle j_\mu^{uu} j_\nu^{uu} \rangle + \frac{1}{9} \langle j_\mu^{ss} j_\nu^{ss} \rangle - \frac{2}{9} \langle j_\mu^{ss} j_\nu^{uu} \rangle + \frac{4}{9} \langle j_\mu^{dd} j_\nu^{uu} \rangle \right\}$$

A closer look - the vector 2pt function

EM current $j_\mu(x) \equiv \frac{2}{3} j_\mu^{uu}(x) - \frac{1}{3} j_\mu^{dd}(x) - \frac{1}{3} j_\mu^{ss}(x)$

$\Pi_{\mu\nu}(q) =$ 

$$= \frac{1}{(2\pi)^2} \int d^4x\, e^{-iqx} \langle j_\mu(x) j_\nu(0) \rangle$$

$$\overset{\text{iso-spin}}{=} \frac{1}{(2\pi)^2} \int d^4x\, e^{-iqx} \left\{ \frac{5}{9} \langle j_\mu^{uu} j_\nu^{uu} \rangle + \frac{1}{9} \langle j_\mu^{ss} j_\nu^{ss} \rangle - \frac{2}{9} \langle j_\mu^{ss} j_\nu^{uu} \rangle + \frac{4}{9} \langle j_\mu^{dd} j_\nu^{uu} \rangle \right\}$$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss} \rangle$ decomposes into connected and disconnected piece as follows:

conn. piece $\rightarrow \quad \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\} \right\rangle$

$\overset{\rightarrow SU(4|1)}{=} \quad \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu r(0) \bar{r}(x)\gamma_\mu s(x) \right\rangle \quad = \langle j^{sr}_\mu j^{rs}_\nu \rangle$

disc. piece $\rightarrow \quad \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\} \mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\} \right\rangle$

$= \quad \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu s(0) \bar{r}(x)\gamma_\mu r(x) \right\rangle \quad = \langle j^{ss}_\mu j^{rr}_\nu \rangle$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss} \rangle$ decomposes into connected and disconnected piece as follows:

conn. piece $\rightarrow$ $\frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_r(0,x)\gamma_\mu \right\} \right\rangle$

$SU(3) \rightarrow SU(4|1)$ with PQ r-quark, mass degenerate to the s-quark

$$\stackrel{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x) \right\rangle = \langle j^{sr}_\mu j^{rs}_\nu \rangle$$

disc. piece $\rightarrow$ $\frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\} \mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\} \right\rangle$

$$= \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x) \right\rangle = \langle j^{ss}_\mu j^{rr}_\nu \rangle$$

Vacuum polarization in $SU(2)$ χPT

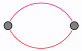
M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss} \rangle$ decomposes into connected and disconnected piece as follows:

conn. piece $\rightarrow$ $\frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\} \right\rangle$

$SU(3) \rightarrow SU(4|1)$ with PQ r-quark, mass degenerate to the s-quark

$$\overset{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x) \right\rangle = \langle j_\mu^{sr} j_\nu^{rs} \rangle$$



disc. piece $\rightarrow$ $\frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\} \mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\} \right\rangle$

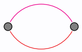
$$= \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x) \right\rangle = \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss}\rangle$ decomposes into connected and disconnected piece as follows:

conn. piece
$$\rightarrow \quad \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu\right\}\right\rangle$$
$$\stackrel{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle \quad = \quad \langle j^{sr}_\mu j^{rs}_\nu\rangle$$



disc. piece
$$\rightarrow \quad \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{S_s(x,x)\gamma_\nu\right\}\mathrm{Tr}\left\{S_s(0,0)\gamma_\mu\right\}\right\rangle$$
$$= \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle \quad = \quad \langle j^{ss}_\mu j^{rr}_\nu\rangle$$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss} \rangle$ decomposes into connected and disconnected piece as follows:

conn. piece $\rightarrow \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \Big\langle \mathrm{Tr}\Big\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \Big\} \Big\rangle$

$$\stackrel{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \Big\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x) \Big\rangle = \langle j_\mu^{sr} j_\nu^{rs} \rangle$$

disc. piece $\rightarrow \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \Big\langle \mathrm{Tr}\Big\{ S_s(x,x)\gamma_\nu \Big\} \mathrm{Tr}\Big\{ S_s(0,0)\gamma_\mu \Big\} \Big\rangle$

$$= \frac{1}{(2\pi)^2} \int d^4x \, e^{iqx} \Big\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x) \Big\rangle = \langle j_\mu^{ss} j_\nu^{rr} \rangle$$

Vacuum polarization in $SU(2)\ \chi$PT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss} \rangle$ decomposes into connected and disconnected piece as follows:

conn. piece
$$\rightarrow \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\} \right\rangle$$
$$\stackrel{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x) \right\rangle = \langle j^{sr}_\mu j^{rs}_\nu \rangle$$

disc. piece
$$\rightarrow \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu \right\} \mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\} \right\rangle$$
$$= \frac{1}{(2\pi)^2} \int d^4x\, e^{iqx} \left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x) \right\rangle = \langle j^{ss}_\mu j^{rr}_\nu \rangle$$

$$\langle j^{ss}_\mu j^{ss}_\nu \rangle = \langle j^{sr}_\mu j^{rs}_\nu \rangle + \langle j^{ss}_\mu j^{rr}_\nu \rangle$$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss}\rangle$ decomposes into connected and disconnected piece as follows:

conn. piece

$$\rightarrow \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu\right\}\right\rangle$$

$$\stackrel{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle \quad = \quad \langle j_\mu^{sr} j_\nu^{rs}\rangle$$

disc. piece

$$\rightarrow \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{S_s(x,x)\gamma_\nu\right\}\mathrm{Tr}\left\{S_s(0,0)\gamma_\mu\right\}\right\rangle$$

$$= \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle \quad = \quad \langle j_\mu^{ss} j_\nu^{rr}\rangle$$

$$\langle j_\mu^{ss} j_\nu^{ss}\rangle = \langle j_\mu^{sr} j_\nu^{rs}\rangle + \langle j_\mu^{ss} j_\nu^{rr}\rangle$$

- flavour-diagonal two-point correlator can be written as the sum of a purely connected correlator and a purley disconnected correlator
- the u- and d-sector is easier - iso-spin can be used: $\langle j_\mu^{uu} j_\nu^{uu}\rangle = \langle j_\mu^{ud} j_\nu^{du}\rangle + \langle j_\mu^{uu} j_\nu^{dd}\rangle$

Vacuum polarization in $SU(2)$ χPT

M. Della Morte, AJ arXiv:1009.3783

$\langle j^{ss} j^{ss}\rangle$ decomposes into connected and disconnected piece as follows:

conn. piece

$$\rightarrow \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{ S_s(x,0)\gamma_\nu S_s(0,x)\gamma_\mu \right\}\right\rangle$$

$$\overset{\rightarrow SU(4|1)}{=} \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu r(0)\bar{r}(x)\gamma_\mu s(x)\right\rangle = \langle j^{sr}_\mu j^{rs}_\nu\rangle$$

disc. piece

$$\rightarrow \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \mathrm{Tr}\left\{ S_s(x,x)\gamma_\nu\right\} \mathrm{Tr}\left\{ S_s(0,0)\gamma_\mu \right\}\right\rangle$$

$$= \frac{1}{(2\pi)^2}\int d^4x\, e^{iqx}\left\langle \bar{s}(0)\gamma_\nu s(0)\bar{r}(x)\gamma_\mu r(x)\right\rangle = \langle j^{ss}_\mu j^{rr}_\nu\rangle$$

$$\langle j^{ss}_\mu j^{ss}_\nu\rangle = \langle j^{sr}_\mu j^{rs}_\nu\rangle + \langle j^{ss}_\mu j^{rr}_\nu\rangle$$

- flavour-diagonal two-point correlator can be written as the sum of a purely connected correlator and a purley disconnected correlator
- the u- and d-sector is easier - iso-spin can be used: $\langle j^{uu}_\mu j^{uu}_\nu\rangle = \langle j^{ud}_\mu j^{du}_\nu\rangle + \langle j^{uu}_\mu j^{dd}_\nu\rangle$
- expressions for both can be derived in ChiPT

A closer look - the vector 2pt function

describe VP (full, conn, disc) correlators in $SU(4|1)$ partially quenched chiral perturbation theory

Gasser & Leutwyler Ann. Phys. 158 (1984) 142 , Nucl. Phys. B250 (1985) 465, Sharpe & Shoresh Phys. Rev. D62 (2000) 094503 Bernard & Golterman Phys. Rev. D46 (1992) 853857

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left\{D_\mu U^\dagger D^\mu U\right\} - \frac{BF^2}{2}\mathrm{Tr}\left\{MU^\dagger + M^\dagger U\right\}$$

$$U = e^{-i\frac{2\Phi}{F}} \qquad D_\mu U = \partial_\mu U + iv_\mu U - iUv_\mu$$

A closer look - the vector 2pt function

describe VP (full, conn, disc) correlators in $SU(4|1)$ partially quenched chiral perturbation theory

Gasser & Leutwyler Ann. Phys. 158 (1984) 142 , Nucl. Phys. B250 (1985) 465,
Sharpe & Shoresh Phys. Rev. D62 (2000) 094503
Bernard & Golterman Phys. Rev. D46 (1992) 853857

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left\{D_\mu U^\dagger D^\mu U\right\} - \frac{BF^2}{2}\mathrm{Tr}\left\{MU^\dagger + M^\dagger U\right\}$$

$$U = e^{-i\frac{2\Phi}{F}} \qquad D_\mu U = \partial_\mu U + iv_\mu U - iUv_\mu$$

- express $\langle j^{ss} j^{rr}\rangle$ in terms of the $SU(4|1)$ generators:

$$j^{rr}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar{\psi}(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} - \tfrac{3}{2}T^{15}\right)\psi(x)$$
$$j^{ss}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar{\psi}(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} + \tfrac{1}{2}T^{15} - \sqrt{2}T^8\right)\psi(x)$$

A closer look - the vector 2pt function

describe VP (full, conn, disc) correlators in $SU(4|1)$ partially quenched chiral perturbation theory

Gasser & Leutwyler Ann. Phys. 158 (1984) 142 , Nucl. Phys. B250 (1985) 465, Sharpe & Shoresh Phys. Rev. D62 (2000) 094503 Bernard & Golterman Phys. Rev. D46 (1992) 853857

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left\{D_\mu U^\dagger D^\mu U\right\} - \frac{BF^2}{2}\mathrm{Tr}\left\{MU^\dagger + M^\dagger U\right\}$$

$$U = e^{-i\frac{2\Phi}{F}} \qquad D_\mu U = \partial_\mu U + i v_\mu U - i U v_\mu$$

- express $\langle j^{ss} j^{rr}\rangle$ in terms of the $SU(4|1)$ generators:

$$j^{rr}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar{\psi}(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} - \tfrac{3}{2}T^{15}\right)\psi(x)$$
$$j^{ss}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar{\psi}(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} + \tfrac{1}{2}T^{15} - \sqrt{2}T^8\right)\psi(x)$$

- $\mathcal{L}^{(2)}$ - remains unchanged

A closer look - the vector 2pt function

describe VP (full, conn, disc) correlators in $SU(4|1)$ partially quenched chiral perturbation theory

Gasser & Leutwyler Ann. Phys. 158 (1984) 142 , Nucl. Phys. B250 (1985) 465, Sharpe & Shoresh Phys. Rev. D62 (2000) 094503 Bernard & Golterman Phys. Rev. D46 (1992) 853857

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left\{D_\mu U^\dagger D^\mu U\right\} - \frac{BF^2}{2}\mathrm{Tr}\left\{MU^\dagger + M^\dagger U\right\}$$

$$U = e^{-i\frac{2\Phi}{F}} \qquad D_\mu U = \partial_\mu U + iv_\mu U - iUv_\mu$$

- express $\langle j^{ss} j^{rr}\rangle$ in terms of the $SU(4|1)$ generators:

$$j^{rr}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar\psi(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} - \tfrac{3}{2}T^{15}\right)\psi(x)$$
$$j^{ss}_\mu(x) = \sqrt{\tfrac{2}{3}}\bar\psi(x)\gamma_\mu\left(T^0 - \tfrac{1}{2}T^{24} + \tfrac{1}{2}T^{15} - \sqrt{2}T^8\right)\psi(x)$$

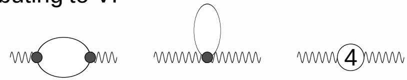
- $\mathcal{L}^{(2)}$ - remains unchanged
- $\mathcal{L}^{(4)}$ - needs to be modified *R. Kaiser, Phys.Rev.D63:076010,2001*

$$\mathcal{L}^{(4)} = (L_{10} + 2H_1)\,\mathrm{Str}\left\{\hat v_{\mu\nu}\hat v_{\mu\nu}\right\} + H_s\,\mathrm{Str}\left\{v_{\mu\nu}\right\}\mathrm{Str}\left\{v_{\mu\nu}\right\}$$

where $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$ and the X_i are Gasser-Leutwyler LEC's and where $\hat{}$ means that the trace has been subtracted

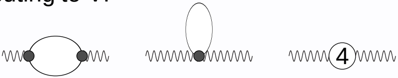
Predictions

- diagrams contributing to VP



Predictions

- diagrams contributing to VP



- result for $N_f = 2+1$ M. Della Morte, AJ arXiv:1009.3783 :

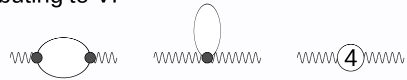
$$\Pi^{(4|1)}_{\rm Full}(q^2) = -2\,(L_{10}(\mu)+2H_1(\mu)) - 4i\Big(\bar{B}_{21}(\mu^2,q^2,M_\pi^2) + \bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

$$\Pi^{(4|1)}_{\rm Conn}(q^2) = -2\,(L_{10}(\mu)+2H_1(\mu)) - 4i\Big(\tfrac{10}{9}\bar{B}_{21}(\mu^2,q^2,M_\pi^2) + \tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_{ss}^2) + \tfrac{7}{9}\bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

$$\Pi^{(4|1)}_{\rm Disc}(q^2) = -4i\Big(-\tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_\pi^2) - \tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_{ss}^2) + \tfrac{2}{9}\bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

Predictions

- diagrams contributing to VP



- result for $N_f = 2+1$ M. Della Morte, AJ arXiv:1009.3783 :

$$\Pi^{(4|1)}_{\rm Full}(q^2) = -2\,(L_{10}(\mu)+2H_1(\mu)) - 4i\Big(\bar{B}_{21}(\mu^2,q^2,M_\pi^2) + \bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

$$\Pi^{(4|1)}_{\rm Conn}(q^2) = -2\,(L_{10}(\mu)+2H_1(\mu)) - 4i\Big(\tfrac{10}{9}\bar{B}_{21}(\mu^2,q^2,M_\pi^2) + \tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_{ss}^2) + \tfrac{7}{9}\bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

$$\Pi^{(4|1)}_{\rm Disc}(q^2) = -4i\Big(-\tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_\pi^2) - \tfrac{1}{9}\bar{B}_{21}(\mu^2,q^2,M_{ss}^2) + \tfrac{2}{9}\bar{B}_{21}(\mu^2,q^2,M_K^2)\Big)$$

- conn and disc receive unphysical contribution from a “meson” containing two strange quarks
- these cancel in the sum
- absence of disconnected diagrams in $SU(3)$-limit is reproduced
- $SU(3)$-symmetry does not allow for LECs to contribute to the disconnected diagram → parameter-free prediction for the disconnected diagram

Analytical prediction for the quark disconnected diagram

$$a_\mu^{\rm LHV} \propto \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 K(q^2)\big(\Pi(q^2) - \Pi(0)\big)$$

i.e. only the difference $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ is relevant for us

- $N_f = 2$ in $SU(2)$: $\frac{\hat{\Pi}_{\rm disc}(q^2)}{\hat{\Pi}_{\rm conn}(q^2)} = -\frac{1}{10}$

Analytical prediction for the quark disconnected diagram

$$a_\mu^{\rm LHV} \propto \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 K(q^2)\big(\Pi(q^2) - \Pi(0)\big)$$

i.e. only the difference $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ is relevant for us

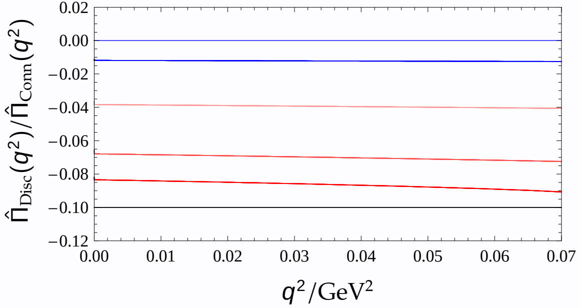
- $N_f = 2$ in $SU(2)$: $\frac{\hat{\Pi}_{\rm disc}(q^2)}{\hat{\Pi}_{\rm conn}(q^2)} = -\frac{1}{10}$
- $N_f = 2 + 1^{PQ} \to SU(4|2)$ expressions similar to the $N_f = 2 + 1$-case

Analytical prediction for the quark disconnected diagram

$$a_\mu^{\mathrm{LHV}} \propto \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 K(q^2)\big(\Pi(q^2) - \Pi(0)\big)$$

i.e. only the difference $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ is relevant for us

- $N_f = 2$ in $SU(2)$: $\frac{\hat{\Pi}_{\mathrm{disc}}(q^2)}{\hat{\Pi}_{\mathrm{conn}}(q^2)} = -\frac{1}{10}$
- $N_f = 2 + 1^{PQ} \rightarrow SU(4|2)$ expressions similar to the $N_f = 2 + 1$-case
- $N_f = 2 + 1$:



Analytical prediction for the quark disconnected diagram

- relying on this NLO prediction it suffices to compute the connected diagram and predict the (subdominant) disconnected diagram
- alternatively we can quantify how precisely we would like to know the quark-disconnected contribution
- but: vector-d.o.f.s may dominate and modify the expressions found

(work in progress)

Analytical prediction for the quark disconnected diagram

Comments:

- our method: quark-disconnected diagrams can be predicted for arbitrary hadronic *n*-point functions for which an effective theory description exists (in particular all possible 2-point and 3-point mesonic and baryonic correlators)
- applicable to many interesting physical processes
- interesting in particular for
  - observables without vector contributions
  - predictions should improve as $m_\pi \to m_\pi^{\mathrm{phys}}$ (where computation of disc. diagrams becoms more and more expensive)

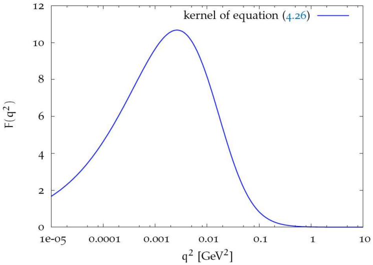
Outline:

- predicting quark-disconnected diagrams ✓
- improving momentum resolution
- preliminary numerical results

Accessing small momenta

$1/L < k < a^{-1}$ use lattice for small momenta and PT for large momentum transfers

$$a_\mu^{\mathrm{LHV}} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2)\left(\Pi(q^2) - \Pi(0)\right)$$



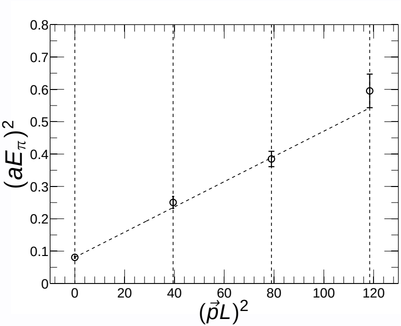
- lattice momenta $q = 2\pi(\frac{n_t}{T}, \frac{n_x}{L}, \frac{n_y}{L}, \frac{n_z}{L})$

Twisted boundary conditions

quark boundary conditions → hadron boundary conditions

de Divitiis et al. PLB 595 (2004) 408, Bedaque PLB 593 (2004) 82, Sachrajda, Villadoro PLB 609 (2005) 73, Flynn, J., Sachrajda PLB 632 (2006) 313

$$E_{\pi^\pm} = \sqrt{m_{\pi^\pm}^2 + (\vec{n}\frac{2\pi}{L})^2}$$



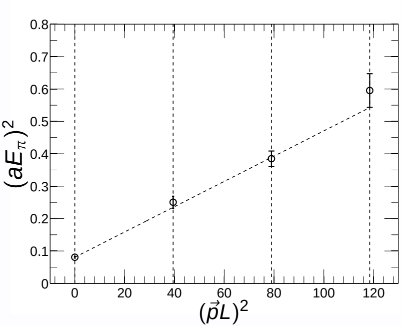
Twisted boundary conditions

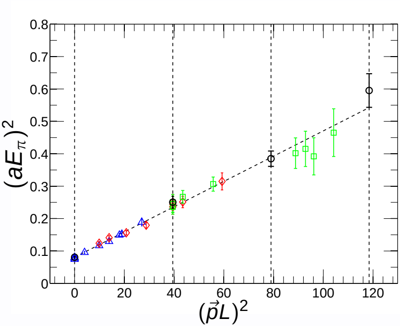
quark boundary conditions → hadron boundary conditions

de Divitiis et al. PLB 595 (2004) 408, Bedaque PLB 593 (2004) 82, Sachrajda, Villadoro PLB 609 (2005) 73, Flynn, J., Sachrajda PLB 632 (2006) 313

$$E_{\pi^\pm} = \sqrt{m_{\pi^\pm}^2 + (\vec{n}\frac{2\pi}{L})^2}$$

$$E_{\pi^\pm} = \sqrt{m_{\pi^\pm}^2 + (\vec{n}\frac{2\pi}{L} + \frac{\vec{\theta}_u - \vec{\theta}_d}{L})^2}$$



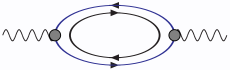


Twisted boundary conditions for the VP

- in flavour-neutral pions the effect of the twist always cancels:

$$E_{q_1 q_2} = \sqrt{m_{12}^2 + \left(\vec{\theta}_2 - \vec{\theta}_1\right)^2}$$

- similarly for the VP



  effect of valence twist cancels

- However, using the decomposition $\langle j^{uu} j^{uu} \rangle = \langle j^{ud} j^{du} \rangle + \langle j^{uu} j^{dd} \rangle$ the connected part can be computed for arbitrary momenta
- interpolate disc. diagram between Fourier-momenta or use EFT prediction

Outline:

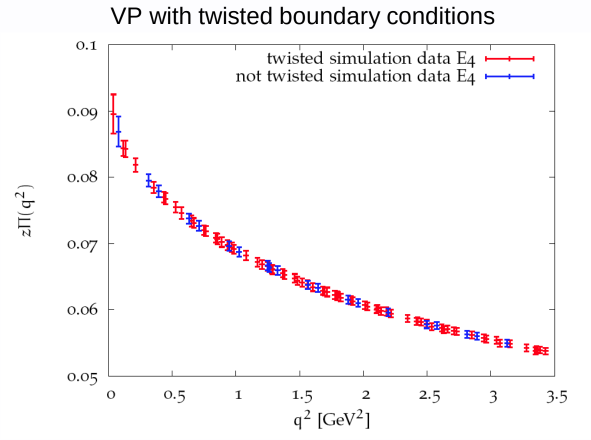
- predicting quark-disconnected diagrams ✓
- improving momentum resolution ✓
- preliminary numerical results

Mainz simulations on CLS lattices

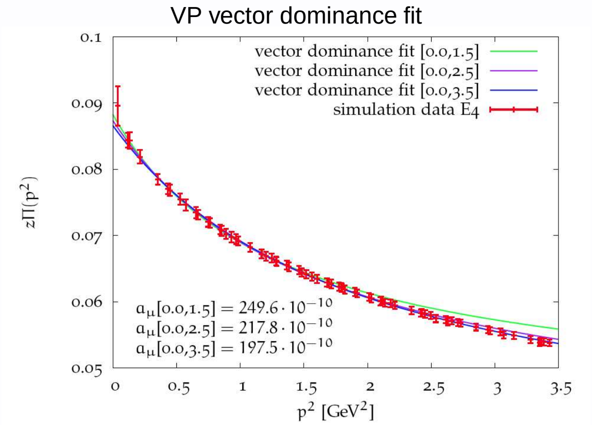
- simulations on Wilson Cluster, Mainz, Germany
- NP improved $N_f = 2$ Wilson fermions
- all simulations and fits on the following pages carried out by Benajmin Jäger as part of his final year thesis
- connected part only
- simulation paramters:

name	κ_{sea}	L [fm]	N_{cfg}	m_π [MeV]	κ_s
D1	0.13550	1.7	104	957.4(10.9)	0.13713
D2	0.13590	1.7	149	696.5(10.3)	0.13632
D3	0.13610	1.7	168	550.5(5.2)	0.13605
D4	0.13620	1.7	168	490.3(5.2)	0.13591
D5	0.13625	1.7	169	428.6(3.7)	0.13574
E2	0.13590	2.2	158	696.5(10.3)	0.13632
E3	0.13605	2.2	156	593.4(1.1)	0.13609
E4	0.13610	2.2	162	550.5(5.2)	0.13605
E5	0.13625	2.2	168	428.6(3.7)	0.13574
F6	0.13635	3.3	200	297.8(0.9)	0.13575

Numerical simulation - preliminary



Numerical simulation - preliminary



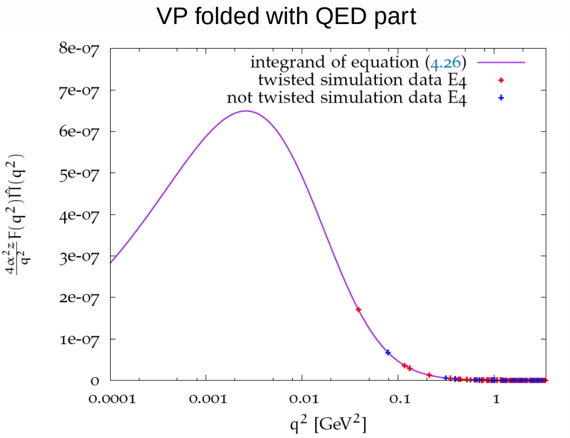
- vector dominance
- model for cross section ratio *Shifman, Vainshtein, Zakharov, Nucl. Phys. B 147 (1979) 448*

$$R(s) = \sum_f e_f^2 \left(A\delta(s - m_V^2) + B\theta(s - s_0) \right)$$

$$\Pi(q^2) = B \ln(a^2 q^2 + a^2 s_0) - \frac{A}{q^2 + m_V^2} + K$$

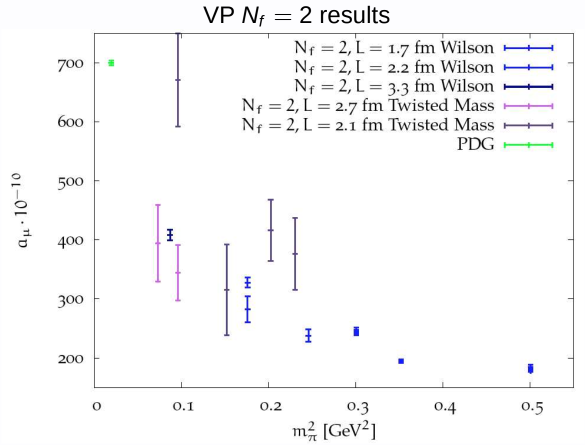
- polynomial
- Padé

Numerical simulation - preliminary



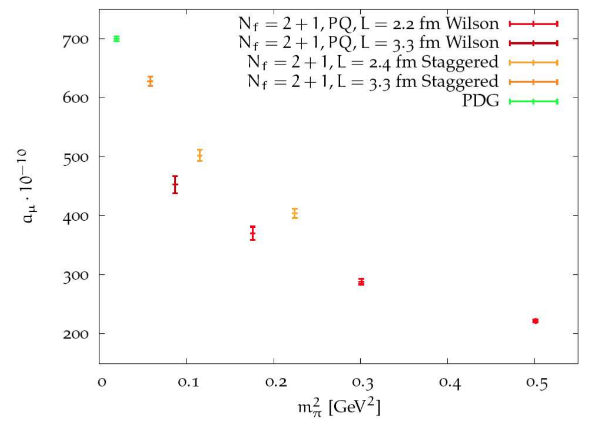
- twisting clearly stabilizes fits
- results from different fit-ansätze spread much less
- spread is taken as estimate for systematic uncertainty from fit

Numerical simualation - preliminary



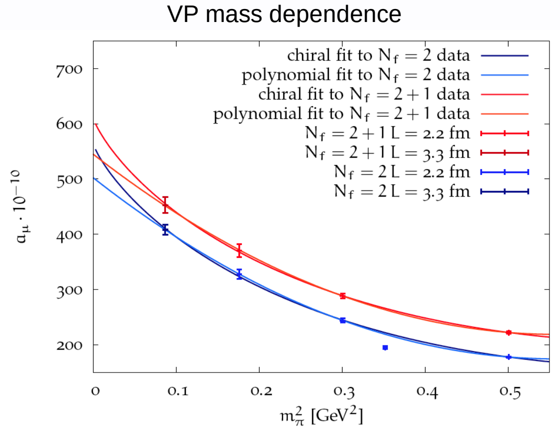
Numerical simulation - preliminary

VP $N_f = 2+1$ results



error bars on Wilson results take into account spread over fit-ansaetze

Numerical simulation - preliminary



- again various ansätze for the extrapolation in m_π^2
- spread used to estimate systematic uncertainty
- $N_f = 2 \rightarrow N_f = 2 + 1^{PQ}$ lifts results (also seen by *T. Blum, C. Aubin, PRD 75, 114502 (2007)* for $N_f = 0 \rightarrow N_f = 2_1$)
- big question mark - ρ

Summary/Outlook

- we have developed new ideas for lattice QCD simulations for a_μ^{LHV}:
  - predict/estimate quark disconnected diagrams
  - improved momentum resolution using partially twisted boundary conditions

- our approach to predicting quark-disconnected diagrams in chiral effective theory is applicable to any quark-multi-linear *n*-pt function

  $\rightarrow$ many applications beyond VP (work on this has been started)

- we will keep on working towards a reliable prediction for a_μ^{LHV} - we are convinced that brute force alone won't lead to the desired precision - this is a interesting playground for interesting field-theoretic questions