Charmonium physics on the lattice with highly improved staggered quarks

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HPQCD collaboration

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Outline

- Motivation.
- Improved Staggered quarks.
- Heavy quarks.
- Charm and bottom meson spectrum.
- Extracting parameters of QCD: charm and bottom quark masses.
- Outlook.
Motivation

- Low-energy QCD is a strongly-coupled QFT. We need non-perturbative tools to deal with it.
  - Other strongly-coupled sectors BSM?

- Lattice QCD provides a non-perturbative definition of QCD. It also provides a quantitative calculational tool. And lately it is also becoming a precise tool.

- To make precise calculations in QCD.
  - Test lattice field theory as a tool for studying strongly coupled field theories. (CLEO-с, $f_D$, $f_{D_s}$)
  - To calculate theoretical quantities needed in the analysis of experimental data, for example, in the determination of elements of the CKM matrix.
  - To further test QCD as the theory of strong interactions.

- To deepen our understanding of the physics of QCD, for example, confinement.
LQCD: Quenched vs Unquenched

- Fermions are numerically very hard to include.
- Ignore fermion pair production \(\Rightarrow\) quenched QCD.

\[ n_f = 2+1 \]

Plus the successful prediction of \( m_{B_c} \) (I. Allison et al).
(Some) systematic errors

- **Finite volume**: $m^{-1}_\pi \ll L$. In practice, $L \approx 2.5, 3\text{fm}$
- **Finite lattice spacing**: we need simulations at different values of $a$, to extrapolate to the continuum limit $a \to 0$.
  - To simulate at small values of $a$, while keeping the physical $L$ constant is very expensive.
  - Typically, error $\propto a, a^2$
  - Improved actions (and operators) decrease the error, making the extrapolation from a given set of lattice spacings more precise.
- **Chiral extrapolation**: In practice, we are not able to simulate at physical values of the light quark masses $m_{u,d}$.
- **Lattice spacing determination**: Error in the determination of the lattice spacing in physical units ($r_1$).
Improved Staggered Quarks

- The staggered action describes 4 tastes (in 4D). The spectrum on the lattice has a multiplicity of states corresponding to the same continuum state. There are unphysical taste-changing interactions that lift the degeneracy between such states.
- These effects are lattice artifacts, of order $a^2$, and vanish in the continuum limit $a \to 0$. They involve at leading order the exchange of a gluon of momentum $q \approx \pi/a$.
- Taste-changing interactions perturbative for typical values of the lattice spacing, and can be corrected systematically a la Symanzik.

Smear the gauge field to remove the coupling between quarks and gluons with momentum $\pi/a$.

- In an unquenched simulation, $\sqrt[4]{\text{det.}} \to "\text{Rooting trick}"$. 
Improved Staggered Actions

- **ASQTAD** (S. Naik, The MILC collaboration, P. Lepage)

  ![Diagram of ASQTAD action]

  - Discretization errors \( \approx O(\alpha_s a^2, a^4) \).
  - Substantially reduced taste-changing with respect to the unimproved action.

- **HISQ** (E.F., Q. Mason, C. Davies, K. Hornbostel, P. Lepage, H. Trottier.)

  - Discretization errors \( \approx O(\alpha_s a^2, a^4) \).
  - Substantially reduced taste-changing with respect to ASQTAD.

- **HISQ2, HISQ3, ...**

  - Taste-changing interactions can be further reduced.
Heavy Quarks

- The discretization errors grow with the quark mass as powers of $am$.
- For a direct simulation, we need:
  
  \[ am_h \ll 1 \] (heavy quarks)
  \[ La \gg m_\pi^{-1} \] (light quarks)

- Two scales. Difficult to do directly.
- Instead take advantage of the fact that $m_h$ is large: $\Rightarrow$ effective field theory (NRQCD, HQET). Very successful for b quarks.
Charm Quarks

- The charm quark is in between the light and heavy mass regime.

- Quite light for an easy application of NRQCD.

- Quite large for the usual relativistic quark actions, $am_c \sim 1$.

- However, if we use a very accurate action (HISQ) and fine enough lattices (MILC), it is possible to get accurate results.
  - Errors for HISQ: $O((am)^4, \alpha_s(am)^2)$.
  - Non-relativistic system: can be tuned for further suppression by factors of $(v/c)$. Can reduce the errors to the few percent level.
  - Simple: use the same action in the heavy and the light sector. Conserved currents for c quarks (non-renormalization.)

- We will use this action both for heavy-heavy and heavy-light systems $\Rightarrow$ consistency check.
Fixing the parameters

The free parameters in the lattice formulation are fixed by setting a set of calculated quantities to their measured physical values.

- Scale: lattice spacing $a$: Fixed through the upsilon ($b\bar{b}$) spectrum, $m_{\Upsilon(2S)} - m_{\Upsilon(1S)}$.

- Quark masses: $m_u, m_d, m_s, m_c, m_b$. Fixed by $m_\pi, m_K, m_{\eta_c}, m_{\Upsilon(1S)}$.

- In the HISQ charm quark formulation: improvement parameter $\epsilon$. Fixed by requiring relativistic dispersion relation, $c^2 = 1$. 
Configurations

- MILC ensembles: Improved gluon action and 2 + 1 ASQTAD sea quarks: \((m_l, m_l, m_s)\)

- \(a\) ranges between 0.06 and 0.16 fm.

- Physical \(m_s\).

- \(m_l = m_s/10\) to \(m_s/2.5\)
Taste splitting

- Doubling $\rightarrow$ 16 lattice mesons.
- Taste-breaking interactions lift degeneracy $\rightarrow$ taste-splitting.
- Can be reduced further with HISQ2, ...
Meson spectrum

The gold-plated meson spectrum from lattice QCD - HPQCD 2008

UNFLAVORED

FLAVORED

Expt
Fix parameters
Postdictions
Predictions

Meson Mass (GeV)
Charm and bottom quark mass

Work in collaboration with:

K.G. Chetyrkin (Universität Karlsruhe)
J.H. Kühn (Universität Karlsruhe)
M. Steinhauser (Universität Karlsruhe)
C. Sturm (Brookhaven National Laboratory)

- Bare lattice $m_c$ (fixed through $m_{\eta_c}$) + lattice PT (2-loops). Very demanding. Combination of diagrammatic + high-$\beta$ PT. Preliminary result: $m_c^{\overline{MS}}(3\text{GeV}) = 0.983(23)\text{GeV}$.

- Lattice current-current correlators + high-order cont. PT. $m_c^{\overline{MS}}(3\text{GeV}) = 0.986(10)\text{GeV}$. Most precise determination to date.
  Experimental $e^+e^-$ data + cont. PT (Chetyrkin et al): $m_c^{\overline{MS}}(3\text{GeV}) = 0.986(13)\text{GeV}$
Method of moments

\[ G(t) \equiv a^6 \sum_x (am_{0c})^2 \langle 0| j_5(x, t)j_5(0, 0)|0 \rangle \]

\[ j_5 = \bar{\psi}_c \gamma_5 \psi_c \]

- Mass factors \( \rightarrow \) independent of the cutoff in the continuum limit (PCAC).
- \( G_{\text{cont}}(t) = G_{\text{lat}}(t) + \mathcal{O}(a^2) \)

\[ G_n = \sum_t (t/a)^n G(t) \]

Low \( n \) moments perturbative \( (m_c \text{ large}) \).

\[ G_n = g_n (\alpha_{\overline{MS}}(\mu), \mu/m_c) \]

\[ \frac{1}{(am_c(\mu)))^{n-4}} \]
Moments

Better to use \textit{reduced} moments:

\[ R_n \equiv \begin{cases} G_4 / G_4^{(0)} & n = 4 \\ \frac{am_{\eta_c}}{2am_{(0)}^{pole,c}} \left( G_n / G_n^{(0)} \right)^{1/(n-4)} & n \geq 6 \end{cases} \]

\[ R_n = \begin{cases} r_4(\alpha_{\overline{MS}}(\mu), \mu / m_c) & n = 4 \\ \frac{r_n(\alpha_{\overline{MS}}(\mu), \mu / m_c)}{2m_c(\mu) / m_{\eta_c}} & n \geq 6 \end{cases} \]

\[ m_c(\mu) = \frac{m_{\eta_c}^{exp} r_n^{PQCD}}{2} \frac{r_n^{LQCD}}{R_n^{LQCD}} \]

\[ R_4^{LQCD} = r_4(\alpha_{\overline{MS}}, \mu / m_c) \]
Continuum and sea quark mass extrapolation.

\[ R_n(a) = R_n(0) \left( 1 + c_1 \alpha_s(\alpha m_c)^2 + c_2 \alpha_s(\alpha m_c)^4 + \cdots \right) \]
\[ \left( 1 + d_1(2m_u/d + m_s)/mc + \cdots \right) \]
Results for $m_c$

$m_c(3\text{GeV}) = 0.986(10)\text{GeV}$

$m_c(m_c) = 1.268(9)\text{GeV}$

Continuum determination from vector current and experimental $R(e^+e)$:

$mc(3\text{GeV}) = 0.986(13)\text{GeV}$

$\triangleright$ Same method can be used for $m_b$. Work in progress.

$m_c(3\text{GeV})$ error budget from $R_6$ in %

- $a^2$ extrapolation 0.2
- perturbation theory 0.4
- $\alpha_{\overline{MS}}$ uncertainty 0.3
- gluon condensate 0.3
- statistical errors 0.1
- $m_0c$ errors from $r_1/a$ 0.5
- $m_0c$ errors from $r_1$ 0.6
- $m_{u/d/s}$ extrapolation 0.2
- finite volume 0.1

Total 1.0
Light quark masses

- Leverage the precision on $m_c$ to get to $m_s$ and $m_l$.
- Feasible because we use the same action for both charm and light quarks.

![Graph showing the relationship between bare $m_c$ and $a^2 / fm^2$]
Conclusions and outlook

- The use of a highly improved quark action and fine enough lattices provides a very good way of studying systems with charm quarks from first principles.
- Direct determination of \( m_c \) from the lattice and from current-current correlators + continuum PT.
- Accurate \( m_c/m_s \).
- Semileptonic form factors: \( D \rightarrow \pi \ell \nu, D \rightarrow K \ell \nu \)
- Leptonic decay width \( \psi \rightarrow e^+e^- \). Known accurately from experiment (\( \sim 2\% \)).
Results for $\alpha_s$

\[ \alpha_{\overline{\text{MS}}}(M_Z, n_f = 5) = 0.1174(12) \]

World average: 0.1189(10)
(S. Bethke, Prog.Part.Nucl.Phys.58,2007)

error budget from
\[ R_4 \text{ in } \% \]

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Total: 1.0