

# $J/\psi$ -N interaction and $J/\psi$ - nuclei

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*High-energy hadron physics with hadron beams*

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1. The case of the  $D\text{-}\bar{N}$  interaction.
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5.  $J/\psi$  nuclei (brief)
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# Quark masses\*

$$m_u \approx 1.5 \sim 3.3 \text{ MeV}$$

$$m_d \approx 3.5 \sim 6.0 \text{ MeV}$$

$$m_s \approx 66 \sim 126 \text{ MeV}$$

$m \gg \Lambda_{\text{QCD}} \approx (1\sim 4) \cdot 100 \text{ MeV}$ :

perturbative treatments of  $m_q$  become possible

→ Effective field theoretical (EFT) treatments of QCD  
(even nonrelativistic QCD) become possible\*\*

$$m_c \approx 1.16 \sim 1.34 \text{ GeV}$$

\*PDF: The u-, d-, and s-quark masses are estimates of so-called "current-quark masses," in a mass-independent subtraction scheme such as  $\overline{\text{MS}}$ . The ratios  $m_u/m_d$  and  $m_s/m_d$  are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splittings in hadron masses. We have normalized the  $\overline{\text{MS}}$  masses at a renormalization scale of  $\mu = 2 \text{ GeV}$ . Results quoted in the literature at  $\mu = 1 \text{ GeV}$  have been rescaled by dividing by 1:35.

\*\*cf. N. Brambilla & A. Vairo, *Rev. Mod. Phys.* 77, 1423 (2005).

# Lightest charmed mesons

(Lightest) charmed mesons:  $\Gamma \approx 10^{-4}$  eV.

$$m_c + m_d \approx 1.2 \text{ GeV}$$

$D^+$  (c d),  $D^0$  (c u),  $D^-$  (c d),  $D^0$  (c u):  $I(J^P) = \frac{1}{2}(0^-)$ ,  $m_D \approx 1.87 \text{ GeV}$

$D_s^+$  (c s),  $D_s^-$  (c s) :  $I(J^P) = 0(0^-)$ ,  $m_{D_s} \approx 1.97 \text{ GeV}$

(Lightest) CC mesons: CC in 1S states.

$$2m_c \approx 2.3 \sim 2.7 \text{ GeV}$$

$\eta_c$  (c c) :  $I(J^P) = 0(0^-)$ ,  $m_{\eta_c} \approx 2.98 \text{ GeV}$ ,  $\Gamma \approx 27 \text{ MeV}$

$J/\psi$  (c c) :  $I(J^P) = 0(1^-)$ ,  $m_{J/\psi} \approx 3.10 \text{ MeV}$ ,  $\Gamma \approx 93 \text{ eV}$

Note:  $2m_u + m_d \approx 6.5 \sim 13 \text{ MeV}$ , while  $M_p \approx 938.3 \text{ MeV}$ .

# 1. The case of D-bar-N interaction

Attractive. Probably strong enough to form  $\underline{D}$ -N bound states.

One  $\pi$ -exchange interaction: S.Yasui & K.Sudoh, PRD(2009).

Bound state:  $I = 0$  and  $J^P = 1/2^-$

$$E_B = 1.4 \text{ MeV} \rightarrow a = +4.7 \text{ fm}$$

$$\text{and } R = 3.8 \text{ fm}$$

with the coupled channels of

$$|\bar{D}N\rangle = c_0^D(|D^- p\rangle - |\bar{D}^0 n\rangle) + c_1^D(|D^{*-} p\rangle - |\bar{D}^{*0} n\rangle)$$

$$m_{\underline{D}}^0 - m_{\underline{D}}^- \approx 5 \text{ MeV}$$

$$m_{\underline{D}}^* - m_{\underline{D}} \approx 140 \text{ MeV}$$

$\rightarrow$  D-bar nuclear bound states:

Coupled atomic + nuclear bound states of charmed mesons

with the meson-nucleon bound-states of  $I = 0$  and  $J^P = 1/2^-$

$\rightarrow$  nuclear shell structure possibly destroyed?

But hold the horse.

Quark-meson coupling model: (Guichon, Fleck-Benz-Shimizu-Yazaki,Saito-Thomas)

→K.Saito, K.Tsushima, A.W.Thomas, PPNP(2007):

Non-interacting N bag model in Rel. Mean Field (QHD) model,

$\sigma$ ,  $\omega$ ,  $\rho$  coupling to  $q$ ,  $Q$ , ~self-consistently at the N and nuclear levels.

Tsushima,Lu,Thomas,Saito,Landau, PRC(1999)

$D \cdot$  bar mass shift at ~ nuclear matter (at the center of Pb):  $\delta m_{\underline{D}} \approx -45 \text{ MeV}$

QCD sum rule: A.Hayashigaki, PLB(2000) on  $D$ , noting  $\delta m_D \approx \delta m_{\underline{D}} \approx -50 \text{ MeV}$

Yasui - Sudoh  $\underline{D}N$  scattering length roughly corresponds to:

$$a = +4.7\text{fm} \rightarrow \delta m_D = -17.4\text{MeV} (\Lambda=1.27\text{GeV})$$

$$-22.4\text{MeV} (\Lambda=1.00\text{GeV})$$

However, at the LO meson-baryon interaction,

Chiral SU(3)  $\rightarrow$  SU(4) model:

A.Mishra, Bratkovskaya, Shaffer-Bielich, Schramm, Stöcker, PRC(2004)

$$\delta m_{\underline{D}} = \text{even down to } \sim -150 \text{ MeV}$$

Hofmann-Lutz (coupled-equation) model:

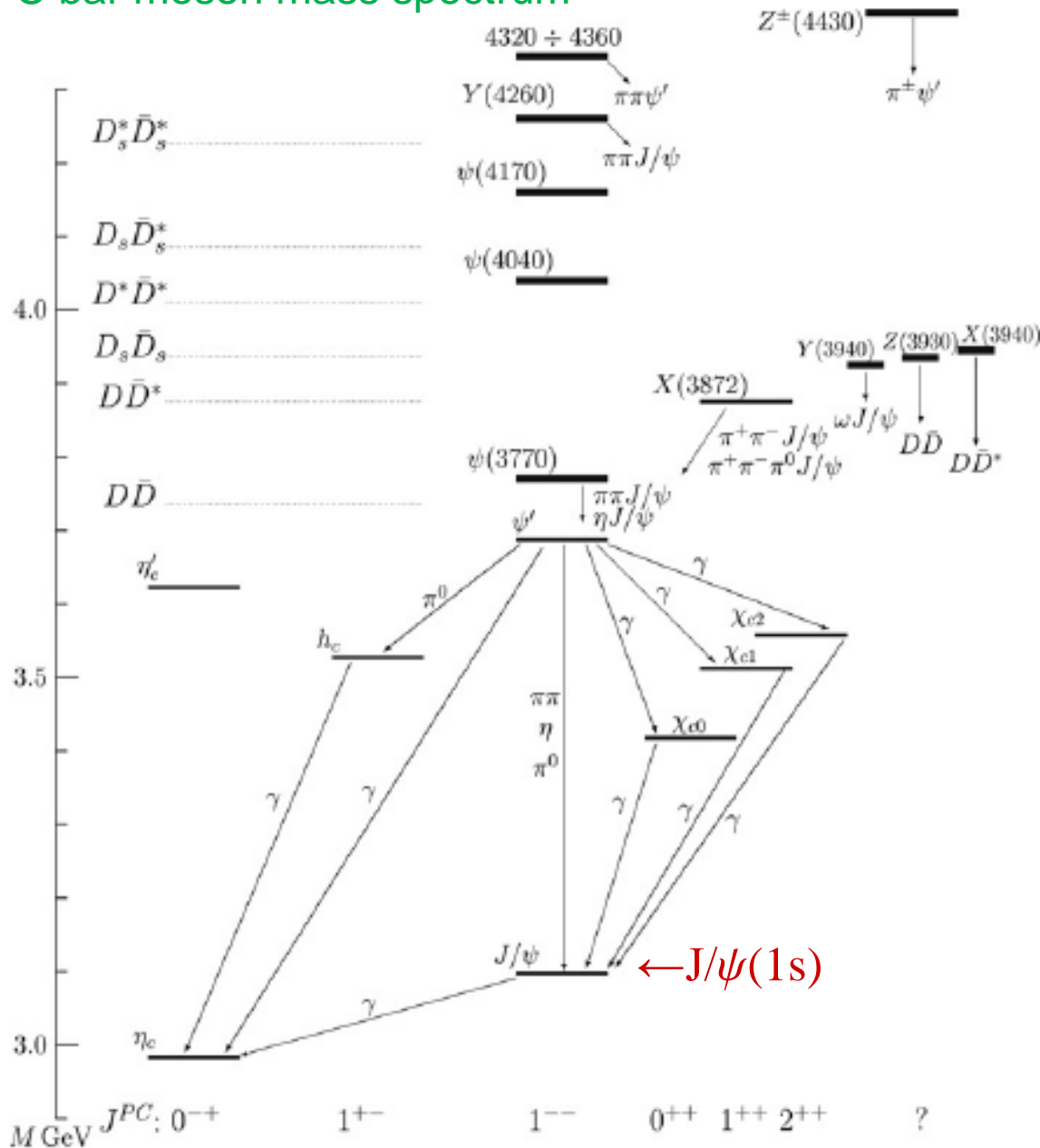
Includes all  $J^P = \frac{1}{2}^-$  pseudoscalar-baryon states consisting of u, d, s, c with s-wave interactions using the universal vector coupling strength:

M.F.M.Lutz & C.L.Korpa, PL(2006); J.Hofmann & Lutz, NPA(2005)

$$\delta m_{\underline{D}} = + 17 \text{ MeV}$$

Strong coupling of various channels makes the strength of the D-N interaction much uncertain.

C-C-bar meson mass spectrum



## 2. $J/\psi$ and $J/\psi - N$ mass spectrum, and $J/\psi$ size

$J/\psi(1s)$ :

$I=0, G=-, J=1, PC=- -$   
 $M=3097\text{MeV},$   
 $\Gamma=0.093\text{MeV}$

Like  $\phi(1020)$  of  $s\bar{s}$  but with  $\Gamma=4.3\text{MeV}$   
 Decaying to K's.

$\eta_c(1s)$ :

$\Gamma=27\text{MeV}$

May not be pure  $c\bar{c}$ ?



$N + J/\psi$  couples little to other hadronic channels:

$J/\psi$  have a lonely life with  $N$ .

1) The  $J/\psi$  mass is below the  $D - D\text{-bar}$  mass by more than 60 MeV.

2) The proton -  $J/\psi$  mass is below the  $\Lambda_c - D\text{-bar}$  mass by about 120 MeV:

$$\text{Mass} (\Lambda_c + D\text{bar}) \approx 4.1561 \pm 0.0002 \text{ GeV}$$

$$\text{Mass} (p + J/\psi) \approx 4.0352 \pm 0.0001 \text{ GeV}$$

All other OZI-allowed hadronic states with the same quantum number of  $N + J/\psi$  have higher masses.

3) The  $J/\psi$  mass is above the  $\eta_c(1s)$  mass by 117 MeV, but a transition to  $\eta_c(1s)$  involves a spin flip of  $c$  or  $c\text{-bar}$ . Because of the large mass, the transition is expected to be rare.

4) Coupling to OZI-non-allowed channels is expected to be small:

*S.J. Brodsky & G. A. Miller, PL B412, 125 (1997).*

- \*  $N + J/\psi \rightarrow N + J/\psi + \pi + \pi$  Isospin allowed via two-pion exchanges  
but down by the order of  
 $(m_\pi / 4\pi f_\pi)^2 \approx 1 \%$ .
- \*  $J/\psi \rightarrow e^+ + e^-$   $5.94 \pm 0.06 \%$   
 $\mu^+ + \mu^-$   $5.93 \pm 0.06 \%$   
 $\rho + \pi$   $1.69 \pm 0.15 \%$ : the largest two-body hadronic branching  
ratio ( $\rho \pi$  puzzle:  $\rho$  has  $c\text{-}\bar{c}$  component?)  
but  $N + J/\psi \rightarrow (N + \rho + \pi) \rightarrow N + J/\psi$  unlikely with the order  
of  $10^{-4} \%$  interaction strength.
- \* Also  $N + J/\psi \rightarrow (N + D + D\text{-bar}) \rightarrow N + J/\psi$  unlikely with about  $0.4 \%$   
interaction strength.

$J/\psi$  is small:

At short distances, the non-relativistic C and C·bar in  $J/\psi$  interact through the color singlet ( $P_0$ ) part of the Coulombic potential

$$V(r) = -\frac{4}{3} \frac{\alpha_s(1/r)}{r} P_0 + \frac{2}{3} \frac{\alpha_s(1/r)}{r} P_8 .$$

The Bohr radius of this C-C·bar bound state is

$$a_0 = (3/2) [m_c \alpha_s(1/a_0)]^{-1} .$$

Numerically

$$\alpha_s(\Lambda_Q) \approx 0.5 \sim 0.6 \quad [\alpha_s(q) = g(q)^2/4\pi]$$

$$a_0 \approx 0.3 \text{ fm} \quad (1/a_0 \equiv \Lambda_Q \approx 0.75 \sim 0.64 \text{ GeV})$$

N.B.  $\Lambda_Q \equiv 1/a_0$ .

Note:  $-4/3 = t_1^a \cdot t_2^a$  for the color singlet pair.

## 4. $J/\psi$ -N interaction is definitely attractive

through QCD van der Waals (two gluon exchange) interaction:

The  $J/\psi$ -N amplitude at the threshold in Born approximation is expressed in the second order of the color dipole coupling  $\hat{H}_{\text{int}} = -(\mathbf{t}_1^a - \mathbf{t}_2^a)\mathbf{r} \cdot \mathbf{E}^a$  :

$$f_{\text{B}} = -\frac{m_{\text{red}}}{2\pi} \frac{2\pi}{3} \alpha_s \left\langle \varphi \left| r^i \frac{1}{H^{(8)} + \epsilon} r^j \right| \varphi \right\rangle_{J/\psi} \langle K_2 | E_i^a E_j^a | K_1 \rangle_{\text{N}} (K_1 = K_2)$$

$$= -\frac{m_{\text{red}}}{2\pi} 4\alpha_{J/\psi} \left\langle \frac{1}{2} \mathbf{E}_a \cdot \mathbf{E}_a \right\rangle_{\text{N}}$$

Here,

$J/\psi$  chromo-polarizability:  $\alpha_{J/\psi} \approx \mathbf{d}_2 a_0^3/4$  [ $\mathbf{d}_2 = 7 \cdot (4\pi/27)$ , Wilson coefficient (1S) ;

c-c-bar octet state propagator:  $(H^{(8)} + \epsilon)^{-1}$ . (Peskin,  $N_c \rightarrow \infty$ )]

$f_{\text{B}}$  is expressed in the forms of multi-pole expansion\*,  
of operator product expansion,  
and (QCD) EFT/HQ formulation \*\*

\*K.Gottfried (1978), M.B.Voloshin (1979), M.Peshkin (1979); G.Bhanot & Peshkin (1979), T.M.Yan (1980), A.B.Kaidalov & P.E.Volkovitsky (1992).

\*\*M. Luke, A. V. Manohar, & M. J. Savage (1992), S.J.Brodsky and G. A. Miller (1997).

Approximation (thus **the source of uncertainties**)

on the two major factors evaluated at  $\Lambda_Q$  :

1) The  $J/\psi$  chromo-polarizability:

$$\begin{aligned}\alpha_{J/\psi} &\approx d_2 a_0^3/4 \approx 0.9 \text{ GeV}^{-3} && (\text{N}_c \rightarrow \infty; \text{M.Peshkin}(1979)) \\ &\approx 2.0 \text{ GeV}^{-3} \text{ and up} && (\text{A.Sibirtsev \& M.B.Voloshin (2005)}).\end{aligned}$$

2) The N matrix element of the gluon operator:

$$\begin{aligned}\langle \frac{1}{2} \mathbf{E}^a \cdot \mathbf{E}^a \rangle_N &\approx (4\pi^2/9) \langle T_\mu^\mu \rangle_N + 2\pi \alpha_s (1/a_0) \langle T_G^{00} \rangle \\ &\approx (3V_2/8 + \pi/9\alpha_s) m_N \approx m_N\end{aligned}$$

Here,  $T(\Lambda_Q)$  = the energy-momentum tensor

$V_2(\Lambda_Q) \approx 0.5$ , the gluon momentum fraction in N

Note, however, neglect of the second term (formally in NLO) leads to

$$\approx (1/4) m_N \quad (\text{A.B.Kaidalov \& P.E.Volkovitsky}(1992))$$

So, we have found the Born approximation of the  $J/\psi$  - N scattering amplitude at the threshold. It is

$$a_{j/\psi}^B = 2m_{red} \int d^3r V(r)$$

in terms of the local  $J/\psi$  - N potential  $V(r)$ .

By writing  $V(r) = V_0 \cdot [V_N \delta^3(r)] = \delta m [V_N \delta^3(r)]$

obtain 
$$\delta m = \frac{a_{j/\psi}^B}{2m_{red}} \frac{1}{V_N} \rightarrow \frac{a_{j/\psi}^B}{2m_{red}} \rho_0 \quad \text{with } \rho_0 = 0.172 \text{ fm}^{-3}$$

by neglecting the Fermi motion and possible modification of  $V(r)$  in the presence of other nucleons.

$a_{J/\psi-N}^B \rightarrow a_{J/\psi-N}$  or  $\sigma_{J/\psi}$  is a different matter.

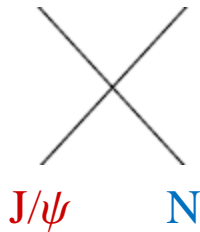
How to  $a_{J/\psi-N}^B \rightarrow a_{J/\psi-N}$

1) Assume the form of  $V(r)$ :  $V(r) = V_0 \exp(-r^2/R^2)$

$R \sim$  the nucleon size, 0.8 fm (Brodsky & Miller, 1997)

$a_{J/\psi-N}^B = -0.19$  fm  $\rightarrow a_{J/\psi-N} = -0.24$  fm by solving Sch. Eq.

2) Apply the LO NR  $J/\psi$ -N EFT:



$$V(r) = \frac{\pi^2}{m_{red}\Lambda} c_0(\Lambda) \delta^3(r)$$

Since  $a_{J/\psi-N}^B = \frac{\pi}{2\Lambda} c_0(\Lambda)$ ,  $a_{J/\psi-N} = \left( \frac{1}{a_{j/\psi}^B} + \frac{\Lambda}{\pi} \right)^{-1} = -0.22$  fm  
 for  $\Lambda = 1/R = 0.25$  GeV

The EFT treatment reveals the regularization dependence of  $a_{J/\psi-N}^B$ ,

$a_{J/\psi-N}^B = a_{J/\psi-N}^B(\Lambda)$ , or the intrinsic ambiguity with  $\delta m$ :

For the  $J/\psi$  scattering amplitude, write

$$V_N \delta^3(r) = \frac{1}{\Lambda^3} \delta^3(r) \quad \text{in} \quad V(r) = V_0 \cdot [V_N \delta^3(r)] = \delta m [V_N \delta^3(r)]$$

The point is simply that  $a_{J/\psi-N}$  is an observable and independent of  $\Lambda$ ,  
but  $a_{J/\psi-N}^B$  is not.

The difference between  $a_{J/\psi-N}$  and  $a_{J/\psi-N}^B$  in our  $J/\psi$ -N case is expected to be small as the interaction being weak:

$$a_{J/\psi-N}^B = -0.19 \text{ fm} \quad \rightarrow \quad a_{J/\psi-N} = -0.24 \text{ fm} \quad (\text{Brodsky \& Miller, 1997}) \\ = -0.22 \text{ fm} \quad (\text{EFT, R.S.})$$



# $\delta m$ (MeV) and the spin-averaged $a_{J/\psi-N}$ (fm) calculations (All with $\sim$ ):

## pQCD

$-(8 \sim 11)$		Luke, Manohar & Savage (1992)
	$-0.24$	Brodsky, Miller (1997)
$-8 \pm 3$		Lee, Ko (2003)
$-11$	$-0.2$	deTeramond, Epinoza, & Ortega-Rodriguez (1998)*
$-3$	$-0.06$	Kaidalov & Volkovitsky (1992)
$\leq -21$		Sibirtsev, Voloshin (2005)

## Sum rule

$-7$	$-0.2$	Klingl, Kim, Lee, Morath, Weise, (1999)
$-(4 \sim 7)$	$\approx -0.1$	Hayashigaki (1999)

## Lattice\*\*

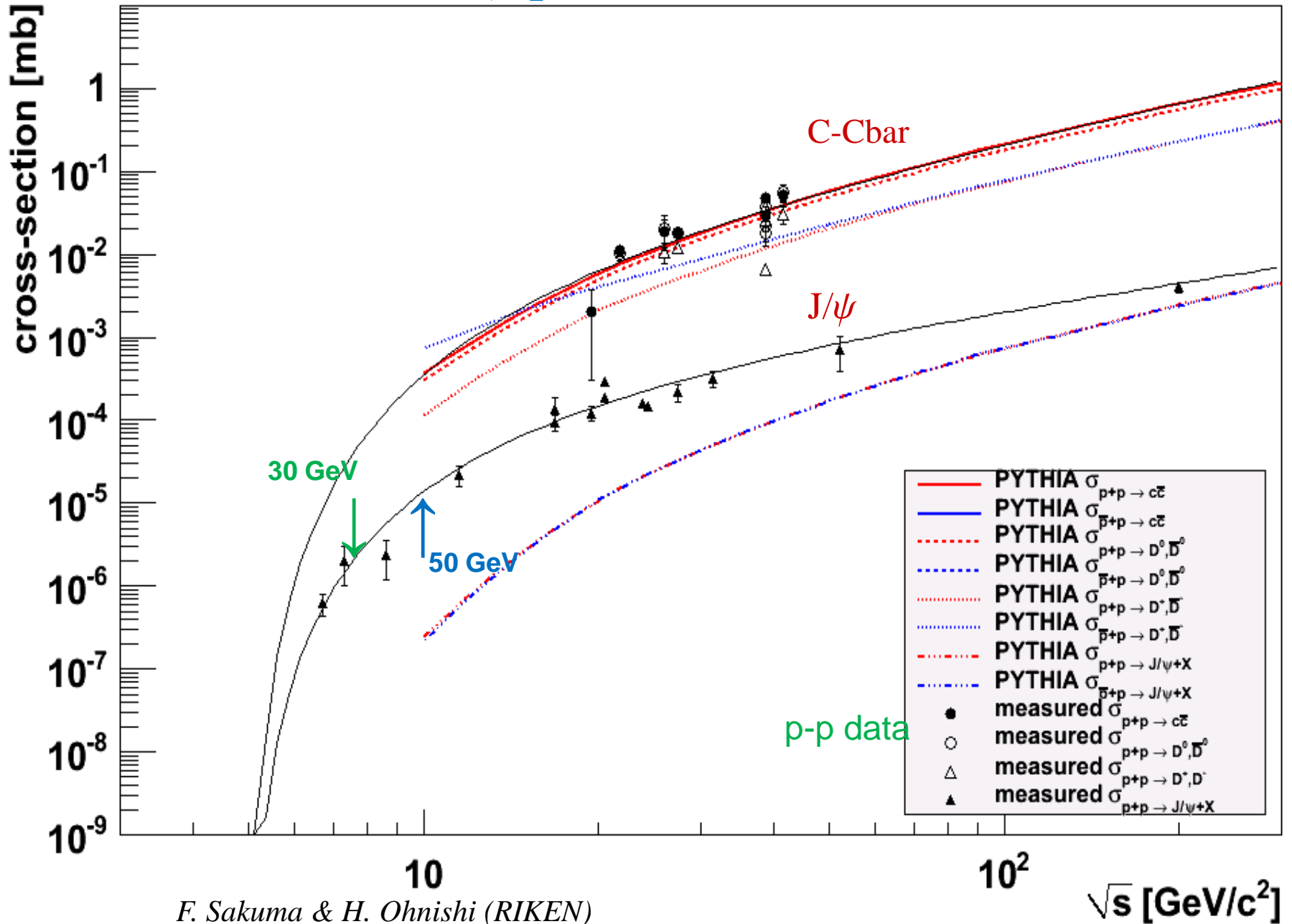
	$-0.71 \pm 0.48$	Yokokawa, Sasaki, Hatsuda, Hayashigaki (2006)
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## exp. analysis

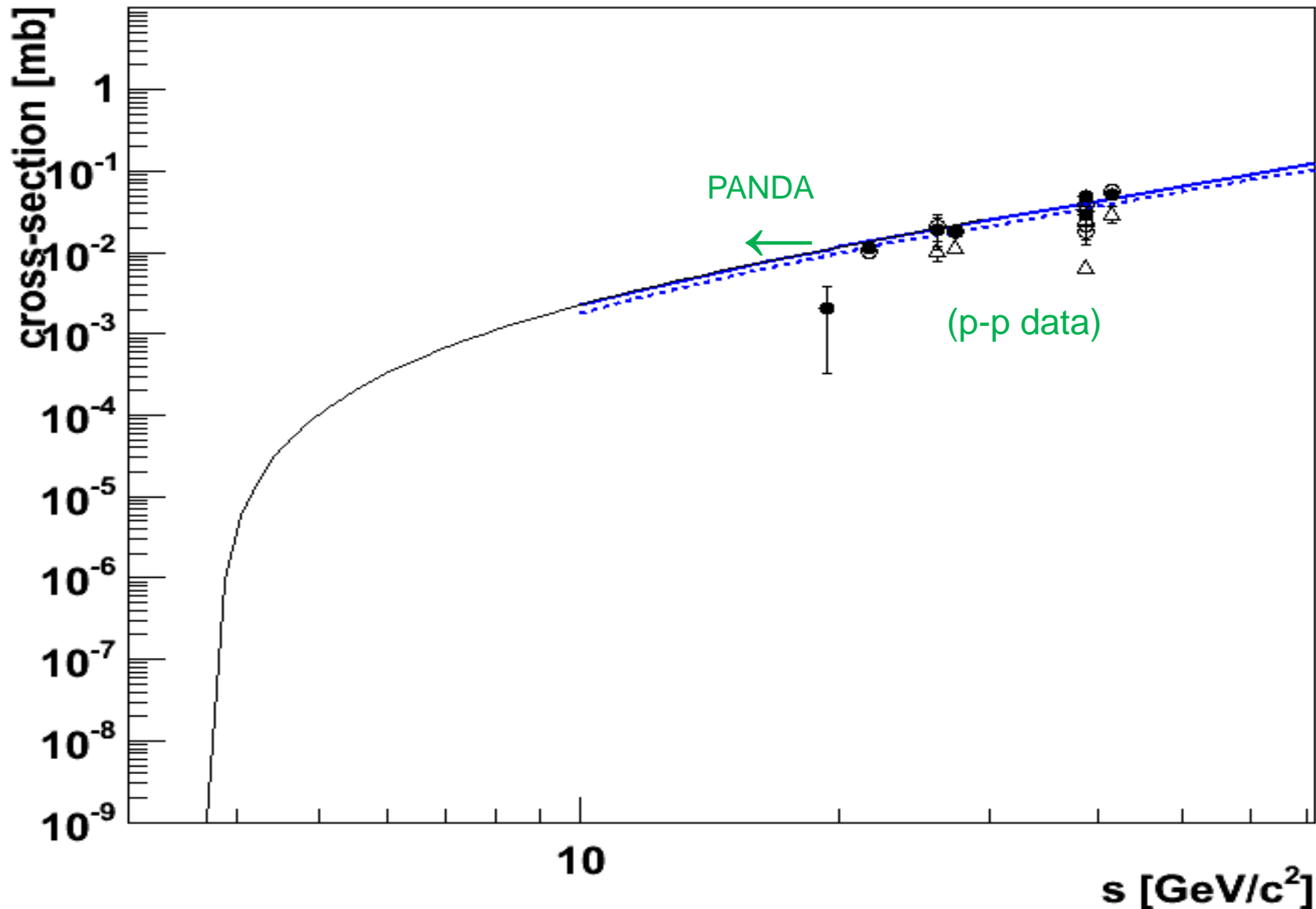
	$\sim 0.1$ (magnitude)	$(\gamma \text{ prod.})$ Hüfner et al. (2000)
$-11$	$-0.2$	*(p-p $A_{NN}$ )

\*\*A new, more precise lattice calculation is underway (private comm. S. Sasaki)

# 4. $J/\psi$ production



pbar-p



- $p\text{-bar } p \rightarrow \pi^0 J/\psi$  cross sections from  $J/\psi \rightarrow p\text{-bar } p \pi^0$

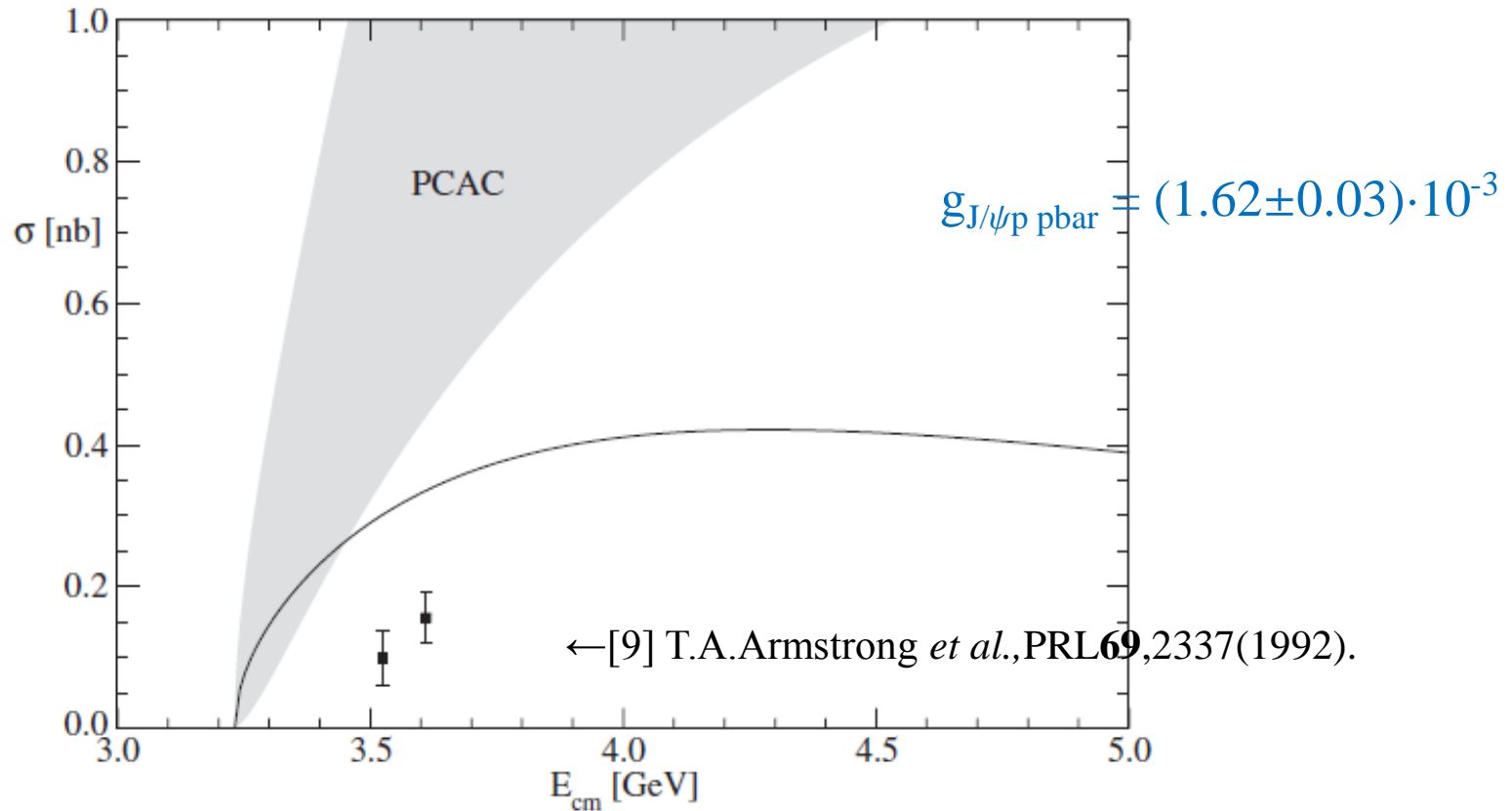


FIG. 4. Theoretical and experimental cross sections for  $p\bar{p} \rightarrow \pi^0 J/\psi$ . The theoretical predictions are the constant amplitude result Eq. (7) (solid) and the range of PCAC cross sections, from Eq. (8) (filled). The experimental points are from E760 [9].

## Some possible experiments

TABLE II. Kinematics for the production of  $\eta_c$ -nucleus bound states. All quantities are given in GeV.

Process	$\epsilon$	$p_{\text{c.m.}}$	$p_1^{\text{lab}}$
$\gamma \ ^3\text{He} \rightarrow (\ ^3\text{He} \ \eta_c)$	0.020	2.20	4.52
$p d \rightarrow (\ ^3\text{He} \ \eta_c)$	0.020	2.48	7.64
$\bar{p} \ ^4\text{He} \rightarrow (\ ^3\text{H} \ \eta_c)$	0.020	1.48	2.30
$\gamma \ ^4\text{He} \rightarrow (\ ^4\text{He} \ \eta_c)$	0.120	2.24	3.96
$n \ ^3\text{He} \rightarrow (\ ^4\text{He} \ \eta_c)$	0.120	2.60	6.09
$dd \rightarrow (\ ^4\text{He} \ \eta_c)$	0.120	2.71	9.51

S.J.Brodsky, I.Schmidt, & G.F.deTera mond, PRL 64, 1011(1990)

$$\pi^+ d \rightarrow J/\psi p_1 p_2$$

S.J.Brodsky & G.A.Miller, PLB 412, 125 (1997)

$$p\text{-bar} \ ^4\text{He} \rightarrow \pi(\text{all charges}) + 3\text{nucleons}_{J/\psi}$$

R.S.

## 5. $J/\psi$ -nuclei

How attractive is the nuclear interaction to form a bound state with nuclei?:

### A simple estimate

For a mass  $\mu$  particle in a square-well potential  $V$  of the radius  $R$ :

$$V \leq -\frac{1}{2\mu} \left( \frac{\pi}{2R} \right)^2 \quad \text{The equality is for the unitary limit (E}_B = 0\text{)}.$$

Let's

$$R = (1.12 \text{ fm}) A^{1/3} = \left( \frac{3}{4\pi\rho_o} \right)^{1/3} A^{1/3} \quad \text{with } \rho_o = 0.172 \text{ fm}^{-3}$$

We get

$$\begin{aligned} V &\leq - (20.4 \text{ MeV}) A^{-2/3} && \text{for } \mu = m_{\underline{D}} \approx 1.87 \text{ GeV} \\ &\leq - (12.3 \text{ MeV}) A^{-2/3} && \text{for } \mu = m_{J/\psi} \approx 3.10 \text{ GeV} \end{aligned}$$

## $J/\psi$ -nucleus folded potential

The approximation is

$$V_{J/\psi-A}(r) \approx \int d^3 r' V_{J/\psi-N}(r-r') \rho_A(r').$$

If the  $J/\psi$  interaction is weak, the standard “lowest-order impulse approximation” in nuclear physics is

$$V_{J/\psi-A}(r) = \frac{2\pi}{m_{red}} a_{J/\psi-N} \rho_A(r).$$

Note that if the interaction should be strong enough to form a bound state,  $a_{J/\psi}$  diverges and then changes its sign,  $- \rightarrow +$ , as the strength increases.  $a_{J/\psi} \rightarrow \pm\infty$  diverges;  $\sigma_{J/\psi} \rightarrow \infty$  unless inelastic process also occurs.

## $J/\psi$ nuclei using folded potentials

- \* *Nuclear-Bound Quarkonium*, S.J.Brodsky, I.Schmidt, & G.F.deTera mond,PRL(1990).  
*Comment*: D.A.Wasson, PRL(1991).
- \* *Heavy-Quarkonia interactions with nucleons and nuclei*,  
A.B.Kaidalov & P.E.Volkovitsky, PRL(1992)
- \* *Is  $J/\psi$ -nucleon scattering dominated by the gluonic van der Waals interaction?*  
S.J.Brodsky & G.A.Miller, PLB(1997)
- \* *Proton-proton spin correlations at charm threshold and quarkonium bound to nuclei* ,  
G.F.deTera mond, R.Espinoza, & M.Ortega-Rodriguez, PRD(1998).

## Quark-meson coupling model

- \* *Nucleon and hadron structure changes in the nuclear medium and the impact on observables*, K.Sato, K.Ysushima, & A.W.Thomas, PPNP(2007).
- \* *Binding of  $D, D$  and  $J/\psi$  mesons in nuclei*, K.Tsushima, arXiv:0907.0244v1 [nucl-th].



## New nuclear structure with $J/\psi$ ?

A small (0.2 ~0.3 fm), heavy ( $\sim 3m_N$ ), spin 1 and negative parity  $J/\psi$  placed in a nucleus with

$$V_N > V_{J/\psi} : \sim \text{Nucleons move around } J/\psi \text{ (Born-Oppenheimer)}$$

For example, a  $J/\psi$  would sit in the middle of four nucleons in  ${}^4\text{He}_{J/\psi}$ , squeezing the  ${}^4\text{He}$  structure, while the nuclear compressibility coefficient is

$$b_{\text{comp}} \equiv \rho^2 \partial^2 (E/A) / \partial \rho^2 \equiv K/9 \approx 13 \text{ MeV}.$$

The spin/parity  $1^-$ : Effects on shell structure in heavier nuclei?

The real picture depends on the magnitude of  $V_{J/\psi-N}$ ,  $E_{B,J/\psi}$ ,

$$\text{and } K.E._{J/\psi} = V_{J/\psi-N} + E_{B,J/\psi}.$$

## 6. NLO effect and nuclear interaction of the excited c-c-bar mesons

**TABLE 1.** Charmonium Mass shift in nuclear matter in MeV

Charmonium	$J^{PC}$	QCD 2nd order Stark Effect	QCD sum rules	Effects of $D\bar{D}$ loop
$\eta_c$	$0^{-+}$	- 8 MeV	-5 MeV	No effect
$J/\psi$	$1^{--}$	-8 MeV	-7 MeV	< 2 MeV
$\chi_{0,1,2}$	$0, 1, 2^{++}$	- 40 MeV	-60 MeV	No effect on $\chi_1$
$\psi(3686)$	$1^{--}$	-100 MeV		< 30 MeV
$\psi(3770)$	$1^{--}$	-140 MeV		< 40 MeV

S.H.Lee, arXiv:nucl-th/0310080

T.Song & S. H. Lee, PRD 72, 034002 (2005).

The issue of the coupling to other channels including inelastic ones.

## Strong coupling of $\psi'$ to $J/\psi$

$$V_{\psi'} \approx -21 \text{ MeV} \times \left[ \frac{1+C}{2} \frac{\alpha_{\psi'}}{2 \text{ GeV}^{-3}} \right]$$

$$\begin{aligned} \Gamma_{\psi' J/\psi} &= |\mathcal{T}_{\psi' J/\psi}|^2 \frac{p_f}{32\pi(M_{\psi'} + m_N)M_{\psi'}m_N} \rho_N \\ &\approx 70 \text{ MeV} \left[ \frac{1+C}{2} \right]^2 \frac{[\alpha_{\psi'}]^2}{2 \text{ GeV}^{-3}} |F(q^2)|^2 \end{aligned}$$

$$\begin{aligned} \sigma(\psi' + N \rightarrow J/\psi + N) &= \frac{1}{p_i} \frac{|\mathcal{T}_{\psi' J/\psi}|^2 p_f}{16\pi(M_{\psi'} + m_N)^2} \\ &\approx 16 \text{ mb} \left[ \frac{1 \text{ GeV}}{p_i} \right] \left[ \frac{1+C}{2} \right]^2 \\ &\quad \times |F(q^2)|^2, \end{aligned}$$

A. Sibirtsev & M.B.Voloshin, PRD 71, 076005(2005)