Matrix model formulations of superstring theory

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Plan of the lectures

I. Superstring theory and matrix models
   (1/11 10:45-12:00)

II. The birth of our universe
    (1/12 10:45-12:00)

III. Confirmation of gauge/gravity duality
     (1/14 10:45-12:00)

Rem.) I will be here until 1/14 morning.
     Please ask me questions before I leave.
I. Superstring theory and matrix models
Plan of the 1\textsuperscript{st} lecture: Superstring theory and matrix models

1. What is superstring theory?
2. Matrix model for superstring theory
3. Summary
I-1 What is superstring theory?
1. What is superstring theory?

- Why superstring theory?
  A: To go beyond Einstein’s theory of general relativity.

- What’s wrong with Einstein’s theory of general relativity?
  A: Singularities appear at the center of a black hole at the beginning of the Universe.
Singularities
(space-time curvature diverges)

Black hole

Big bang

singularity (curvature diverges)
General Relativity becomes invalid!
(Quantum effects become non-negligible.)
The scale at which quantum effects of gravity become non-negligible

3 fundamental constants of physics

\[ L_{\text{pl}} = \left( \frac{\hbar G}{c^3} \right)^{1/2} \sim 10^{-33} \text{cm} \]

When the curvature radius of space-time becomes Planck length, one cannot use Einstein’s theory of general relativity!
Why strings?

- The case of other 3 forces

Quantum Field Theory
(Particles are treated as points.)

Electromagnetism
Weak interaction
Strong interaction

Heisenberg’s uncertainty principle

Intermediate states with infinite energy (momentum) contribute

can be infinitely short time

UV divergence
Renormalization theory

(Tomonaga, Feynman, Schwinger)

- The results for physical quantities (mass spectrum, scattering amplitudes, etc.) can be made finite by redefining the parameters that describe elementary processes
Coupling constant

- Dimensionless in the case of gauge theory
  “renormalizable theory”
- Newton’s gravitational constant

\[[G] \sim M^{-2}\]

\[F_{\text{Newton}} = G \frac{m_1 m_2}{r^2}\]

\[F_{\text{Coulomb}} = \alpha \frac{q_1 q_2}{r^2}\]

In naïve quantum extension of Einstein’s theory:

UV divergence becomes worse at higher orders in the expansion w.r.t. the coupling constant!
String theories do not have UV divergence

Cannot be closer than the string scale

No UV divergences!
Crucial differences from particle theory based on quantum field theory

- propagation
  strings can vibrate!

  • A single string can represent various particles. (Fermions as well as bosons appear from superstrings.)

- interactions
  joining and splitting of strings

  • There is no particular “interaction point” (hence no UV div.).
  • There is no freedom to introduce ad hoc interactions.
Historical remark

- Closed string inevitably includes massless spin-2 particle

  This was a crucial defect as a theory for hadrons (Nambu’s idea)
  since there is no such states,
  but was turned into a virtue (“graviton”) in the context of quantum gravity
  (1974 Sherk-Schwarz, Yoneya)

- The scale of the theory had to be changed from the scale of hadrons to the Planck scale, though.
  (a few 100 GeV) (10^{19} \text{GeV})
Superstring theory

Various vibration modes correspond to various particles.

- photon
- gluon
- $W^\pm, Z$ etc.
- graviton, in particular

Unified description of 4 forces including gravity

1974 Sherk-Schwarz, Yoneya
1984 Green-Schwarz
The goals of superstring theory

- space-time dimensionality puzzle
  - critical dimension is (9+1), but we live in (3+1)d
- particle contents
  - gauge group: $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$
  - matter contents: 3 generations ($q$ and $\ell$) + Higgs(?)
- coupling constants in the Standard Model

- the birth of our Universe and “inflation”
- the fate of our Universe
  - (dark energy, cosmological constant problem)
- the interior structure of a black hole
A big obstacle: non-perturbative definition is not yet established!

Comparison: QCD

- Quark confinement
  \[ V(r) \propto r \]
  lattice gauge theory (Wilson, 1974)

- Nonperturbative calculations

- Perturbative calculations
  \[ V(r) \propto -\frac{1}{r} \]
Compactification

- Superstring theory is naturally defined in \((9+1)\)dim. 
  Unitarity + Lorentz invariance

- \((3+1)\)-dimensional space-time is expected to appear due to some nonperturbative dynamics. 
  not known, at least, until recently.

- Search for perturbative vacua with compactified 6d.
  Good : One can obtain SM-like models.
  Bad : Too many vacua. ("Landscape")

Understanding the nonperturbative dynamics of compactification is crucial to understand our real world!
“Landscape”

- Tremendously many vacua we are living in one of them due to statistical reasons or just because of “anthropic principle”

- Pessimism that appeared from studies based on perturbative string theory + D-branes

- It remains to be seen what happens if full nonperturbative effects are taken into account!
I-2 Matrix model for superstring theory
Matrix model as a nonperturbative formulation of string theory

- 't Hooft (1974)

Feynman diagrams in U(N) gauge theory

large-\(N\) limit
with \(\lambda = g_{\text{YM}}^2 N\) fixed

\(\frac{1}{N}\)-expansion

(discretized)

string worldsheet

tree diagrams

(approximation)

(perturbative expansion)

Note: gauge theory is well-defined for finite \(N\).

One may hope to obtain a nonperturbative formulation of string theory by using matrix degrees of freedom!
An explicit example of nonperturbative string theory

- Brezin-Kazakov, Douglas-Shenker, Gross-Migdal (1990)

\[ Z = \int d\phi \ e^{-S} \]

\[ S = N \ \text{tr} \left( \frac{1}{2} \phi^2 - \frac{\kappa}{3} \phi^3 \right) \]

\[ \kappa \to \kappa_{cr} \text{ as } N \to \infty \]

\[ \text{with } |\kappa - \kappa_{cr}|^p N \text{ fixed} \]

All the diagrams of higher orders equally contribute.

nonperturbative formulation of string theory with 0d target space
Matrix model for superstring theory in 10d

- **IKKT model (1996)** Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

\[
S_b = -\frac{1}{4g^2} \text{tr}([A_\mu, A_\nu][A^\mu, A^\nu])
\]

\[
S_f = -\frac{1}{2g^2} \text{tr}(\psi_\alpha (C \Gamma^\mu)_{\alpha\beta}[A_\mu, \psi_\beta])
\]

\(\mathcal{N} \times \mathcal{N}\) Hermitian matrices

- \(A_\mu\) \((\mu = 0, \cdots, 9)\) Lorentz vector
- \(\psi_\alpha\) \((\alpha = 1, \cdots, 16)\) Majorana-Weyl spinor

raised and lowered by the metric

\(\eta = \text{diag}(-1, 1, \cdots, 1)\)

The action has manifest SO(9,1) symmetry.
Connection to the worldsheet formulation

- **worldsheet action**

\[
S = \int d^2 \xi \sqrt{g} \left( \frac{1}{4} \{X^\mu, X^\nu\}^2 + \frac{1}{2} \bar{\Psi} \gamma^\mu \{X^\mu, \Psi\} \right)
\]

\[
\{X, Y\} \equiv \frac{1}{\sqrt{g}} \epsilon^{ab} \frac{\partial X}{\partial \xi^a} \frac{\partial Y}{\partial \xi^b}
\]

Poisson bracket (regarding $\xi_1$ and $\xi_2$ as $p$ and $q$ in Hamilton dynamics)

**quantization** $\implies$ **IKKT** ($\hbar \sim \frac{1}{N}$)

\[
\{X^\mu(\xi), X^\nu(\xi)\} \rightarrow -i[A^\mu, A^\nu]
\]
$\mathcal{N} = 2$ supersymmetry

\begin{align*}
\delta^{(1)} A_\mu &= i \bar{\epsilon}_1 \Gamma_{\mu} \psi \\
\delta^{(1)} \psi &= \frac{i}{2} \Gamma^{\mu \nu} [A_\mu, A_\nu] \epsilon_1 \\
\delta^{(2)} A_\mu &= 0 \\
\delta^{(2)} \psi &= \epsilon_2
\end{align*}

Take a linear combination:

\begin{align*}
\bar{Q}^{(1)} &= Q^{(1)} + Q^{(2)} \\
\bar{Q}^{(2)} &= i (Q^{(1)} - Q^{(2)})
\end{align*}

\[ [\bar{\epsilon}_1 \bar{Q}^{(i)}, \bar{\epsilon}_2 \bar{Q}^{(j)}] A_\mu = -2 \delta^{ij} \bar{\epsilon}_1 \Gamma_{\mu} \epsilon_2 1_{N \times N} \]

“translation” is realized by \[ \delta A_\mu = c_\mu 1_{N \times N} \]

consistent with identification of $A_\mu$ as ”coordinates”
Natural realization of 2\textsuperscript{nd} quantization

Each of these blocks \longleftrightarrow disconnected worldsheet

Many-body states of strings are naturally included!
Emergence of gravitons

Integrate out the off-diagonal elements to obtain the effective action

\[ \sim \frac{1}{r^8} \]
Dynamical generation of Euclidean space-time

Wick rotation

\[ A_0 = i A_{10} \quad \Gamma^0 = -i \Gamma_{10} \]

Does our 4-dimensional space-time appear?

10 \( N \times N \) Hermitian matrices

\[ (A_{\mu})_{ij} = (x_{i})_{\mu} \delta_{ij} + (a_{\mu})_{ij} \]

\[ \mu = 1, \ldots, 10 \]

\[ i, j = 1, \ldots, N \]

Euclidean model

Finite without cutoff

Krauth-Nicolai-Staudacher ('98),
Austing-Wheater ('01)
Dynamical generation of Euclidean space-time (cont’d)

- Derivation of low-energy effective theory branched-polymer-like system  Aoki-Iso-Kawai-Kitazawa-Tada (’99)

- Explicit calculations by the Gaussian expansion method to study SSB of SO(10)
  Nishimura-Sugino (’02), Nishimura-Okubo-Sugino, Kawai, Kawamoto, Kuroki, Matsuo, Shinohara, Aoyama, Shibusa,…

- Recent observation  Nishimura-Okubo-Sugino(’11)
  1. free energy of SO(d) symmetric vacua  (d=2,3,4,5,6,7) minimum at d=3
  2. extent of space-time  finite in all directions
Results of the Gaussian expansion method
J.N.-Okubo-Sugino (arXiv:1108.1293)

Minimum of the free energy occurs at $d=3$

Extent of space-time finite in all directions

SSB of SO(10) : interesting dynamical property of the Euclidean model, but is it really related to the real world?
I-3 Summary
Summary of the 1st lecture

Superstring theory

- severeness of **UV divergence in quantum gravity** naturally hints at **extended objects**
- **unified theory** of all particles (both forces and matters)
- however, **too many vacua** ("landscape") due to variety of **compactifications from 10d to 4d**
- **fully nonperturbative formulation** is crucial

Matrix models

- analogous to **lattice gauge theory for QCD**
- **IKKT model**: nonperturbative formulation of superstrings
- the **Euclidean version** has **interesting dynamics** but **not quite realistic**… (motivates **Lorentzian version**)