Matrix model formulations of superstring theory

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Plan of the lectures

I. Superstring theory and matrix models  
   (1/11 10:45-12:00)

II. Birth of our universe  
    (1/12 10:45-12:00)

III. Confirmation of gauge/gravity duality  
     (1/14 10:45-12:00)

Rem.) I will be here until 1/14 morning.  
     Please ask me questions before I leave.
II. Birth of our Universe
Plan of the 2nd lecture: Birth of our universe

1. Review of previous works
2. Lorentzian matrix model
3. Summary
II-1 Review of previous works
1. Review of previous works

- Big Bang cosmology

Our universe is expanding since it was born as an invisibly tiny point 13.7 billion years ago.
3 evidences for standard Big Bang cosmology

- Discovery of cosmic expansion (Hubble 1929)
- Theory of nucleosynthesis (Alpher-Bethe-Gamov 1948)
- Discovery of Cosmic Microwave Background (Penzias-Wilson 1965)
History of our Universe

- Birth of our Universe
- Inflation
- Pair annihilation of electrons and positrons
- Neutrinos decoupling from thermal equilibrium
- Nucleosynthesis
- Recombination
- Structure formation
- Growth of density fluctuation
- Transparent to radiation
- Cosmic Microwave Background

Unknown Region!
Singularities (space-time curvature diverges)

Black hole

Big bang

Singularity (curvature diverges)
General Relativity becomes invalid!
(Quantum effects become non-negligible.)

We need to go beyond Einstein!
Quantum Cosmology in 1980s

- “Creation of Universes from nothing”
  Vilenkin (’82,’84)
  Tunneling effects discussed with imaginary time

- “Wave function of the Universe”
  Hartle-Hawking (’83)
  “no boundary” proposal in path-integral formulation

The problem of UV divergence ignored by restriction to a uniform isotropic universe.
More recent works

- Causal Dynamical Triangulation
  
  Ambjorn-Jurkiewicz-Loll ('05)

  Path-integral over the metric
  $\implies$ summation over simplicial manifolds

  Lorentzian nature implemented by introducing causal structure.

- Smooth (3+1)-dimensional space-time emerges dynamically.
- Restoration of full Lorentz symmetry is nontrivial. (c.f., Horava gravity)
II-2 Lorentzian matrix model

Kim-J.N.-Tsuchiya
PRL 108 (2012) 011601
[arXiv:1108.1540]
What we do here:

- Study the Lorentzian matrix model nonperturbative formulation of superstring theory in (9+1) dimensions.
- Highly non-trivial time-evolution of space obtained from dominant matrix configurations.
- SO(9) symmetry of space is broken spontaneously to SO(3) at some point in time, strongly suggesting the birth of our Universe.
- No free parameter in the theory except for 1 scale parameter (c.f., $\Lambda_{\text{QCD}}$). In particular, there is no “initial condition” problem.
Wick rotation is not obvious at all in gravitational theory!

- quantum field theory in flat space, Wick rotation can be justified by analytic continuation for Green’s functions.

That’s not the case when we have gravity.

- Amjorn, Jurkiewicz, Loll (’05) Within dynamical triangulation approach, Lorentzian gravity is quite different from Euclidean gravity.

- Kawai-Okada (’10) Coleman’s multiverse proposal for the cosmological constant problem revisited using Lorentzian gravity.
Matrix model with SO(9,1) symmetry

\[ S_b = -\frac{1}{4g^2} \text{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) \]
\[ S_f = -\frac{1}{2g^2} \text{tr}(\Psi_\alpha (C \Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta]) \]

\( N \times N \) Hermitian matrices

\( A_\mu \) (\( \mu = 0, \ldots, 9 \)) Lorentz vector
\( \Psi_\alpha \) (\( \alpha = 1, \ldots, 16 \)) Majorana-Weyl spinor

\( \eta = \text{diag}(-1, 1, \ldots, 1) \)

Wick rotation \((A_0 = -iA_{10}, \Gamma^0 = i\Gamma_{10})\)

Euclidean model with SO(10) symmetry
Difference between Euclidean and Lorentzian (I) bosonic action

- **Euclidean model**
  \[ S_b \propto \text{tr} (F_{\mu\nu})^2 \quad F_{\mu\nu} = -i[A_{\mu}, A_{\nu}] \]
  positive definite

  Classical flat direction is lifted up by quantum effects.
  The model is well defined without any cutoff.
  Krauth-Nicolai-Staudacher ('98), Austing-Wheater ('01)

- **Lorentzian model**
  \[ S_b \propto \text{tr} (F_{\mu\nu} F^{\mu\nu}) = -2 \text{tr} (F_{0i})^2 + \text{tr} (F_{ij})^2 \]
  opposite sign!

Looks extremely unstable!

Hence, no one ever dared to study this model seriously!
Difference between Euclidean and Lorentzian (II) Pfaffian (obtained by integrating out fermions)

\[ \int d\Psi \ e^{iS_f} = \text{Pf} \mathcal{M}(A) \]

- **Euclidean model** \( \text{Pf} \mathcal{M}(A) \in \mathbb{C} \)

  The phase plays a crucial role in SSB of SO(10),
  But it makes Monte Carlo studies extremely difficult.

  J.N.-Vernizzi ('00), Anagnostopoulos-J.N.('02)

- **Lorentzian model** \( \text{Pf} \mathcal{M}(A) \in \mathbb{R} \)

  Good news for Monte Carlo studies, but we lose a source of SSB.
The definition of the partition function

- **Euclidean model**

\[ Z = \int dA \, d\psi \, e^{-S} = \int dA \, e^{-S_b} \text{Pf} \mathcal{M}(A) \]

- **Lorentzian model**

\[ Z = \int dA \, d\psi \, e^{iS} = \int dA \, e^{iS_b} \text{Pf} \mathcal{M}(A) \]

connection to the worldsheet theory

\[ S = \int d^2\xi \sqrt{g} \left( \frac{1}{4} \{X^\mu, X^\nu\}^2 + \frac{1}{2} \bar{\psi} \gamma^\mu \{X^\mu, \psi\} \right) \]

\[ \xi_0 \equiv -i\xi_2 \quad \text{(We need to Wick rotate the worldsheet coordinate, too.)} \]
Regularizing the Lorentzian model

- We need to introduce IR cutoffs in both temporal and spatial directions.
  \[
  \frac{1}{N} \text{tr} (A_0)^2 \leq \kappa \\
  \frac{1}{N} \text{tr} (A_i)^2 \leq L^2
  \]

- It turns out that we can remove them as we take the large-N limit. (highly nontrivial dynamical property)

- Lorentz symmetry and supersymmetry is broken explicitly, but this explicit breaking is expected to disappear in the large-N limit. (need to be checked by studying Ward identities.)
How to deal with the phase factor \( e^{iS_b} \) in

\[
Z = \int dA \, d\psi \, e^{iS} = \int dA \, e^{iS_b} \text{Pf}\mathcal{M}(A)
\]

Under the scale transformation \( A_\mu \rightarrow \rho A_\mu \)

\[
S_b \quad \mapsto \quad \rho^4 S_b
\]
\[
dA \quad \mapsto \quad \rho^{10(N^2-1)} dA
\]
\[
\text{Pf}\mathcal{M}(A) \quad \mapsto \quad \rho^{8(N^2-1)} \text{Pf}\mathcal{M}(A)
\]

Integrating over the scale factor first, we get \( \delta(S_b) \)

and \[
\frac{1}{N} \text{tr}(A_i)^2 = L^2
\]
The model we put on a computer

\[ Z = \int dA \delta \left( \frac{1}{N} \text{tr} (F_{\mu \nu} F^{\mu \nu}) \right) \text{Pf} \mathcal{M}(A) \]

\[ \times \delta \left( \frac{1}{N} \text{tr} (A_i)^2 - 1 \right) \theta \left( \kappa - \frac{1}{N} \text{tr} (A_0)^2 \right) \]

Monte Carlo simulation: Rational Hybrid Monte Carlo algorithm
no sign problem unlike in the Euclidean model

c.f.) Yoneya’s model (’97)

\[ Z = \int dA \delta \left( \frac{1}{N} \text{tr} (F_{\mu \nu} F^{\mu \nu} - C) \right) \text{Pf} \mathcal{M}(A) \]

\[ C < 0 : \text{a model motivated by space-time uncertainty principle} \]
Extracting the time-evolution from matrices

- Eigenvalue distribution of $A_0$
  
  Extends smoothly as $\kappa \to \infty$
  
  Thanks to supersymmetry

- C.f.) Bosonic model
  
  Eigenvalues of $A_0$ attract each other
  
  The distribution has finite extent even in the $\kappa \to \infty$ limit

SUSY plays a crucial role in generating the time!
We observe band-diagonal structure

\[ A_0 = \begin{pmatrix} t_1 & * & * & \cdots & * \\ t_2 & n & t_{\nu+1} & \cdots & t_n \\ & n & n & \cdots & n \\ & & t_{\nu+n} & \cdots & t_N \end{pmatrix} \]

\[ A_i = \begin{pmatrix} n \\ n \end{pmatrix} \]

\[ \overline{A}_i(t) \]

\[ \delta t \]

\[ t = \frac{1}{n} \sum_{a=1}^{n} t_{\nu+a} \]

\[ \nu = 0, 1, \ldots, N - n \]

\( t \) represents space structure at fixed time \( t \)
Determination of the block size

\[ N = 16 \]

\[
\left| \frac{Q_{ij}}{Q_{N2,N2}} \right|^{1/2}
\]

\[
|l-j|
\]

\[ (l+j)/2 = 2 \]
\[ (l+j)/2 = 4 \]
\[ (l+j)/2 = 6 \]
\[ (l+j)/2 = 8 \]

\[ (A_i)^2 = \begin{cases} 
(1 + j)/2 = 2 \\
(1 + j)/2 = 4 \\
(1 + j)/2 = 6 \\
(1 + j)/2 = 8 
\end{cases} \]

We take \( n = 4 \)
The size of the space v.s. time

\[ R(t)^2 \equiv \frac{1}{n} \text{tr} \, A_i(t)^2 \]

peak at \( t = 0 \) starts to grow for \( \kappa > \kappa_{cr} \)

\[ N = 16 , \quad n = 4 \]

Symmetric under \( t \to -t \)

We only show \( t < 0 \)
Spontaneous breaking of SO(9)

\[ T_{ij}(t) = \frac{1}{n} \text{tr} \{ \bar{A}_i(t) \bar{A}_j(t) \} \]

\[ \text{SO}(9) \xrightarrow{\text{SSB}} \text{SO}(3) \]

\[ N = 16, \quad \kappa = 4.0 \]

“critical time”
Mechanism of SSB

\[ \text{tr}(F_{\mu\nu}F^{\mu\nu}) = 0 \quad \leftrightarrow \quad 2\text{tr}(F_{01})^2 = \text{tr}(F_{ij})^2 \]

\[ F_{\mu\nu} = -i[A_\mu, A_\nu] \]

large \( \kappa \) \quad \rightarrow \quad \text{tr}(F_{0i})^2 \quad \text{become large, and so does} \quad \text{tr}(F_{ij})^2

\[ \frac{1}{N}\text{tr}(A_i)^2 = 1 \quad \rightarrow \quad \text{It is more efficient to maximize} \quad \text{tr}(F_{ij})^2 \quad \text{at some fixed time} \]

Middle point \( t = 0 \) is chosen, because the eigenvalue distribution of \( A_0 \) is denser around \( t = 0 \) so that enhancement to \( \text{tr}(F_{0i})^2 \) is the least.

\[ \text{Peak of} \quad R(t)^2 = \frac{1}{n}\text{tr}(\bar{A}_i)^2 \quad \text{at} \quad t = 0 \quad \text{grows as} \quad \kappa \quad \text{increases} \]
Mechanism of SSB (cont’d)

maximize $\frac{1}{N} \text{tr} (F_{ij})^2$
with fixed $\frac{1}{N} \text{tr} (A_i)^2 = 1$

maximize $G = \text{tr} (F_{ij})^2 - \lambda \text{tr} (A_i)^2$

$2 [A_j, [A_j, A_i]] - \lambda A_i = 0$

solution: $A_i = \chi L_i$ for $i \leq d$
$A_i = 0$ for $d < i \leq 9$

representation matrices of a compact semi-simple Lie algebra
with $d$ generators

$L_i = \begin{pmatrix} \frac{1}{2} \sigma^i \\ 0 \end{pmatrix}$

d=3!
An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.

The set-up is invariant under rotation by $90^\circ$

This problem has rotational symmetry.
An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.

\[ \ell = 2\sqrt{2} \approx 2.828 \]

Wrong!
An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.

Becomes a different figure if we rotate it by 90 degrees.

Breaks rotational symmetry

\[ \ell = 1 + \sqrt{3} \]
\[ \sim 2.732 \]

Correct!
An example of spontaneous breaking of rotational symmetry

Connect the 4 corners by a path in such a way that its total length becomes minimum.

This can be obtained by rotating the previous figure.

Correct, as well!
Examples of spontaneous symmetry breaking in physics

- Bardeen-Cooper-Schrieffer theory of superconductor formation of Cooper pairs breaks $U(1)$ symmetry

- Nambu: the origin of hadron mass in QCD formation of chiral condensate breaks chiral symmetry pions can be considered as Nambu-Goldstone bosons.

- Higgs mechanism: the origin of mass in Standard Model

our conjecture:

The SSB of SO(9,1) in the Lorentzian matrix model: the origin of our Universe
Removing the two cutoffs $\kappa$ and $L$ in the $N \to \infty$ limit

\[
\begin{align*}
1) \ & \kappa \to \infty \text{ with } N \to \infty \quad \text{(continuum limit)} \\
\ & \kappa = \beta N^{1/4}
\end{align*}
\]

\[
\begin{align*}
2) \ & L \to \infty \text{ with } \beta \to \infty \quad \text{(infinite volume limit)} \\
\ & \text{fix the scale by } R(t_C)
\end{align*}
\]

The theory thus obtained has no parameters other than one scale parameter!

A property that nonperturbative superstring theory is expected to have!
Large-N scaling

Clear large-N scaling behavior observed with $\kappa = \beta N^{1/4}$ (continuum limit)
Infinite volume limit

The extent of time increases and the size of the universe becomes very large at later time. (infinite volume limit)
I-3 Summary
Summary of the 2\textsuperscript{nd} lecture

- A new proposal for the nonperturbative formulation of type IIB superstring theory in ten dimensions.

  instead of making Wick rotation, we introduce the IR cutoffs
  for both temporal and spatial directions

  \begin{align*}
  (\kappa) & \quad (L)
  \end{align*}

- The two cutoffs can be removed in the large-N limit.

- The theory thus obtained has no parameters other than one scale parameter.
• Integrating over the scale factor first, we obtain a model without sign problem.

c.f.) Monte Carlo studies of Euclidean model difficult due to sign problem

• Monte Carlo simulation revealed SSB of SO(9) down to SO(3) at some critical time. mechanism is totally different from that for the SSB in the Euclidean model

  the size of the 3d space increases with time

• Cosmological singularity is naturally avoided due to noncommutativity.
Crucial role played by SUSY
c.f.) bosonic model
eigenvalues of $A_0$ attract each other
the distribution does not extend as $\kappa \to \infty$
no expansion and no SSB!

Complementary studies based on classical equations of motion

Kim-J.N.-Tsuchiya arXiv:1110.4803

A class of SO(3) symmetric solutions

- the time-dependence compatible with the expanding universe
- noncommutativity of space time : OK
Discussion

- It is likely that we are seeing the birth of our Universe in the Monte Carlo results.
- **We need larger N to see late times.**
- The mechanism of SSB $\text{SO}(9) \rightarrow \text{SO}(3)$ relies crucially on noncommutative nature of space.
- Does commutative space-time, we observe now, appear at late time?
Classical solutions

- At later times, we naively expect a classical solution to dominate because the action gets larger due to expansion.

- As a complementary approach, we therefore study classical solutions.

- It turns out that there are infinitely many solutions in the large-N limit (reminiscent of the Landscape).

- We need to find which one is connected to Monte Carlo result.

- We find some interesting examples of solutions which represent expanding universe with commutative space-time.
Classical solutions (cont’d)

EOM

\[ \frac{\delta}{\delta A_\mu} \left( -\frac{1}{4} \text{tr}([A_\mu, A_\nu]^2) - \frac{\lambda}{2} \text{tr}(A_i)^2 - \frac{\tilde{\lambda}}{2} \text{tr}(A_0)^2 \right) = 0 \]

Example

\[ A_0 = \sqrt{-\lambda} T_0, \quad A_1 = c_1 \sqrt{-\lambda} T_1, \quad A_2 = c_2 \sqrt{-\lambda} T_1, \quad A_3 = c_3 \sqrt{-\lambda} T_1 \]

\[ SL(2, R) \] algebra \quad \text{commutative space-time}

\[ [T_0, T_1] = i T_2 \quad [T_0, T_2] = -i T_1 \quad [T_1, T_2] = -i T_0 \]

\[ a(t) = A \sqrt{t^2 + t_0^2} \]

\[ H = \frac{\dot{a}}{a} \sim a^{-\frac{3}{2}(1+w)} \quad w = -\frac{1}{3} \left( \frac{2t_0^2}{t^2} + 1 \right) \]

| \( t = t_0 \) | \( w = -1 \) |
| \( t \to \infty \) | \( w = -\frac{1}{3} \) |

\[ A = 1 \quad t_0 = 5 \]
Speculations

Space-space noncommutativity

Monte Carlo simulation

SO(9)  symmetry of space  SO(3)

t_{cr}  present time

classical solution

accelerating expansion

size of the space

Space-space NC disappears for some dynamical reason.
My main message:

- The birth of our Universe might be considered as an issue that goes far beyond the reach of science.

- Intriguingly, the issue might be the first “real physics” that can be addressed by superstring theory.

- This is not so surprising since superstring theory is supposed to be a fundamental theory at the Planck scale.

- Reproducing Standard Model at the TeV scale may be more challenging, but it is totally well-defined. That will prove that superstring theory/matrix model is indeed describing our Nature.
Future prospect (I)

**Inflation (1981)**

A rapid accelerating expansion believed to have occurred just after the birth of our universe.

Solves various puzzles in the Big Bang cosmology naturally.

Explains detailed properties of the Cosmic Microwave Background

Can we clarify the mechanism of inflation?
Future prospects (II)

Accelerating expansion in the present epoch

Observation of type Ia supernovae → Nobel Prize in Physics, 2011
Precise measurement of CMB (WMAP etc.) etc.

The existence of dark energy
Mysterious energy that does not dilute as the universe expands
Occupies more than 70% of the total energy of the Universe

Can we explain it by quantum gravitational effects in superstring theory?
Future prospects (III)

Remaining puzzles in particle physics

- **dark matter**
  Strongly suggested by cosmic observations
  No good candidate in **Standard Model** of particle physics
  *(possibly)* superpartners, excited modes in extra dimensions

- **Higgs particle**
  The only particle in Standard Model that is still undiscovered.
  Introduced for a mechanism to give mass to particles.
  Unnatural from the viewpoint of particle physics including gravity
  Needs clarification from both experimental and theoretical sides.

Can we clarify all these puzzles using superstring theory?
Important issues to be addressed in Lorentzian matrix model

- Does a local field theory on a commutative space-time appear at later time?

- How do 4 fundamental interactions and the matter fields appear at later time?

Monte Carlo simulation AND Studies of classical solutions (+ quantum corrections) Some approach like “renormalization group”

We hope the Lorentzian matrix model provides a new perspective on particle physics beyond the standard model cosmological models for inflation, modified gravity, etc..
BACKUP SLIDES
Regularizing the Lorentzian model

In order to separate space and time, we “gauge fix” the boost invariance.

the gauge-fixed config. \( \{ \tilde{A}_\mu \} \) with \( \tilde{A}_\mu = O_{\mu\nu} A_\nu \) is chosen such that \( \frac{1}{N} \text{tr} (\tilde{A}_0)^2 \) is minimized with respect to \( O \in \text{SO}(9, 1) \)

**SO(9) symmetry** is still manifest.

(1) IR cutoff in the temporal direction

\[
\frac{1}{N} \text{tr} (A_0)^2 \leq \kappa \frac{1}{N} \text{tr} (A_i)^2
\]

(invariant under \( A_\mu \rightarrow \rho A_\mu \))
Regularizing the Lorentzian model (cont’d)

Regularizing oscillating functions

\[ Z = \int dA \, e^{iS_b - \epsilon |S_b|} \, Pf\mathcal{M}(A) \]

\[ \epsilon \to 0 \text{ limit after path integral} \]

\[ Z = \int dA \int_0^\infty dr \, \delta \left( \frac{1}{N} \text{tr} (A_i)^2 - r \right) e^{iS_b - \epsilon |S_b|} \, Pf\mathcal{M} \]

inserting unity

rescaling variables \( A_\mu \to r^{1/2} A_\mu \)

\[ \int_0^\infty dr \, r^{\frac{18}{2} (N^2 - 1) - 1} e^{r^2 (iS_b - \epsilon |S_b|)} \propto \frac{1}{|S_b|^{\frac{18}{4} (N^2 - 1)}} \]

diverges at \( S_b = 0 \)
Regularizing the Lorentzian model (cont’d)

This divergence comes from integrating over the scale factor \( r \)

Cure this divergence by imposing:

(2) IR cutoff in the spatial direction

\[
\frac{1}{N} \text{tr} (A_i)^2 \leq L^2
\]

\[
\int_0^{L^2} dr \ r^{\frac{18}{2}(N^2-1)-1} e^{r^2(iS_b-\epsilon|S_b|)} \propto \left( \frac{1}{L^4} + c|S_b| \right)^{-\frac{18}{4}(N^2-1)}
\]

for sufficiently large \( L \) and \( N \) \( \delta(S_b) \)
Thus we arrive at

\[
Z = \int dA \delta \left( \frac{1}{N} \text{tr} (F_{\mu\nu}F^{\mu\nu}) - C \right) \text{PfM}(A) \times \delta \left( \frac{1}{N} \text{tr} (A_i)^2 - 1 \right) \theta \left( \kappa - \frac{1}{N} \text{tr} (A_0)^2 \right)
\]

\[C = 0, \text{ according to our derivation}\]

\[
C < 0 : \text{ a model motivated by space-time uncertainty principle} \quad \text{Yoneya ('97)}
\]

Monte Carlo simulation: Rational Hybrid Monte Carlo algorithm

no sign problem unlike in the Euclidean model