High order Wilson loops from Numerical Stochastic Perturbation Theory

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in collaboration with
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Outline

1. Introduction
2. What is NSPT?
3. Results for high-order Wilson loops
   - Loop expansion coefficients $W_{NM}^{(n)}$
   - A model for series summation on finite lattices
   - Boosted perturbation theory
   - Gluon condensate
4. Summary

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Motivation

- Lattice gauge theory provides a promising tool to calculate the non-perturbative gluon condensate $\langle \frac{\alpha}{\pi} G\, G \rangle$ from Wilson loops

  Banks, Horsley, Rubinstein, Wolff (1981), Di Giacomo, Rossi (1981), Kripfganz, Kirschner, Ranft, AS (1982), Ilgenfritz, Müller-Preussker (1982), ...

  precise knowledge of perturbative tail necessary
- Study of the large order behavior of perturbative series on the lattice - factorial behavior or (still) not (see. e.g. investigations of Meurice (2006), Burgio, Di Renzo, Marchesini, Onofri (1998))

$$O \sim \sum_n c_n \lambda^n \,, \lambda : \text{generic coupling}$$

Widely believed: perturbative QCD is an asymptotic theory factorial growth of the coefficients

$$c_n \overset{?}{\sim} C_1\, C_2^n\, \Gamma(n + C_3)$$

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- Used framework for high loop calculations in lattice perturbation theory: NSPT
- Investigation of high-order perturbative Wilson loops of different sizes
  Test up to 20'th loop order – at least on finite lattices – where a possible set-in of an assumed asymptotic behavior might occur
  Earlier work for high-order plaquette
  10 loops Di Renzo, Scorzato (2001) and up 16 loops Rakow (2005)
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Stochastic quantization and Langevin equation

Starting point of NSPT given by ***stochastic quantization*** Parisi, Wu (1981)

Main ingredients

- Introduction of a *stochastic time t* in a Euclidean field theory

$$\phi(x) \to \phi(x, t)$$

- *Langevin equation* with *Gaussian noise* η

$$\frac{\partial \phi(x,t)}{\partial t} = -\frac{\partial S[\phi]}{\partial \phi(x,t)} + \eta(x,t)$$

$$\langle \eta(x,t)\eta(x',t')\rangle = 2\delta(x-x')\,\delta(t-t')$$

- Solution results in

$$\langle O[\phi_1(x_1,t), \phi_2(x_2,t), \ldots] \rangle_\eta \xrightarrow{t\to\infty} \frac{1}{Z}\int [D\phi]\, O[\phi_1(x_1), \phi_2(x_2), \ldots]\, e^{-S[\phi]}$$

Average on noise converges to average on Gibbs measure

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Langevin equation for lattice QCD

$$\frac{\partial}{\partial t} U_{x,\mu}(t) = \mathrm{i}\left\{ \nabla_{x,\mu} S_G[U] - \eta_{x,\mu}(t) \right\} U_{x,\mu}(t)$$

S_G $SU(3)$ lattice gauge action, $\nabla_{x,\mu}$ left Lie derivative

Solve Langevin equation by discretizing $t = n\,\epsilon$

Solution at next time step $n+1$ in peculiar version of *Euler scheme*

$$U_{x,\mu}(n+1) = \mathrm{e}^{\mathrm{i}\,F_{x,\mu}[U,\eta]}\; U_{x,\mu}(n)\,, \quad F_{x,\mu}[U,\eta] = \epsilon\,\nabla_{x,\mu} S_G[U] + \sqrt{\epsilon}\,\eta_{x,\mu}$$

Within that scheme $U_{x,\mu}$ stays in the group manifold

Non-perturbative application:
Langevin simulations of lattice QCD Zwanziger, Stamatescu, Wolff (1983), Zwanziger, Seiler, Stamatescu (1984), Batrouni et al., 1985,... including stochastic gauge fixing instead of standard Monte Carlo (MC)

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Perturbation theory

Perturbation theory
introduced by means of a *formal* expansion of the U's and F's

$$U_{x,\mu}(n) \to 1 + \sum_{l>0} \beta^{-l/2} U^{(l)}_{x,\mu}(n)\,, \quad \beta^{-1/2} = g/\sqrt{2N_c}$$

The Langevin equation at finite $\varepsilon = \beta\epsilon$ transforms into a hierarchical system of updates for each order $U^{(l)}$

$$\begin{aligned} U^{(1)}(n+1) &= U^{(1)}(n) - F^{(1)}(n) \\ U^{(2)}(n+1) &= U^{(2)}(n) - F^{(2)}(n) + \frac{1}{2}(F^{(1)}(n))^2 - F^{(1)}(n)\,U^{(1)}(n) \\ &\dots \end{aligned}$$

The system is than truncated to $l_{\max}$ and numerically integrated:
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Practice of NSPT

- choose maximal loop order number $l_{max}/2$
  $\rightarrow$ keep in storage l_{max} link fields at given Langevin step
- For stabilization add stochastic gauge fixing and zero mode subtraction to update
- Construct Wilson loops W_{NM} of size $N \times M$ from expanded $U^{(l)}$

$$W_{NM}(n^\star) = 1 + \sum_{n=1}^{n^\star} W_{NM}^{(n)} (g^2)^n$$

- Perform limits:
  long Langevin trajectories in equilibrium
  runs at several ε and extrapolation to $\varepsilon \rightarrow 0$
  $V \rightarrow \infty$ (?)
- Computation on Linux/HP-clusters (Leipzig), at HLRN (Hannover-Berlin), NEC SX-9 of RCNP (Osaka)

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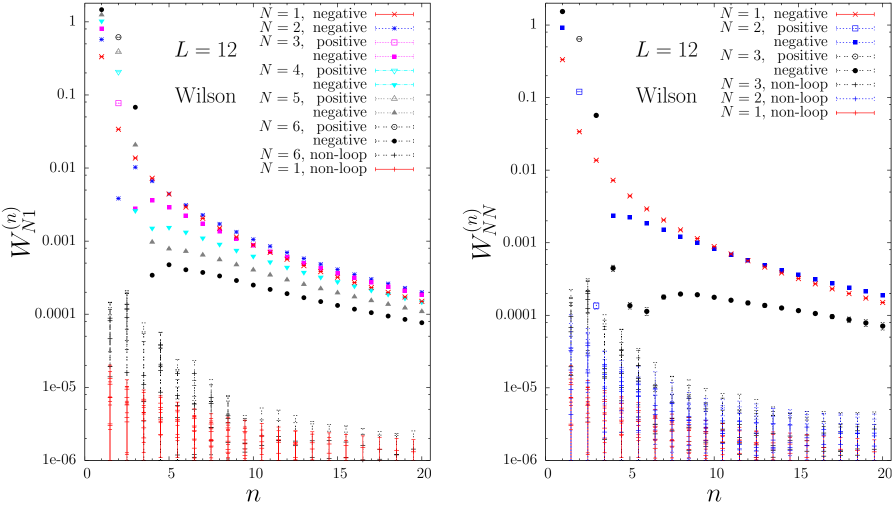
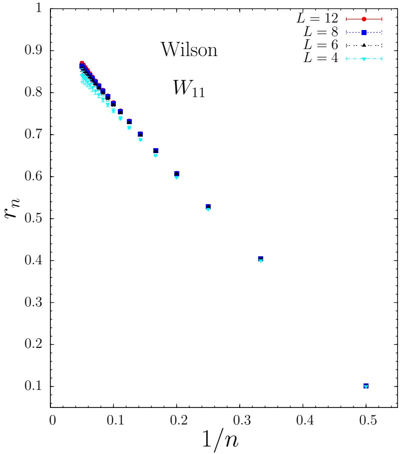
Expansion coefficients: signal/noise ratio



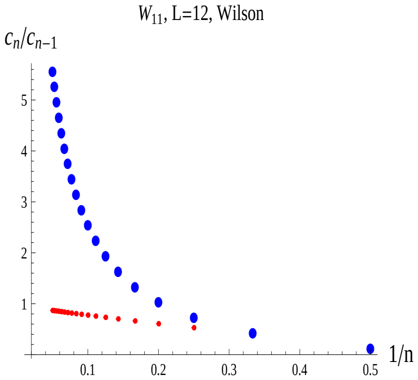
Series behavior of data $r_n = c_n / c_{n-1}$

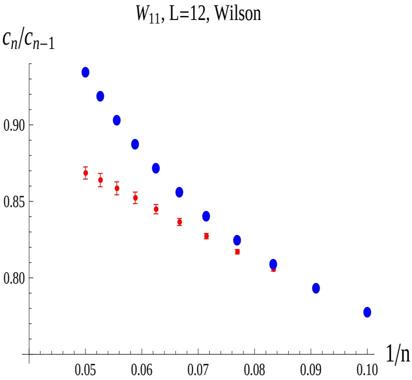
Coefficients c_n from $W(g^2) = \sum c_n \, (g^2)^n$ (for each (N, M))

No sign of dramatic change with lattice volume for W_{11}



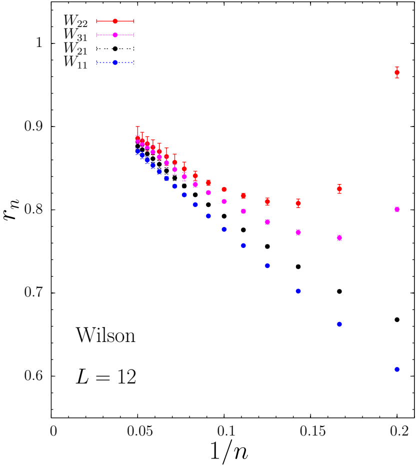
Data and (absence of ?) factorial behavior





Red: NSPT data
Blue: Factorial behavior: starting with n=2 (left), n=10 (right)
Up to loop-order $n = 20$ no factorial behavior found!

Series behavior for different loop sizes



Hypergeometric model

- Ratio r_n can be surprisingly well described for moderate Wilson loop sizes by

$$r_n = \frac{c_n}{c_{n-1}} = u\left(1 - \frac{1+\gamma}{n}\right) + \frac{p}{n(n+s)}, \quad n > n_0$$

$$u\left(1 - \frac{1+\gamma}{n}\right) \rightarrow W(g^2) \sim (1 - u\, g^2)^{\gamma}$$

  in purple: describe curvature for lower loop number n

- Convergence radius $g^2 < 1/u$
  $\rightarrow$ summable series (hypergeometric function)

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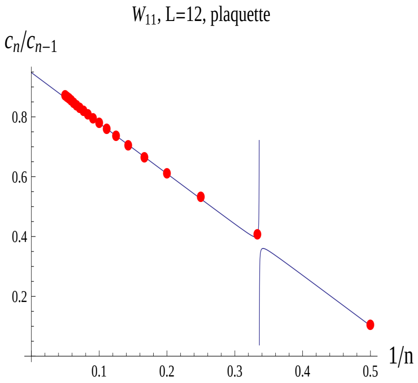
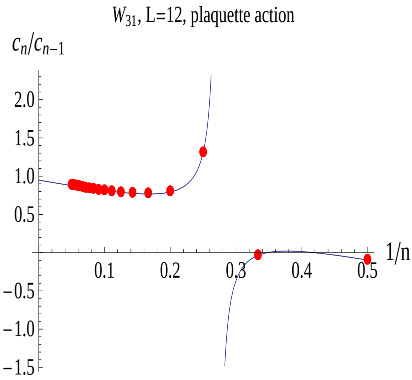
Example Domb-Sykes plots

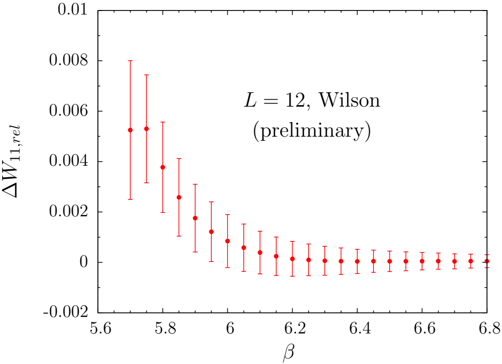




Wilson loops and weak coupling regime

Question: To what extend are Wilson loops dominated by perturbation theory?

$$\Delta W_{11,rel}(\beta) = \left| \frac{W_{11,MC} - W_{11,PT}}{W_{11,MC}} \right| (\beta)$$



Boosted perturbation theory

- Bare lattice coupling g^2 - bad expansion parameter
- Use instead boosted coupling

$$g_b^2 = \frac{g^2}{W_{11,\mathrm{pert}}}$$

with $W_{11,\mathrm{pert}}$ from NSPT

- Reordering of perturbative coefficients $c_n \to c_n^{(b)}$

$$\left.\begin{array}{l} g_b^2 > g^2 \\ |c_n^{(b)}| \ll |c_n| \end{array}\right\} \text{improved convergence behavior}$$

- First successful application: Rakow (2005)
- Further advantage of boosted PT: no model assumption necessary

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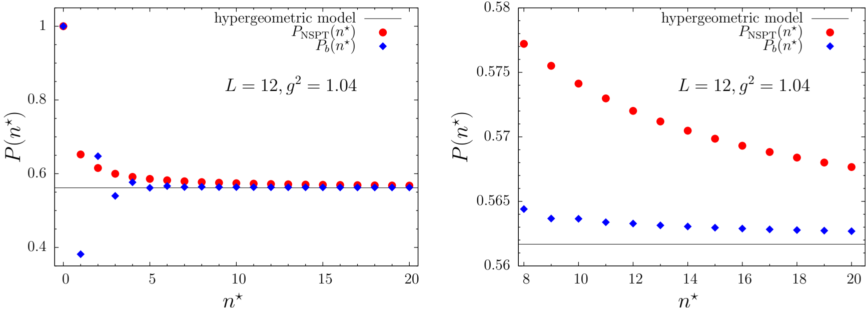
Example plot for $P \equiv W_{11}$



$P(n^\star)$: perturbative series summed up to $n^\star$
Chosen coupling g^2 at convergence limit

Gluon condensate ?

- $\langle GG \rangle$ (introduced by SVZ) is a dimensionful OPE quantity
- $\rightarrow$ on the lattice one expects

$$a^4 \langle GG \rangle \sim \Delta P(n^\star) = P_{MC} - P_{PT}(n^\star) \propto c_4\, a^4$$

($n^\star$: order of lattice perturbation theory)

- $a \leftrightarrow \beta$
  use MC data at different β
  choose different lattice sizes to stay in confinement
- Narison/Zakharov conjecture (2009):

$$\Delta P(n^\star) \propto c_2(n^\star)\, a^2 + c_4(n^\star)\, a^4$$

$c_2(n^\star)\, a^2$ due to small $n^\star$
$c_2(n^\star) \rightarrow 0$ for large $n^\star$

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$$a^4 \langle GG \rangle \sim \Delta P(n^\star) = P_{MC} - P_{PT}(n^\star) \propto c_4\, a^4$$

  ($n^\star$: order of lattice perturbation theory)

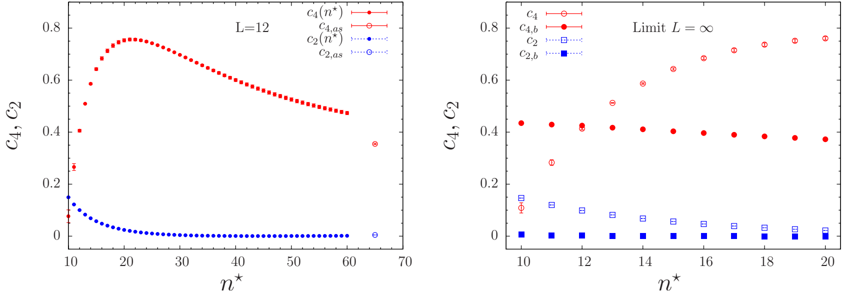
- $a \leftrightarrow \beta$
  use MC data at different β
  choose different lattice sizes to stay in confinement
- Narison/Zakharov conjecture (2009):

$$\Delta P(n^\star) \propto c_2(n^\star)\, a^2 + c_4(n^\star)\, a^4$$

  $c_2(n^\star)\, a^2$ due to small $n^\star$
  $c_2(n^\star) \rightarrow 0$ for large $n^\star$

Dominance of dimension 4 operator ?

- c_4 and c_2 as function of maximal loop number



- Naive LPT: $n^\star \leq 20$: NSPT data, $n^\star > 20$: hypergeometric model series expansion
- $c_{i,as}$: values for the total sum of hypergeometric model
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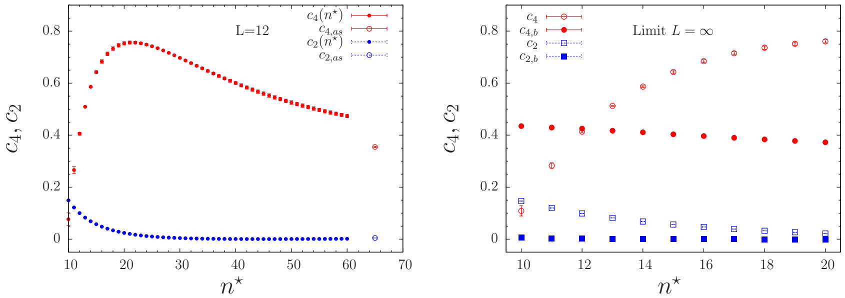

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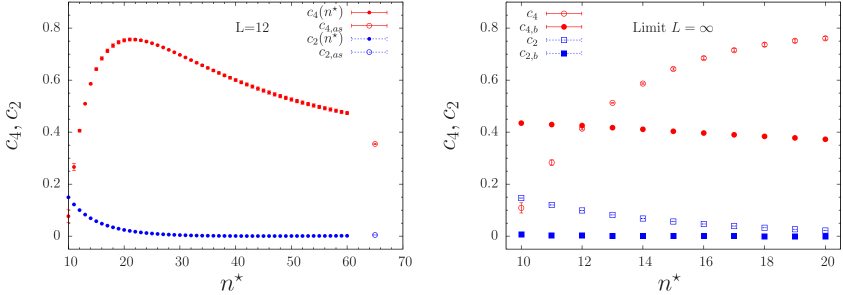

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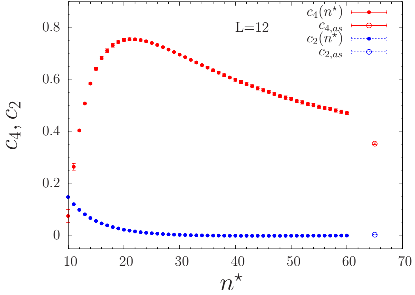

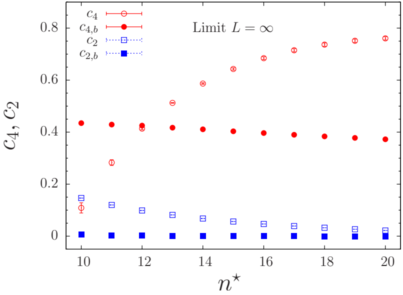

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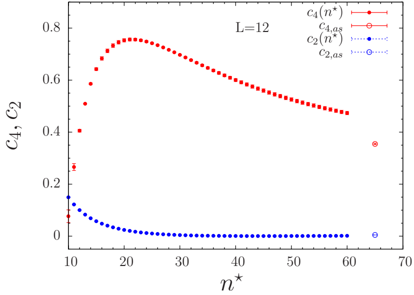

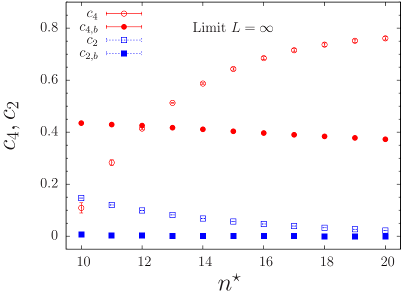

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$$a^4 \frac{\pi^2}{36} \frac{b_0 g^2}{\beta(g)} \langle \frac{\alpha}{\pi} GG \rangle = \Delta P$$

Systematic uncertainties:

- Naive PT with model for summation or **boosted** PT
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  perturbative plaquette at $L = \infty$ extrapolation from boosted perturbation theory
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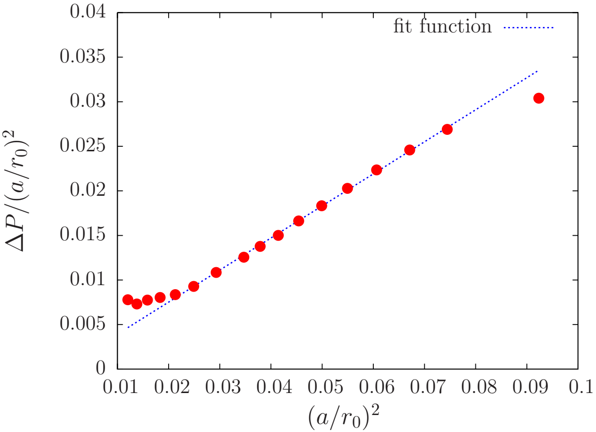

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($r_0 = 0.467\,\mathrm{fm}$)

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