Excited Charmonium Spectrum and $\eta_c - \eta'$ - Mixing

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Outline

1. Charmonia on the lattice
2. The Spectrum
3. $\eta_c - \eta'$ - Mixing
4. Vector Mixing
5. Conclusion & Outlook

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Goals

- confirmation and prediction of the experimental spectrum
- precise calculation of decay constants, EM transition rates, ...
- gaining insight in the quark and gluon content of the physical states
- standard approach (in Euclidian time):

\[ \langle O(t)O^\dagger(0) \rangle = A_0 e^{-m_0 t} + A_1 e^{-m_1 t} + \ldots \]
Fermion action

- crucial question: Which fermion action to choose?
- chiral symmetry (mostly) plays minor role in charm systems: chiral actions like overlap or domain wall are overkill
- effective actions like NRQCD:
  - no true continuum limit
  - large radiative and relativistic corrections (∝ v² ≈ 0.3)
- happy medium: (Clover-)Wilson action with fine lattices
Variational Method

- Choose basis of operators $O_i, \ i = 1, \ldots, N$ and construct a CCM:

\[
C_{ij}(t) = \langle O_i(t)O_j^\dagger(0) \rangle = \sum_n v_i^n v_j^{n*} e^{-tE_n}
\]

- Solve symmetric generalized eigenvalue problem:

\[
C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} \psi^\alpha = \lambda^\alpha(t, t_0) \psi^\alpha
\]

- Eigenvalues behave like:

\[
\lambda^\alpha(t, t_0) \propto e^{-(t-t_0)E_\alpha} [1 + O(e^{-(t-t_0)\Delta E_\alpha})]
\]

- Eigenvectors behave like:

\[
\psi^\alpha(t, t_0) \approx v^n + \sum_{l>N} e^{-(E_l-E_n)t} F(n, l, N; t, t_0)
\]
All-to-All Propagators - Why?

- disconnected part of 2-point functions

- 3-point functions

- static light mesons
All-to-All Propagators - Theoretical framework

- create $N$ random noise vectors
  \[ \eta^i_{\alpha,a,x} = \frac{1}{\sqrt{2}}(v + iw) \quad v, w \in \{\pm 1\}, i = 1, \ldots, N \]

- define the random contraction
  \[ \frac{1}{N} \sum_i \eta^i_{\alpha,a,x} \eta^i_{\beta,b,y} = \delta_{x,y} \delta_{a,b} \delta_{\alpha,\beta} + O\left(\frac{1}{\sqrt{N}}\right) \]

- by inverting the Dirac Operator $M$ on these sources we obtain $N$ solution vectors
  \[ s^i = M^{-1} \eta^i, \quad i = 1, \ldots, N. \]

- naive estimate for A2AP
  \[ \sum_i s^i \eta^{i\dagger} = \sum_i M^{-1} \eta^i \eta^{i\dagger} = M^{-1} \left(1 + O\left(\frac{1}{\sqrt{N}}\right)\right) \]
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Effective masses

\[ \beta = 5.20 \]

\[ a^{-1} \approx 1730 \text{MeV} \]

\[ \#\text{conf} = 100 \]
Charmonia on the lattice

The Spectrum

$\eta_c - \eta'$ - Mixing

Vector Mixing

Conclusion & Outlook

Spectrum

$\Delta m_{1S} = 73(2)$MeV

$\Delta m_{2S} = 47(6)$MeV

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Motivation

1S Hyperfine splitting for charmonia consistently underestimated by lattice calculations

Possible reasons:

- missing continuum limit
- wrong sea quark content
- glueball mixing
- mixing with lighter mesons
The Mixing

- term “$\eta_c - \eta'$ - mixing” is a bit cheated in our case
- simulations performed on $N_f = 2$ lattice without valence strange quarks
- so we are actually in the flavor SU(2) $\rightarrow \eta_c - \eta_2$ - mixing
- build up a cross correlator matrix containing both light and charm meson interpolating fields
- eigenvectors will give information about magnitude of mixing
- “unperturbed” states: $(c\gamma_5\bar{c})_0$, $(c\gamma_5\bar{c})_{10}$, $(c\gamma_5\bar{c})_{80}$, $(u\gamma_5\bar{u})_0$, $(u\gamma_5\bar{u})_{5}$, $(u\gamma_5\bar{u})_{40}$
The Mixing Matrix

\[
\begin{pmatrix}
2 \circ \circ & 2 \circ \circ \\
2 \circ \circ & 2 \circ \circ - 4 \circ \circ
\end{pmatrix}
\]

\[
(t)
\]
Light Spectrum

![Graph showing mixing effects between \( \eta_c \) and \( \eta' \) in the light spectrum.](image)

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Excited Charmonium Spectrum and \( \eta_c - \eta' \) - Mixing
Submatrices Spectrum

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Full Matrix Spectrum
$\eta'$ eigenvectors

![Graph showing eigenvectors](image_url)

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$\eta_c$ eigenvectors

![Graph showing eigenvectors of $\eta_c$]
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Psi(2S) and Psi(3770) very close in mass

Question: How do their wavefunctions look like?

How good a quantum number is angular momentum?

Operator basis: $(c\gamma_i \bar{c})_0$, $(c\gamma_i \bar{c})_{20}$, $(c\gamma_i \bar{c})_{80}$, $(c s_{ijk} \gamma_j D_k \bar{c})_0$, $(c s_{ijk} \gamma_j D_k \bar{c})_{80}$
$J/\psi$ eigenvalues

\begin{figure}
\centering
\includegraphics[width=\textwidth]{jpsi_eigenvalues.png}
\end{figure}

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Excited Charmonium Spectrum and $\eta_c - \eta'$ - Mixing
1S eigenvector

The graph shows the behavior of different quantities as a function of $t/a$ for $\eta_c - \eta'$ mixing. The quantities plotted include $(\gamma_i)_0$, $(\gamma_i)_{20}$, $(\gamma_i)_{80}$, $(s_{ijk} \gamma_j D_k)_0$, and $(s_{ijk} \gamma_j D_k)_{80}$. The graph illustrates how these quantities vary over the range of $t/a$ from 4 to 14.
2S eigenvector

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1D eigenvector
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Conclusion & Outlook

- precise calculation of the charmonium spectrum possible
- however, hyperfine splitting still an open issue
- no significant $\eta_c - \eta'$ - mixing visible (on the used lattices)
- same analysis on $24^3 \times 48$ lattices with $m_\pi \approx 400$ MeV in progress
- include 4-quark-operators in the variational basis to extend study of configuration mixing