Coupled-channel complex scaling法を用いたΛ(1405)の研究

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1. **Introduction**
   - *Recent status of theoretical study of K^-pp*

2. **Application of ccCSM to Λ(1405)**
   - *Coupled-channel complex scaling method (ccCSM)*
   - *Energy-independent $K^{bar}N$ potential*

3. **ccCSM with an energy-dependent $K^{bar}N$ potential for Λ(1405)**

4. **Summary and Future plan**

KEK理論センター研究会『ストレンジネス核物理 2010』
'10.12.03 @ KEK 小林ホール

1. Introduction

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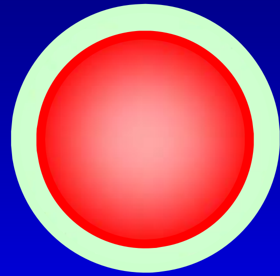
K^{bar} *nuclei* = *Exotic system !?*



I=0 $K^{bar}N$ potential … very attractive

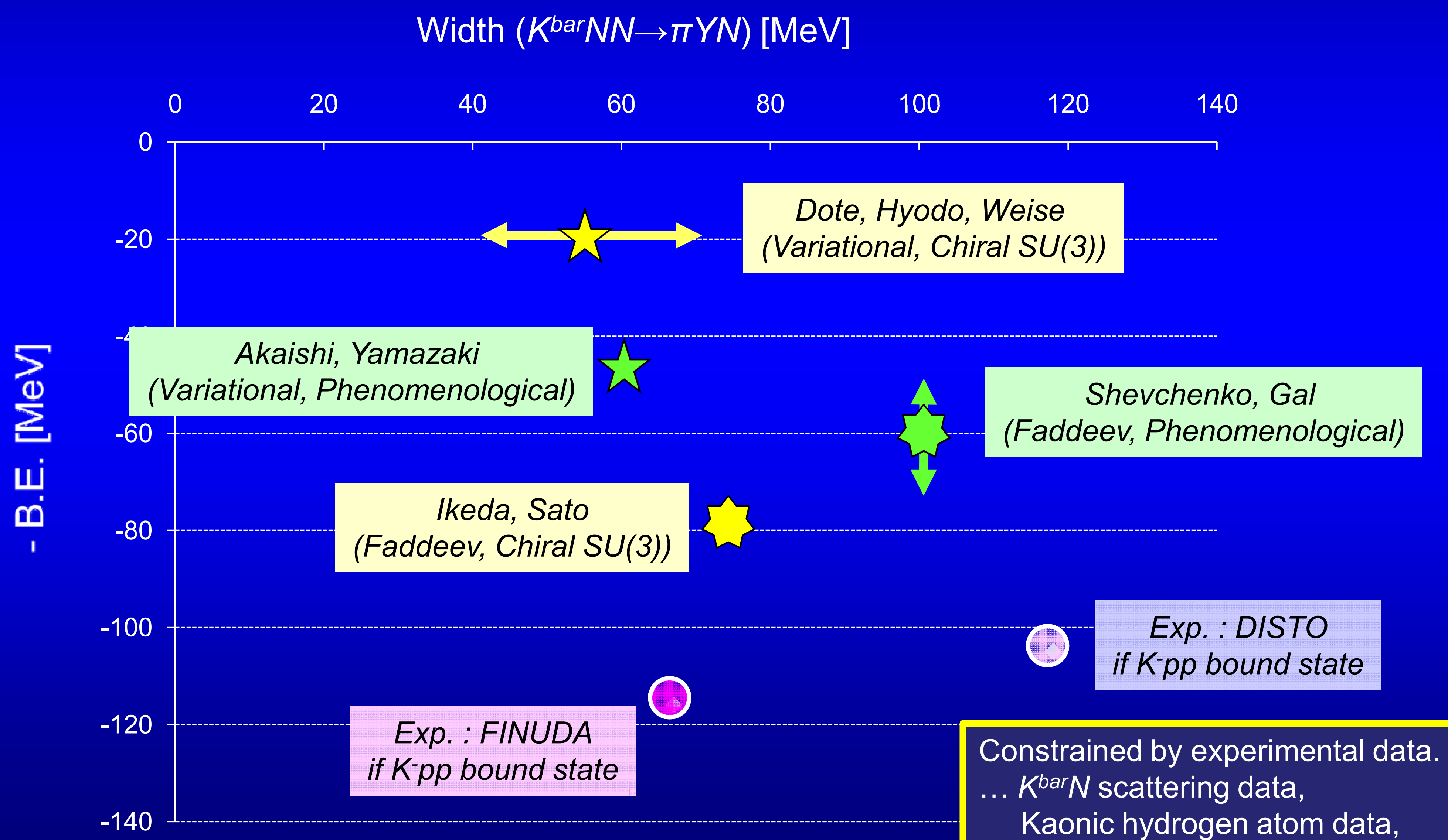
→ *Highly dense state formed in a nucleus*
Interesting structures that we have never seen in normal nuclei…

- *Recently, ones have focused on*

$K^{-}pp$= *Prototye of* K^{bar} *nuclei*

1. Introduction

Recent results of calculation of K⁻pp and related experiments



Constrained by experimental data.
… $K^{bar}N$ scattering data,
Kaonic hydrogen atom data,
"Λ(1405)" etc.

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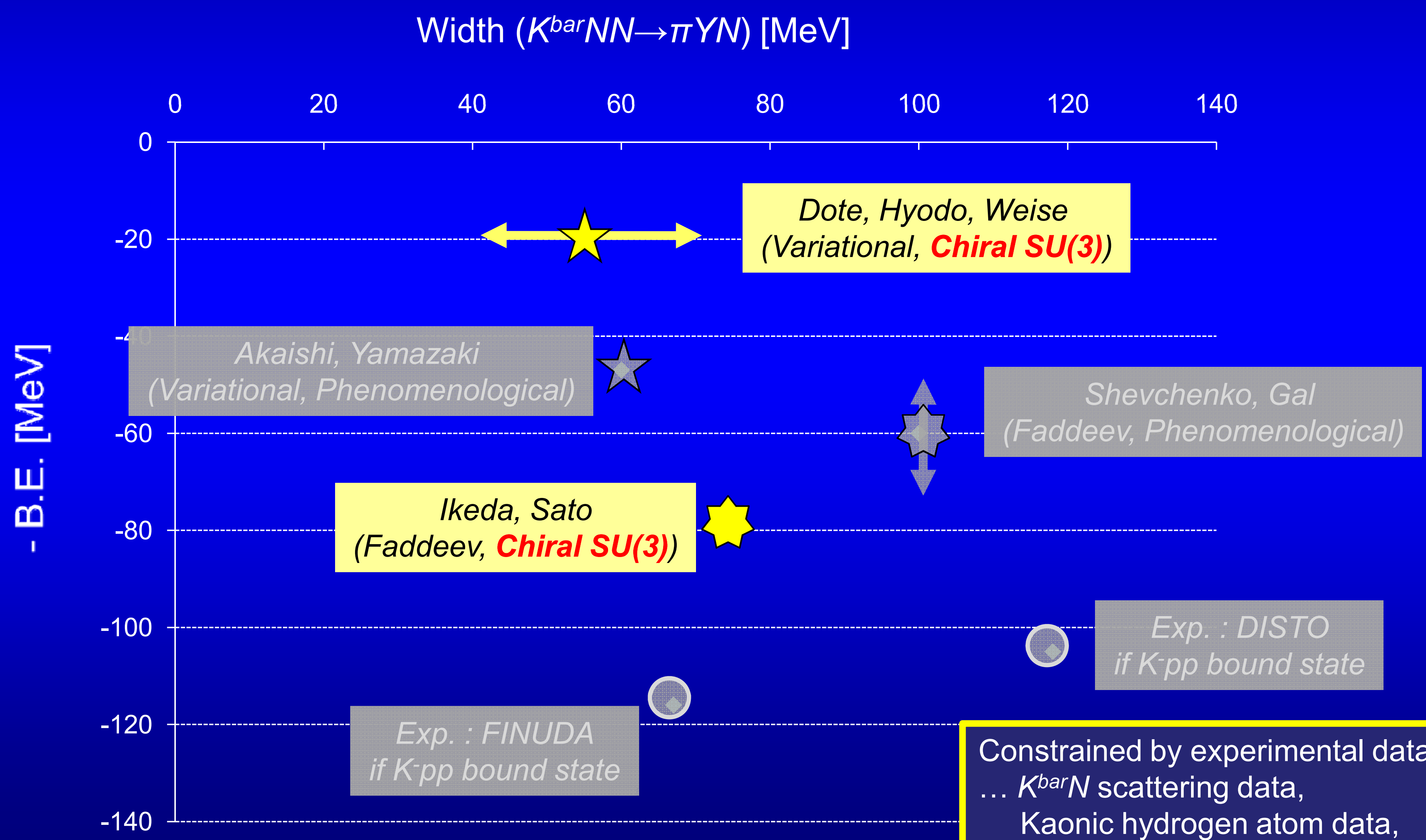

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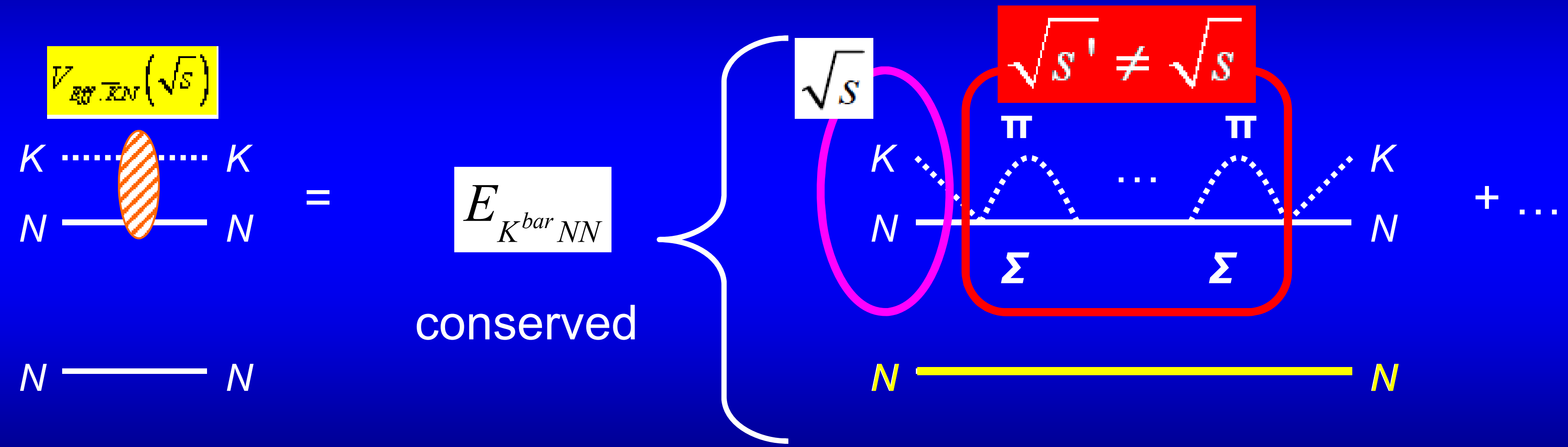
1. Introduction

Recent results of calculation of K^-pp and related experiments

Width ($K^{bar}NN \to \pi YN$) [MeV]

πΣN thee-body dynamics

Three-body system calculated with the effective $K^{bar}N$ potential



-120

-140

Exp. : FINUDA
if K^-pp bound state

Constrained by experimental data.
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Kaonic hydrogen atom data,
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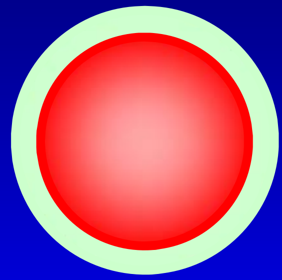
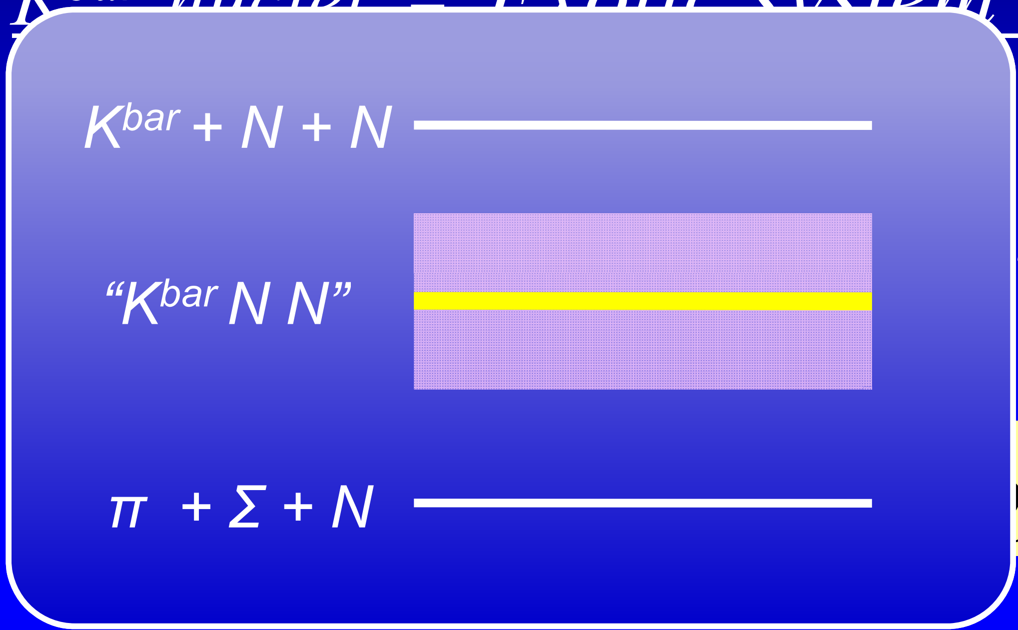
1. Introduction

K^{bar} nuclei = Exotic system !?





…d in a nucleus

…t we have never seen in normal nuclei…

…f K^{bar} nuclei

- In the study of K^{-}pp, it was pointed out that the

πΣN three-body dynamics might be important.

- Based on the variational approach, and explicitly treating the πΣN channel, we try to investigate K^{bar}NN-πΣN resonant state with …

coupled-channel Complex Scaling Method

Kaonic nuclei sdtudied with Complex Scaling Method

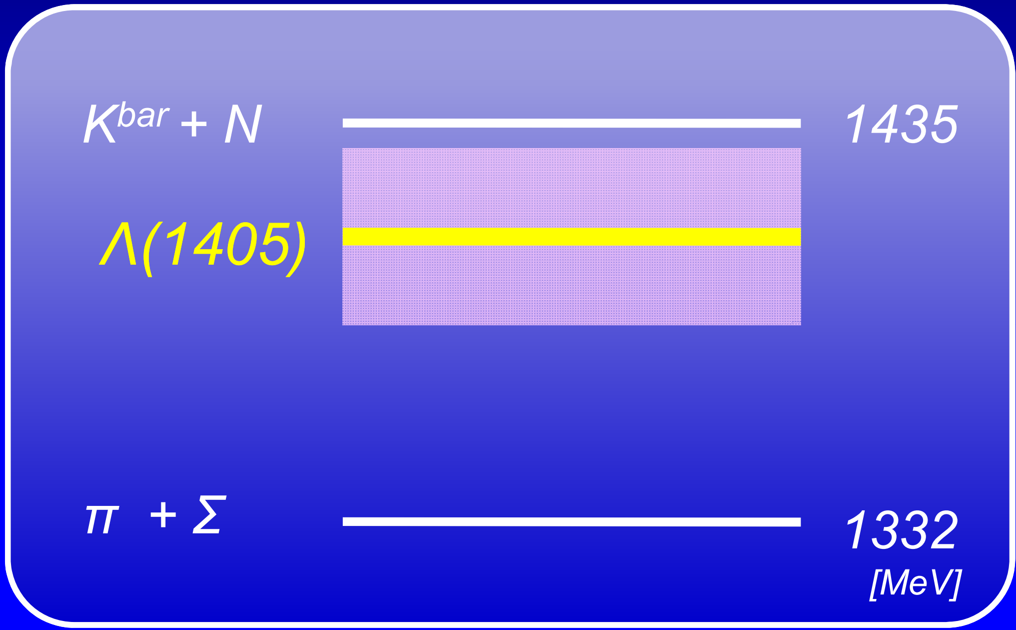
Before K^-pp, …

$\Lambda(1405)$: $I=0$ quasi-bound state of K^-p
... two-body system

2. Application of CSM to Λ(1405)

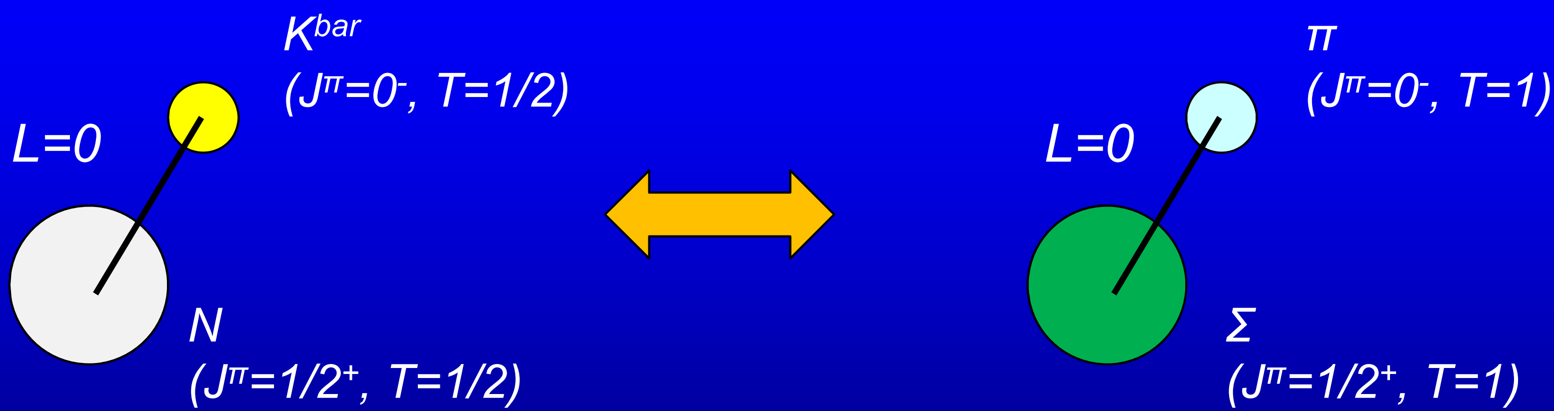
- Coupled-channel Complex Scaling Method (ccCSM)
- Energy-independent $K^{bar}N$ potential

Λ(1405) with c.c. Complex Scaling Method



B. E. (K^{bar}N) = 27 MeV
Γ (πΣ) ~ 50 MeV

J^{π} = $1/2^-$
I = 0



K^{bar}N-πΣ coupled system with s-wave and isospin-0 state

Λ(1405) with c.c. Complex Scaling Method

Schrödinger equation to be solved

$$\hat{H}\,|\Phi_{\bar{K}N-\pi\Sigma}\rangle = E\,|\Phi_{\bar{K}N-\pi\Sigma}\rangle$$

$$\hat{H} = \sum_{\alpha}\Delta M_{\alpha} + \frac{\hat{\mathbf{p}}^2}{2\mu_{\alpha}} + \hat{V}$$

$m_K = 496$ MeV, $M_N = 939$ MeV
$m_\pi = 138$ MeV, $M_\Sigma = 1193$ MeV

$$\mu_\alpha = \frac{m_{\alpha 1} m_{\alpha 2}}{m_{\alpha 1} + m_{\alpha 2}}$$

$$\Delta M_\alpha = \begin{cases} 0 & \text{for } \alpha = \pi\Sigma \\ (m_K + M_N) - (m_\pi + M_\Sigma) & \text{for } \alpha = K^{bar}N \end{cases}$$

$\hat{V}$:

- Phenomenological potential
  = ***Energy independent potential***
  Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005
- Chiral SU(3) potential
  = ***Energy dependent potential***
  N. Kaiser, P. B. Siegel and W. Weise, NPA 594, 325 (1995)

Wave function expanded with *Gaussian base*

$$|\Phi_{\bar{K}N-\pi\Sigma}\rangle = \varphi_{\bar{K}N}(r)\,|\bar{K}N\rangle + \varphi_{\pi\Sigma}(r)\,|\pi\Sigma\rangle$$

$$\varphi_\alpha(r) = \sum_{p=1}^{N_\alpha} C_p^\alpha \exp\left[-\left(r/b_p^\alpha\right)^2\right]$$

$$b_p^\alpha = b_{ini}^\alpha \times \left(\frac{b_{fin}^\alpha}{b_{ini}^\alpha}\right)^{\frac{p-1}{N_\alpha - 1}}$$

$\{C_p^\alpha\}$: ***complex*** parameters to be determined

➡ ***Complex-rotate $\hat{H}$, then diagonalize $\hat{H}_\theta$ with Gaussian base.***

Λ(1405) with c.c. Complex Scaling Method

Schrödinger equation to be solved

$$\hat{H}\,|\Phi_{\bar{K}N-\pi\Sigma}\rangle = E\,|\Phi_{\bar{K}N-\pi\Sigma}\rangle$$

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$m_K = 496$ MeV, $M_N = 939$ MeV
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$$\Delta M_\alpha = \begin{cases} 0 & \text{for } \alpha = \pi\Sigma \\ \ldots + M_\Sigma) & \text{for } \alpha = K^{bar}N \end{cases}$$

$\hat{V}$

... and T. Yamazaki, ... (2002) 044005

... iegel and W. Weise, ... 325 (1995)

Complex rotation of coordinate

$$U(\theta):\quad \mathbf{r} \to \mathbf{r}\,e^{i\theta},\quad \mathbf{k} \to \mathbf{k}\,e^{-i\theta}$$

$$H_\theta \equiv U(\theta)\,H\,U^{-1}(\theta),\quad |\Phi_\theta\rangle \equiv U(\theta)|\Phi\rangle$$

Wave fu...

$|\Phi_{K}$ ...

ABC theorem

The energy of bound and resonant states is independent of scaling angle *θ*.

J. Aguilar and J. M. Combes, Commun. Math. Phys. 22 (1971),269
E. Balslev and J. M. Combes, Commun. Math. Phys. 22 (1971),280

⇨ ***Complex-rotate H, then diagonalize H_θ with Gaussian base.***

2. Application of CSM to Λ (1405)

- Coupled-channel Complex Scaling Method (ccCSM)
- Energy-independent $K^{bar}N$ potential

Phenomenological potential (AY)

Y. Akaishi and T. Yamazaki, PRC 52 (2002) 044005

Energy-independent potential

$K^{bar}N$ $\pi\Sigma$

$$\hat{V}_{AY,I=0} = \begin{pmatrix} -436 & -412 \\ -412 & 0 \end{pmatrix} \exp\left[-\left(r / 0.66\ \mathrm{fm}\right)^2\right] \quad [\mathrm{MeV}]$$

1. *free $K^{bar}N$ scattering data*
2. *1s level shift of kaonic hydrogen atom*
3. *Binding energy and width of Λ(1405)*

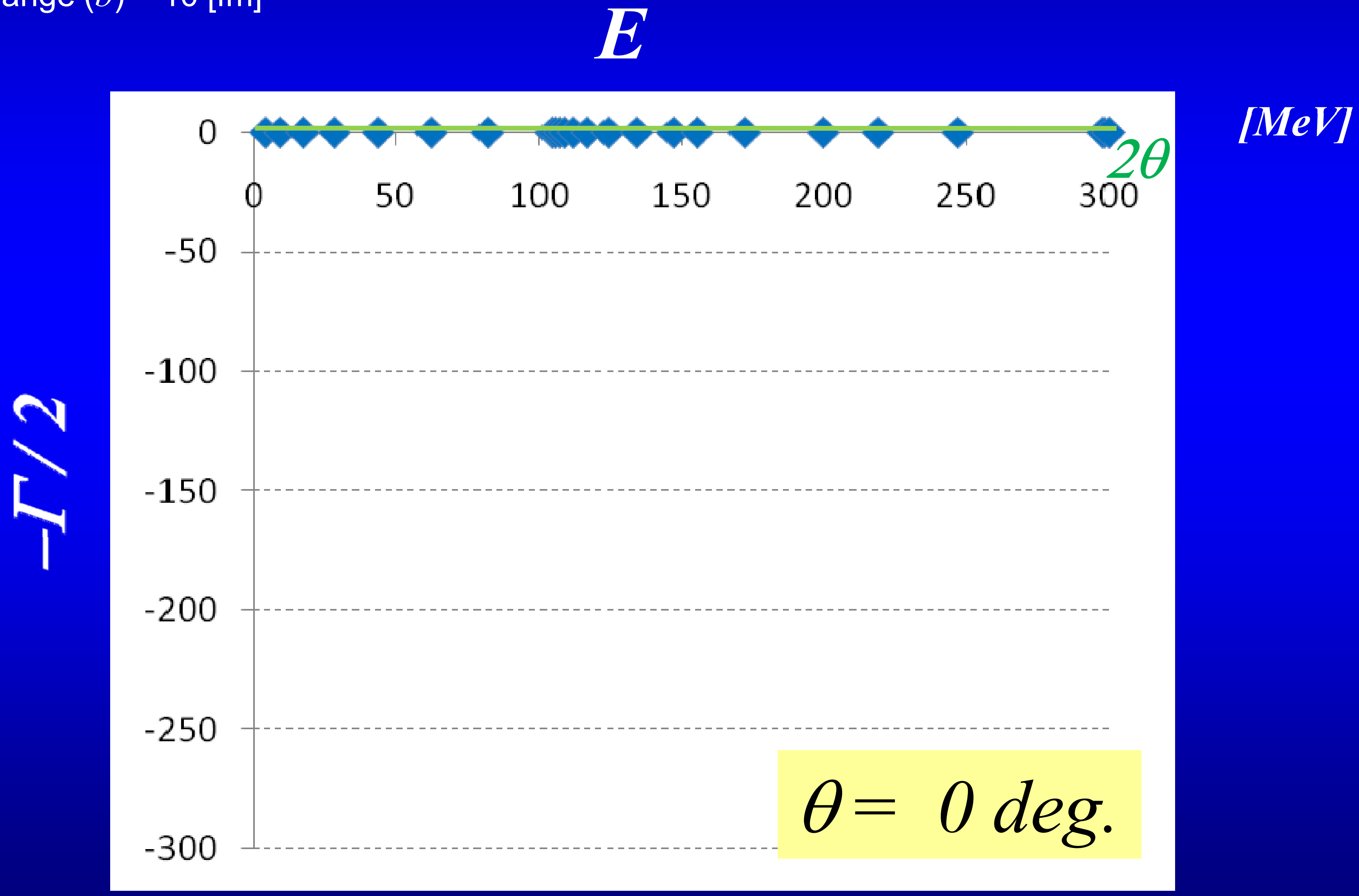
= K^- + proton

※注意:このポテンシャルを作る際に実はccCSMが使われている。
論文には明記されていないが。
なので、このポテンシャルを使用した以下の計算は検証にすぎない。

Λ(1405) with c.c. Complex Scaling Method

θ trajectory

- # Gauss base (n) = 30
- Max range (b) = 10 [fm]



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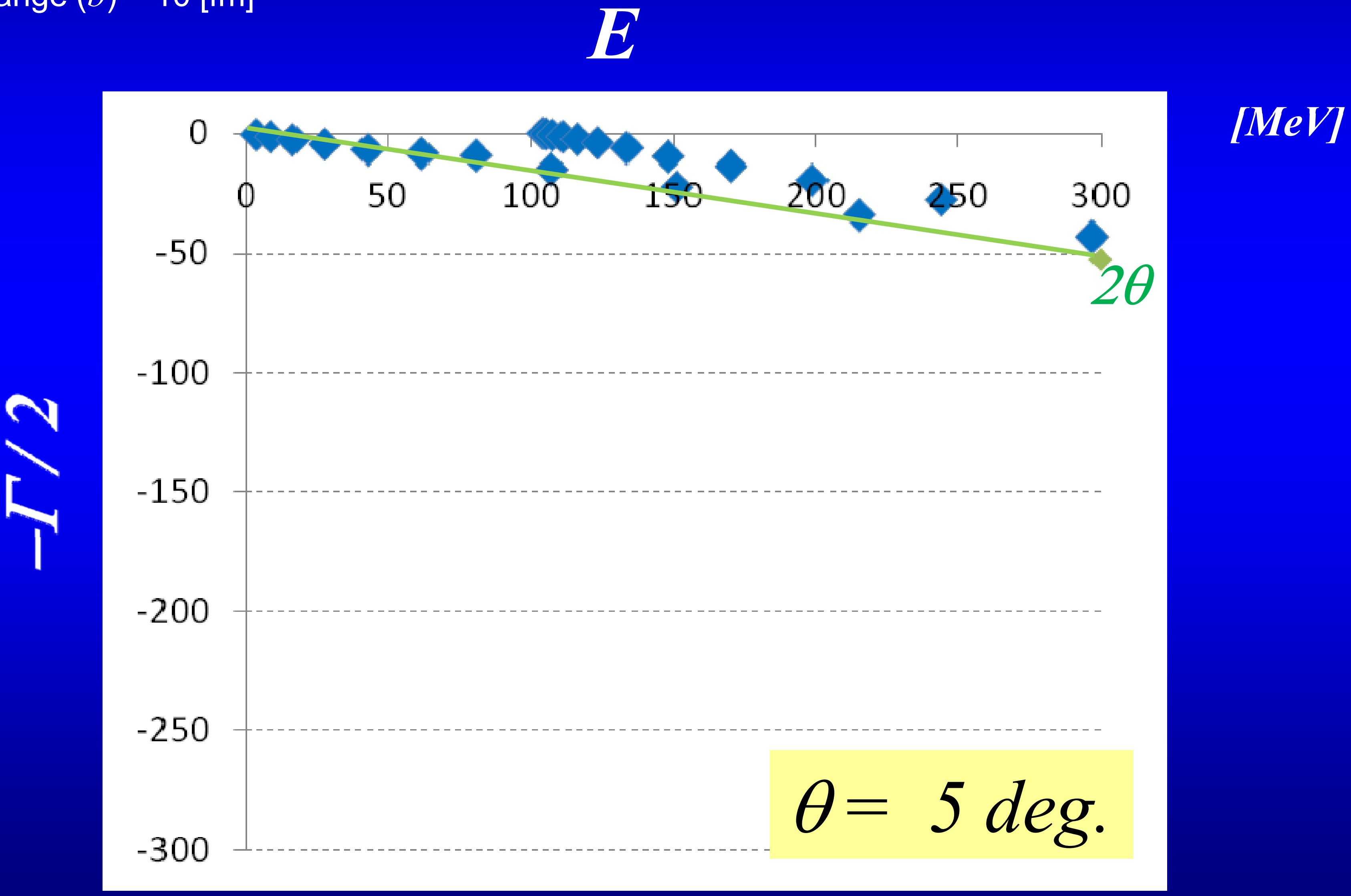

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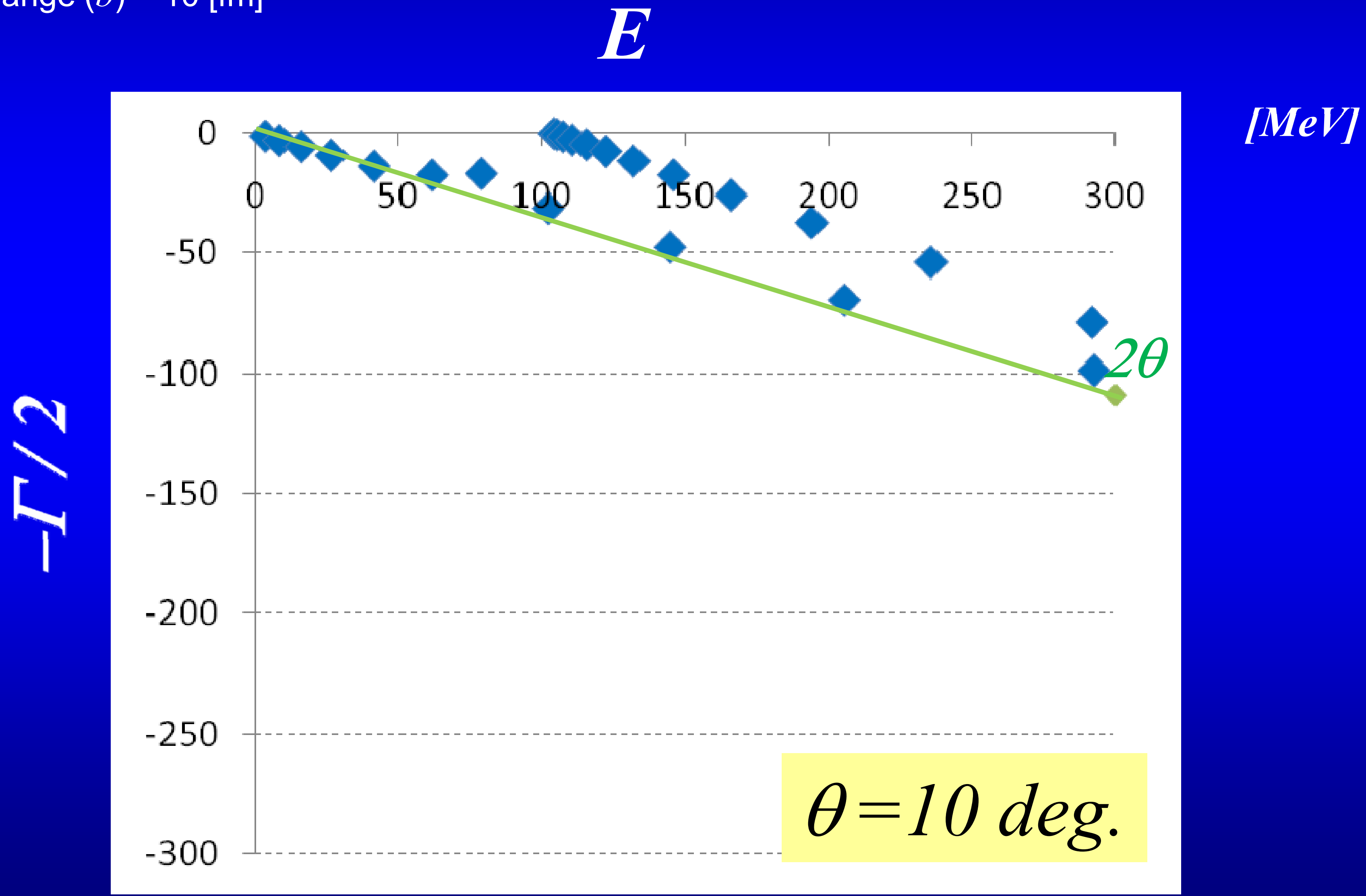

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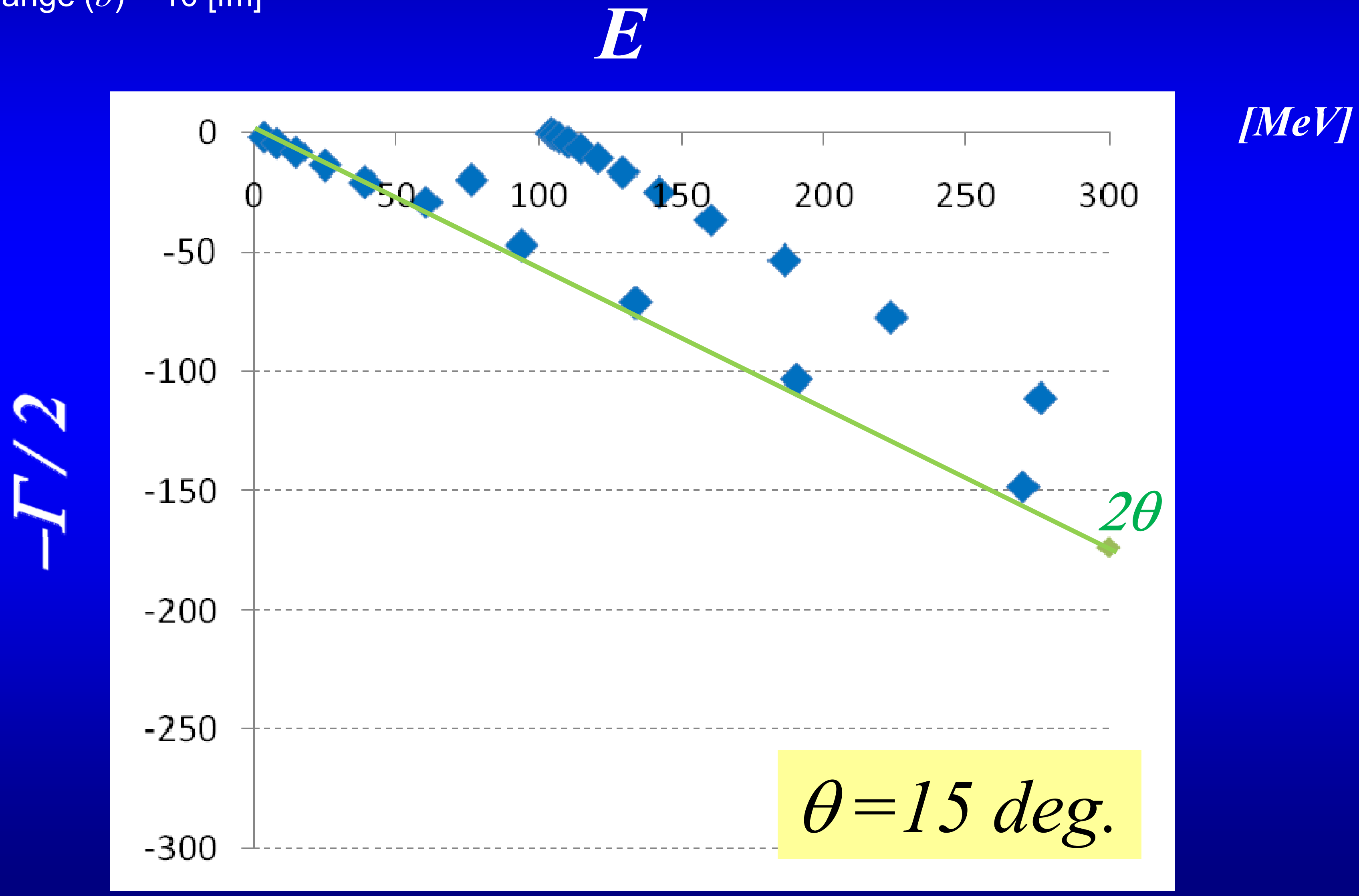

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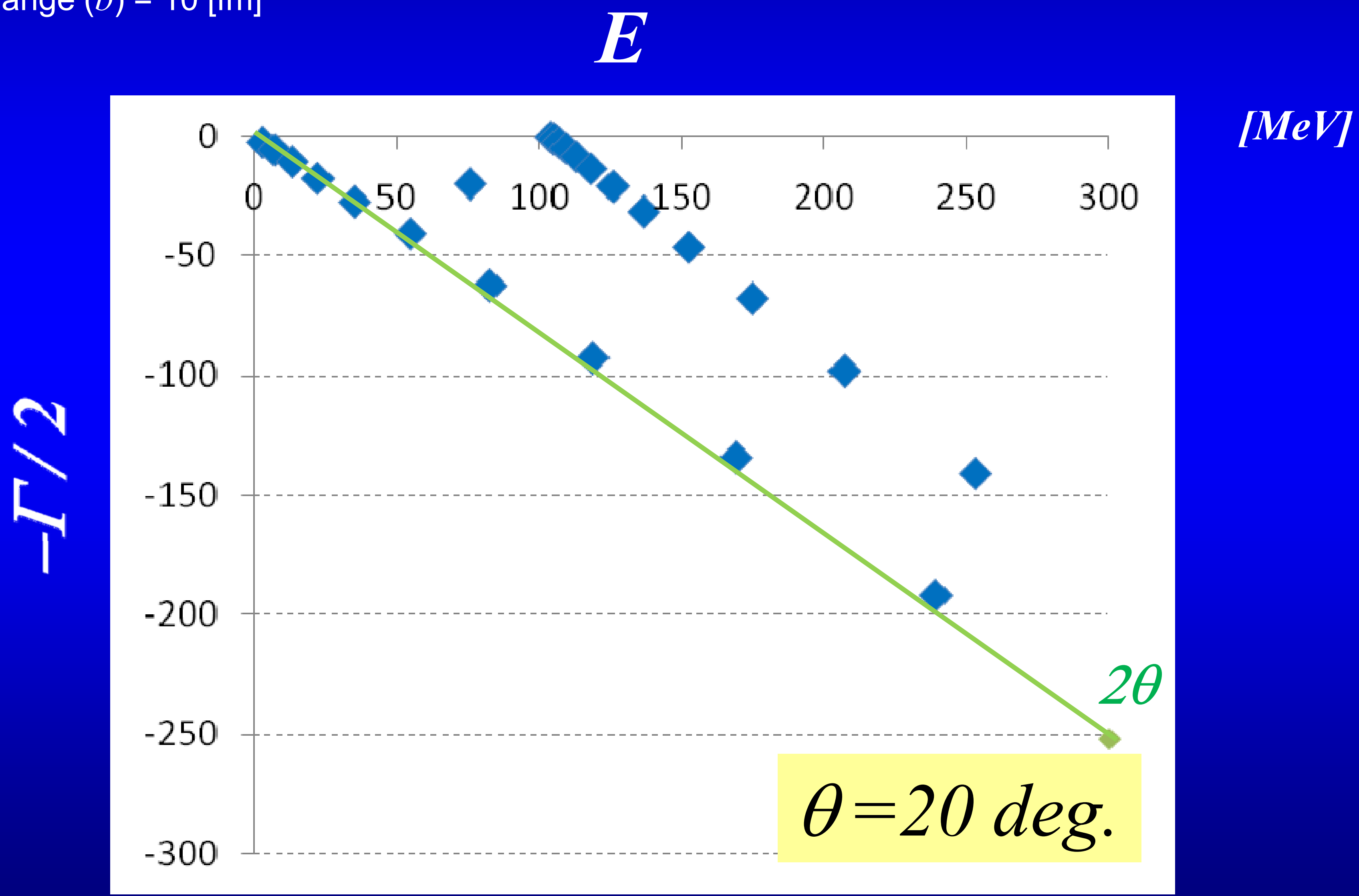

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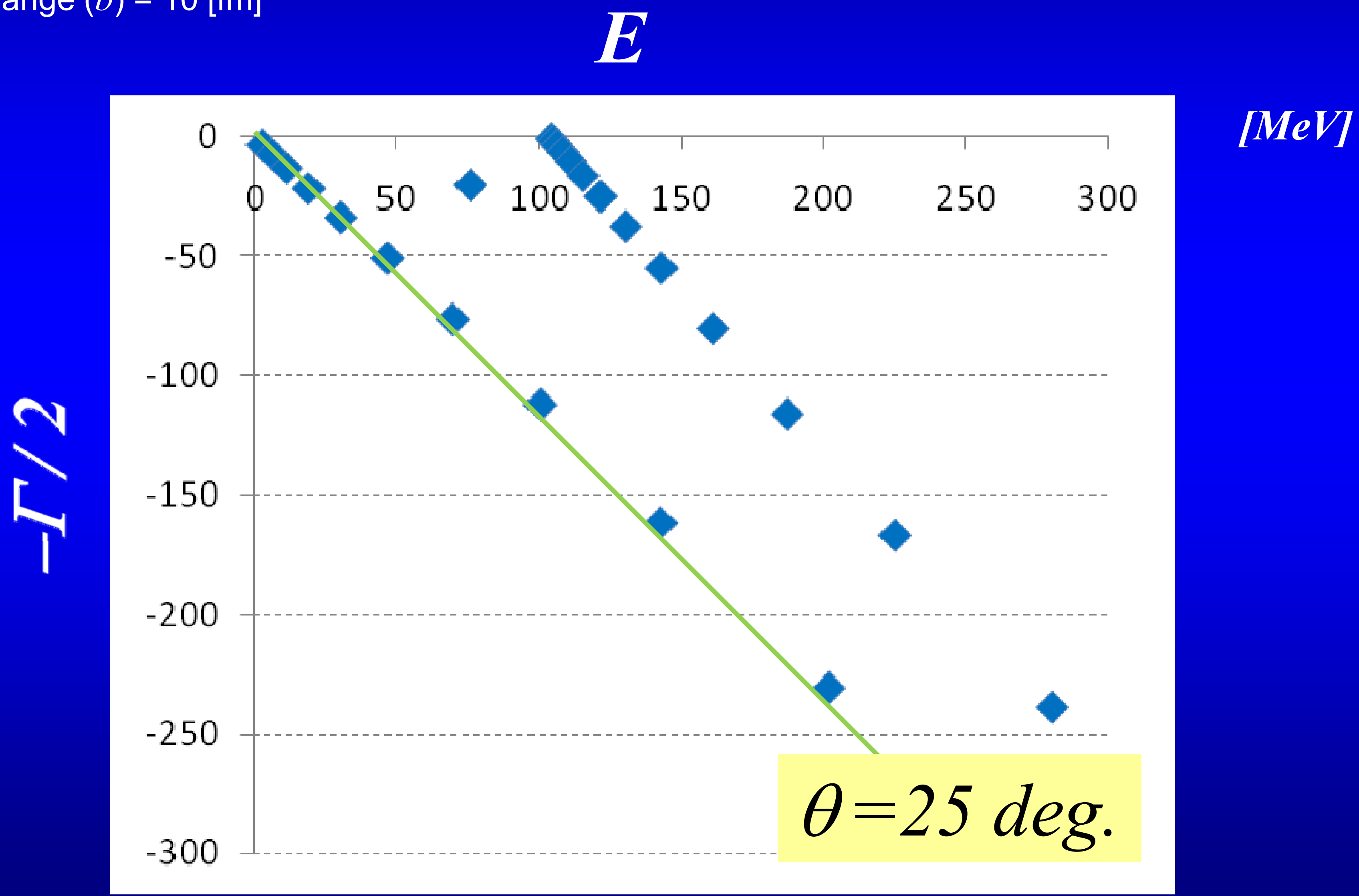

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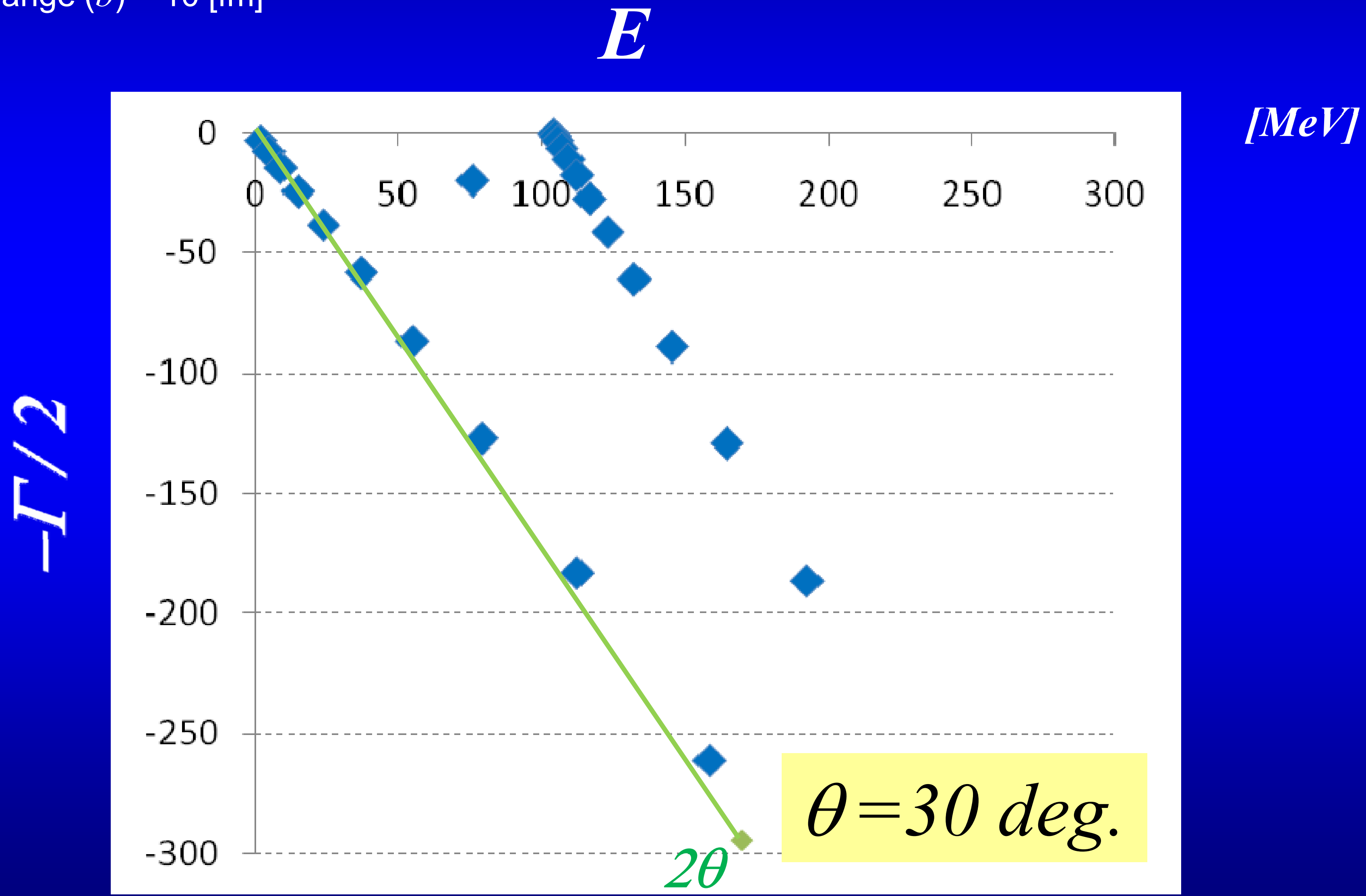

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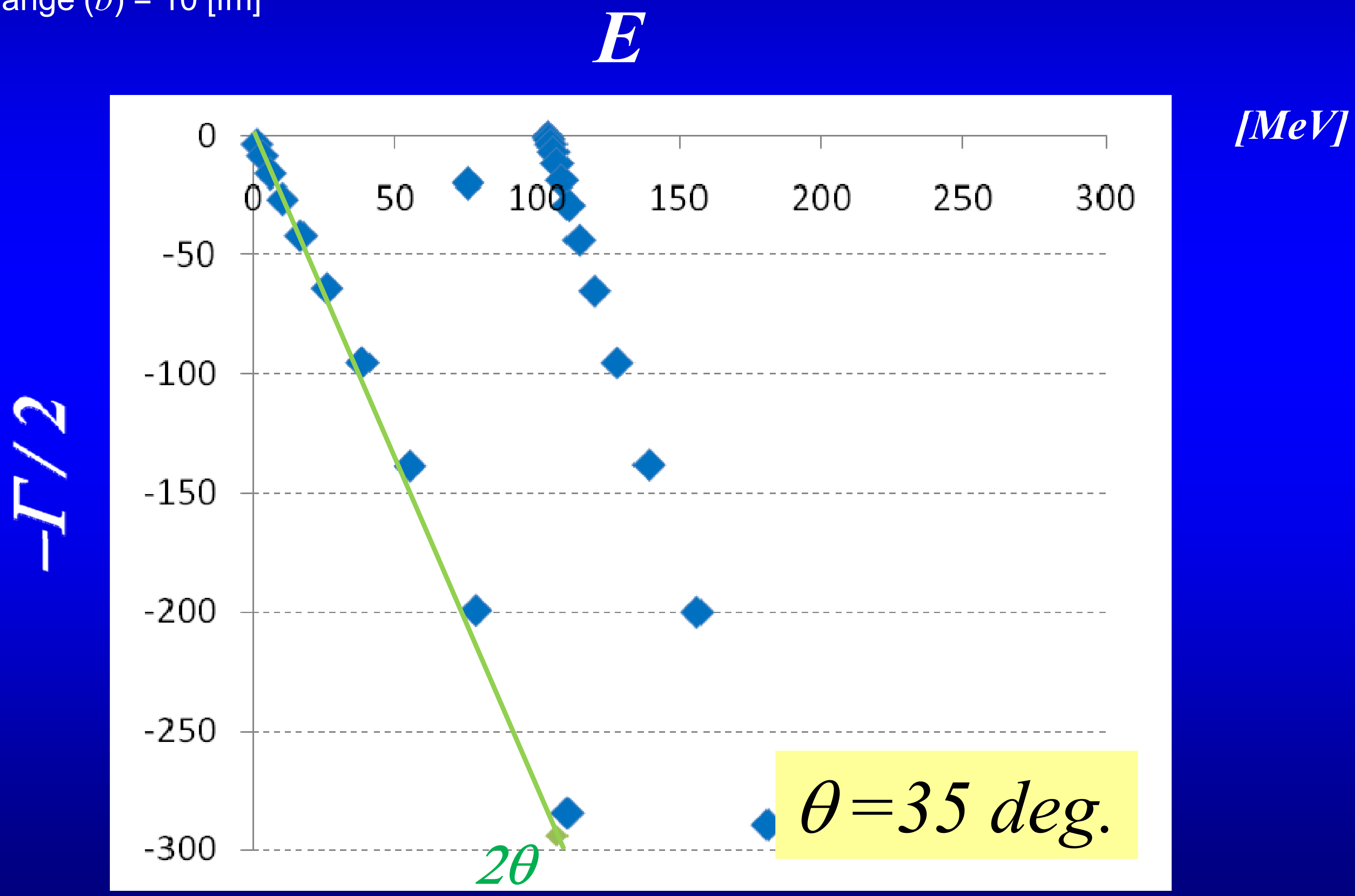

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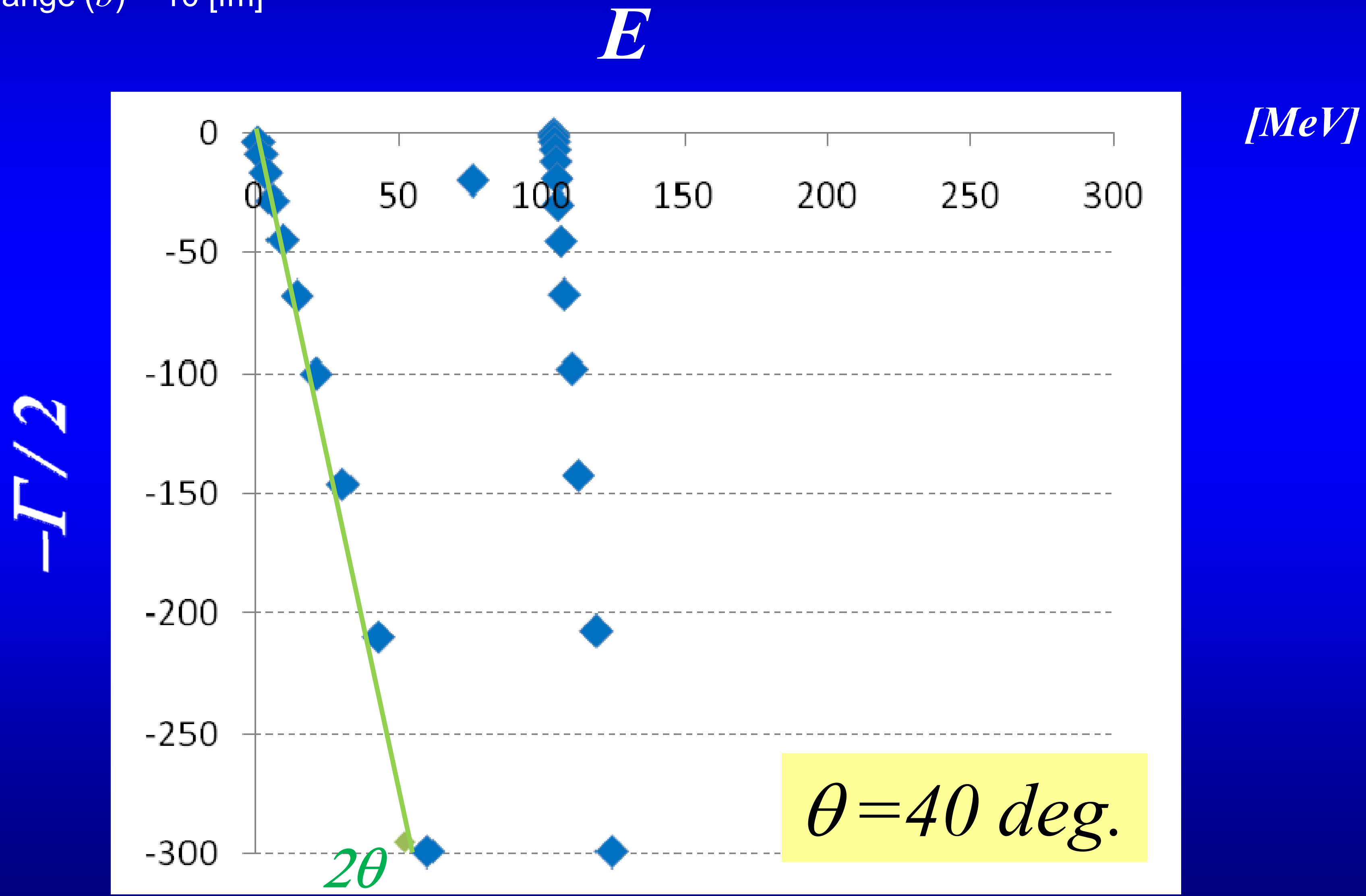

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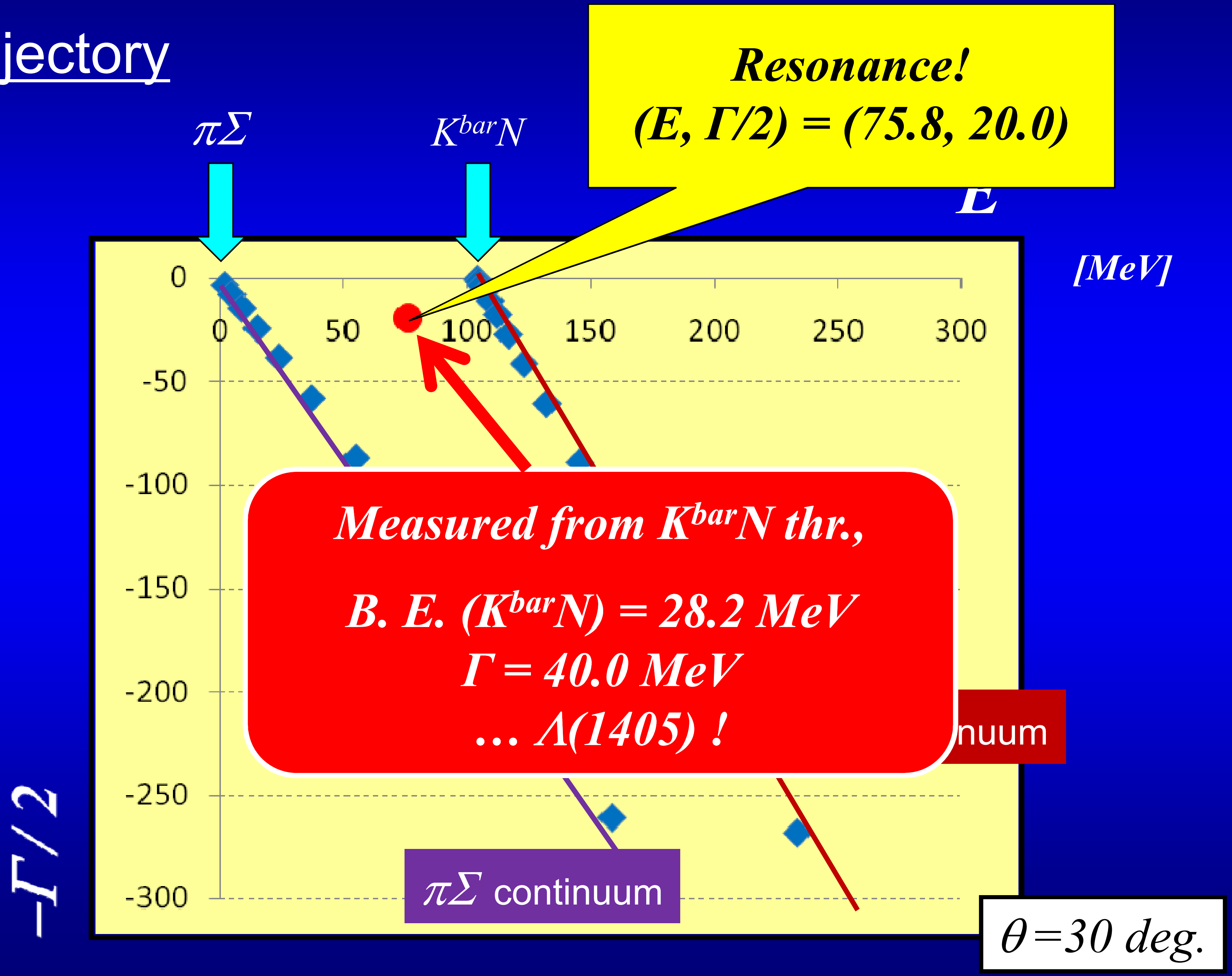


3. ccCSM with an energy-dependent potential for Λ(1405)

Chiral SU(3) potential (KSW)

N. Kaiser, P. B. Siegel and W. Weise, NPA 594, 325 (1995)

Original: δ-function type

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\tilde{\omega}_i\tilde{\omega}_j}}\,\delta(r)$$

Energy dependence is determined by Chiral low energy theorem.
← Kaon: Nambu-Goldstone boson

Present: Normalized Gaussian type

$$V_{ij}^{(I=0)}(r) = -\frac{C_{ij}^{(I=0)}}{8f^2}(\omega_i + \omega_j)\sqrt{\frac{M_i M_j}{s\,\tilde{\omega}_i\tilde{\omega}_j}}\,g(r)$$

$$g(r) = \frac{1}{\pi^{3/2}a^3}\exp\left[-(r/a)^2\right]$$

a: range parameter [fm]

M_i , m_i : Baryon, Meson mass in channel i

E_i : Baryon energy, ω_i : Meson energy

$$E_i = \frac{(\sqrt{s})^2 - m_i^2 + M_i^2}{2\sqrt{s}}, \quad \omega_i = \frac{(\sqrt{s})^2 + m_i^2 - M_i^2}{2\sqrt{s}}$$

Reduced energy: $\tilde{\omega}_i = \frac{\omega_i E_i}{\omega_i + E_i}$

Flavor SU(3) symmetry

$K^{bar}N$ $\pi\Sigma$

$$C_{ij}^{(I=0)} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ & 4 \end{pmatrix}$$

Energy dependence of V_{ij} is controlled by CM energy $\sqrt{s}$.

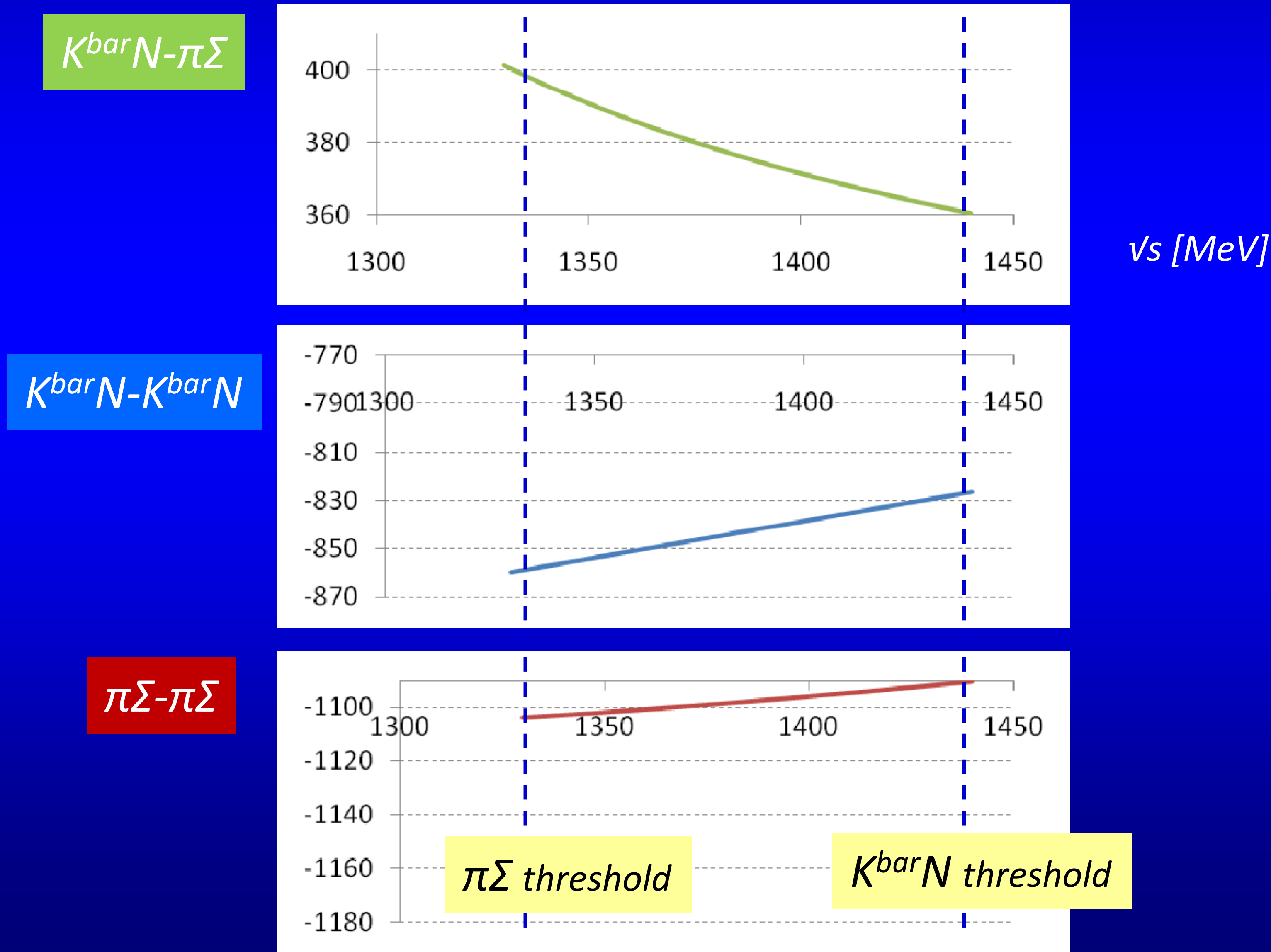
$$V_{ij}^{(I=0)}(r, \sqrt{s})$$

Chiral SU(3) potential (KSW)

N. Kaiser, P. B. Siegel and W. Weise, NPA 594, 325 (1995)

Energy dependence

$$V_{ij}^{(I=0)}\left(r=0,\sqrt{s}\right) \;@\; a=0.5\,fm,\;\; f_\pi=100\,MeV$$



Calculational procedure

Chiral SU(3) potential = Energy-dependent potential

→ ***Self consistency for the energy!***

Assume the values of the CM energy √s.

$\sqrt{s}_{\,Assumed}$ → $\hat{H} = \hat{T} + \hat{V}_{MB}\left(\sqrt{s}\right) +$

Perform the Complex Scaling method.

Then, find a pole of resonance or bound state. $\sqrt{s}_{\,Calculated}$

Check $\sqrt{s}_{\,Calculated} = \sqrt{s}_{\,Assumed}$

If No

Calculational procedure

Chiral SU(3) potential = Energy-dependent potential

→ *Self consistency for the energy!*

Assume the values of the CM energy $\sqrt{s}$.

$$\sqrt{s}_{Assumed} \rightarrow \hat{H} = \hat{T} + \hat{V}_{MB}\left(\sqrt{s}\right) + \hat{M}$$

↓

Perform the Complex Scaling method.

Then, find a pole of resonance or bound state. $\sqrt{s}_{Calculated}$

↓

Check $\sqrt{s}_{Calculated} = \sqrt{s}_{Assumed}$ → *If Yes* → *Finished !*

Calculational procedure

Chiral SU(3) potential = Energy-dependent potential

→ *Self consistency for the energy!*

Assume

$\sqrt{}$

Self consistency is considered
only for the real energy!

$$\sqrt{s}_{Calculated} = \mathrm{Re}\, E$$

Perform

Then, find a pole of resonance or bound state. $\sqrt{s}_{Calculated}$

If No

Check $\sqrt{s}_{Calculated} = \sqrt{s}_{Assumed}$

If Y

Finished !

Result

Case: self consistency for real energy

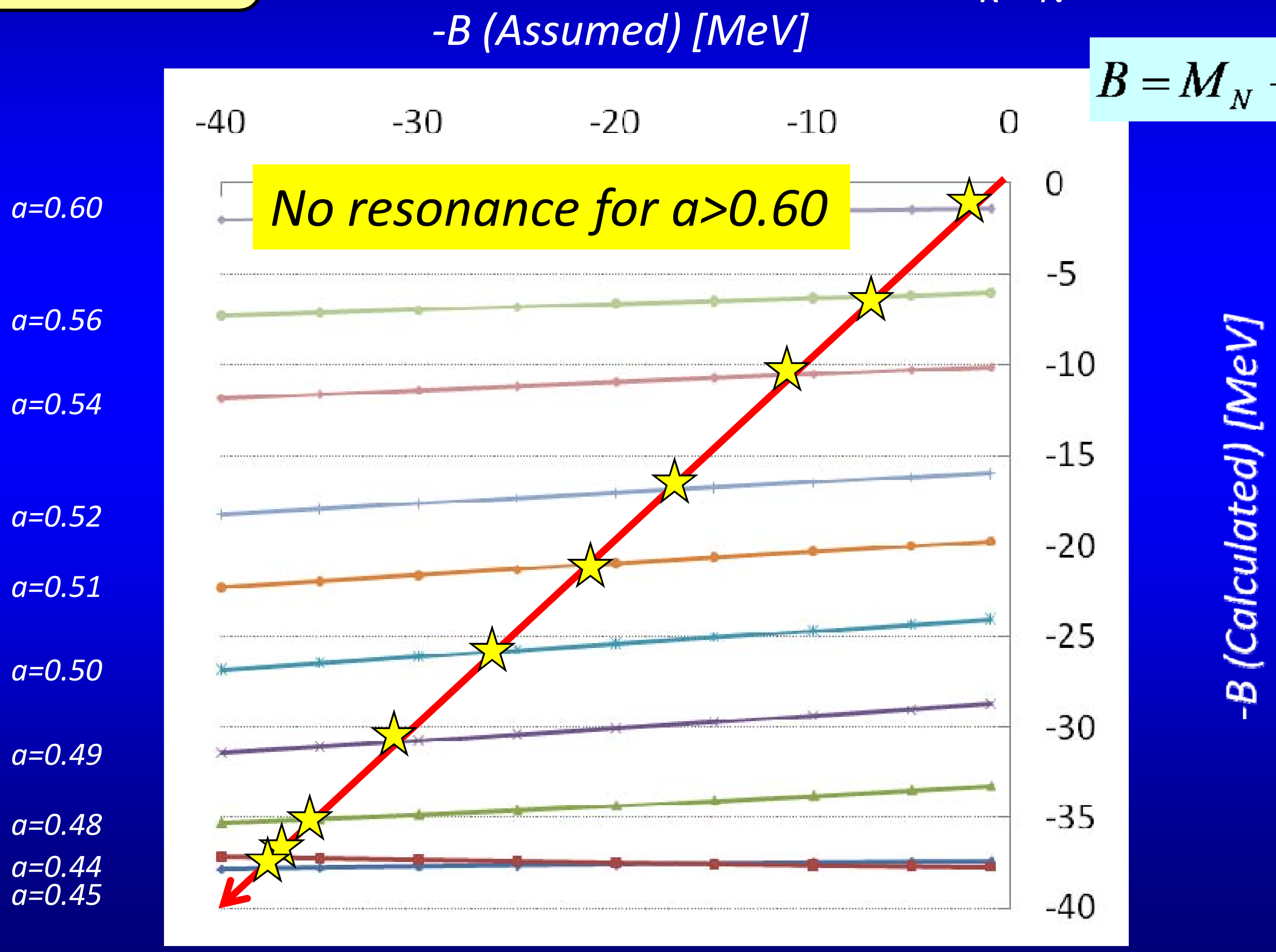
Range parameter (a) and
pion-decay constant f_π are ambiguous
in this model.
Various combinations (a, f_π) are tried.

$f_\pi = 95 \sim 105$ MeV

Self consistency for energy

$f_\pi = 100\ MeV$

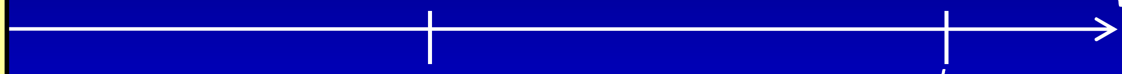
$B = M_N + m_K - \sqrt{s}$



Self consistency for energy

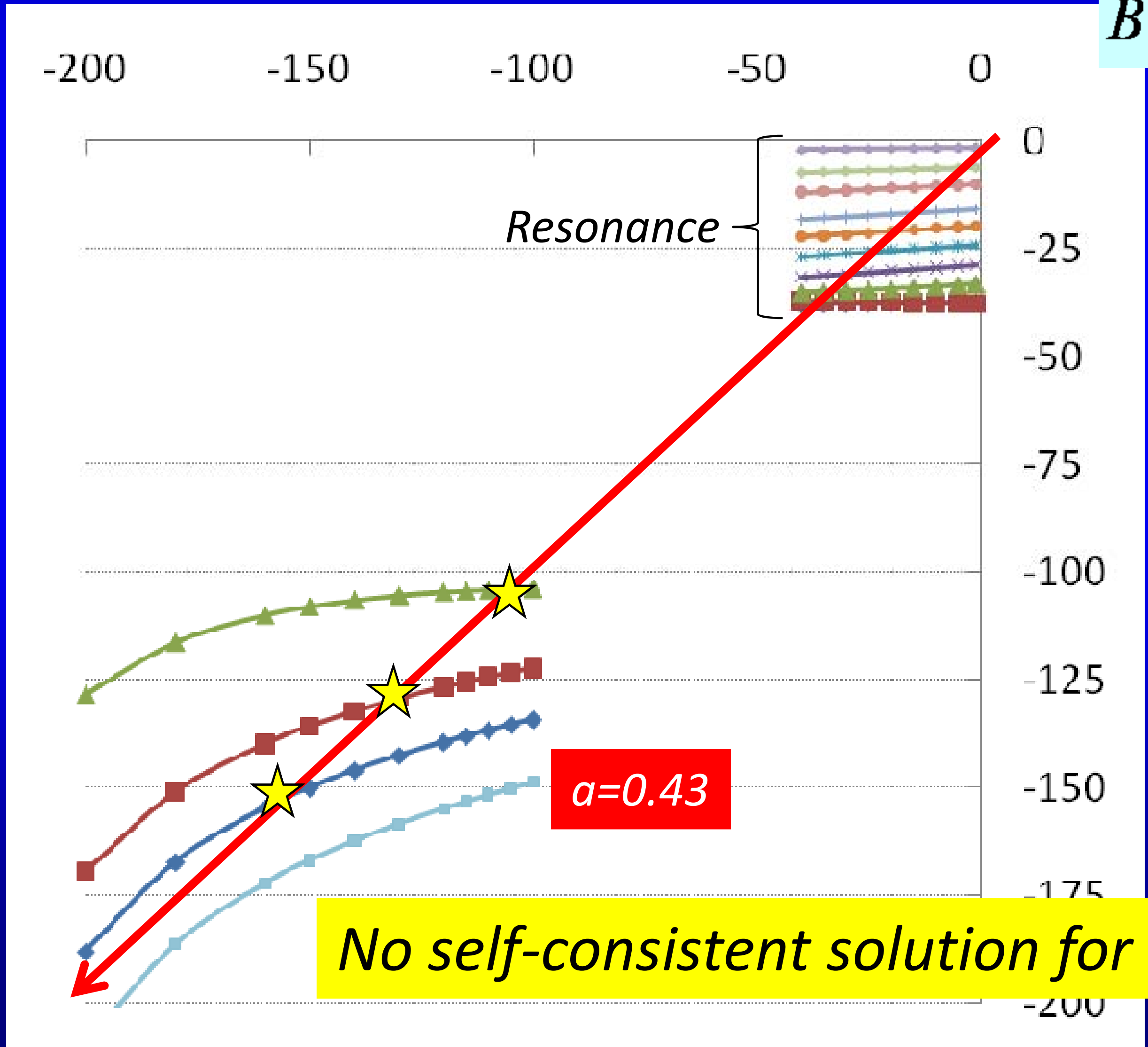
$f_\pi = 100\ MeV$

πΣ bound state



-B (Assumed) [MeV]

$$B = M_N + m_K - \sqrt{s}$$



-B (Calculated) [MeV]

a=0.48

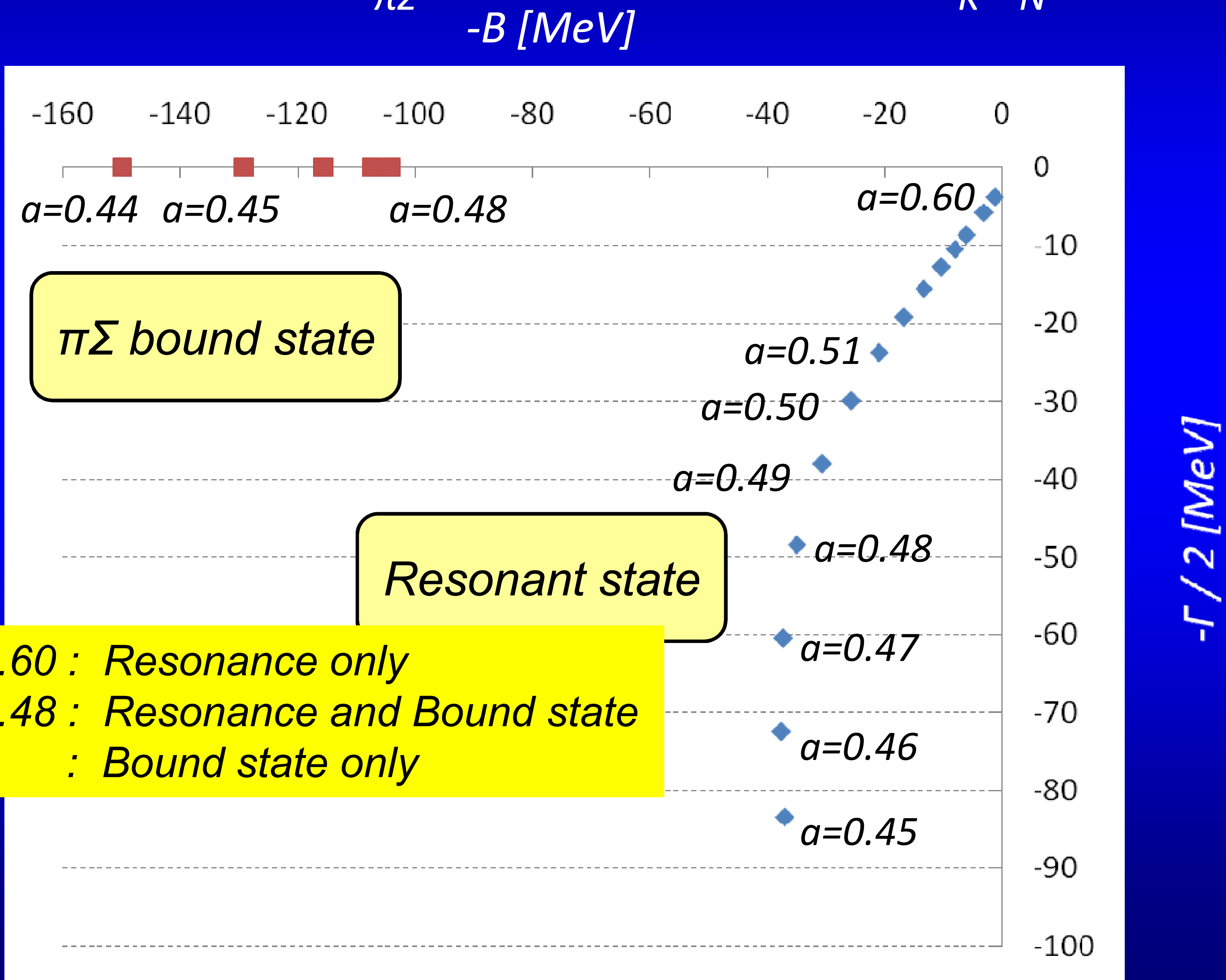
a=0.45

a=0.44

No self-consistent solution for a<0.44

Self consistent solutions (KSW)

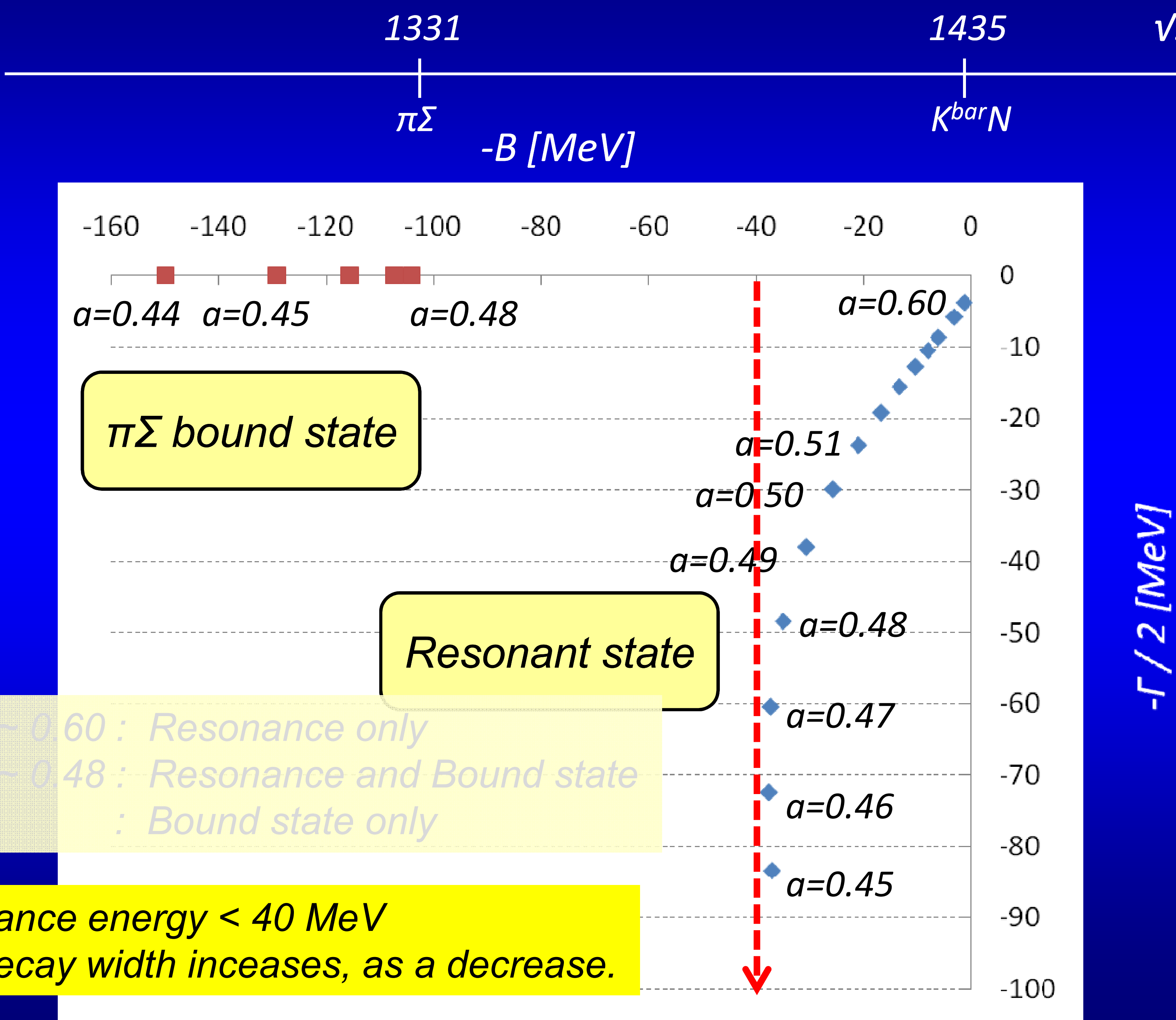
$f_\pi = 100\ MeV$



a=0.49 ~ 0.60 : Resonance only
a=0.45 ~ 0.48 : Resonance and Bound state
a= 0.44 : Bound state only

Self consistent solutions (KSW)

$f_\pi = 100\ MeV$



a=0.49 ~ 0.60 : Resonance only
a=0.45 ~ 0.48 : Resonance and Bound state
a= 0.44 : Bound state only

Resonance energy < 40 MeV
But , decay width inceases, as a decrease.

Root-mean-square radius

	AY	KSW		KSW		KSW	
f_π		95		100		105	
a	0.66	0.52	0.53	0.49	0.50	0.46	0.47
B	28.2	30.1	26.8	30.8	25.8	31.9	25.1
$\Gamma/2$	20.0	41.6	34.0	38.1	29.9	35.8	27.1
Rrms (Re)	1.35	1.05	1.17	1.13	1.23	1.18	1.25
Rrms (Im)	−0.39	−0.09	−0.17	−0.11	−0.23	−0.15	−0.28

Chiral SU(3) potential gives smaller Λ(1405), in comparison with a phen. potential.

Self-consistency for complex energy

$$\sqrt{s}_{Calculated} = \mathrm{Re}\, E$$

$$\Rightarrow \quad \sqrt{s}_{Calculated} = E \quad (\text{complex energy})$$

Search for such a solution that both of real and imaginary parts of energy are identical to assumed ones.

$$(B.E., \Gamma)_{Calculated} = (B.E., \Gamma)_{Assumed}$$

More reasonable?

✓*Pole search of T-matrix is done on complex-energy plane.*

$$T(Z) = V(Z) + V(Z)\, G_0(Z)\, T(Z)$$

Z: complex energy

Self consistency for complex energy

KSW

$f_\pi = 100\ MeV$

a	0.45	0.46	0.47	0.48	0.49	0.50	0.51
−BE (ass)	−25.5	−27.0	−27.5	−27.0	−25.0	−22.0	−19.0
−Γ /2 (ass)	−78.5	−68.5	−58.5	−48.0	−39.0	−31.0	−25.0
−BE	−25.6	−27.0	−27.6	−27.1	−25.1	−22.2	−18.8
−Γ /2	−78.8	−68.6	−58.4	−48.4	−39.1	−31.3	−25.1

Assumed

Calc.

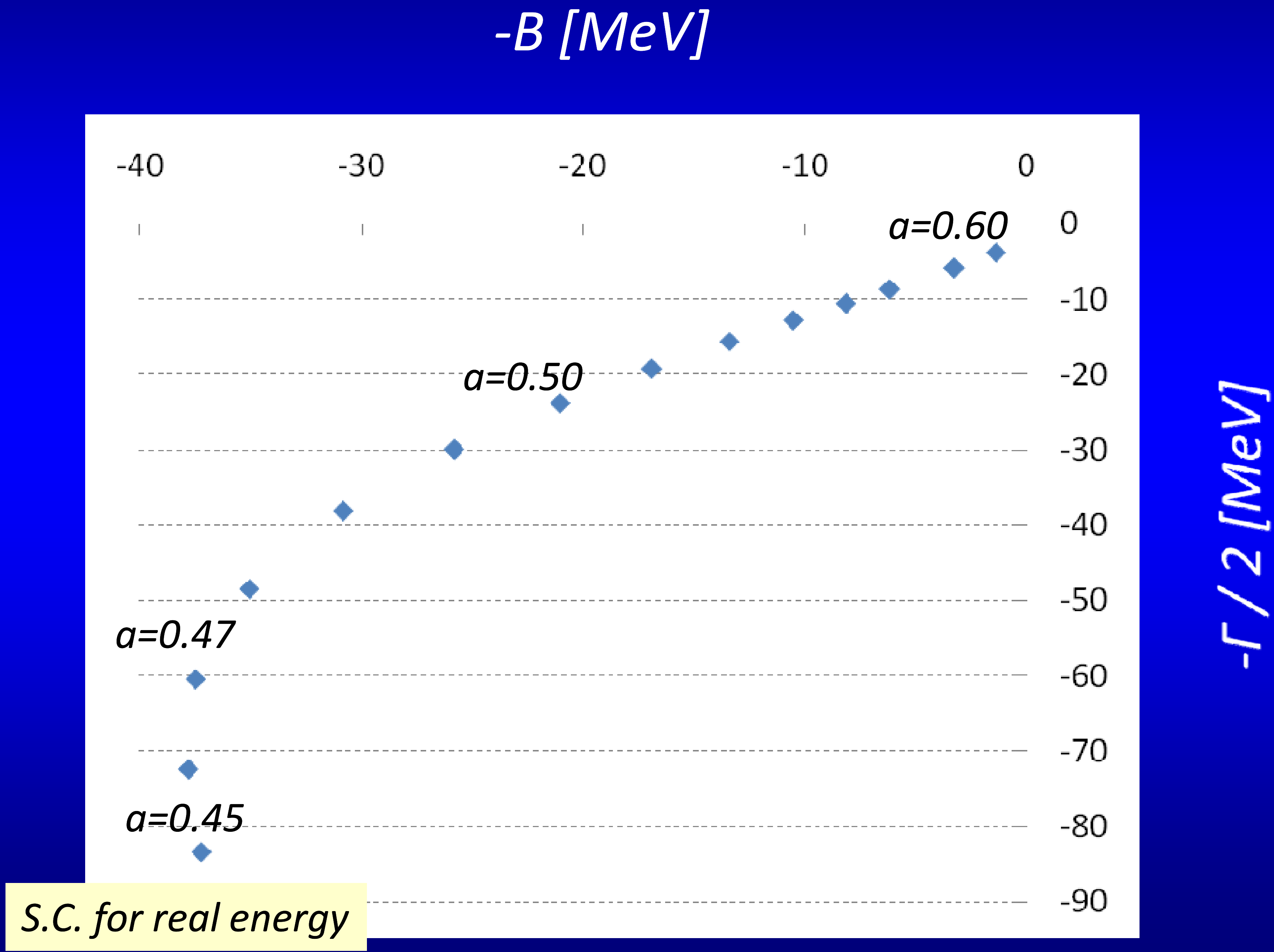
a	0.52	0.53	0.54	0.55	0.56	0.58	0.60
−BE (ass)	−15.5	−12.5	−10.0	−7.5	−6.0	−3.0	−1.5
−Γ /2 (ass)	−20.0	−16.5	−13.5	−11.0	−9.0	−6.0	−4.0
−BE	−15.5	−12.5	−9.9	−7.8	−5.9	−3.2	−1.3
−Γ /2	−20.2	−16.4	−13.4	−10.9	−8.9	−6.0	−3.9

Assumed

Calc.

Self consistency for complex energy

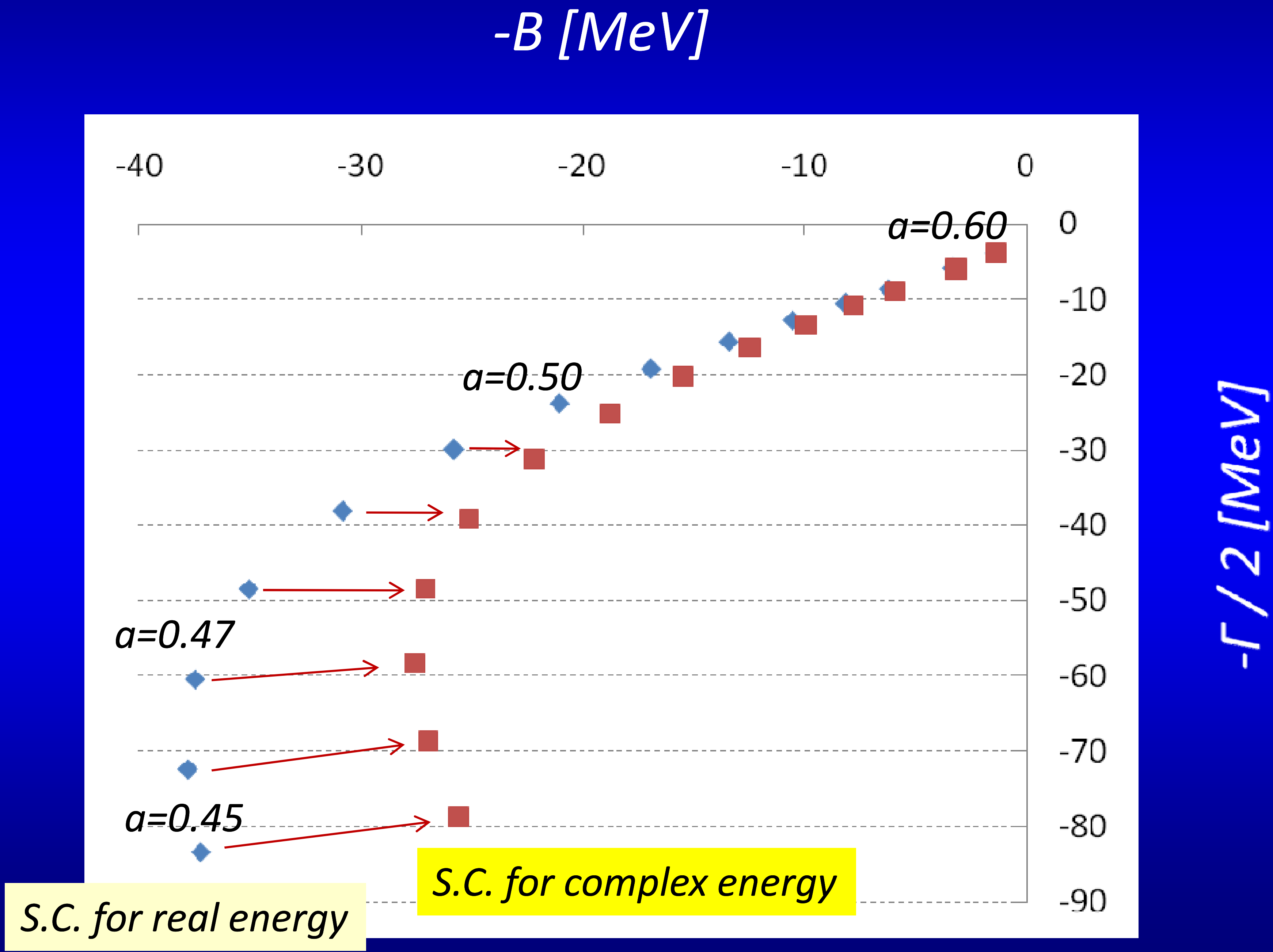
KSW
$f_\pi = 100$ MeV



Self consistency for complex energy

KSW

$f_\pi = 100\ MeV$



4. Summary and Future plan

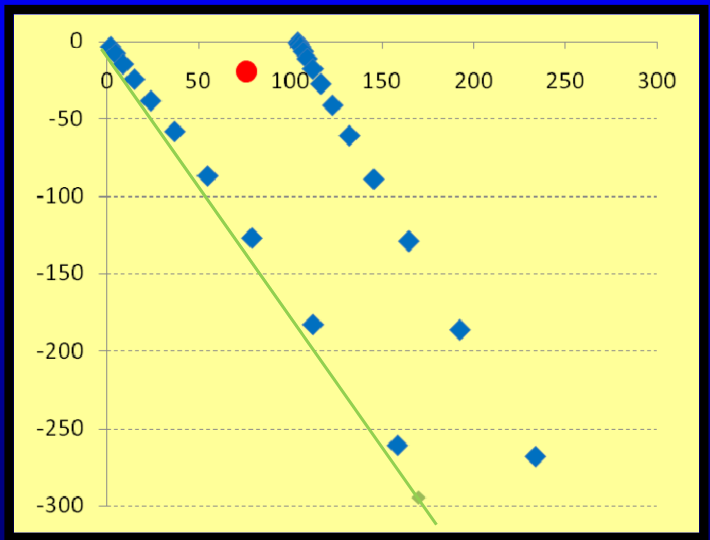
4. Summary

Λ(1405) studied with coupled-channel Complex Scaling Method using energy independent / dependent potentials

- ✓ **Coupled Channel problem = $K^{bar}N + \pi\Sigma$**
- ✓ Solved with *Gaussian base*

Energy-independent case

- ✓ A phenomenological potential (AY) is used.
- ✓ AY result is correctly reproduced: (B.E., Γ) = (28, 40) MeV



Resonance found at

B. E. ($K^{bar}N$) = 28.2 MeV

Γ = 40.0 MeV

4. Summary

Energy-dependent case

✓ A Chiral SU(3) potential (KSW) with Gaussian form is used.

✓ Take into account the self consistency for the real/complex energy

➡ ➢ Self consistent solutions are found, also for the complex energy case.

➢ Only for the restricted range parameter *a* of Gaussian form, self consistent solutions can exist.

➢ As for resonant states in the self consistent solutions for real energy, their binding energy don't exceed 40 MeV for $f_\pi = 95 \sim 105$MeV.

➢ Self-consistency for the complex energy seems to contribute repulsively to the binding energy.

4. Future plan

1. Two-body system … $K^{bar}N$-$\pi\Sigma$ system corresponding to $\Lambda(1405)$

- *For the case of energy dependent potential, further investigation is needed.*

  - *Fix the combination of (a, f_π)*
    … experimental value such as $I=0$ $K^{bar}N$ scattering length.

  - *Another pole ???*
    … Double pole problem suggested by chiral unitary model

D. Jido, J. A. Oller, E. Oset, A. Ramos and U. -G. Meissner, NPA725, 181 (2003)

- *Analyze the obtained wave function*

2. Three-body system … $K^{bar}NN$-$\pi\Sigma N$ system corresponding to "K^-pp"

Effect of $\pi\Sigma N$ three-body dynamics

…

Thank you very much!